Code No. 3392 / S

FACULTY OF ENGINEERING

B.E. I – Semester (Suppl.) Examination, June / July 2017

Subject: Engineering Mathematics – I

Time: 3 Hours **Max.Marks : 70**

Note: Answer all questions from Part A and any five questions from Part B.

PART – A (20 Marks)

1 Define rank of a matrix. Give an example of a 2x3 matrix whose rank is 2. 2

2 Write the symmetric matrix for the quadratic form $Q = x^2 + 2y^2 + 3z^2 - 2xy + 4yz + 6zx$. 2

3 Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{1}{n^2 + n} + \frac{1}{n} \right)$. 2

4 State Leibnitz's test. 2

5 Find the radius of curvature at the origin of the curve $x^4 - 4x^3 - 18x^2 - y = 0$. 2

6 Obtain the equation of envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters a and b are connected by the relation ab = 4. 2

7 Evaluate $\lim_{(x,y) \to (1,2)} \frac{x^2 y}{x + y^2}$. 2

8 Find $\frac{dw}{dt}$ if $w = x^2 + y^2$, $x = \cos^2 t$, $y = \sin^2 t$ at $t = \frac{\text{®}}{4}$. 2

9 Find the normal vector and unit normal vector to the surface $z^2 = x^2 - y^2$ at $(2, 1, \sqrt{3})$. 2

10 Show that the vector $e^{x+y-2z} \left(\hat{i} + \hat{j} + \hat{k} \right)$ is solenoidal. 2

PART – B (5x10 = 50 Marks)

11 a) Find the values of a and b such that the equations x + y + z = 6, x + 2y + 3z = 10, x + 2y + az = 6 have (i) no solution, ii) unique solution and (iii) infinite solutions. 5

b) Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and hence find the matrix represented by $A^8 - 5A^7 - A^6 - 5A^5 - A^4 + 6A^2 + I$. 5

12 a) Test the convergence or divergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \frac{4^4}{5^5} + ----$. 5

b) Prove that the series $\sum (-1)^{n-1} \frac{\sin\ nx}{n^2}$ converges absolutely. 5

13 a) Using Lagrange's mean value theorem, prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$, $0 < a < b$. 5

b) Sketch the graph of the curve $y = \frac{x}{\sqrt{x^2+1}}$. 5

14 a) If $f(x, y) = \begin{cases} \frac{x^2 y(x-y)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$, show that $\frac{\partial^2 f}{\partial x\, \partial y} \neq \frac{\partial^2 f}{\partial y\, \partial x}$ at (0, 0). 5

b) Obtain the Taylor series expansion of the function $f(x, y) = e^{2x+y}$ about (0, 0) upto third degree terms. 5

15 a) Prove that $\nabla(\ell n r) = \frac{\vec{r}}{r^2}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. 5

b) Evaluate $\iint_S \vec{F}.\hat{n}\, ds$, where $\vec{F} = 6z\hat{i} - 4\hat{j} + y\hat{k}$ and S is the portion of the plane $2x + 3y + 6z = 12$ in the first octant. 5

16 Find the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{pmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}.$$ 10

17 a) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$, $x > 0$. 5

b) Find the value of $\oint_C (3x^2 - 8y^2)\, dx + (4y - 6xy)\, dy$, where C is the boundary of the region defined by $x = 0$, $y = 0$, $x + y = 1$ by Green's theorem. 5

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