Code No. 2876/AICTE/S

FACULTY OF ENGINEERING

B.E. (AICTE) I-Semester (Suppl.) Examination, December 2020

Subject : Mathematics - I

Time : 2 Hours **Max. Marks: 70**

Note: (Missing data if, any can be assumed suitable).

PART – A

Answer any five questions. **(5 x 2 = 10 Marks)**

1 Define convergent series. Give an example.

2 Test the convergence of the series $\frac{1}{2} - \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} + \ldots\ldots$

3 Discuss the applicability of Lagrange's mean value theorem for f(x) = 2x + 3 in [2, 4].

4 Find the radius of curvature of the curve x = 2 cost, y = 2 sint at any t.

5 Show that $\lim_{(x,y) \to (1,1)} x^2 + y = 2$.

6 If x = r cosθ, y = r sinθ, z = z , find $\frac{\partial(x, y, z)}{\partial(r, \theta, z)}$.

7 Find the area of the region bounded by y = x and y = x^2.

8 Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} \, dx \, dz \, dy$.

9 Find the gradient of $\log_e r$, where $r = |\vec{r}|, rxi + yj + zk$.

10 State Gauss's divergence theorem.

PART – B

Answer any four questions. **(4 x 15 = 60 Marks)**

11 (a) Discuss the convergence of the series $\sum \left[\sqrt[3]{n^3 + 1} - n\right]$.

(b) Test for convergence of the series $\sum \frac{3.6.9\ldots\ldots 3n}{7.10.13\ldots\ldots(3n+4)} x^n, x > 0$.

12 (a) State and prove Rolle's theorem.

(b) Find the Taylor series expansion of f(x) = $x^3 + 3x^2 + 2x + 3$ about x = 1.

(c) Find the envelope of the family of lines y = a x + a^2.

13 (a) If z = f(x, y), x = u cosα - v sinα, y = u sinα + v cos α, α is a constant, then show that

$$\left(\frac{\partial t}{\partial u}\right)^2 + \left(\frac{\partial t}{\partial v}\right)^2 = \left(\frac{\partial t}{\partial x}\right)^2 + \left(\frac{\partial t}{\partial y}\right)^2$$

(b) Find the points on the curve $x^2 + xy + y^2 = 16$ which one nearest and fourthest from the origin using Lagrange multipliers method.

14 (a) Evaluate $\iint_R (x^2 + y^2)\,dxdy$, where R is the region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant.

(b) Find the volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the coordinate planes.

15 Verify Green's theorem for $\oint_C (x^2 - 2xy)\,dx + (x^2 y + 3)\,dy$, where C is the boundary of the region defined by $y^2 = 8x$ and $x = 2$.

16 (a) Show that the series $\sum \frac{\cos^2 nx}{n\sqrt{n}}$ is absolutely convergent.

(b) Find the centre of circle of curvature of the curve $y = e^x$ at (0, 1).

17 (a) If $f(x, y) = \begin{cases} \frac{x^2 y(x - y)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

(b) Prove that $\nabla^2 r^n = n(n + 1)\, r^{n-2}$, where $r = |\vec{r}|, \vec{r} = xi + yj + zk$.

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