Code No. 3002/BL

FACULTY OF ENGINEERING & INFORMATICS

B.E. I Year (Backlog) Examination, June 2017

Subject: Mathematics – I

Time: 3 Hours **Max.Marks: 75**

Note: Answer all questions from Part A and any five questions from Part B.

PART – A (25 Marks)

1 Discuss the convergence of the series $2+\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\dots$ 2

2 Prove that the series $\sum(-1)^{n+1}\frac{1}{\sqrt{n}}$ is conditionally convergent. 3

3 Find a point on the curve $f(x) = x^2 - 2x$ in [0, 2] at which the tangent is parallel to x-axis. 2

4 Obtain the envelope of the family of curves $y = cx+\frac{3}{2c}$, c is a parameter. 3

5 If $u = 2xy$, $v = x^2 - y^2$, $x = r\cos\theta$, the evaluate $\frac{\partial(u,v)}{\partial(r,\theta)}$. 2

6 Find the Taylor's series for $f(x, y) = x^2 + 4xy + 2y^2 + 6x + 4y + 1$ about (1, - 2). 3

7 If $\bar{F} = x^2yi + xy^2zj - yz^2k$, find grad div $\bar{F}$. 2

8 Evaluate $\oint_c \bar{F}\,d\bar{r}$, where $\bar{F} = (4xy - 3x^2z^2)i + 2x^2j - 2x^3zk$ and C is $y=x^2$ from x=0 to x=1. 3

9 Find the values of k such that the rank of

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & k & 4 \\ -1 & 2 & 5 \end{pmatrix} \text{ is 3.}$$ 2

10 Verify that $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigen vector of $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$ corresponding to the eigen value 5. 3

PART – B (5x10 = 50 Marks)

11 a) Test the convergence of the series $\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^2}x^3+\dots\ (x > 0)$ 5

b) Examine the convergence or divergence of the series $\sum\left(\sqrt{n^3+1}-\sqrt{n^3}\right)$. 5

12 a) State and prove Cauchy's mean value theorem. 5

b) Find all the asymptotes to the curve $x^3 + 3x^2y – 4y^3 – x + y + 3 = 0$. 5

13 a) If z = f(x, y), x = u cos~ - v sin~ , y = u sin~ + v cos~ prove that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}.$$ 5

b) Find the shortest and fastest distances from the point (1, 2, -1) to the sphere $x^2 + y^2 + z^2 = 24$ using Lagrange's method of multipliers. 5

14 a) Find the constants a, b, c such that

$\bar{F}$ = (2x + 3y + az)i + (bx + 2y + 3z)j + (2x + cy + 3z)k is irrotational and find a scalar function f such that $\bar{F} = \nabla f$. 5

b) Evaluate $\oint_C x\,dy – y\,dx$, where c is the triangle with verticdes at (0, 0), (2, 0) and (0, 1) using Green's theorem. 5

15 a) Test the consistency of the equations x +y + z = 6, x-y+2z = 5, 3x+y+z = 8 and 2x-2y+3z = 7 and hence solve. 5

b) Find the characteristic equation of the matrix A = $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and hence find the matrix

$A^8 - 5A^7 - A^6 - 5A^5 - A^4 + 6A^2 + I$. 5

16 a) Find the curvature and radius of curvature of the curve r = a(1 + cos¯) at any¯ . 5

b) Evaluate $\int_0^\infty \int_0^x x e^{\frac{-x^2}{y}} \, dy\, dx$ by change the order of integration. 5

17 Reduce the quadratic form $Q = 3x^2 + 3y^2 + 3z^2 + 2xy + 2xz – 2yz$ to canonical form and hence find its nature, index and signature. 10

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