FACULTY OF ENGINEERING

B.E. (AICTE) I – Semester(Common for All Branches) (Main & Backlog)
Examinations, March / April 2022
Subject: Mathematics - I

Time: 3 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each Question carries 14 Marks.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1.

(a) Discuss the convergence of the series $\sum \frac{1}{n^2}$.

(b) Obtain the fourth degree Taylor's polynomial approximation to $f(x) = e^{2x}$ about $x = 0$.

(c) Show that the following function $f(x,y)$ is continuous at the point $(0,0)$.

$$f(x,y) = \begin{cases} \frac{2x(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}.$$

(d) Evaluate the double integral $\iint_R e^{x^2} dx\, dy$, where the region *R* is given by $R: 2y \leq x \leq 2$ and $0 \leq y \leq 1$.

(e) Find the directional derivative of $f(x,y,z) = xy^2 + 4xyz + z^2$ at the point (1,2,3) in the direction of *3**i**+4**j**-5**k***.

(f) If $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ and $\boldsymbol{r} = |\boldsymbol{r}|$ show that $div\left(\frac{\boldsymbol{r}}{r^3}\right) = 0$.

2. (a) Examine the convergence or divergence of the following series: $\sum \frac{x^{n+1}}{(n+1)\sqrt{n}}$.

(b) Test the convergence of the series $\sum \frac{(-1)^{n-1}}{(2n-2)!}$.

3. (a) Obtain the Taylor's polynomial approximation of degree n to the function $f(x) = \mathrm{e}^x$ about the point $x = 0$.

(b) Using Lagrange mean value theorem, show that $1 + x < e^x < 1 + xe^x$.

4. (a) If $f(x,y) = \tan^{-1}(xy)$, find an approximate value of $f(1.1, 0.8)$ using the Taylor's series (i) linear approximation and (ii) quadratic approximation.

(b) Find the shortest distance between the line $y = 10 - 2x$ and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

5. (a) Evaluate the integral $\iiint_R (x^2 y) dx\, dy\, dz$, where the boundary R: $x^2 + y^2 \leq 1, 0 \leq z \leq 1$.

(b) Evaluate the integral $\iiint_R (2x - y - z) dx\, dy\, dz$, where the boundary R: $0 \leq x \leq 1, 0 \leq y \leq x^2, 0 \leq z \leq x + y$.

6. (a) Find the work done by the force $\boldsymbol{F} = (x^2 - y^3)\boldsymbol{i} + (x + y)\boldsymbol{j}$ in moving a particle along the closed path C containing the curves $x + y = 0, x^2 + y^2 = 16$ and $y = x$ in the first and fourth quadrants.

   (b) Let *D* be the region bounded by the closed cylinder $x^2 + y^2 = 16, z = 0$ and $z = 4$. Verify the divergence theorem if $\mathbf{v} = 3x^2\boldsymbol{i} + 6y^2\boldsymbol{j} + z\boldsymbol{k}$.

7. (a) Evaluate the surface integral $\iint_S \boldsymbol{F}.\boldsymbol{n}\, dA$ where $\boldsymbol{F} = 6z\,\boldsymbol{i} + 6\boldsymbol{j} + 3y\boldsymbol{k}$ and S is the portion of the plane $2x + 3y + 4z = 12$, which is in the first octant.

   (b) Evaluate the integral $\iint_S (\nabla \times \mathbf{v}).\boldsymbol{n}\, dA$ by Stoke's theorem where $\mathbf{v} = (x^2 - y^2)\boldsymbol{i} + (y^2 - x^2)\,\boldsymbol{j} + z\boldsymbol{k}$ and *S* is the portion of the surface $x^2 + y^2 - 2by + bz = 0, b$ constant, whose boundary lies in the *x*-*y* plane.

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(Common to All Branches)

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(Missing data, if any, may be suitably assumed)

PART – A

Note: Answer all questions. **(10 x 2 = 20 Marks)**

1 Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

2 State Cauchy's n^{th} root test.

3 State Lagrange's mean value theorem.

4 Find the coordinates of the centre and radius of the circle of curvature of the curve $xy = 1$ at (1,1).

5 Find $\frac{\partial(u,v)}{\partial(x,y)}$ where $u = x^2 - y^2, v = 2xy$.

6 Find $\frac{df}{dt}$ at $t = 0$ where $f(x,y,z) = x^3 + xz^2 + y^3 + xyz, x = e^t, y = \cos t, z = t^3$.

7 Change the order of integration is $\int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$.

8 Evaluate $\int_0^1 \int_0^x e^{y/x}\, dy\, dx$.

9 Compute the gradient of the scalar function $\phi(x,y,z) = \sin(xyz)$ at $(1,\ 1,\ \barwedge)$.

10 If $\vec{r} = xi + yi + zk$ and $r = |\vec{r}|$, then show that $div\left(\frac{\vec{r}}{r^3}\right) = 0$.

PART – B

Note: Answer any five questions. **(5 x 10 = 50 Marks)**

11 (a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$.

(b) Discuss the convergence of the series $\sum \frac{4.7....(3n+1)}{1.2.....n} x^n$.

12 (a) Prove that $\frac{b-a}{b} < \log\left(\frac{b}{a}\right) < \frac{b-a}{a}$ for $0 < a < b$ and hence show that $\frac{1}{4} < \log\left(\frac{4}{3}\right) < \frac{1}{3}$.

(b) Find the equation of the evolute of the parabola $x^2 = 4ay$.

13 (a) If $u = x\sqrt{1-y^2} + y\sqrt{1-x^2},\ v = \sin^{-1} x + \sin^{-1} y$ then show that u, v are functionally dependent and find the relation between them.

(b) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction.

14 (a) Change the order of integration and hence evaluate $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4 - a^2x^2}}\, dx\, dy$.

(b) Evaluate $\int_0^\infty \int_0^\infty e^{-\left(x^2+y^2\right)}\, dx\, dy$ by changing to polar coordinates.

15 Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\, i - yz^2\, j - y^2zk$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy-plane.

16 (a) In a plane triangle, find the maximum value of cosA cosB cosC.

(b) Examine the convergence of the series $\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + ...$

17 (a) Show that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$ is $4a\cos\frac{\theta}{2}$.

(b) Find the total work done in moving a particle is a force field given by $\vec{F} = 3xyi - 5zj + 10xk$ along the curve $x = t^2 + 1,\ y = 2t^2,\ z = t^3$ from $t = 1$ to $t = 2$.

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