EMI-01-03

Measurement the quantitative comparison of a physical quantity, with that of a same kind, taken as a Standard or a unit. the Standard can be compared in multiples or submultiples of it.

the device or implement used for this measurement is called a measuring Instrument.

Lord Kelvin : When you know something, and if it can be measured and expressed in numbers, then it is Science.

the Quantity to be measured is called the measurand and to a small extent, our physical senses can estimate the value. However physical senses can be subjected to illusions and hence estimates may be erroneous.

Most often, the measurand is either too big or too small, to be capable of ready measurement Hence the need for conversion into adaptable means. this conversion is done through an intermediate element known as Transducer.

EMI-01-04

"Measurements" are classified as
- Direct measurements
- Indirect measurements

Instruments can be classified based on various attributes like:-

1) Absolute Instruments and Secondary Instruments

2) Based on Construction:-
   a) Mechanical b) Electrical c) Electronic

3) a) Direct measuring instruments b) Comparison Instruments

4) Active and Passive instruments.

5) Deflection type and NULL type

6) Monitoring and Controlling instruments

7) Analog and Digital instruments

8) Based on function
   a) Indicating instrument b) Recording instrument
   c) Controlling instrument d)

EMI-01-05

Characteristics of Instruments & Measuring Systems

Measurand can have a Variable Value or a Constant Value with time. first Case is a Dynamic Case and the later a Static Case. Accordingly the instrument too can exhibit Static and Dynamic Characteristics.

Static characteristics are obtained through Static Calibration.

Static

1) Accuracy Relates to the instrument Output with reference to the true value of the measurand. It refers to the degree of closeness of the Output to the True Value.

Accuracy is of 3 types — 1) point accuracy such as melting point or boiling point, 2) percentage of True value — $\text{Error\%} = \frac{\text{Measured value} - \text{True Value}}{\text{True value}} \times 100$

and 3) percentage of full scale deflection

$$\text{Error\%} = \frac{\text{Measured Value} - \text{True Value}}{\text{True Scale Value}} \times 100$$

2) Precision Degree of freedom of the instrument from random errors. It is a measure of consistency and repeatability through Successive measurements

3) **Bias** Constant error which exists Throughout the full range of the instrument. Bias can be positive or negative and can be removed by turnup the thumbwheel to make the pointer read zero, when the instrument is unloaded.

4) **Repeatability** : the closeness of outputs when the same input is applied many no. of Times over a short period of time. It is affected by internal noise and drift.

5) **Reproducability** :- closeness of repeating same value of input quantity when measured at different times.

6) **Tolerance** : the maximum error which can be expected. which can be positive / negative

7) **Reliability** and **Maintainability**

Reliability is the probability of the performance of the system for its assigned functions. for a specified period under given conditions

Maintainability the probability of making good a defective system under conditions of breakdown

8) **Deviation** the departure or difference between measured value and True value.

9) **Scale Range** and **Scale Span**.

Range indicates the extent of which measurements can be read. ex: 0-200, -15 to +60, etc.

Span indicates the difference between max & min values of the range.

for the above ex: Span = 200-0 = 200 and 60-(-15) = 75.

10) Scale Readability: the scale of an analog instrument can have uniform or nonuniform (logarithmic) divisions depending on the type of the instruments

11) **Resolution or discrimination**

Smallest increment of the input quantity to which the measuring system responds.

12) Threshold : the minimum value of input above which there will be an output recorded/ indicated on the measuring instrument is called threshold.

13). Drift: the slow variation of the output of the measuring system which is not due to any change in input to the system is called drift.

14) Instrument efficiency: the ratio of the measured quantity to full scale, and the power absorbed. is called Instrument efficiency.

15) Instrument Hysterisis the different outputs which are shown by the instrument system, when it is loaded and unloaded by same increments.

16) Dead Time and Dead Zone

Dead time is the time required ~~fo~~ by the measuring system to respond to the change in measurand.

Dead Zone: the largest change of input for which there is no corresponding output

EMI-01-09

17) **Overshoot** the momentum developed by the input on the measuring system, causes the pointer mass to move beyond the equilibrium position and cover a period of time settles to final steady position . . the moving beyond is called **Overshoot**.

18) **loading effect** A measuring element, when introduced into the system consumes energy and hence will not/cannot faithfully reproduce the measurement value. the difference in true value and value reported is the distortion in the form of attenuation, waveform distortion, phase shift. etc.

19)

20)

ERRORS

① **Absolute error**: the difference in the Values of the measurand as taking from an Absolute(A) instrument and the measuring instrument(A_m) is the Absolute error.

$$\delta_A = A_m - A.$$

② **Relative error** and **Percentage error**

$$\text{Relative error} = \frac{\delta_A}{A} = \frac{A_m - A}{A}.$$

$$\% \text{ error} = \frac{A_m - A}{A} \times 100$$

③ **Limiting error** or **Guarantee error**:

the limit of deviations from the specified values is known as limiting error.

ex:- A meter has a limiting error ±2% on full scale, which means that for a full scale of 25A (say) the reading can vary by

$$\frac{25 \times 2}{100} = 0.5A$$

EMI-01-11

Types of errors

① Gross error because of human mistake.

② Systematic error which are constant and are well defined by a mathematical expressi they are classified as (i) Instrumental error (ii) environmental error and (iii) observational error.

③ Random or accidental errors. accumulation of small defects.

Sources of errors

Poor design, change in process parameters poor maintenance, design limitations, Insufficient knowledge, errors due to lack of Skill

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EMI-01-12

Units & Dimensions

Quantities are of many kinds like – physical, Chemical, mechanical, Thermal, electrical, etc

In order to measure or Compare, some magnitude is to be taken as standard.

Basic Units which Can be used to describe all other phenomena are Fundamental Units like Mass (M) Length (L), Time (T). the other units expressed in terms of fundamental Units are Called Derived Units.

Absolute Units are those units which Can be derived experimentally without Comparing with any other fundamental Units.

Dimensional equations expressing derived quantities in terms of fundamental dimensions of M, L & T.
used to Check accuracy of an equation and for Conversion of units from one system to other

Seven Basic Units of SI System

Meter — length equal to 1650763.73 wavelengths of Orange line in the spectrum of internationally specified Krypton discharge lamp.

Kilogram: the mass of platinum Iridium Cylinder preserved at International Bureau of Weights and Standards.

EMI-01-13

Second : the interval occupied by 9192631770 cycles of radiation of transition of Cesium-133 atom.

Ampere· that Current, which if maintained in two straight parallel ~~wires~~ Conductors of infinite length of negligible Circular Cross section and placed 1 meter apart, in vaccum, would produce a force of 2×10^{-7} Newtons/meter.

Kelvin $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water

Candela· luminous intensity in the perpendicula direction of a Surface of $\frac{1}{6,00,000}$ Squaremeter of black body at the temperature of freezing platinu under Standard atmospheric pressure.

Mole the amount of Substance of a system which Contains as ~~an~~ many atoms as in 0.012 kg of Carbon 12.

Supplementary Units

plane angle — radian | rad
Solid angle — Steredians | sr

EMI-01-14

Standards of measurement a) absolute Standards

b) i) International Standards.
(ii) primary Standard.
(iii) Secondary Standard
(iv) working Standard

International Standard — by international agreem

Primary Standards — maintained by National Standards laboratory in different parts of the world.

Secondary Standards : Basic reference standar used in Industrial measurements

Working Standards principal tools of measureme in a laboratory.

International and Absolute Units

International Units

1) International Ohm — resistance offered by a column of mercury of mass 14.4521 g of uniform cross section and of length 106.300 cm to passage of an unvarying current at the temperature of melting ice.

EMI-01-15

International Ampere – that unvarying current which when passed through a solution of silver nitrate ($AgNO_3$) in water, will deposit silver at the rate of 0.00111800 g/sec.

International Volt : 1V = when applied to international resistance of 1Ω, will pass current of 1A.

International watt : electrical energy expended per sec, when unvarying current of one International Ampere flows through one international ohm.

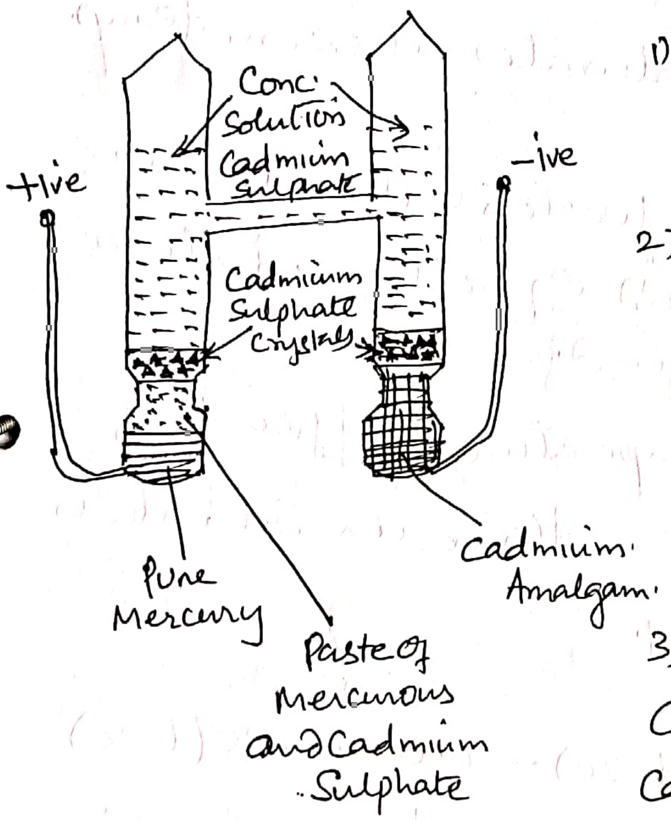
EMI-01-17

Voltage Standard – Standard Cell.

WESTON Standard cell — Has the voltage across terminals Constant for a very long time, when no appreciable Currents are drawn from the Cell. These are Used as a Secondary Standard of Voltage.

WESTON Cadmium Cell has a ref. voltage of 1.0183V

- Saturated Cadmium Cell
- Unsaturated Cadmium Cell



1) H shaped glass vessel, each limb is of about 25 mm in diameter

2) One leg contains, pure mercury, paste of mercurous and Cadmium Sulphate, Cadmium Sulphate crystals and Con. Cadmium Sulphate Solution, in that order from the bottom of the leg. This is the +ive terminal

3) The other terminal Contains Cadmium amalgam Cadmium Sulphate crystals and Conc. Cadmium Sulphate Solution from bottom, in that order. This is the –ive terminal.

4) The H link is also filled with Conc. Cadmium Sulphate Solution

5) the limbs are hermitcally sealed.

Cadmium Sulphate solution — electrolyte

Mercury — +ive electrode

Cadmium amalgam (1 part Cadmium, 7 parts mercury) — -ive electrode

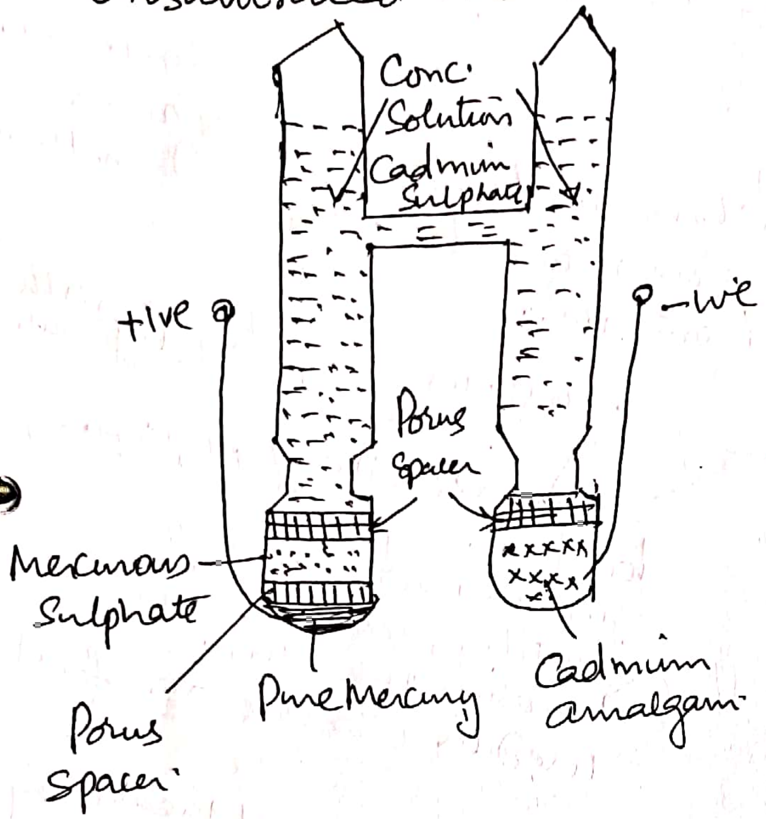
6) the cells differ in construction as Saturated and unsaturated cells. Saturated cells have crystals of cadmium sulphate. the other cell is un saturated at room temp

7) Saturated (Cadmium) Weston Cell is used as the primary standard of voltage. Materials of high purity are used. Cells have high temperature Coefficient of -40μV per °c rise. Hence an oil bath is used for temp. Control.

$E_t = E_{20} - 0.000046(t-20) - 0.00000095(t-20)^2 + 0.00000001(t-20)^3$ is the expression for voltage at any other temp.

EMI-01-19

Unsaturated Cadmium Cell.



1) there are no $CdSO_4$ Crystals. Solutions gets saturated at 4°c and unsaturated at room temperature

2) low temperature Coefficient - 10μV per deg. rise. No temp. Correction is required.

3) each cell manufactured gives a different voltage and hence manufacturer certifies by Comparing with a primary Standard

Precautions

1) Current Cannot be drawn from the Cell. A very high protective ~~Cur~~ resistance is Connected in series; in a circuit

2) Voltmeter Should not be used to measure Voltage as it draws Current

3) Cell Should not be short circuited and then it looses its value as a Standard

4) Store in places with a temp of 15°c-20°c.

EMI-01-20

Standard Resistance

Circuit Components can be classified as
- energy dissipators Resistors
- energy Storage Components : Capacitors & Inductors.

All the 3 Component characteristics are present in all Circuit Components but one Characteristic may dominate.

Resistance materials
- High resistivity (Primary alloys of ≠ diff materials
- low resistivity (Al, Copper)

low temp. coefficient
good mechanical strength
ductility, solderability,
Corrosion resistance.
used in Conductors for electricity

High resistivity
- ① precision electrical measuring instruments. (Standard, substandard)

  Standard resistances and resistance boxes.
- ② resistance elements for rheostats
- ③ high temperature elements for electric furnaces, heating devices.

Important characteristics for precision instruments of resistance :-
① Stability of resistance
② Resistance to Oxidation, moisture and Corrosion

EMI-01-21

Material used for Resistance

1) **Platinum** high cost, high temp. coefficient of resistance,

2) **Manganin** Alloy of 84% Copper, 12% Manganese and 4% Nickel.

Specific resist - 0.42 - 0.74 μ-Ω-m.
Temp. coeff. of Resist - 0.00003 $\Omega/\Omega/^\circ C$
Density - 8,400 kg/m³
Tensile strength - 40-55 kg/mm²
Max. working temp - ≤ 60°C.

Most widely used material

3) **Constantin** — alloy of 60% Copper, 40% Nickel

Resistivity - 49×10^{-8} Ω-m.
Density 8,900 kg/m³
Tensile strength 40-50 kg/mm²
Max working temp 400 - 450°C.

4) **Nichrome** alloy of Nickel & Chromium

Resistivity 1.0 μ-Ω-m.
Temp. coefficient of resistance 0.00044 $\Omega/\Omega/^\circ C$
Density 8,400 kg/m³.
Tensile strength 70 kg/mm²
Working Temp 1000°C.

Resistance Coils

Characteristics — Resistance value and power dissipations

Coils formed on wooden | ceramic | metal formers (of brass)

- wooden ↓ Absorb moisture
- ceramic ↓ poor heat conductivity
- metal formers (of brass) ↓ dissipate heat Not desirable in ac applications. (for ac application former to be non metallic and non inductive)

Insulation — Cotton | silk

Mixture of Cotton & silk fibres known as ESCO is used as an insulation

Material affected by humidity in opposite ie Cotton lengthens while silk contracts. thereby nullifying the effect

Mechanical fatigue while forming coils is overcome by annealing.

Inductance & Capacitance effects of Resistances windings are reduced by Special winding methods — Bifilar winding, Chaperon winding, Ayrton-Perry winding, Curtis & Wenner winding etc

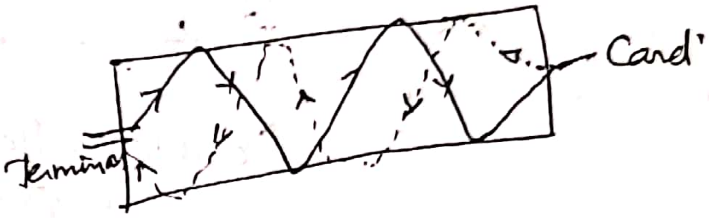
Bifilar winding

largely reduces inductance effect
used for resistances upto 0.1, to 1.0Ω.

Chaperon winding

wound in even no of layers. after each layer the direction is reversed

Ayrton-perry winding

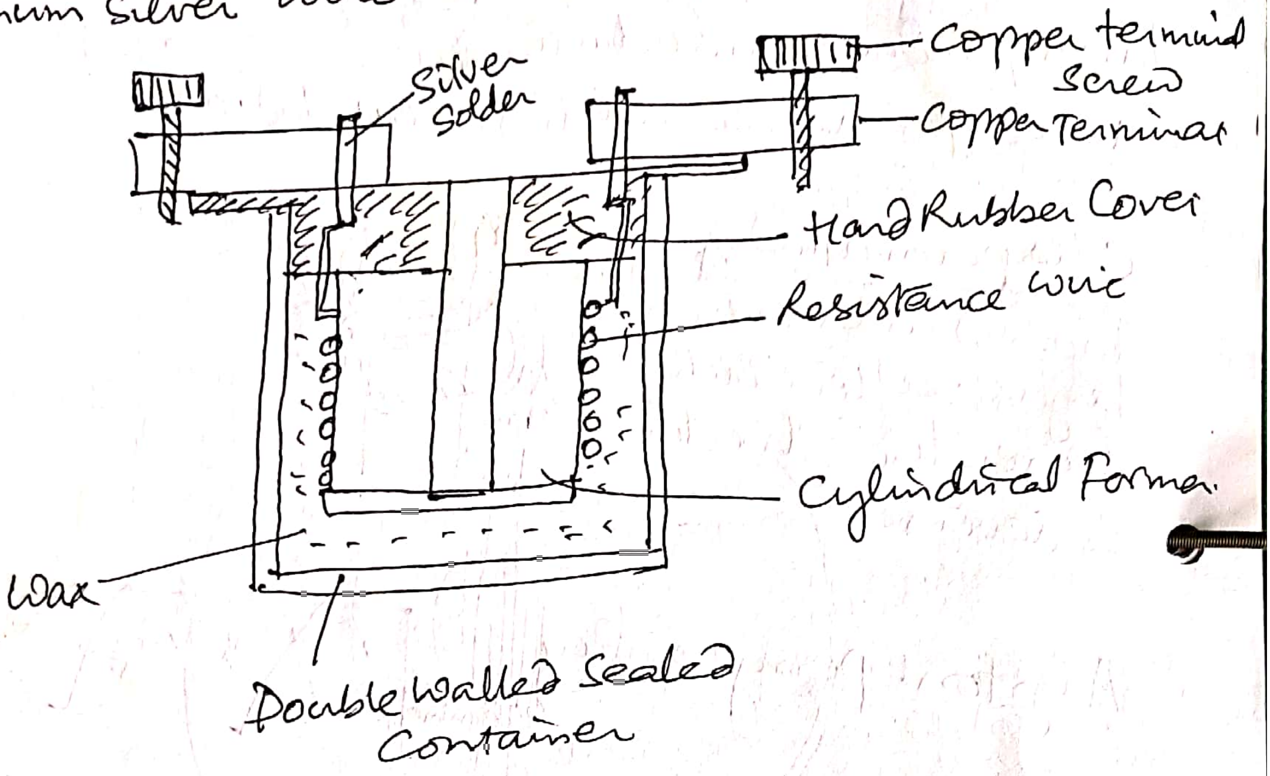


Resistance Standard

: the material used in Standard resistors should have :-

1) High resistivity to get a compact resistor
2) Small temp. coefficient, for corrections in temperature variations.
3) Greater permanence ie, long life with Stand
Value, or no appreciable Change in Valu
over long period of time
4) Low thermo electric effect, not easily Oxidi
5) easily workable, formable and joinable.

Construction :- Bifilar winding of the coil of platinum silver wire, wound non inductively.



Precision of few parts in 10^7 over many years

the coil, after one layer is baked with insulation at approx. 140°C and annealed. the former has a silk layer, with shellac coating.

the space between coil and outer case is filled with wax

Secondary Standard — available in multiples of

$10\,\Omega$,

$$R_t = R_{25} + \alpha(t-25) + \beta(t-25)^2$$

α, β - temperature coefficients

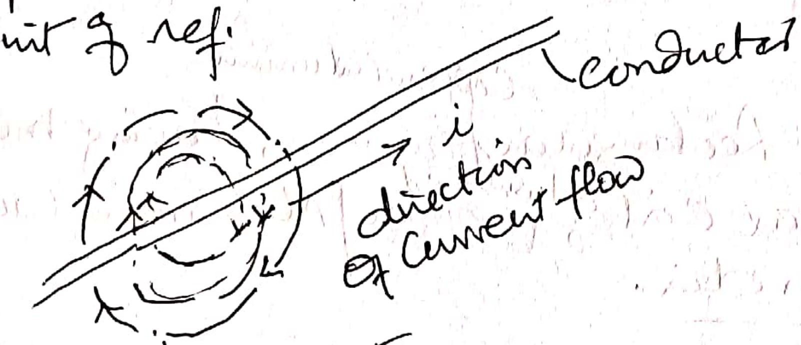
EML-01-25

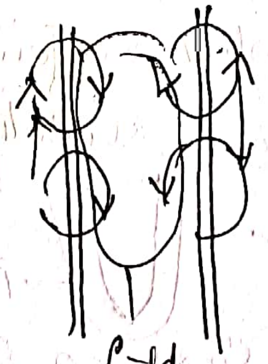
Measuring Instruments

- Analog
  - Indicating
  - Recording
  - Integrating
- Digital

Indicating Instruments have the following essenti
Components :- 1) A moving element 2) A pointer
3) Graduated scale 4) A pivoted system. 5) A prote
Case 6) A set of terminals 7) A magnetic field

Principle of operation : A current Carrying Conductor, when placed in a magnetic field, experiences a force, towards or away from the point of ref.





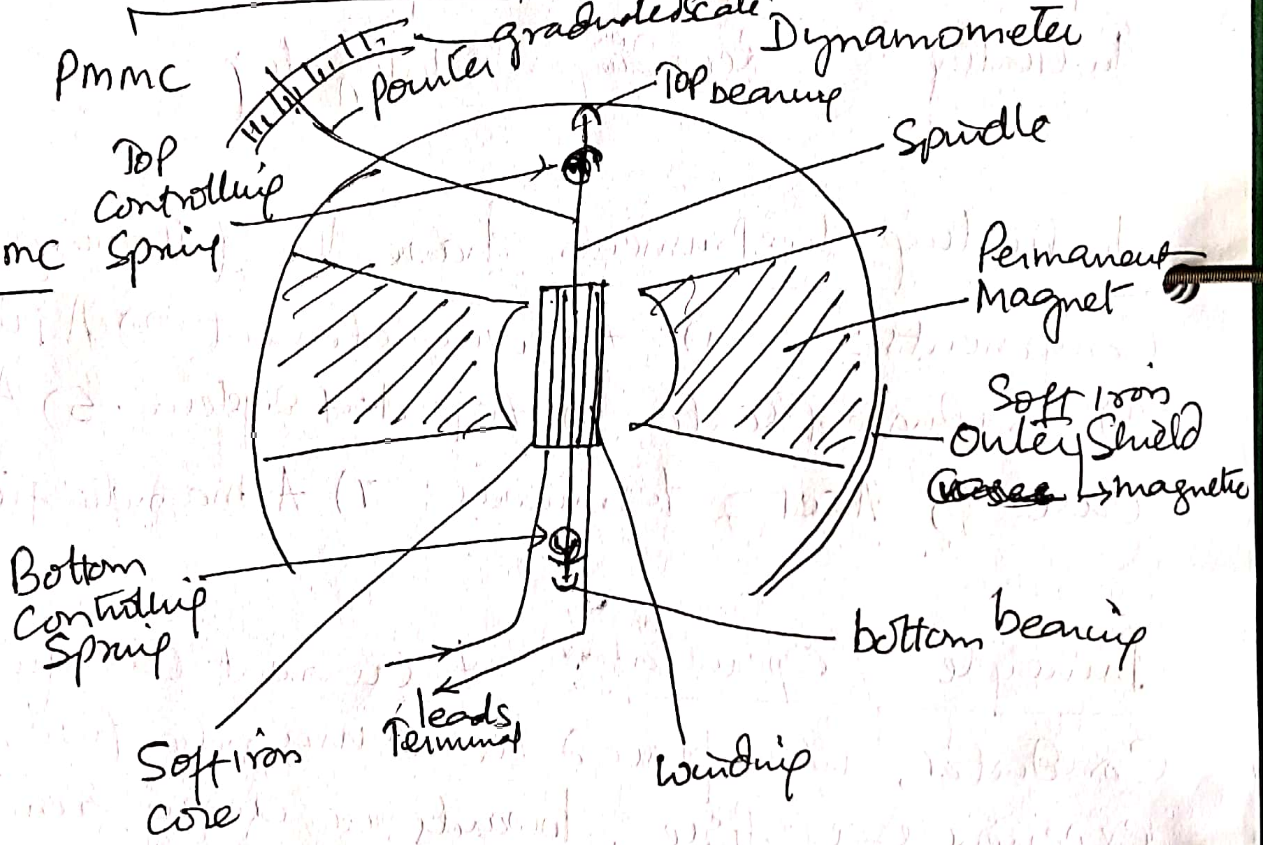
EMI-01-26.

Construction of a Permanent Magnet Moving Coil (PMMC) type of instrument:-

Moving Coil

- PMMC
- Dynamometer

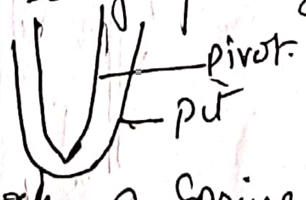
PMMC



Description

moving system :- Rectangular copper or aluminium former, having multi turn rectangular coil of insulated Copper/Aluminium wire of 0.02 to 0.2 mm dia.

the system is pivoted on Jewel bearings of 'V' shape. the pit has radius slightly larger than the pivot for circular contact area.



To protect the pivoted system from shock, a spring can be used in the top and bottom bearing for lateral movement of the spindle.

Magnet System, Alnico magnets, sector shaped moving system and Magnet System Comprise the Deflection System

Control System

└ Controlling Torque produced by top & bottom, phospher bronze hair Springs of helical or Coiled type. the top & bottom Springs are Coiled in clockwise and anticlock wise directions.

the springs Carry the Current to & fro of the moving coil.

Damping System Eddy current damping, Air friction damping, oil dash pot damping

Scale and Pointer System : Light aluminium flat, Carried by the spindle over a graduated Scale.

the flat forms a vertical knife edge.

the Scale markings for PMMC are uniform.

To avoid Parallax error, the Scale Carries a mirror.

Torque equation

Magnetic flux density — B

Current through conductor — i

length of the conductor — l

Angle moved — θ

~~force~~ No of conductors — N

force on conductor $F = Bil$ Newtons

Total force on N conductors $= F_1 = BilN$ Newton

Since the coil is pivoted and has a radius 'r'

then Deflection Torque applied $T_d = F_1 \cdot r$

$T_d = BilNr$ Newton-m.

for a given system, B, l, N, r are constants

hence $T_d \propto i$

the controlling torque T_c is proportional to the deflection 'θ' because of springs.

hence $T_c \propto \theta$.

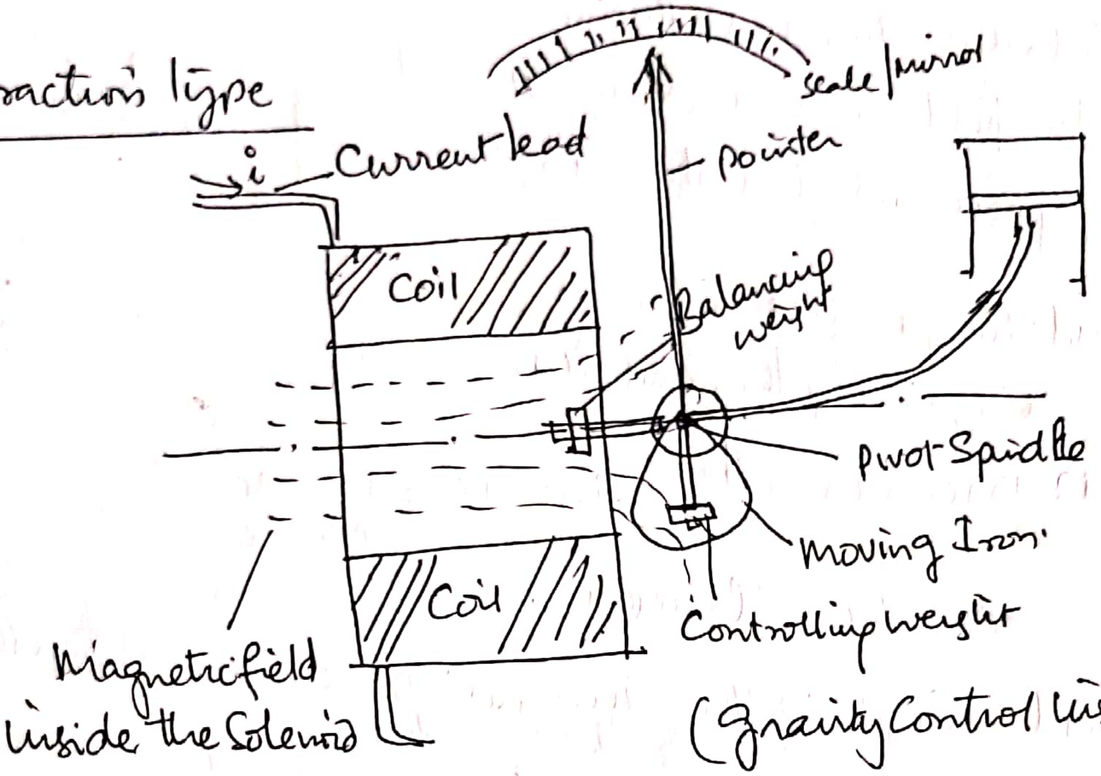
for steady indication $T_d = T_c$

or $\underline{\underline{i \propto \theta}}$

Moving Iron Instruments

- Attraction type
- Repulsion type

① Attraction type



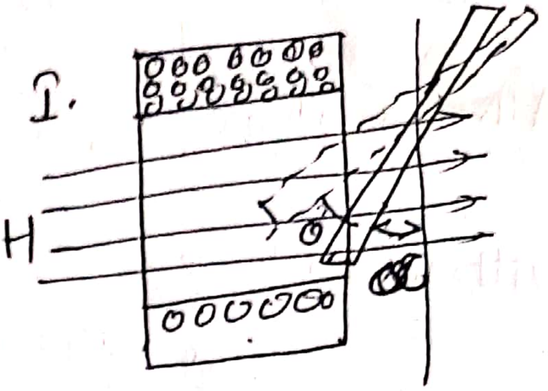
(Gravity Control instrument)

Construction is as shown :

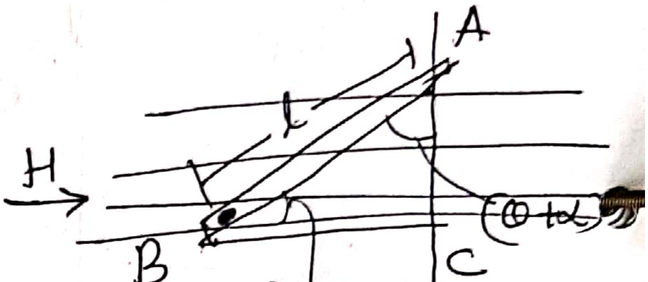
Principle of operation :- Magnetic field set up by the Solenoid influences the moving Iron and sets up a force of attraction [magnetic field induces eddy current in the Iron piece, which sets up a magnetic field and the Interaction is attraction from weaker field side to Stronger field]

The movement of the Iron piece is facilitated by the pivot. the pointer moves along the Graduated Scale.

Torque equation of Attraction type



the magnetic piece is inclined at an angle 'α' to the vertical initially. Due to influence of the field of the coil of strength H the piece is deflected by 'θ'.



Field strength of attraction as the disc, along the central axis is ~~BC~~ proportional to $\sin(\theta+\alpha) = \frac{BC}{AB} = \frac{BC}{l}$ (90-(θ+α))

or $BC = l\sin(\theta+\alpha)$.

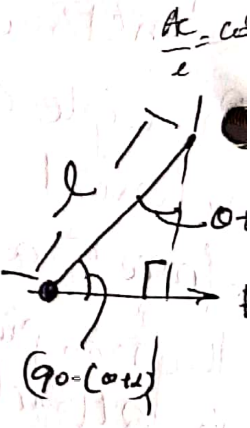
Field strength of the magnetic piece $\propto H\sin(\theta+\alpha)$

Force of attraction $\propto [H\sin(\theta+\alpha)]H$.

$\propto H^2\sin(\theta+\alpha)$

Since $H \propto I$

$\therefore$ force $\propto I^2\sin(\theta+\alpha)$



Deflecting Torque $T_d \propto$ Force × distance moved

$\propto I^2\sin(\theta+\alpha) \cdot l\cos(\theta+\alpha)$

$\propto I^2\sin 2(\theta+\alpha)$

Controlling Torque $T_c \propto \theta$

at balance

$\therefore$ $T_d = T_c$ $\therefore$ $\theta = I^2\sin 2(\theta+\alpha)$

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Unit III.

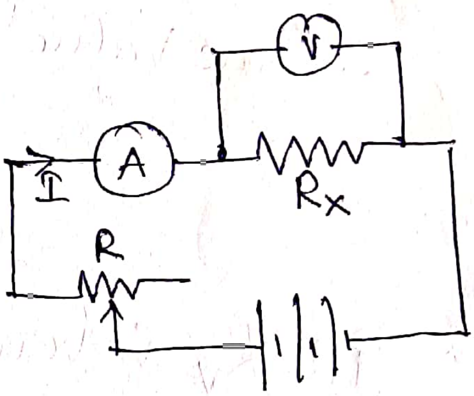
Bridge Methods.

Resistance can be classified as Low, Medium and High resistance, in terms of Measurement Procedures.

Low resistance	— upto 1Ω	armature & Series field winding etc
Medium resistance	— 1Ω to 100 KΩ	electrical appara
High resistance	— above 100KΩ.	Insulating mater

Low resistance

Ammeter-Voltmeter method :-



In this method Current through the resistance under test R_x and the voltage across it both are measured.

Under ideal Condition resistance of Ⓐ and Ⓥ do not have any effect on the Current through the unknown R_x. Hence $R_x = \frac{V_{reading}}{A_{reading}}$

In practise, let I_V be the Current through Ⓥ and I_x — Current through unknown resistance

then $I = I_x + I_V$.

$I_x = I - I_V$.

b

True value of $R_x = \frac{V}{I_x} = \frac{V}{I - I_V}$

Current through Voltmeter $= \frac{V}{R_V}$.

$\therefore$ True Value of $R_x = \frac{V}{I - \frac{V}{R_V}} = \frac{V}{I\left(1 - \frac{V}{IR_V}\right)}$.

the measured value of Resistance $= R_m = \frac{V}{I}$.

$\therefore$ True Value of $R_x = \frac{V}{I\left(1 - \frac{R_m}{R_V}\right)} = \frac{R_m}{\left(1 - \frac{R_m}{R_V}\right)}$

If R_V is very large then $R_V \to \infty$

hence $\frac{R_m}{R_V} = 0$ $\therefore$ $\underline{R_x \simeq R_m.}$

the unknown Value is equal to the measured Value.

True value of $R_x = \frac{V}{I_x} = \frac{V}{I - I_V}$

Current through Voltmeter $= \frac{V}{R_V}$.

$\therefore$ True Value of $R_x = \frac{V}{I - \frac{V}{R_V}} = \frac{V}{I\left(1 - \frac{V}{IR_V}\right)}$.

the measured value of Resistance $= R_m = \frac{V}{I}$.

$\therefore$ True Value of $R_x = \frac{V}{I\left(1 - \frac{R_m}{R_V}\right)} = \frac{R_m}{\left(1 - \frac{R_m}{R_V}\right)}$

If R_V is very large then $R_V \to \infty$

hence $\frac{R_m}{R_V} = 0$ $\therefore$ $\underline{R_x \simeq R_m}$.

the unknown Value is equal to the measured value.

c

Let the galvanometer sensitivity be given as S_V deg/ie Volt.

then the deflection for 'e' = $\theta = S_V e$

ie $\theta = S_V \cdot \frac{E S \Delta R}{(R+S)^2}$.

or $S_V = \frac{\theta (R+S)^2}{E S \Delta R}$.

Bridge Sensitivity $S_B = \frac{\theta}{\left(\frac{\Delta R}{R}\right)}$ defined as deflection of galvanometer per fractional change in unknown resistance

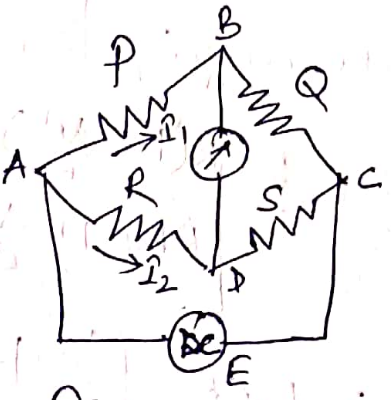
$$\therefore S_B = \frac{S_V \cdot E S \Delta R}{(R+S)^2} \times \frac{R}{\Delta R} = \frac{S_V E S R}{(R+S)^2}.$$

$$\text{or } S_B = \frac{E S_V}{\frac{(R+S)^2}{RS}} = \frac{E S_V}{\frac{R^2 + S^2 + 2RS}{RS}} = \frac{E S_V}{\left(\frac{R}{S} + \frac{S}{R} + 2\right)}.$$

Sensitivity is max. when $\frac{R}{S} = 1$

Medium Resistance

Wheat stone bridge.



$I_1P = I_2R$ & $I_1Q = I_2S$ at balance.

$$\frac{I_1P}{I_1Q} = \frac{I_2R}{I_2S} \quad \text{ie} \quad \frac{P}{Q} = \frac{R}{S}.$$

'R' is the unknown Resistance $\therefore R = \frac{PS}{Q}$.

Sensitivity of Wheatstone bridge.

the effect of a small change in unknown resistance which will lead to deflection is to be studied.

$R \rightarrow$ changes to $R+\Delta R$.

Voltage accross Ⓐ $= V_{AD} - V_{AB} = I_2(R+\Delta R) - I_1P$.

$$I_2 = \frac{E}{(R+\Delta R)+S}$$ III^ly $$I_1 = \frac{E}{P+Q}$$

$$\therefore e = V_{AD} - V_{AB} = \left(\frac{E}{R+\Delta R+S}\right)(R+\Delta R) - \left(\frac{E}{P+Q}\right)(P)$$

$$= E\left[\frac{R+\Delta R}{R+\Delta R+S} - \frac{P}{P+Q}\right]$$

Since $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{P}{P+Q} = \frac{R}{R+S}$.

$$\therefore e = E\left[\frac{R+\Delta R}{R+\Delta R+S} - \frac{R}{R+S}\right] = E\xcancel{\left[\frac{\frac{R}{R+S} + \frac{\Delta R}{R+S}}{\frac{R+S}{R+S} + \frac{\Delta R}{R+S}} - \frac{R}{R+S}\right]}$$

rearranging terms

$$e = E\left[\frac{R\left(1+\frac{\Delta R}{R}\right)}{R+S\left(1+\frac{\Delta R}{R+S}\right)} - \frac{R}{R+S}\right]$$

$$= \frac{E\cdot R}{R+S}\left[\frac{1+\frac{\Delta R}{R}}{1+\frac{\Delta R}{R+S}} - 1\right]$$

the term $\frac{1+\frac{\Delta R}{R}}{1+\frac{\Delta R}{R+S}} - 1$ is expanded as

$$\left[\left(1+\frac{\Delta R}{R}\right)\left(1-\frac{\Delta R}{R+S}+\frac{(\Delta R)^2}{(R+S)^2}+\cdots\right)-1\right]$$

neglecting higher order ΔR^2 etc.

$$\left\{\left(1+\frac{\Delta R}{R}\right)\left[\cancel{1}-\frac{\Delta R}{R+S}\right]\right\}-1$$

$$= \cancel{1}+\frac{\Delta R}{R}-\frac{\Delta R}{R+S}-\frac{\Delta R^2}{R(R+S)}\cancel{-1}$$

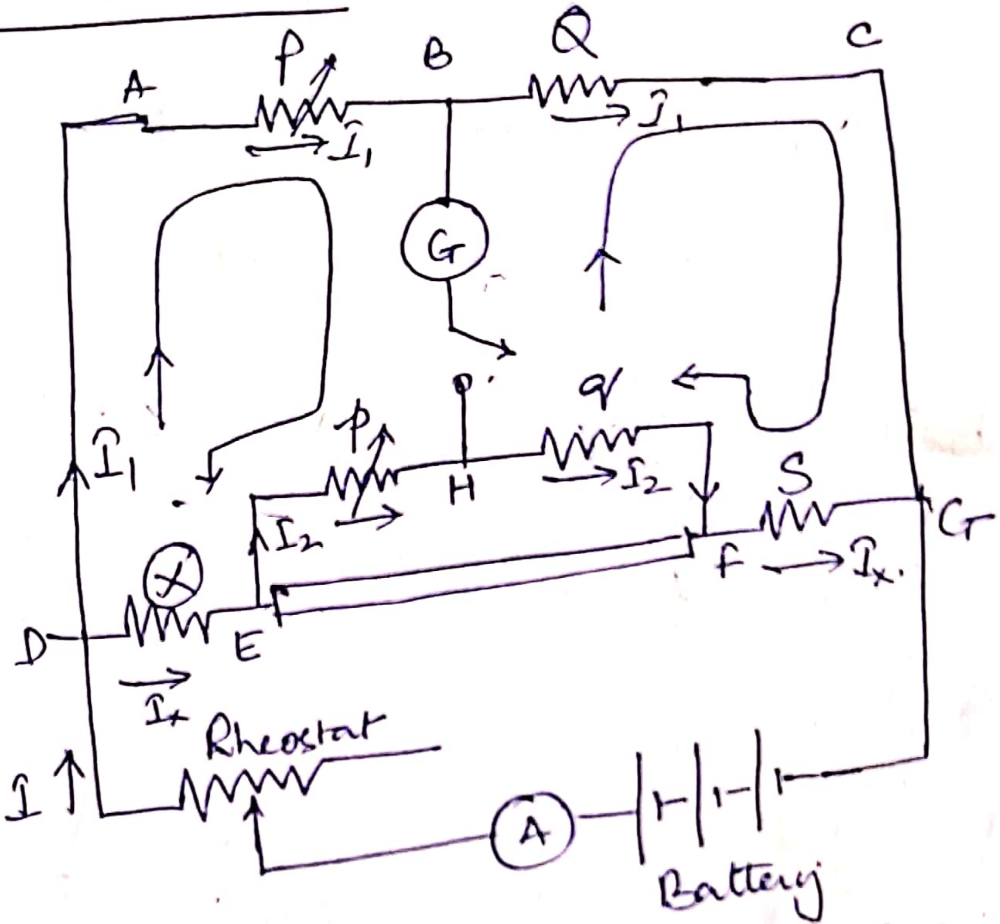
$$\frac{\Delta R(R+S)-R\Delta R-\Delta R^2}{R(R+S)} = \frac{\Delta R\left[\cancel{R}+S-\cancel{R}\right]-\Delta R^2}{R(R+S)}$$

$$\doteqdot \frac{S\cdot\Delta R}{R(R+S)}$$ neglecting higher order

$$\therefore e = \left(\frac{E\cdot\cancel{R}}{R+S}\right)\left(\frac{S\cdot\Delta R}{\cancel{R}(R+S)}\right) = \frac{E\,S\cdot\Delta R}{(R+S)^2}$$

c

Kelvin's Double Bridge:



Ⓧ is the resistance under test and S is a standard resistance of similar range of x.

Ⓧ and Ⓢ is connected by a very low resistance link EF:

P, Q, p, q are known, non inductive resistances

when P, p are Variable resistances.

the ratio of P/Q and p/q is kept same till the galvanometer reads zero.

d

In the loops as shown.

$$I_1 \cdot P - I_2 \cdot p - I_x \cdot X = 0$$

or $I_x X = I_1 P - I_2 \cdot p$ —(1)

Similarly: $I_1 Q - I_2 q + I_x S = 0$

or $I_x \cdot S = I_1 Q - I_2 q$ —(2).

Dividing 1 by 2

$$\frac{I_x \cdot X}{I_x \cdot S} = \frac{I_1 \cdot P - I_2 \cdot p}{I_1 \cdot Q - I_2 \cdot q} = \frac{P\left[I_1 - I_2 \cdot \frac{p}{P}\right]}{Q\left[I_1 - I_2 \cdot \frac{q}{Q}\right]}$$

Since $\frac{p}{P} = \frac{q}{Q}$. from $\frac{P}{Q} = \frac{p}{q}$.

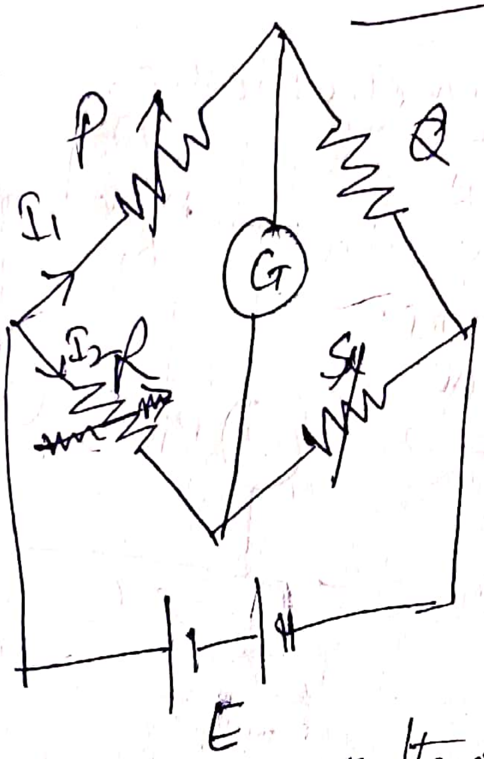
$$\frac{X}{S} = \frac{P}{Q}.$$

hence $\underline{X = \frac{P}{Q} \cdot S.}$

e.

Medium Resistance

Wheatstone Bridge



R = unknown

At balance, ie no current through galvanometer

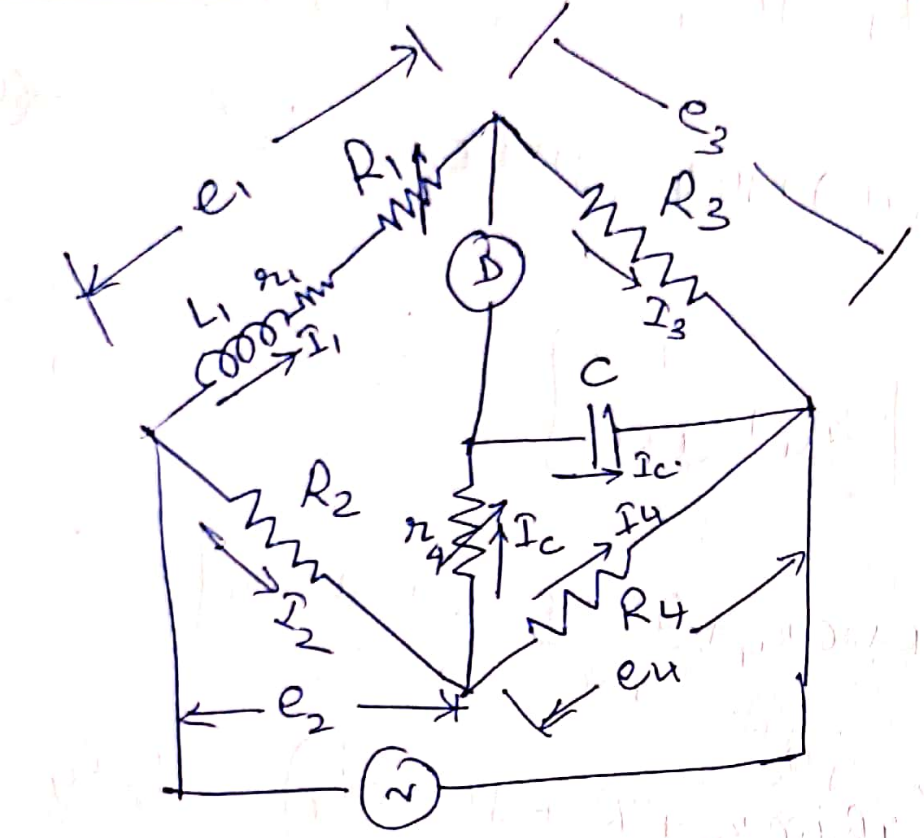
$I_1 P = I_2 R$ and $I_1 Q = I_2 S$.

or $\frac{I_1 P}{I_1 Q} = \frac{I_2 R}{I_2 S}$ or $\frac{P}{Q} = \frac{R}{S}$.

~~$\boxed{S = \frac{QR}{P}}$~~

$$\boxed{R = \frac{PS}{Q}}$$

ANDERSON'S BRIDGE



$V = V_m \sin\omega t$

$i = C\frac{dv}{dt} = CV_m\omega \cos\omega t$

$\cos\omega t = \sin(\omega t + 90)$

$i = CV_m\omega \sin(\omega t +$

$Z = \frac{V}{I} = \frac{V_m \angle 0}{C\omega V_m \angle 90}$

$= \frac{1}{j\omega C}$

Maxwell, Anderson, Wein, Heaviside, Campbells, Desauty, Schering, Kelvin Double bridge,

Unknown Inductor $r_1 + j\omega L_1$, Connected to a known Resistance R_1.

at balance $I_1 = I_3$; $I_2 = I_c + I_4$;

equating Voltages:-

$$I_1 \cdot R_3 = \frac{I_c}{j\omega C} \quad \therefore I_c = j I_1 R_3 \omega C. \quad — ①$$

$$I_1\left((r_1 + R_1) + j\omega L_1\right) = I_2 \cdot R_2 + I_c \cdot r_4. \quad — ②$$

also $$I_c\left(r_4 + \frac{1}{j\omega C}\right) = (I_2 - I_c) R_4. \quad — ③$$

Substituting for I_c in 2 & 3.

$I_c [r$

2

$$\hat{I}_1\left((r_1+R_1)+j\omega L_1\right) = \hat{I}_2 R_2 + j\hat{I}_1 R_3 \omega C r_4$$

$$\text{or}\quad \hat{I}_1\left[(r_1+R_1)+j\omega L_1 - jR_3\omega C r_4\right] = \hat{I}_2 R_2 \quad — ④$$

from 3

$$j\hat{I}_1 R_3\omega C\left[r_4 + \frac{1}{j\omega C}\right] = \left[\hat{I}_2 - j\hat{I}_1 R_3 \omega C\right]R_4.$$

$$\text{or}\quad I_1\left[jR_3\omega C r_4 + \frac{jR_3\omega C}{j\omega C} + jR_3\omega C R_4\right] = I_2 R_4$$

$$\text{ie}\quad I_1\left(jR_3\omega C r_4 + R_3 + jR_3\omega C R_4\right) = I_2 R_4 \quad - ⑤$$

Dividing ~~eqs~~ from 4, 5

$$\frac{(r_1+R_1)+j\omega L_1 - jR_3\omega C r_4}{R_2} = \frac{jR_3\omega C r_4 + j\omega C R_3 R_4 + R_3}{R_4}$$

ie Seperating real & imaginas

$$\frac{r_1+R_1}{R_2} = \frac{R_3}{R_4}. \qquad \therefore r_1 = \frac{R_2R_3}{R_4} - R_1$$

$$\frac{\cancel{j\omega}L_1}{R_2} - \frac{\cancel{j}R_3\cancel{\omega}Cr_4}{R_2} = \frac{\cancel{j}R_3\cancel{\omega}Cr_4 + \cancel{j\omega}CR_3R_4}{R_4}$$

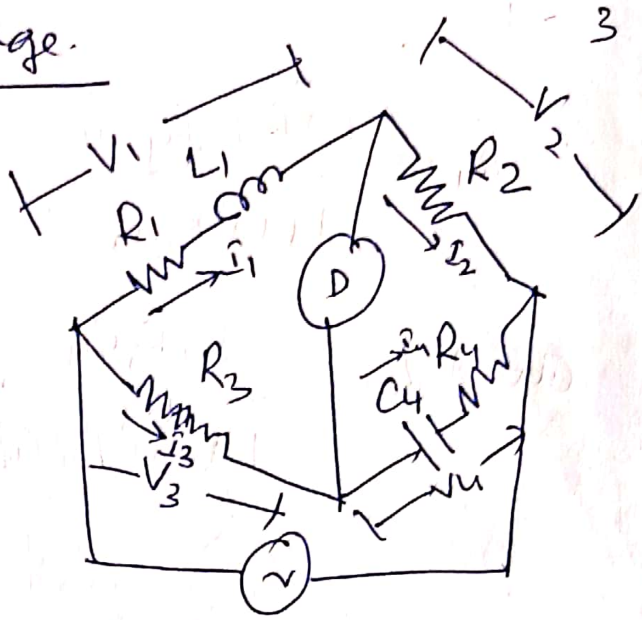
$$\frac{L_1}{R_2} = \frac{R_3Cr_4 + CR_3R_4}{R_4} + \frac{R_3Cr_4}{R_2}$$

$$\therefore L_1 = \cancel{R_2}\left(R_2R_3Cr_4 + R_2R_3R_4C + R_3R_4Cr_4\right)$$

$$\text{ie}\quad L_1 = \frac{CR_3}{R_4}\left[R_2r_4 + R_2R_4 + R_4r_4\right]$$

$$\frac{CR_3}{R_4}\left[R_2(r_4+R_4) + R_4r_4\right]$$

Hay's bridge.



$L_1, R_1 \rightarrow$ Inductor under test

$Z_1 Z_4 = Z_2 Z_3.$

$(R_1 + j\omega L_1)\left(R_4 + \frac{1}{j\omega C_4}\right)$

$= R_2 R_3.$

$$\frac{R_1 + j\omega L_1}{R_2} = \frac{R_3}{R_4 + \frac{1}{j\omega C_4}} = \frac{R_3}{R_4 - \frac{j}{\omega C_4}} = \frac{R_3 \cdot \omega C_4}{(\omega C_4 R_4 - j)}$$

~~or $\frac{R_1 + j\omega L_1}{R_2} = \frac{R_3 \omega C_4 (\omega C_4 R_4 + j)}{(\omega^2 C_4^2 R_4^2 + 1)}$~~

~~$\frac{R_1}{R_2} + \frac{j\omega L_1}{R_2} = \frac{R_3 \omega^2 C_4^2 R_4}{\omega^2 C_4^2 R_4^2 + 1} + \frac{j R_3 \omega C_4}{\omega^2 C_4^2 R_4^2 + 1}$~~

$$\frac{R_1}{R_2} + \frac{j\omega L_1}{R_2} = \frac{R_3}{R_4 - \frac{j}{\omega C_4}} = \frac{R_3 \omega C_4}{\omega R_4 C_4 - j}$$

$$= \frac{R_3 \omega C_4 (\omega R_4 C_4 + j)}{\omega^2 R_4^2 C_4^2 + 1}$$

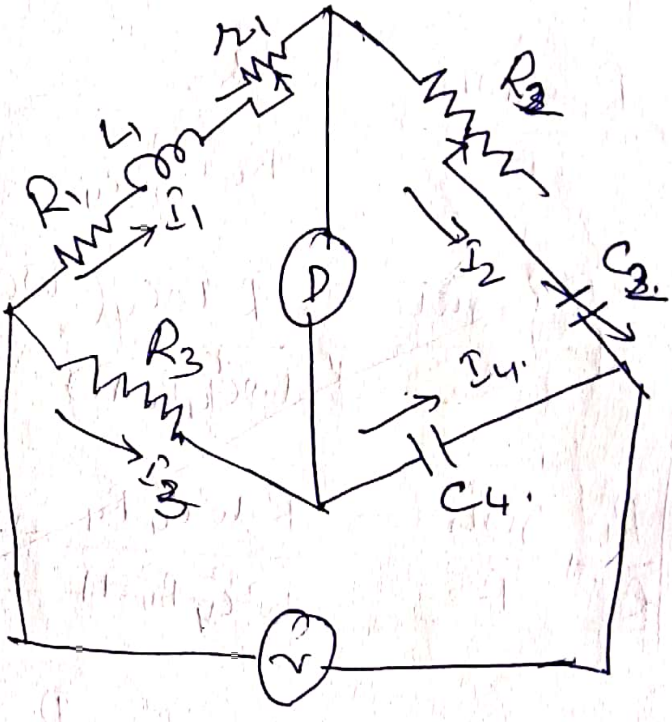
$$\therefore \frac{R_1}{R_2} = \frac{R_3 \omega C_4 (\omega R_4 C_4)}{1 + \omega^2 R_4^2 C_4^2} \quad \& \quad \frac{\omega L_1}{R_2} = \frac{R_3 \omega C_4}{1 + \omega^2 R_4^2 C_4^2}$$

$$\therefore R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1+\omega^2 R_4^2 C_4^2}$$

$$\text{and } L_1 = \frac{R_2 R_3 C_4}{1+\omega^2 R_4^2 C_4^2}$$

Owen's Bridge

R_1, L_1 unknown.



$Z_1 Z_4 = Z_2 Z_3$.

$$[(R_1+r_1)+j\omega L_1]\left[\frac{1}{j\omega C_4}\right] = ~~R_2R_3~~$$

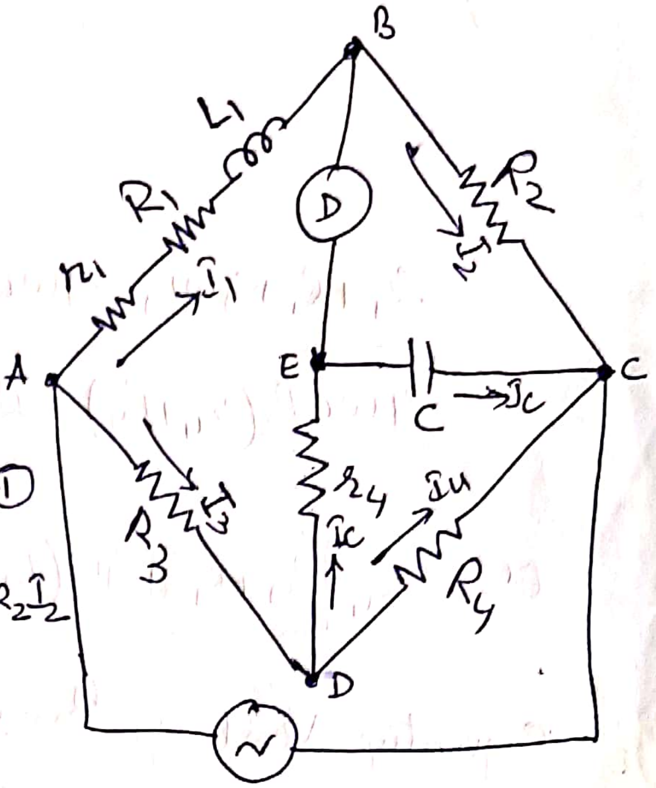
$$R_3\left(R_2+\frac{1}{j\omega C_2}\right)$$

$$\frac{[(R_1+r_1)+j\omega L_1]}{R_3} = \frac{\left(R_2+\frac{1}{j\omega C_2}\right)}{\frac{1}{j\omega C_4}} = j\omega C_4\left(R_2+\frac{1}{j\omega C_2}\right)$$

$$\frac{\cancel{j\omega} L_1}{R_3} = \cancel{j\omega} C_4 R_2 \qquad \therefore \boxed{L_1 = R_2 R_3 C_4}$$

$$\frac{R_1+r_1}{R_3} = \frac{C_4}{C_2} \qquad \boxed{R_1 = \frac{C_4 R_3}{C_2} - r_1}$$

Anderson's Bridge.



At balance.

$I_1 = I_2$; $I_3 = I_c + I_4$.

$$I_1(r_1 + R_1 + j\omega L_1) = I_3 R_3 + I_c \cdot r_4 \quad ①$$

$$I_2 R_2 = I_c \cdot \frac{1}{j\omega C} \quad ② \quad \therefore I_c = j\omega C R_2 I_2$$

$$I_c\left(r_4 + \frac{1}{j\omega C}\right) = I_4 R_4 \quad ③$$

Sub. ② in ① and equate $I_1 = I_2$

$$\therefore I_1\left[r_1 + R_1 + j\omega L_1\right] = I_3 R_3 + j\omega C R_2 I_1 \cdot r_4.$$

or $$I_1\left[r_1 + R_1 + j\omega L_1 - j\omega C R_2 r_4\right] = I_3 R_3 \quad — Ⓐ$$

Sub. ② in ③ and $I_4 = I_3 - I_c$ and subst. for I_c in terms of $I_2 = I_1$.

$$j\omega C R_2 I_2\left(r_4 + \frac{1}{j\omega C}\right) = (I_3 - I_c) R_4$$

$$= I_3 R_4 - j\omega C R_2 I_2 R_4.$$

or $$I_2\left[j\omega C R_2 r_4 + R_2 + j\omega C R_2 R_4\right] = I_3 R_4.$$

or $$I_1\left[j\omega C R_2 r_4 + R_2 + j\omega C R_2 R_4\right] = I_3 R_4 \quad \because I_2 = I_1.$$

— Ⓑ

Divide A/B and cross multiply.

Scanned with CamScanner

$$\frac{r_1+R_1+j\omega L_1 - j\omega C R_2 r_4}{j\omega C R_2(r_4+R_4)+R_2} = \frac{R_3}{R_4}$$

or $R_4\left[r_1+R_1+j\omega(L_1-CR_2r_4)\right] = R_3\left[j\omega CR_2(r_4+R_4)+R_2\right]$

equating real & Imag. parts.

$R_4r_1+R_4R_1 = R_3R_2$

$$\therefore \boxed{R_1 = \frac{R_3R_2-R_4r_1}{R_4} = \frac{R_3R_2}{R_4} - r_1}$$

$R_4\left[L_1 - CR_2r_4\right] = CR_2(r_4+R_4)R_3.$

~~$\therefore L_1 = CR_2r_4 + CR_2(r_4+R_4)$~~

~~$= CR_2\left[r_4(1+\right.$~~ $CR_2\left[\right.$~~$r_4+(r_4$~~$\left.+R_4)\right]$

~~$L_1 = R_4CR_2r_4 + \frac{CR_2R_3(r_4+R_4)}{R_4}$~~

$R_4L_1 - CR_4R_2r_4 = CR_2(r_4+R_4)R_3$

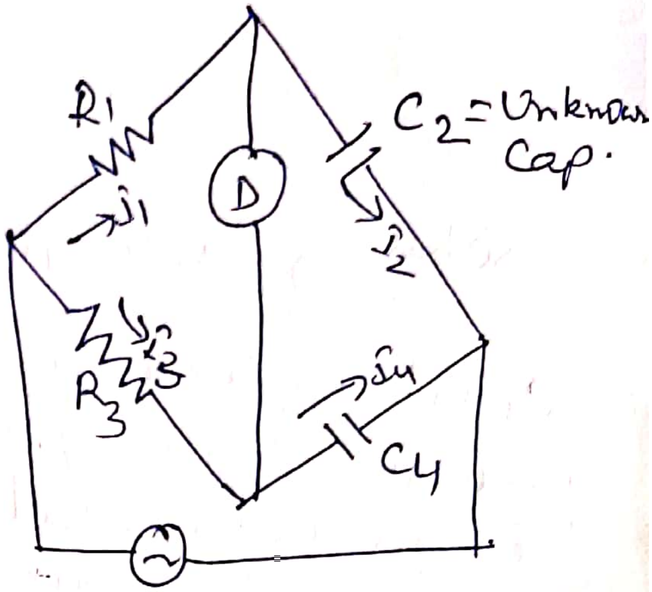
$R_4L_1 = CR_4R_2r_4 + CR_2R_3(r_4+R_4)$

$= CR_2\left[R_4r_4 + R_3(r_4+R_4)\right]$

$\therefore L_1 = \frac{CR_2}{R_4}\left[R_4r_4+R_3(r_4+R_4)\right]$

Capacitance Measurements

① De Sauty's Bridge.



at balance $Z_1Z_4 = Z_2Z_3$.

$Z_1 = R_1$; $Z_2 = \frac{1}{j\omega C_2}$

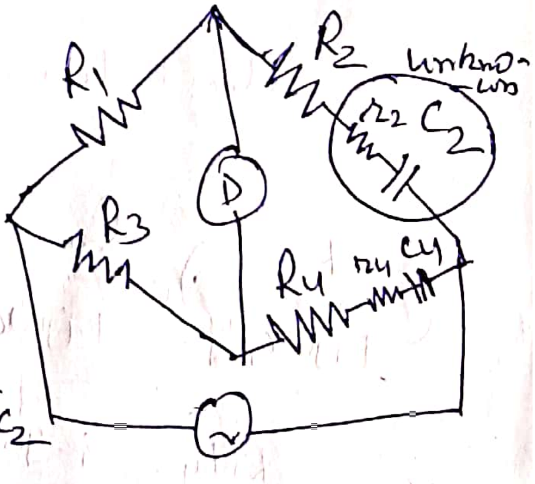
$Z_3 = R_3$; $Z_4 = \frac{1}{j\omega C_4}$

$$\therefore \frac{R_1}{j\omega C_4} = \frac{R_3}{j\omega C_2}$$

$$\therefore \boxed{C_2 = \frac{R_3}{R_1}.C_4}$$

this is true only when C_2 & C_4 are pure capacitances. However the practical capacitors have dielectric loss, replace C_2 & C_4 with internal and external resistors let r_2, r_4 be the internal resistances and R_2, R_4 the external resistances.

the circuit is modified as:



At balance

$$R_1\left(R_4+r_4+\frac{1}{j\omega C_4}\right) = R_3\left(R_2+r_2+\frac{1}{j\omega C_2}\right)$$

ie $R_1(R_4+r_4) + \frac{R_1}{j\omega C_4} = R_3(R_2+r_2) + \frac{R_3}{j\omega C_2}$

equating real & imaginary parts

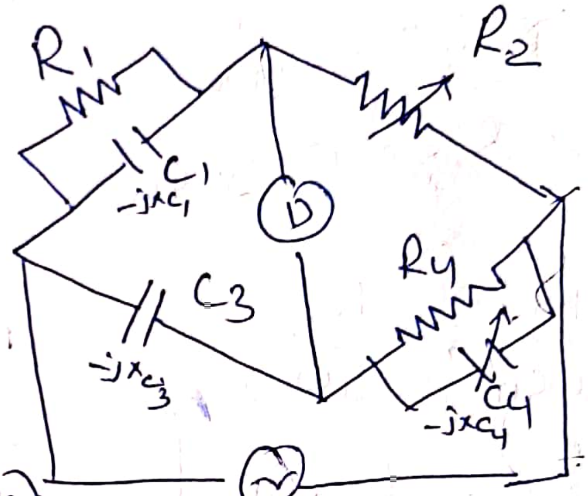
$R_1(R_4+r_4) = R_3(R_2) + R_3r_2$ $\quad \therefore \boxed{r_2 = \frac{R_1(R_4+r_4) - R_3(R_2)}{R_3}}$

also $\frac{R_1}{C_4} = \frac{R_3}{C_2}$ $\quad \therefore \boxed{C_2 = \frac{R_3}{R_1}.C_4}$

loss angle $\tan\delta_2 = \omega r_2 C_2$

& $\tan\delta_4 = \omega r_4 C_4$

Schering Bridge (Parallel)



C1 is the Capacitance under test and R1 represents the equivalent dielectric loss component of resistance

C3, R2, C4, R4 are Standard Capacitors, Resistances. R4 is the dielectric loss component of C4 Capacitor

Impedance $Z_1 = \left[\frac{1}{R_1} + \frac{1}{-jX_{C_1}}\right]^{-1} = \left[\frac{1}{R_1} + j\omega C_1\right]^{-1} = \frac{R_1}{1+j\omega C_1 R_1}$.

$Z_2 = R_2$; $Z_3 = \frac{1}{j\omega C_3}$ $\quad Z_4 = \frac{R_4}{1+j\omega C_4 R_4}$.

Under balanced Condition $\boxed{Z_1 Z_4 = Z_2 Z_3}$

$$\left(\frac{R_1}{1+j\omega C_1 R_1}\right)\left(\frac{R_4}{1+j\omega C_4 R_4}\right) = \frac{R_2}{j\omega C_3}.$$

or $\frac{R_1 R_4}{1+j\omega C_1 R_1} \times \frac{1-j\omega C_1 R_1}{1-j\omega C_1 R_1} = \frac{(1+j\omega C_4 R_4)R_2}{j\omega C_3}$

$= \frac{R_1 R_4(1-j\omega C_1 R_1)}{1+\omega^2 C_1^2 R_1^2} = -\frac{jR_2(1+j\omega C_4 R_4)}{\omega C_3}.$

ie $$\frac{R_1R_4}{1+\omega^2C_1^2R_1^2} - \frac{j\omega C_1R_1^2R_4}{1+\omega^2C_1^2R_1^2} = \frac{-jR_2}{\omega C_3} + \frac{\omega C_4R_2R_4}{\omega C_3}$$

equating real & imaginary parts.

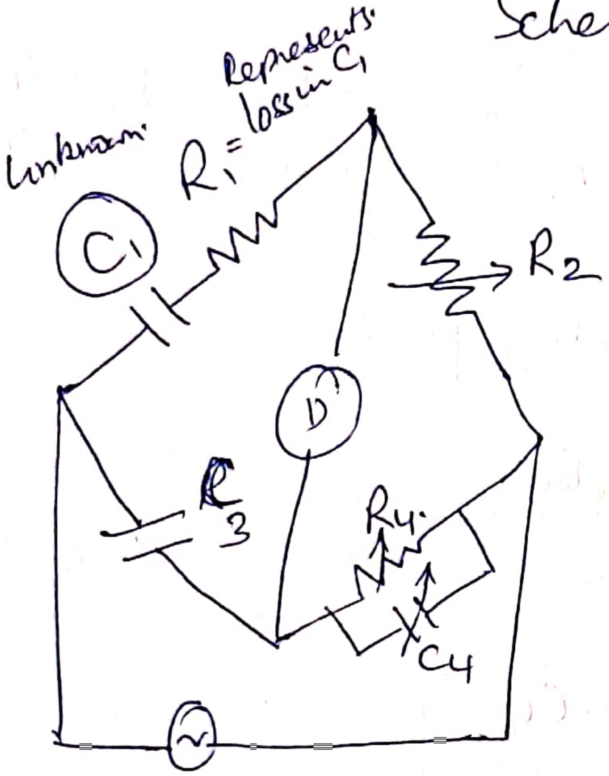
$$\frac{R_1\cancel{R_4}}{1+\omega^2C_1^2R_1^2} = \frac{\cancel{\omega}C_4R_2\cancel{R_4}}{\cancel{\omega}C_3}$$

$$\therefore \boxed{R_1 = \frac{R_2(1+\omega^2C_1^2R_1^2)\cdot C_4}{C_3}}$$

imaginary part $$\frac{\omega^2C_1R_1^2R_4}{1+\omega^2C_1^2R_1^2} = \frac{R_2}{C_3}$$

$$\therefore \boxed{C_1 = \frac{R_2(1+\omega^2C_1^2R_1^2)}{C_3\,\omega^2R_1^2R_4}}$$

Schering bridge (Series ckt)



$Z_1 = R_1 + \frac{1}{j\omega C_1}$

$Z_2 = R_2$

$Z_3 = \frac{1}{j\omega C_3}$

$Z_4 = \left[\frac{1}{R_4} + \frac{1}{\frac{1}{j\omega C_4}}\right]^{-1}$

$= \frac{R_4}{1 + j\omega C_4 R_4}$

balance $Z_1 Z_4 = Z_2 Z_3$.

$$\left(R_1 + \frac{1}{j\omega C_1}\right)\left(\frac{R_4}{1+j\omega C_4 R_4}\right) = \frac{R_2}{j\omega C_3}$$

$$\left(\frac{R_1 j\omega C_1 + 1}{\cancel{j}\cancel{\omega} C_1}\right) R_4 = \frac{R_2(1+j\omega C_4 R_4)}{\cancel{j}\cancel{\omega} C_3}$$

$$\frac{j R_1 \omega \cancel{C_1} R_4}{\cancel{C_1}} + \frac{R_4}{C_1} = \frac{R_2}{C_3} + \frac{j\omega C_4 R_4 \cdot R_2}{C_3}$$

$\frac{R_4}{C_1} = \frac{R_2}{C_3}$ or $\boxed{C_1 = \frac{C_3 R_4}{R_2}}$

$\cancel{j} R_1 \cancel{\omega} \cancel{R_4} = \frac{\cancel{j}\cancel{\omega} C_4 \cancel{R_4} R_2}{C_3}$ $\boxed{\therefore R_1 = \frac{C_4 R_2}{C_3}}$

Anderson bridge

Advantage :- ① fixed Capacitor is used as agai
Variable in maxwell bridge.

② easier to balance in terms of Convergence
when Compared to maxwell bridge.

Disad : 1) More no. of components, hence more diff
in balancing.
2) Shielding is more complicated.

Hay's bridge **Disad** 1) used for inductance when Q f
is very large.
2) Dependant on frequency for very
Q factor, the measurements expressions tend to b
simpler as in Maxwell.

Owen's bridge **Disad** requires variable Stand
Capacitance.. and should be
large value when measuring Coils of lo
Q factor.

Maxwell wien bridge:

Adv.

① permits measmement of inductance in terms of capacitance as capacitance does not have an external field, hence compact and easily shielded.

② When the parallel branch of R,C is variable the balance is independant. they are decade resistance box and decade capacitance box.

③ the bridge balance does not get effected by frequeny.

④ For the coil under test, can have large current rating under balance & as long as the series resistor can carry that value of current safely

⑤ $Q \text{ factor} = \frac{\omega L_1}{R_1} = \frac{\omega C_4 R_2 R_3}{\left(\frac{R_2 R_3}{R_4}\right)} = \omega C_4 R_4$.

by maintaining C_4 constant, the Resistance R_4 can be graduated for Q factor.

Disad. 1) Requires variable standard Resistor & Capacitor which is a very expensive component.

2) good for Q factor $1 \leq Q \leq 10$.

Maxwell wien bridge:

Adv.

① permits measurement of inductance in terms of Capacitance as Capacitance does not have an external field, hence Compact and easily shielded.

② When the parallel branch of R,C is variable the balance is independant. they are decade resistance box and decade capacitance box.

③ the bridge balance does not get effected by frequency.

④ For the Coil under test, Can have large Current rating under balance & as long as the Series resistor, Can carrying that value of Current safely

⑤ $Q \text{ factor} = \frac{\omega L_1}{R_1} = \frac{\omega C_4 R_2 R_3}{\left(\frac{R_2 R_3}{R_4}\right)} = \omega C_4 R_4.$

by maintaining C_4 Constant, the Resistance R_4 Can be graduated for Q factor:

Disad : 1) Requires Variable Standard Resistor & Capacitor which is a very expensive component.

2) good for Q factor $1 \le Q \le 10$.

Comparison of Bridge methods- Advantages, disadvantages and Precautions

AC bridges — Measurement as in Wheatstone bridge balance method.
— More Complex method of calculation.
— Error prone.

AC source — Power supply

Detector — sensitive to ac voltages like earphone. Vibration galvanometer or tunable amplifier detector can be used.

Earphone at 1000 Hz, the Ear sensitivity is very high. But at 50 Hz, 10^{-6} W is only noticable; For inductor measurements, with non linear hysterisis loop, overtones are generated. Only a min tone can be achieved.

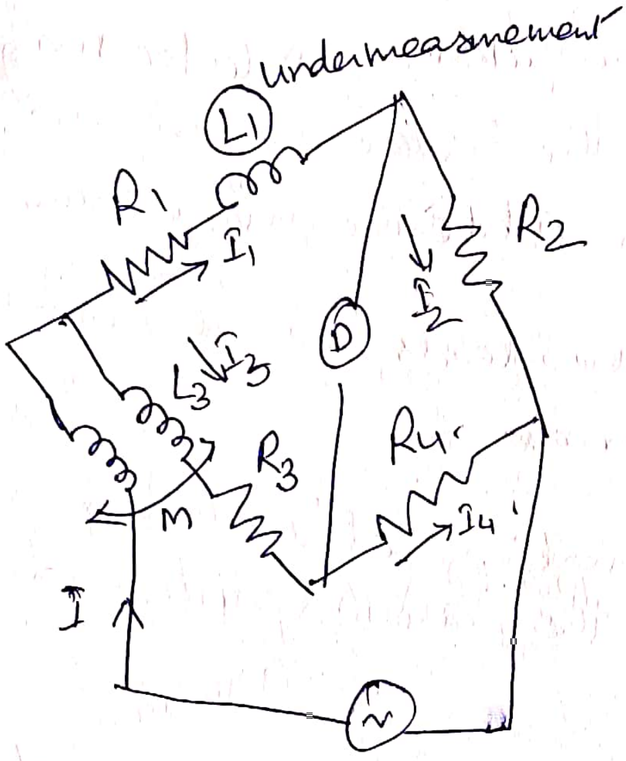
Precautions | Sources of errors:-

1) Stray magnetic fields
2) Leakage because of poor insulation
3) Eddy current errors
4) Frequency & wave form errors.

↓

Change in supply frequency effects balance. balance for fundamental frequency. When wave form changes, for the harmonics, the bridge may not be balanced.

Mutual Inductance   Heaviside Campbell bridge



Balance Condit

$I_1 = I_2$ & $I = I_1 + I_3$.

~~$I_1 R_2 = I_3 R_4$~~

$I_1 R_2 = I_3 R_4$ ①

$I_1(R_1 + j\omega L_1)$

$= I_3(R_3 + j\omega L_3) + j\omega M I$

$= I_3(R_3 + j\omega L_3) + j\omega M(I_1 + I_3)$

ie $I_1(R_1 + j\omega L_1) - j\omega M I_1 = I_3(R_3 + j\omega L_3) + j\omega M I_3$ ②

Divid 2 by 1 $\dfrac{R_1 + j\omega L_1 - j\omega M}{R_2} = \dfrac{R_3 + j\omega L_3 + j\omega M \cancel{I_3}}{R_4}$

equating real & imag

ie $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$ or $R_1 = \dfrac{R_2 R_3}{R_4}$

and $\dfrac{L_1 - M}{R_2} = \dfrac{L_3 + M}{R_4}$

$$\therefore \boxed{L_1 = \frac{R_2(L_3 + M)}{R_4} + M = \frac{R_2(L_3 + M) + R_4 M}{R_4}}$$

Sources of Errors in bridge measurements ⑥

① Stray magnetic fields. Detector may indicate a balance, when the bridge may not be balanced.
use of shields to isolate the bridge arms.

loops formed by the leads are also one of the prime sources of errors.

for electrostatic Coupling, electrostatic shields are used, whenever there are Capacitive elements in the Circuit.

② Leakage errors. resulting from poor electrostatic and electro magnetic insulation.

③ Eddy Current errors — avoid large masses near the bridge circuits.

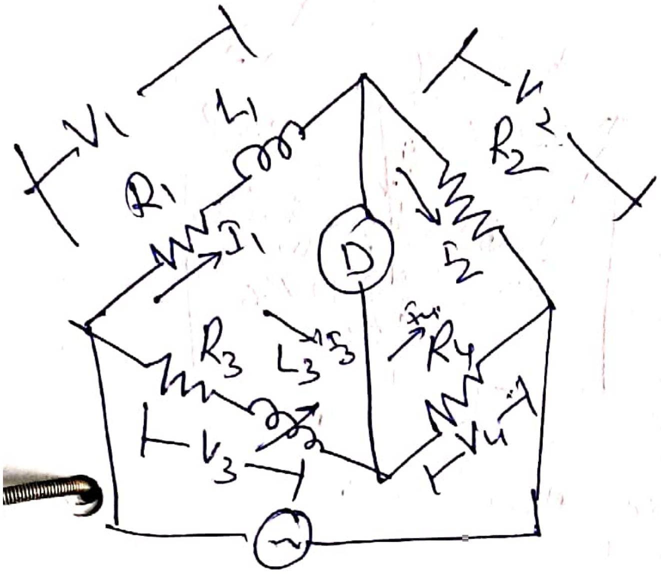
④ Residual errors Inductors / Capacitors / Resistors Cannot be made as only a pure Capacitor / Inductor / resistor.

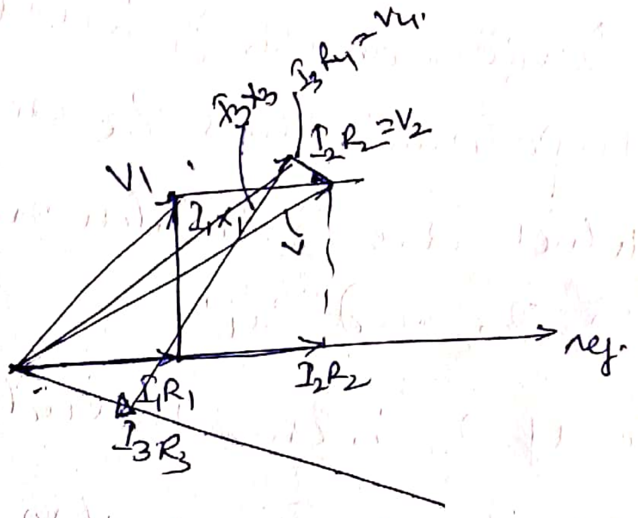
⑤ frequency & wave form errors.
when the results are dependant on frequency element and waveform.

employ wave filters, to eliminate unwanted harmonics

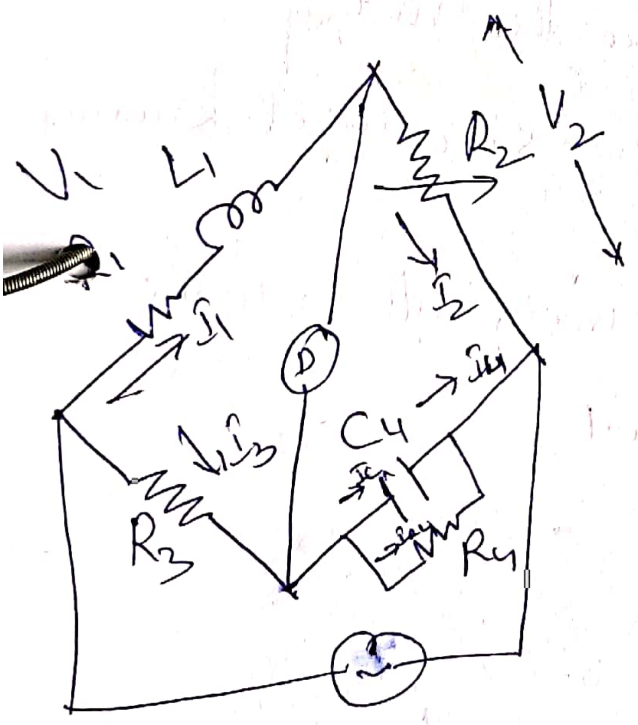
Phasor diagrams for bridg

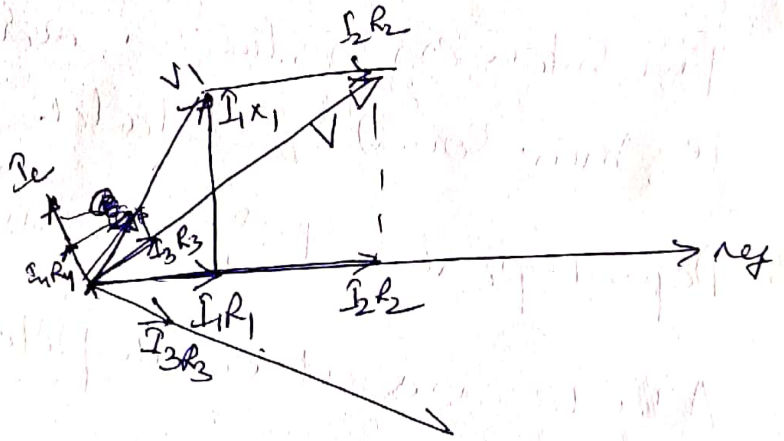
① Maxwells Inductance bridge



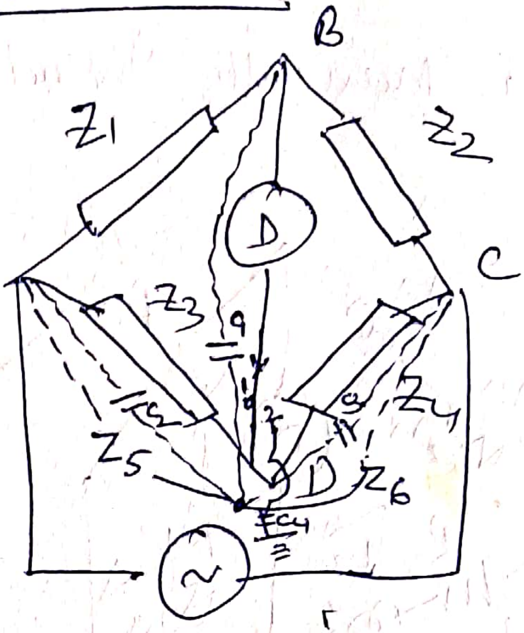


② Maxwells Inductance Capacitance bridge





Wagner's Earthing Device



Device consists of 2 additional impedance arms Z_5 and Z_6 which are of similar nature of Z_3 and Z_4.

ie if Z_3 is and inductance + resistance, Z_5 is also of same nature.

Junction point of Z_5 & Z_6 is earthed.

C_1, C_2, C_3 and C_4 are stray capacitances

Switch on position (1) → balance the bridge.

Put switch on position (2), adjust Z_5, Z_6 to get balance ie min. sound of detector.

Repeat positions 1 & 2 successively till null is obtain

ABCE are at earth potential