Code No. 3003

FACULTY OF ENGINEERING

B.E. I – Year (Suppl.) Examination, November / December 2016

Subject : Mathematics - II

Time : 3 hours **Max. Marks : 75**

Note: Answer all questions from Part-A. Answer any FIVE questions from Part-B.

PART – A (25 Marks)

1 Find an integrating factor of $xdy - ydx + y^2dx = 0$. 2

2 Solve $\frac{dy}{dx} + xy = 2x$. 3

3 Determine whether the functions $x^2, \frac{1}{x^2}$ are linearly independent on $(0,\infty)$. 2

4 Find the complementary function of $y'' - y = x \sin x$. 3

5 Define ordinary and singular points. 2

6 Prove that $P_n(-x) = (-1)^n P_n(x)$. 3

7 Evaluate $\int_0^{\pi/2} \frac{1}{\sqrt{\sin x}} dx$ using Beta and Gamma functions. 2

8 Write the solution of $x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$ in terms of Bessel functions. 3

9 Find $L\{t\, e^{-t}\}$. 2

10 Find $L^{-1}\left\{\frac{e^{-2s}}{s^2 - 4}\right\}$. 3

PART – B (50 Marks)

11 a) Solve $y^2dx + (x^2 - xy - x^2)\, dy = 0$. 5

b) Show that the family of curves $y^2 = 4a\,(a + x)$, a being parameter, is self orthogonal. 5

12 a) Find the general solution of $x^2y'' - 3xy' + 3y = \ell n x$. 5

b) Solve the system equations $\frac{dx}{dt} = y$, $\frac{dy}{dt} = -9x$. 5

13 a) Express $x^3 - 3x^2 - 2x + 7$ in terms of Legendre Polynomials $P_n(x)$. 5

b) Prove that $n P_n(x) + P_{n-1}^1(x) = x P_n^1(x)$. 5

..2

14 a) Prove that $(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$. 5

b) Prove that $J_{-5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{(3 - x^2)\cos x}{x^2} + \frac{3\sin x}{x}\right]$. 5

15 a) Find $L\{f(t)\}$, where $f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$ and f(t + 2) = f(t). 5

b) Apply convolution theorem to find $L^{-1}\left\{\frac{1}{(s-1)\,(s+2)}\right\}$. 5

16 a) A radioactive substance disintegrates at a rate proportional to its mass. When mass is 10 mgm, the rate of disintegration in 0.051 mgm per day. How long will it take for the mass to reduce from 10 to 5 mgm? 5

b) Solve $y'' - y = e^{2x}$ by the method of variation of parameters. 5

17 a) Find the power series solution of $y' - 2y = 0$ about x = 0. 5

b) Using the generating function, prove that 5

i) $\cos(x\sin\theta) = J_0(x) + 2J_2(x)\cos 2\theta + J_4(x)\cos 4\theta + \ldots\ldots\ldots\ldots$ and

ii) $\sin(x\sin\theta) = 2J_1(x)\sin\theta + 2J_3(x)\sin 3\theta + \ldots\ldots\ldots\ldots$

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