CODE NO: 78

FACULTY OF ENGINEERING

B.E 2/4 (ECE) II-Semester (Backlog) Examination, May / June 2018

Subject: Probability Theory & Stochastic Processes

Time: 3Hours **Max. Marks: 75**

Note: Answer all questions from Part-A. Answer any five questions from Part-B

ART-A (25 Marks)

1. A and B alternately throw a pair of dice. A wins if he throws 6 before B throws 7, and B wins if he throws 7 before A throws 6. If A begins, find the probability of A winning the game. (3)
2. X and Y are random variables related by Y=3X+6, X is a zero mean random variable with variance 2. Find the variance of Y. (2)
3. What is a Random variable? Explain different types of Random variables. (3)
4. Determine whether the following function is a valid probability distribution function or not? Write the properties used. (3)

$$G_X(x) = \frac{x}{a}\left[u(x-a) - u(x-2a)\right]$$

5. What are Random Numbers and mention applications of random numbers? (3)
6. Define mean Ergodic process. (2)
7. Autocorrelation function of a stationary random process is $R_{XX}(^2) = 25 + \frac{4}{1+6^{2\,2}}$. Find its variance. (3)
8. Express third order moments about mean of a random variable X in terms of lower order moments like mean, variance etc. (2)
9. If X and Y are independent, then show that E [XY] = E[X]*E[Y]. (2)
10. What is meant by stochastic convergence of probability? (2)

PART-B (50 Marks)

11. (a) State and prove Baye's Theorem. (4)
    (b) Two boxes B1 and B2 contain 100 and 200 light bulbs respectively. B1 and B2 have 15 and 5 defective bulbs respectively.
    (i) Suppose a box is selected at random and one bulb is picked out. What is the probability that it is defective?
    (ii) Suppose we test the bulb and it is found to be defective. What is the probability that it came from B1? (3+3)

12. Discuss the characteristics of Rayleigh, Nakagami-m, Beta, Gamma random variables using relevant expressions and sketches of their distribution and density functions. (10)

13 (a) In a sports event javelin throw distances are well approximated by a Gaussian distribution for which mean is 30m and standard deviation is 5m. In a qualifying round, contestants must throw farther than 27m to qualify. In the main event the record throw is 44m
(i) What is the probability of being disqualified in the first round?

(ii) In the main event what is the probability the record will be broken? (2+3)

(b) Obtain the characteristic function of Poisson random variable. (5)

14. Let X and Y are two random variables with joint Probability Mass Function

$f(x,y) = \frac{x+2y}{18}$, for x=1,2 ; y= 1,2

Find (i) correlation between X and Y (ii) Variance (X/Y=1) (5+5)

15. (a) Check whether the random process X(t)= A cos(ω_0t + θ) is WSS process or not, for A and ω_0 being constant and θ is uniformly distributed between (0,2π). (7)

(b) Write the properties of Autocorrelation function of a random process and prove any two of them. (3)

16. If X and Y are independent random variables and W=X+Y, where the densities of X and Y are assumed to be $f_X(x) = 4e^{-4x}u(x)$ and $f_Y(y) = 5e^{-5y}u(y)$. Find the density function of W. (10)

17. A Gaussian random variable, for which $\overline{X}$ =0.6 and $\pm_X$ =0.8 is transformed to a new random variable by the transformation.

$$Y=T(X) = \begin{cases} 4 & 1.0 \le x \le \infty \\ 2 & 0 \le x \le 1.0 \\ -2 & -1.0 \le x < 0 \\ -4 & -\infty < x < -1.0 \end{cases}$$

(i) Find the density function of Y. (5)

(ii) Find the mean and variance of Y (2+3)

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