Code No. 2876 / AICTE

FACULTY OF ENGINEERING

B.E. I – Semester (AICTE) (Main & Backlog) Examination, December 2019

Subject: Mathematics – I

Time: 3 Hours **Max.Marks: 70**

Note: Answer all questions from Part-A and any five questions from Part-B

PART – A (10x2 = 20 Marks)

1 Examine the convergence of the series $\sum\left(1+\frac{1}{n}\right)^{-n}$ 2

2 Define absolutely and conditionally convergent series. 2

3 Write the geometrical interpretation of Rolle's theorem. 2

4 Find the envelope of the family of curves $y = mx + m^2$, m is a parameter. 2

5 If $f(x,y) = \frac{xy}{x^2+y^2}$, find $\frac{\partial f}{\partial x}$ at (2, 3) 2

6 Find $\frac{\partial(u,v)}{\partial(x,y)}$ if $x+y = u$, $y = uv$ 2

7 Change the order of integration in $\int_0^a\int_y^a f(x,y)\,dx\,dy$ 2

8 Evaluate $\int_0^{\log a}\int_0^{\log b}\int_0^{\log c} dz\,dy\,dx$. 2

9 Show that the vector field $\bar{v} = (\sin y + z)\,i + (x\cos y - z)\,j + (x - y)\,k$ is irrotational 2

10 Find the maximum value of the directional derivative of $f(x, y, z) = x^2 + yz^2$ at (1, -1, 3) 2

PART – B (5x10 = 50 Marks)

11 a) Examine the convergence of the series $1+\frac{1}{2}+\frac{1.3}{2.4}+\frac{1.3.5}{2.4.6}+\ldots\ \ldots$ 4

b) Test the following series for conditional or absolute convergence

i) $\sum(-1)^n\frac{1}{\sqrt{n}}$

ii) $\sum(-1)^{n-1}\frac{1}{n^2+1}$ 6

12 a) State and prove Cauchy's mean value theorem. 5

b) Find the evolute of the curve xy = 1. 5

13 a) Discuss the continuity of the function 5

$$f(x,y) = \begin{cases} (x+y)\sin\left(\dfrac{1}{x+y)}\right), & x+y \neq 0 \\ 0 & , \quad x+y = 0 \end{cases} \quad \text{at } (0, 0)$$

Show that $f_x(0\ 0)$ does not exist.

b) Find the local maximum and minimum values of the function $f(x, y) = 2x^2 - 2y^2 - x^4 + y^4$. 5

14 a) Evaluate $\iint_R e^{-(x+y)} \sin\left(\dfrac{®y}{x+y}\right)$ dx dy, where R is the entire first quadrant in the xy – plane using the transformation x+y = u, y = v. 5

b) Find the volume of the solid common to the two cylinders $x^2+y^2 = 4$ and $x^2+ z^2 = 4$. 5

15 a) Find the directional derivative of $\nabla.(\nabla f)$, where $f(x, y, x) = 2x^3y^2z^4$, at (1, -2, 1) in the direction of the normal to the surface $xy^2z = 3x + z^2$. 5

b) Find the work done in moving a particle in the force field $\vec{F} = 3x^2\, i + (2xz - y)\, j + zk$ along the straight line from (0, 0, 0) to (2, 1, 3). 5

16 a) Discuss the convergence of the series $\sum \dfrac{1}{e^n}\left(1+\dfrac{1}{n}\right)^{2n}$. 5

b) Expand $f(x, y) = e^{x+y}$ in Taylor series about (1, 1). 5

17 a) Find the equation of the circle of curvature of the curve y = sin x at $\left(\dfrac{®}{2}, 1\right)$. 5

b) Using Gauss divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}$ ds where $\vec{F} = 2xyi + yzj + xzk$ and s is the surface of the parallelopiped bounded by x = 0, x = 2; y = 0, y = 1; z = 0, z =3. 5

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