Code No. 5002 / N

FACULTY OF ENGINEERING

B.E. I – Year (New) (Suppl.) Examination, December 2015

Subject: Mathematics – I

Time: 3 Hours **Max.Marks: 75**

Note: Answer all questions from Part A. Answer any five questions from Part B.

PART – A (25 Marks)

1 State *p*-series test. 2

2 Show that the series $\sum \frac{\cos nx}{n^2}$ is absolutely convergent. 3

3 Find the value of C of Cauchy mean value theorem for f(x) = x^2 and g(x) = x^3 in [1,4]. 2

4 Obtain all the asymptotes of the curve $\frac{4}{x^2} + \frac{9}{y^2} = 1$. 3

5 Determine $\lim\limits_{(x,\, y) \to (1, -1)} x^2 - y^2$. 2

6 If f(x, y) = $\frac{x+y}{x-y}$, find $\frac{\partial f}{\partial x}$ at (2, 3). 3

7 Evaluate $\nabla\left(\frac{1}{r}\right)$, where r = $|\bar{r}|$, $\bar{r}$ = x i + yj + z k. 2

8 Evaluate $\int_C \bar{F}.d\bar{r}$, where $\bar{F}$ = x^2i + y^3j and C is the arc of the parabola y = x^2 in the xy plane from (0, 0) to (1, 1). 3

9 Define linear transformation. 2

10 Obtain the symmetric matrix A for the quadratic form Q=x^2+2y^2+3z^2+4xy+8yz+6xz. 3

PART – B (50 Marks)

11 a) Discuss the convergence of the series $\sum\left[\sqrt{n^2+1} - \sqrt{n^2-1}\right]$ 5

b) State Leibnitz test and use it to test the convergence of the series Σ $(-1)^n$ $\frac{n}{2n+1}$. 5

12 a) State and prove Rolle's theorem. 5

b) Find the Taylor series expansion of f(x) = x^3+3x^2+2x+1 about x = -1. 5

13 a) If w = x^2+y^2+z^2, x = cos t, y = ℓn (t + 1), z = e^t, find $\frac{dw}{dt}$ at t = 0. 5

b) Evaluate $\int_o^\infty\int_o^\infty e^{-(x^2+y^2)}$ dx dy by changing to polar coordinates. 5

14 a) Find the values of a and b such that the surface $5x^2-2yz-9z = 0$ intersects the surface $ax^2+by^3=4$ orthogonally at (1, -2, 1). 5

b) Use Stoke's theorem to evaluate $\int_c (x+y)\,dx + (2x-z)dy + (y+z)dz$, where c is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and *0, 0, 6). 5

15 a) Show that the vectors (1, 3, 5), (2, -1, 4), (-2, 8, 2) are linearly dependent. 5

b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$. 5

16 a) Find the radius of curvature of the curve $x^2 = 8y$ at (4, 2). 5

b) Find the extreme values of xyz when x+y+z = 3. 5

17 a) Find the directional derivative of $f(x, y, z) = x^2 + y^2 + 2z^2$ at (1, 1, 2) in the direction of grad f. 5

b) Determine the values of k for which the system of equations $x - ky + z = 0$, $kx + 3y - kz = 0$, $3x + y - z = 0$ has i) only zero solution ii) non-zero solution. 5

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Code No. 5010 / O

FACULTY OF ENGINEERING AND INFORMATICS

B.E. I - Year (Old) Examination, December 2015

Subject : Mathematics - I

Time : 3 hours **Max. Marks : 75**

Note: Answer all questions from Part-A. Answer any FIVE questions from Part-B.

PART – A (25 Marks)

1 Discuss the applicability of Rolle's theorem for f(x)=tax x in [0, π]. 3

2 Find the envelope of the family of lines $y = cx + c^2$. 2

3 Determine $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - y^2}{x^2 + y^2}$, if it exists. 3

4 Find the total derivative of $f(x,y)=xy^3+x^3y$. 2

5 Evaluate $\int_0^\infty\int_0^\infty \bar{e}^{(x^2+y^2)}\,dx\,dy$. 3

6 State Green's theorem. 2

7 Find the value of λ so that the system of equations 2x + y = 2z = 0, x + y + 3z = 0, 4x + 3y + λz = 0 has a non-zero solution. 3

8 Are the vectors (1, 2), (2, 3), (4, 3) linearly dependent? 2

9 Show that the geometric series $1 + r + r^2 + r^3 + \ldots\ldots\ldots\ldots$ is convergent if |r|<1. 3

10 Test the convergence of the series $\frac{1}{2}-\frac{2}{3}+\frac{3}{4}-\frac{4}{5}+$.......... 2

PART – B (50 Marks)

11 a) Find the Taylor series expansion of $f(x) = x^2 \ell n\, x$ about x = 1. 5

b) Find the radius of curvature of the curve x = t – sint, y = 1 – cost at t = π. 5

12 a) If $f(x,y) = \dfrac{x-y}{x+y}$, find f_{xy} and f_{yx} at (1, 1). 5

b) Find the extreme values of xy on the line segment x + 2y = 2, x ≥ 0 , y ≥ 0. 5

13 Verify Gauss's divergence theorem for $\overline{F} = 2x^2y\, i - y^2 j + 4xz^2\, k$ over the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0 and z=1. 10

14 a) If $A = \begin{pmatrix} 2 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, find A^{-1} by Cayley-Hamilton theorem. 5

b) Reduce the quadratic form $Q = x^2 + 4y^2 + z^2 + 4xy + 6yz + 2zx$ to canonical form. 5

15 a) Discuss the convergence of the series $1 + 2x + 3x^2 + 4x^3 + \ldots\ldots\ldots$. 5

b) Test the convergence of the series $\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^3+$.............. 5

16 a) State and prove Cauchy's mean value theorem. 5

b) Find the value of n such that $r^n \bar{r}$ is solenoidal 5

17 a) Find the rank of the matrix $A = \begin{pmatrix} 1 & -1 & 2 & 3 \\ 0 & 2 & 4 & 3 \\ -2 & 1 & 5 & 2 \end{pmatrix}$ 5

b) Test whether the series $-1+\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{4^2}-----$ is absolutely convergent or conditionally convergent. 5

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