Code No. 3002

FACULTY OF ENGINEERING

B.E. I – Year (Suppl.) Examination, December 2016

Subject : Mathematics - I

Time : 3 hours **Max. Marks : 75**

Note: Answer all questions from Part-A. Answer any FIVE questions from Part-B.

PART – A (25 Marks)

1 Examine the convergence of the series $\sum\left(1+\frac{1}{2^n}\right)^{2^n}$. 3

2 State Logarithmic test. 2

3 Expand $f(x)=e^{\sin x}$ in Maclaurin's series upto the term x^2. 3

4 Find the intervals in which the curve $y=x+\frac{1}{x}$ is concave up or concave down. 2

5 Prove that $\lim_{(x,y)\to(0,0)} x^2+y^2=0$. 3

6 If $z=\log(u^2+v), u=e^{x^2+y^2}, v=x^2+y$ find $\frac{\partial z}{\partial x}$ at (1, 0). 2

7 Prove that curl (grad f) = $\bar{0}$, where f is a differentiable scalar field. 3

8 In what direction from (3, 1, -2) is the directional derivative of f(x, y, z) = $xy^2 z^3$ maximum? 2

9 Determine whether the vectors (1, 2, 3), (1, 0, 0), (0, 0, 1) are linearly dependent. 3

10 If $A=\begin{pmatrix}1 & 2 & -1\\ 0 & -1 & 5\\ 0 & 0 & -3\end{pmatrix}$, find its spectrum and spectral radius. 2

PART – B (50 Marks)

11 a) Test the convergence of the series 5

$$\sum_{n=1}^{\infty}\frac{1\cdot4\cdot7\ ....(3n-2)}{2\cdot5\cdot8..........(3n-1)}$$

b) Discuss the convergence of the series $\sum\frac{\cos^2 nx}{n\sqrt{n}}$ 5

12 a) State and prove Rolle's theorem. 5

b) Find the envelope of all ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which have a constant area $A=\pi ab$. 5

13 a) Prove that $f(x,y)=\sqrt{x^2+y^2}$ is not differentiable at (0, 0). 5

b) Find the local maximum and minimum values of $f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$. 5

14 Verify Stoke's theorem for the vector field $\overline{F}=(2x-y)i-yz^2j-y^2zk$ over the upper half surface of $x^2+y^2+z^2=1$ bounded by its projection on the xy plane. 10

15 a) Determine the value of k for which the matrix 5

$$A=\begin{pmatrix}6 & 3 & 5 & 9\\ 5 & 2 & 3 & 6\\ 3 & 1 & 2 & k\end{pmatrix}$$ is of rank 3.

b) Find the values of λ so that the equations $x + y + z = 1$, $2x + y + 4z = \lambda$, $4x + y + 10z = \lambda^2$ have a solution and solve them completely in each case. 5

16 a) Find all the asymptotes to the curve $y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 = 0$. 5

b) Evaluate $\iint_R e^{-\left(x^2+y^2\right)}dx\,dy$, by changing to polar coordinates, where R is the region $x^2 + y^2 \le 9$, by changing to polar coordinates. 5

17 Diagonalize the matrix $A=\begin{pmatrix}1 & -1 & 2\\ 0 & 2 & -1\\ 0 & 0 & 3\end{pmatrix}$ and obtain its modal matrix. 10

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