Code No. 3392

FACULTY OF ENGINEERING

B.E. I - Semester (Main) Examination, December 2016

Subject : Engineering Mathematics - I

Time : 3 Hours **Max. Marks: 70**

Note: Answer all questions from Part-A and answer any five questions from Part-B.

PART – A (20 Marks)

1 Convert the matrix $A = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 0 & 5 \\ -1 & 2 & 3 \end{pmatrix}$ into echelon form. (2)

2 Write any two properties of eigen values. (2)

3 Discuss the convergence of the series $\sum_{n=1}^{\infty}\left(1+\frac{1}{n^2}\right)^{n^2}$. (2)

4 Define the terms : (a) absolutely convergent series and
(b) conditionally convergent series (2)

5 Find a point at which the tangent to the curve y = ℓnx is parallel to the chord joining the point (1, 0) and (e, 1). (2)

6 Expand f(x) = e^x sin x in powers of x upto the term x^2. (2)

7 Show that $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$ doesn't exist. (2)

8 If u = 2xy, v = $x^2 - y^2$, x= r cos θ, y = r sin θ, compute $\frac{\partial(u,v)}{\partial(r,\theta)}$. (2)

9 Compute the gradient of the scalar function $f(x, y, z) = e^{xy}(x+y+z)$ at (2,1, 1). (2)

10 If f is a differentiable scalar field, then show that ∇ x (∇f) = $\vec{0}$. (2)

PART – B (50 Marks)

11 (a) If $A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$. Find the eigen values of $3A^5 - A^4 + A^2 + 3I - A^{-1}$. (5)

(b) Find the symmetric matrix, index and signature of the quadratic form
$Q = 3x^2 + 3y^2 + 3z^2 + 2xy + 2xz - 2yz$ (5)

12 (a) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{1.4.7......(3n-2)}{25.8...........(3n-1)}$. (5)

(b) Examine whether the series $-1+\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{4^2}-\frac{1}{5^2}+....$ is absolutely convergent or conditionally convergent. (5)

13 (a) State Cauchy's mean value theorem and verify it for the functions f(x) = e^{-x} and g(x)=e^x in [a, b]. (5)

(b) Find all asymptote to the curve (5)
$y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 = 0$.

14 (a) Show that the function (5)

$$f(x,y)=\begin{cases}\dfrac{x^2-y^2}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0\quad , & (x,y)=(0,0)\end{cases}$$ is not differentiable at (0, 0)

(b) Find the absolute maximum and minimum values for the function $f(x, y) = x^2 - y^2 - 2y$ in the closed region R ; $x^2 + y^2 \leq 1$ (5)

15 (a) Show that the vector function $\vec{V}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$ is irrotational and find its scalar potential. (5)

(b) Use Green's theorem to evaluate the line integral $\oint_C (xy+x^2)\,dx+(x^2+y^2)dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. (5)

16 Diagonalize the matrix $A=\begin{pmatrix}1 & 2 & -2\\ 2 & 1 & -4\\ 1 & -1 & -2\end{pmatrix}$. (10)

17 (a) Discuss the convergence of the series $\sum\left[\dfrac{\sqrt{n+1}-\sqrt{n}}{n^2}\right]$. (5)

(b) Find the directional derivative of $f(x, y, z) = x^2 + y^2 + z^2$ at (1, 2, 3) in the direction of the vector $2\hat{i}+3\hat{j}+6\hat{k}$. (5)

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