Code No: 11604/A

FACULTY OF ENGINEERING

B.E. I – Semester (Main) Examination, December 2018

Subject: Mathematics – I

Time: 3 Hours **Max. Marks: 70**

Note: Answer all questions from Part A and Five questions from Part B.

PART – A (10x2 = 20 Marks)

1) Determine the nature of the series $\sum_{n=1}^{\infty} \frac{2+5n}{7n-3}$.

2) Determine the nature of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$.

3) Verify Rolle's mean value theorem for the function $f(x) = \frac{\sin x}{e^x}$ on [0,®].

4) Find the envelope of the family of straight lines $x\cos\sim + y\sin\sim = a$ where ~ is the parameter.

5) Discuss the continuity of the function

$$f(x,y) = \begin{cases} \frac{(x-y)^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$ at (0,0)

6) If $x = u(1+v)$, $y = v(1+u)$ then evaluate $\frac{\partial(x,y)}{\partial(u,v)}$

7) Evaluate $\int_0^{®/4} \int_{\sin x}^{\cos x} dy dx$

8) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} 6\, dx\, dy\, dx$

9) Find the unit normal vector to the surface $f(x,y,z) = x^2 y - y^2 z - xyz$ at P(1, -1, 0)

10) If $\vec{a}$ is a constant vector and $\vec{r} = xi + yi + zk$ then evaluate $div(\vec{a} \times \vec{r})$

PART – B (50 Marks)

11\. a) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n.3^n}$. 5

b) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{(n+1)^n} x^n$ 5

12\. a) State and prove Cauchy's mean Value Theorem. 5

b) Find the evolute of the curve $x = \mathrm{a}\cos^3 t,\ y = a\sin^3 t$ 5

13\. a) Find the minimum value of $x+y+z$, subject to the condition $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$. 5

b) Examine for maximum and minimum values of the function
$f(x,y) = x^4 + 2x^{2y} - x^2 + 3y^2$ 5

14\. a) Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-\left(x^2+y\ \right)} dx\, dy$ by changing to polar co ordinates.

b) Find the volume of the unit sphere $x^2 + y^2 + z^2 = 1$ 5

15\. Verify Green's theorem for $\oint_c \left(xy^2 + 2xy\right)dx + x^2 dy$ where C is the boundary of the region enclosing $y^2 = 4x, x = 3$ 10

16\. a) Find the circle of curvature of the curve $xy = 9$ at the point (1,9) 5

b) Find the Taylor series expansion of the function $f(x,y) = \frac{1}{1-x-y}$ around (0,0). 5

17\. a) Evaluate $\int_0^2\int_x^2 2y^2 \sin xy\, dydx$ by changing the order of integration. 5

b) Using Gauss divergence theorem, evaluate $\iint_S x\,dy\,dz + y\,dz\,dx + z\,dx\,dy$ where S is the surface of the sphere $(x-2)^2 + (y-2)^2 + (z-2)^2 = 16$ 5

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