Code No. 15159/AICTE

FACULTY OF ENGINEERING
B.E. I - Semester (AICTE) (Main) Examination, July 2021

Subject: Mathematics - I

Time: 2 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any three questions from the remaining six questions.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1 Answer any four questions. **(4 x 4 = 16 Marks)**

(a) Examine the convergence of the sequence $\{a_n\}$ *where* $a_n = 3 + (-1)^n$.

(b) Determine the nature of the series $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$.

(c) Using Lagrange mean value theorem, show that $|\cos b - \cos a| \le |b - a|$.

(d) If $f(x, y) = x\cos y + e^x \sin y, x = t^2 + 1, y = t^3 + 1$, then find $\frac{df}{dt}$ at $t = 0$.

(e) If $z = x^3 + y^3 - 3axy$ then find $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y^2}$.

(f) Find the area of the region bounded by the curves $x = y^2, \ x + y - 2 = 0$.

(g) If $\vec{r} = xi + yj + zk$ then show that (i) $\nabla \cdot \vec{r} = 3$ (ii) $\nabla x \vec{r} = 0$.

(3x18 = 54 Marks)

2 (a) Determine the nature of the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(2n-1)}$.

(b) Determine the nature of the series $\sum_{n=1}^{\infty} \frac{(n)!}{(n^n)^2}$.

3 (a) Find ‘c’ of Cauchy’s mean value theorem for $f(x) = \log_e x$ and $g(x) = 1/x$ in the interval [1, e].

(b) Find the radius of curvature at any point on the cardioid $r = a$ (1 - cos θ).

4 (a) If $z = f(x + ay) + \phi(x - ay)$, show that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$.

(b) Expand $e^x \log(1 + y)$ in powers of x and y up to the terms of third degree using Taylor’s theorem.

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5 (a) By changing the order of Integration evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$.

(b) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

6 (a) Find the directional derivative of $f(x, y, z) = xy^3 + yz^3$ at the point (2,-1,1) in the direction of the vector i + 2j + 2k.

(b) Verify Green's theorem for $\int_c [(xy + y^2)dx + x^2 dy]$ where c is bounded by $y = x^2$ and $y = x$.

7 (a) Find the envelope of the family of lines $y = mx + \sqrt{1 + m^2}$, m being parameter.

(b) Evaluate $\int\int r^3 drd\theta$ over the area between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$.

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Code No. 15001/AICTE

FACULTY OF ENGINEERING

B.E. (AICTE) I-Semester (Backlog) Examination, July 2021

Subject : Mathematics – I

Time: 2 hours **Max. Marks: 70**

Note: Missing data, if any, may be suitably assumed.

PART – A

Answer any five questions. **(5x2 = 10 Marks)**

1 Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2+1}{n^2}$.

2 State D'Alembert's Ratio Test.

3 Verify Rolle's theorem for the function f(x)=x^2 in [-1, 1].

4 Find the curvature and radius of curvature of the curve $x^2 + y^2 = a^2$ at (x, y).

5 Find the first order partial derivatives for $f(x, y) = x^4 - x^2 y^2 + y^4$ at (-1, 1).

6 Find $\frac{df}{dt}$ at $t = 0$ where $f(x, y) = x \cos y + e^x \sin y$, $x = t^2 + 1$, $y = t^3 + t$.

7 Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z \, dx \, dy \, dz$.

8 Evaluate $\int_0^{\pi} \int_0^{a \sin\theta} r \, dr \, d\theta$.

9 If $\vec{a}$ is a constant vector and $\vec{r} = xi + yj + zk$, then show that curl$(\vec{a} \times \vec{r}) = 2\vec{a}$.

10 Find the angle between surfaces $x \log z = y^2 - 1$ and $x^2 y = 2 - z$ at the point (1, 1, 1).

PART – B

Answer any four questions. **(4x15 = 60 Marks)**

11 (a) Test for the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots..$

(b) Discuss the absolute and conditional convergence of the series

$$\frac{x}{\sqrt{3}} - \frac{x^2}{\sqrt{5}} + \frac{x^3}{\sqrt{7}} + \ldots.. \;\ldots\ldots$$

12 (a) Expand f(x) = $e^{\sin x}$ by Maclaurin series.

(b) Find the envelope of the family of lines $y = mx + \sqrt{1+m^2}$, m being the parameter.

13 (a) Expand $f(x, y) = Tan^{-1}\left(\frac{y}{x}\right)$ is powers of (x – 1) and (y – 1) upto 3rd degree terms.

(b) If the sum of three numbers is constant then prove that their product is maximum when they are equal.

14 (a) Change the order of integration in $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx$ and hence evaluate it.

(b) Find the volume of the region bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

15 (a) Show that div(grad r^n) = $\nabla^2(r^n) = n(n+1)r^{n-2}$.

(b) Evaluate $\int_C \vec{F}\cdot\hat{n}\, ds$ where $\vec{F} = 4xi - 2y^2 j + z^2 k$ and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

16 (a) If u = xyz, $v = x^2 + y^2 + z^2$, w = $x + y + z$ then find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.

(b) Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n!)} x^{2n} .$$

17 (a) Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the Folium $x^3 + y^3 = 3axy$.

(b) Find the directional derivative of $f(x, y) = e^x \sec y$ at $P(0, \pi/4)$ in the direction of $\overrightarrow{PQ}$ where Q is the origin.

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