FACULTY OF ENGINEEERING
B.E. I Semester (AICTE)(Backlog) Examination, October 2021

Subject : Mathematics -I

Time : 2 Hours   **Max. Marks: 70**
Note: Missing data, if any, may be suitably assumed.

PART – A

Note: Answer any five questions.   **(5x2 = 10 Marks)**

1   State Cauchy's n^{th} root test.

2   Examine the convergence of the series $2 - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \ldots$

3   Let $f'(x) = \frac{1}{3 - x^2}$ and f(0)=1. Find an interval in which $f(1)$ lies.

4   Define radius of curvature and circle of curvature.

5   Show that $\underset{(x,y) \to (0,0)}{Lt} \frac{x + \sqrt{y}}{x^2 + y^2}$ do not exist.

6   If $u = f(x - y, y - z, z - x)$ then find the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.

7   Evaluate $\iint\limits_{R} e^{x^2} dx\, dy$ where the region R is given by $R : 2y \le x \le 2$ and $0 \le y \le 1$.

8   Write the applications of double integral.

9   If $\vec{r} = xi + yj + zk,\ |\vec{r}| = r$ then show that $grad\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$

10 Find the directional derivative of $f(x, y) = x^2 y^3 + xy$ at (2,1) in the direction of a unit vector which makes an angle of $\frac{\pi}{3}$ with x-axis.

PART – B

Note: Answer any four questions.   **(4x15 = 60 Marks)**

11 a) Test the convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \frac{4^4}{5^5} + \ldots$

   b) Discuss the convergence of the series $\sum \frac{1.4.7 \ldots\ldots(3n - 2)}{2.5.8 \ldots\ldots(3n - 1)}$

12 a) Using Taylor's series obtain the value of $Cos31^0$ correct to four decimal places.
   b) Find the evolute of the curve $y^2 = 4ax$

13 a) If $z = f(x, y),\ x = u \cos\alpha - v \sin\alpha,\ y = u \sin\alpha + v \cos\alpha$ where α is a constant, then show that

$$\left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$$

   b) Divide a number into three parts such that the product of the first, square of the second and cube of the third is maximum.

14 a) Change the order of integration and evaluate the integral $\int\limits_{y=0}^{1} \int\limits_{x=y}^{\sqrt{2-y^2}} \frac{y}{\sqrt{x^2 + y^2}}\, dxdy$

   b) Find the volume of the solid in the first octant bounded by the paraboloid $z = 36 - 4x^2 - 9y^2$

15 Verify Stoke's theorem for the vector field $\vec{v} = (3x - y)i - 2yz^2 j - 2y^2 zk$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 16, z > 0$.

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16 a) If $f(x, y) = Tan^{-1}(xy)$ find an approximate value of f(1.1, 0.8) using Taylor's series quadratic approximation.

b) Show that the variables $u = x - y + z, v = x + y - z, w = x^2 + xz - xy$ are functionally related. Find the relationship between them.

17 a) Evaluate the integral using the Green's theorem $\int_C (x^2 + y^2) dx + (5x^2 - 3y) dy$, C is the boundary of the region enclosing $x^2 = 4y, y = 4.$

b) Discuss the maxima and minima of $f(x, y) = x^3 y^2 (1 - x - y)$

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