Code No. 9355 / N

FACULTY OF ENGINEERING

B.E. I – Year (New) (Main) Examination, May / June 2015

Subject : Mathematics - I

Time : 3 Hours **Max. Marks: 75**

Note: Answer all questions from Part - A and answer any five questions from Part-B.

PART – A (25 Marks)

1 Test the series $\sum\left(\frac{1}{2^n}+\frac{1}{n}\right)$ for convergence. (3)

2 State Raabe's test. (2)

3 Find the Taylor series expansion of $f(x)= e^x$ about x=1. (3)

4 Obtain the curvature of the curve $x^4 + y^4 = 2$ at (1, 1). (2)

5 If u = f(x-y, y-z, z-x), prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. (3)

6 If $u = x^2 + y^2$, v = 2xy, find $\frac{\partial(u,v)}{\partial(x,y)}$ at (1, 2). (2)

7 Find div $\overline{F}$, where $\overline{F}$ = grad $(x^3+y^3+z^3+3xyz)$. (3)

8 Find the constants a, b, c such that the vector
$\overline{F}$ = (x+2y+az)I + (bx-3y-z)j + (4x+cy+2z)k is irrotational. (2)

9 Determine whether the set of vectors {(1, 0, 0), (0, 1, 0), (0, 0, 1) } forms a basis in IR^3. (3)

10 If the sum of the eigen values of the matrix $A=\begin{pmatrix}1 & 4 & 5\\ 0 & k & 2\\ -1 & 2 & 2k\end{pmatrix}$ is 10, find k. (2)

PART – B (50 Marks)

11 (a) Discuss the convergence of the series $\sum\left(1+\frac{1}{n}\right)^n x^n$, x > 0. (5)

(b) Test the series $\sum(-1)^{n-1}\frac{n}{n^2+1}$ for absolute convergence or conditional convergence. (5)

12 (a) State and prove Lagrange mean value theorem. (5)

(b) Sketch the graph of the curve $y^2(2-x)=x^3$. (5)

13 (a) If f(x, y)=$\begin{cases}\frac{xy}{\sqrt{x^2+y^2}}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$, find $\frac{\partial f}{\partial x}$ *and* $\frac{\partial f}{\partial y}$ at (0,0). (5)

(b) Find the local maximum and minimum values of the function
$f(x)=3x^4 - 2x^3 - 6x^2 + 6x + 1$ in [0,2]. (5)

14 (a) Find the directional derivative of $f(x, y, z) = x^2 - y^2 + 2z^2$ at the print P(1, 2, 3) in the direction of the line PQ where Q in the point (5, 0, 4). (5)

(b) Apply divergence theorem to evaluate $\iint_S \bar{r}.\hat{n}\, ds$, where $\bar{r}$ = xi + yj + zk and S is the surface of the sphere $x^2 + y^2 + z^2 = 9$. (5)

15 (a) Reduce the matrix A = $\begin{pmatrix} 1 & 4 & 5 & 2 \\ 2 & 8 & 6 & 7 \\ 3 & 5 & 2 & 1 \\ -1 & 2 & 3 & 0 \end{pmatrix}$ to echelon form and hence find its rank. (5)

(b) State Cayley-Hamilton theorem and use it to find the inverse of the matrix

$$A = \begin{pmatrix} 2 & 3 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & -1 \end{pmatrix}$$ (5)

16 (a) Find the envelope of the family of lines y= c x + $\sqrt{1+c^2}$, c is the parameter. (5)

(b) Show that the function $f(x,y) = \begin{cases} \dfrac{x^2+y^2}{x^2-y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ is not continuous at (0, 0). (5)

17 (a) If $f(x, y, z) = x + y - 2z^2$, compute grad f and verify that curl grad f = $\bar{0}$. (5)

(b) Obtain the symmetric matrix A for the quadratic form Q = 2xy + 2yz + 2zx and also find the nature of the quadratic form. (5)

*****

Code No. 9002 / M

FACULTY OF ENGINEERING AND INFORMATICS

B.E. I – Year (Main) Examination, May / June 2015

Subject : Mathematics - I

Time : 3 hours **Max. Marks : 75**

Note: Answer all questions from Part-A. Answer any FIVE questions from Part-B.

PART – A (25 Marks)

1 Let $f'(x) = \frac{1}{3-x^2}$ and f(0) = 1. Find an interval in which f(1) lies. (2)

2 Find the equation of the envelope of the family of straight lines $y = cx + c^2$ where c is a parameter. (3)

3 Prove that $f(x,y) = \begin{cases} \frac{x^2+xy+x+y}{x+y}; (x,y) \neq (2,2) \\ 4 \quad ; (x,y) = (2,2) \end{cases}$ (2)

is discontinuous at the point (2, 2).

4 If f(x, y) = $\tan^{-1}$ (x y), find an approximate value of f(1.1, 0.8) using the Taylor series linear approximation. (3)

5 Evaluate the double integral $\iint_R xy \, dx \, dy$, where 'R' is the region bounded by the x-axis, the line y = 2x and the parabola $x^2 = 4ay$. (2)

6 If $\bar{a}$ is a constant vector and $\bar{r} = xi + yj + zk$ then prove that $\nabla X(ax\bar{r}) = 2\bar{a}$. (3)

7 Test whether the vectors (1,0,0), (0,2,0), (0,0,3) are linearly independent or not. (2)

8 Find all values of λ for which rank of the matrix. (3)

$$A = \begin{bmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix}$$

is equal to 3.

9 Test the convergence of the series $\sum \left[\frac{(1+nx)^n}{n^n} \right]$. (2)

10 Show by an example that every convergent series need not be absolute convergent. (3)

PART – B (50 Marks)

11 a) State and prove Lagrange's mean value theorem. (5)
b) Find the evolute of $x^2 = 4ay$. (5)

12 a) Find the shortest distance between the line y = 10 – 2x and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ (5)

b) Show that $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$ does not exist. (5)

13 a) Show that the vector field defined by the vector function $\overline{V} = xyz(yzi + xzj + xyk)$ is conservative.

b) Show that $\int_C (yz-1)dx + (z + xz + z^2)dy + (y + xy + 2yz)dz$ is independent of the path of integration from (1, 2, 2) to (2, 3, 4). Evaluate the integral. (5)

14 a) Prove that eigen values of i) an Hermitian matrix are real (5)
ii) a skew-Hermitian matrix are zero or purely imaginary.

b) Examine $A = \begin{bmatrix} 1 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 3 \end{bmatrix}$ is positive definite. (5)

15 a) Discuss the convergence of the series $\sum\left[\frac{1.35..(2n-1)}{2.46..(2n)}\frac{x^{2n}}{2n}\right]$. (5)
b) Test the convergence of the series $1 + 3x + 5x^2 + 7x^3 + \ldots$ (5)

16 a) State and prove Cayley Hamilton theorem. (5)

b) Find the eigen values and the corresponding eigen vectors. (5)

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

17 a) If x = r cosθ, y = r sinθ, then find $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2$. (5)

b) If $u = \log[x^2 + xy + y^2]$ then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$. (5)

******