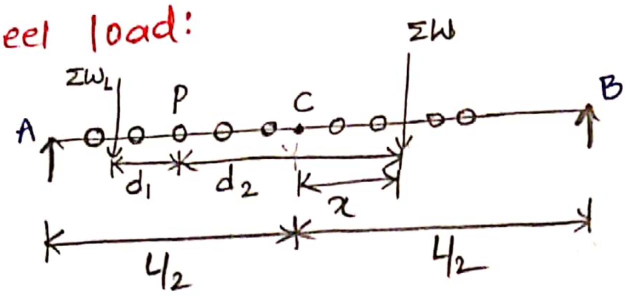
## Maximum Bending moment under a choosen wheel load:




let P be the choosen wheel load of the system of loads.

let $\Sigma W$ be the resultant of all loads of system

let $x$ be distance of $\Sigma W$ from C

let $\Sigma W_L$ be resultant of loads left of P.

$d_1$ and $d_2$ be the distances from P to $\Sigma W_L$ and $\Sigma W$

let $V_A$ and $V_B$ be the reactions at A and B.

$$\Sigma M_B = 0$$

$$V_A \times L = \Sigma W\left(\frac{L}{2}-x\right)$$

$$\therefore V_A = \frac{\Sigma W}{L}\left(\frac{L}{2}-x\right)$$

Bending Moment under choosen load 'P.'

$$BM_P = V_A\left[\frac{L}{2}+x-\frac{d}{2}\right] - \Sigma W_L \times d_1$$

$$= \frac{\Sigma W}{L}\left[\frac{L}{2}-x\right]\left[\frac{L}{2}+x-\frac{d}{2}\right] - \Sigma W_L d_1$$

For maximum moment at P, $\frac{dM}{dx} = 0$

$$M = \frac{\Sigma W}{L}\left[\left[\left(\frac{L}{2}\right)^2 + \left(\cancel{\frac{L}{2}}\right)x - d_2\left(\frac{L}{2}\right)\right] - \left[+\cancel{\left(\frac{L}{2}\right)}\right]x - x^2 + d_2 x\right.$$

$$- \Sigma W_L d_1$$

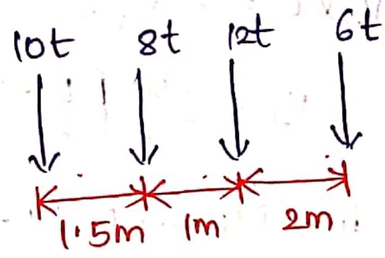
$$\frac{dM}{dx} = 0$$

$$\Rightarrow -2x + d_2 = 0$$

$$x = \frac{d_2}{2}.$$

For maximum BM under a choosen wheel load, the load system should be so placed on beam. Such that resultant of all wheel load system and choosen wheel load should be equidistant from the midpoint of beam.

17) 4 wheel loads 10t, 8t, 12t, 6t moving on a beam of span 12m moving from right to left. Find maximum BM under 8 ton and also the support reactions




Sol] calculating resultant load from 10t load,

$$(10+8+12+6) \times \bar{x} = 10 \times 0 + 8 \times 1.5 + 12 \times 2.5 + 6 \times 4$$

$$\Rightarrow \bar{x} = 1.91m.$$

Distance between choosen load and $\Sigma w$ is

$$= 1.91 - 1.5 = 0.41 m$$

For maximum bending moment, the 8t must be placed $\frac{0.41}{2} = 0.205 m$ from left of c.

<u>Reaction:</u>

$$V_A \times 12 = 36 \times (6 - 0.205) \Rightarrow V_A = 17.385t$$

$$BM = 17.385 \times (6 - 0.205) - 10 \times (1.5)$$

$$= 85.74t$$

## Curves of Maximum Bending Moment and Shear force

For four general cases

i) single point load moving

ii) UDL longer than span

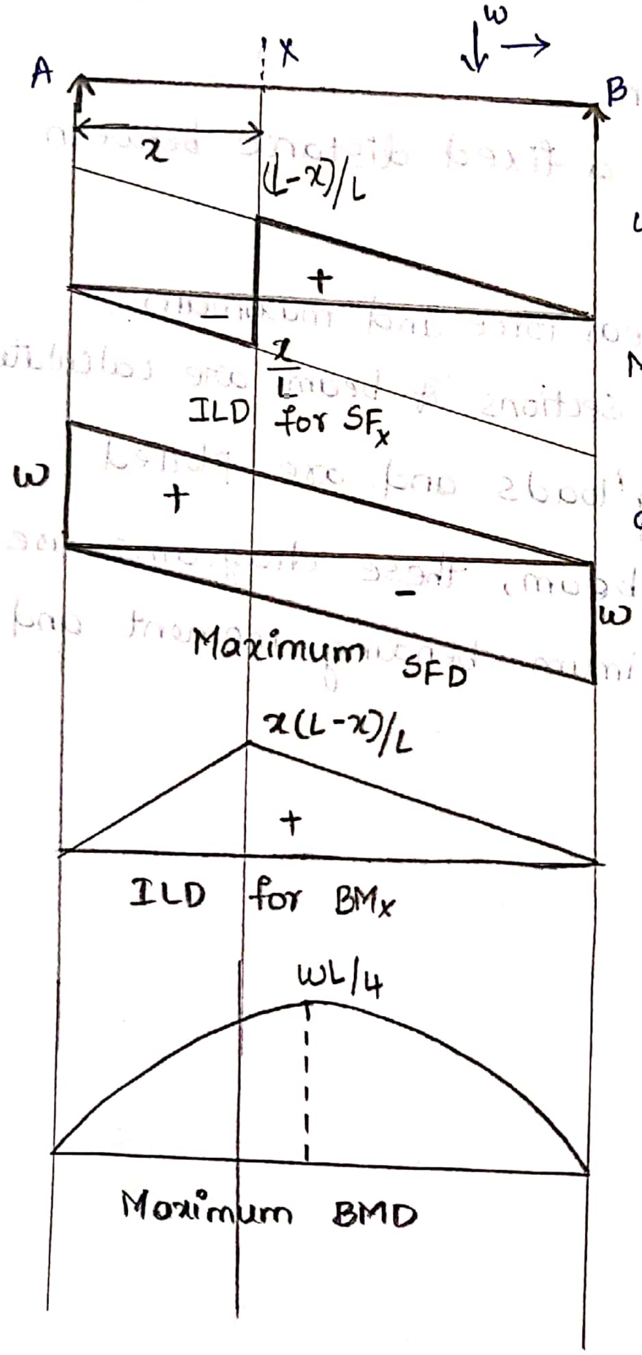
iii) UDL shorter than span

iv) Two point loads at a fixed distance between them.

The maximum shear force and maximum bending moment at all sections of beam are calculated for a system of moving loads and are plotted along the length of beam, these diagrams are called curves of maximum bending moment and shear force.

1. Curves of maximum Bending Moment and Shear force for Single point load.

Consider a simply supported beam AB and load 'w' kN are is moving on beam. Consider a Section 'x' at a distance 'x' m from A.




Maximum −ve SF:

at $x \Rightarrow SF = w \times \frac{x}{L}$

when $x = 0$, $SF = 0$

$x = l$, $SF = -w$

Maximum +ve SF:

at $x \Rightarrow SF = \frac{w(L-x)}{L}$

at $x = 0$, $SF = W$

at $x = l$, $SF = 0$

For maximum BM:

at $x \Rightarrow BM_x = \frac{wx(L-x)}{L}$

at $x = 0$, $BM_x = 0$

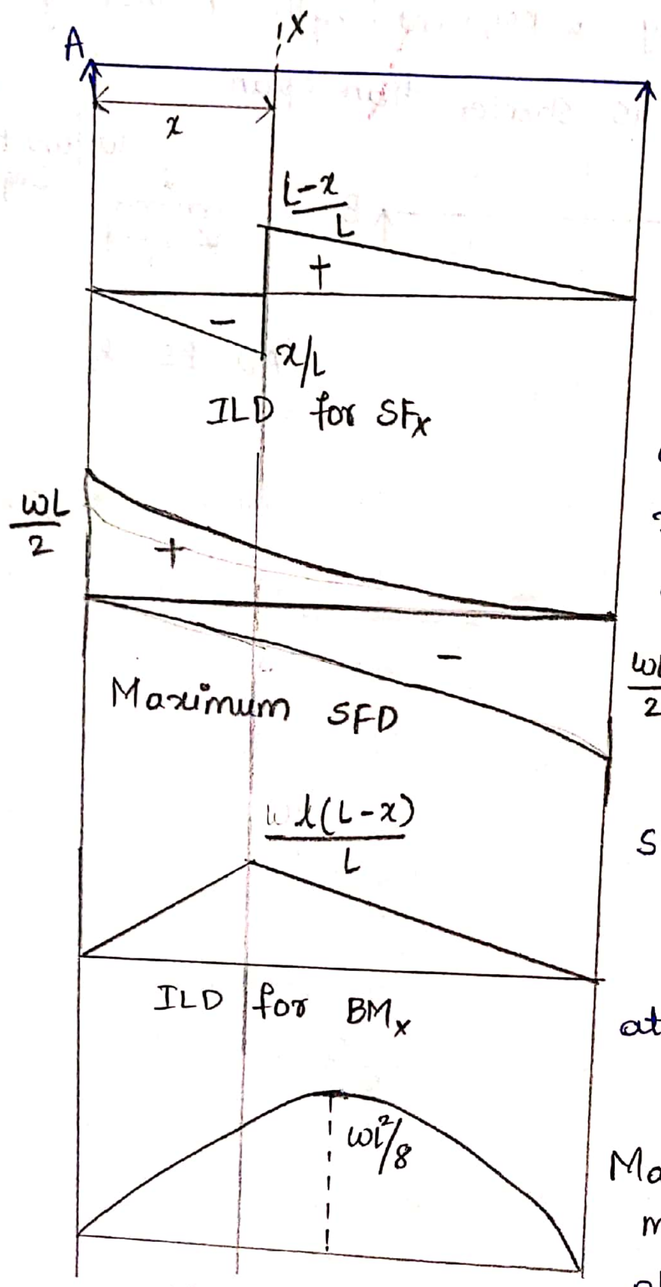
$x = l$, $BM = 0$

$x = \frac{L}{2}$ $BM = \frac{WL}{4}$

## 2. UDL longer than Span:

Consider a Simply supported beam AB and a UDL 'w' kN/m is moving on the beam. consider a Section 'x' at a distance 'x' m from A.

'w' kN/m




Maximum BMD

**Maximum −ve SF:**

From A to x

$SF_x = \left(\frac{1}{2} \times x \times \frac{x}{L}\right) \times -w = \frac{wx^2}{2L}$

$= -\frac{wx^2}{2L}$

at $x=0$ $SF_A = 0$

$x = L$ $SF_B = -\frac{wL}{2}$

$x = \frac{L}{2}$ $SF_x = -\frac{wL}{8}$

**Maximum +ve SF:**

From B to x

$SF_x = \frac{1}{2} \times (L-x) \times \frac{(L-x)}{L} \times w$

$= \frac{w(L-x)^2}{2L}$

at $x=0$, $SF_A = \frac{wL}{2}$

$x = L$, $SF_B = 0$

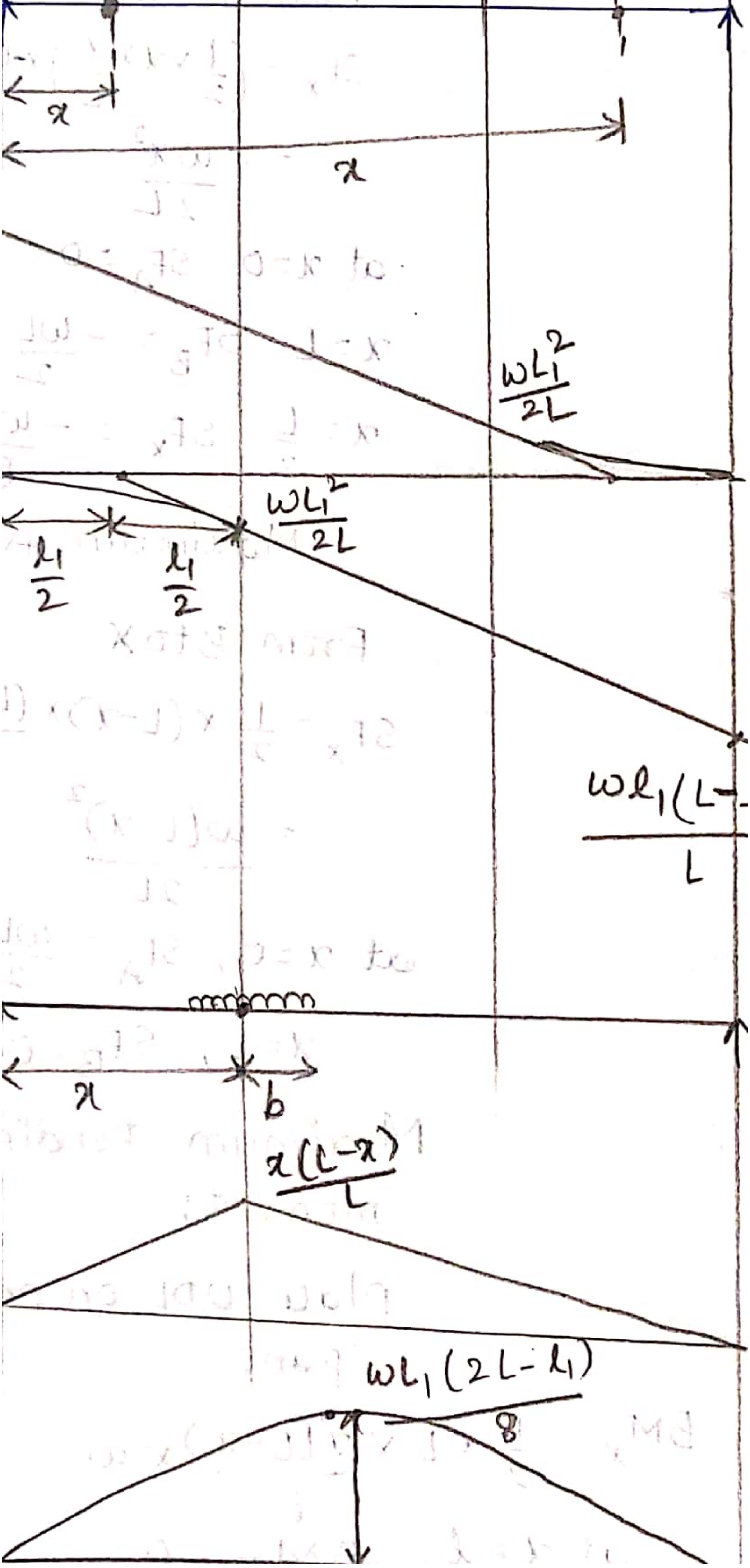
**Maximum Bending moment:**

place UDL on entire Span,

$BM_x = \frac{1}{2} \times L \times \frac{x(L-x)}{L} \times w$

at $x=0$, $BM_A = 0$ , at $x = l$ $BM_B = 0$

at $x = \frac{L}{2}$, $BM_x = \frac{wL^2}{8}$


x
x
$\frac{wL_1^2}{2L}$
$\frac{wL_1^2}{2L}$
$\frac{l_1}{2}$
$\frac{l_1}{2}$
$\frac{wl_1(L-}{L}$
x
b
$\frac{x(L-x)}{L}$
$\frac{wL_1(2L-l_1)}{8}$

$AD = BE = l_1 \quad ; \; l_1 < L$

Maximum -ve SF:

i) when $x$ in AD $\Rightarrow x < l_1$

$SF_x = -V_B$

$\Sigma M_A = 0$

$V_B \times l = w \times x \times \frac{x}{2}$

$\Rightarrow V_B = \frac{wx^2}{2L}$

$SF_x = -V_B$ is valid from A to D

at $x=0$ $\quad SF_A = 0$ , $\quad x = l_1 \quad SF_D = -\frac{wl_1^2}{2L}$

ii) when $x$ is in EB $\Rightarrow x > l_1$

$SF_x = -V_B$

$V_B \times l = wl_1\left(x - \frac{l_1}{2}\right) \Rightarrow V_B = \frac{wl_1\left(x - \frac{l_1}{2}\right)}{l}$

at $x = l_1$ $\quad SF_D = \frac{-wl_1^2}{2l}$

at $x = L$, $\quad SF_B = \frac{-wl_1\left(L - \frac{L_1}{2}\right)}{L}$

Point of inflection: $SF = 0$

$\frac{wl_1\left(x - \frac{l_1}{2}\right)}{l} = 0 \Rightarrow x = \frac{l_1}{2}$.

To have the maximum BM at $x$, UDL must be placed such that

$$b = \frac{l_1}{l}(l - x).$$

$\Sigma M_A = 0$

$V_B \times l = wl_1\left(x + b - \frac{L_1}{2}\right)$

$V_B = \frac{wl_1}{l}\left(x + b - \frac{l_1}{2}\right)$

$$V_b = \frac{wl_1}{l}\left(x+b-\frac{l_1}{2}\right)$$

BM at $x = V_b(L-x) - wb \times \frac{b}{2}$

$= V_b(L-x) - \frac{wb^2}{2}$

$= \frac{wl_1}{l}\left(x+b-\frac{l_1}{2}\right) - \frac{wb^2}{2}.$

(or) $wl_1\left(L-\frac{l_1}{2}\right)x$

$x \times \frac{(L-x)}{L}$

$$BM_x = \frac{wl_1}{2l^2}(2l-l_1) * x(L-x)$$

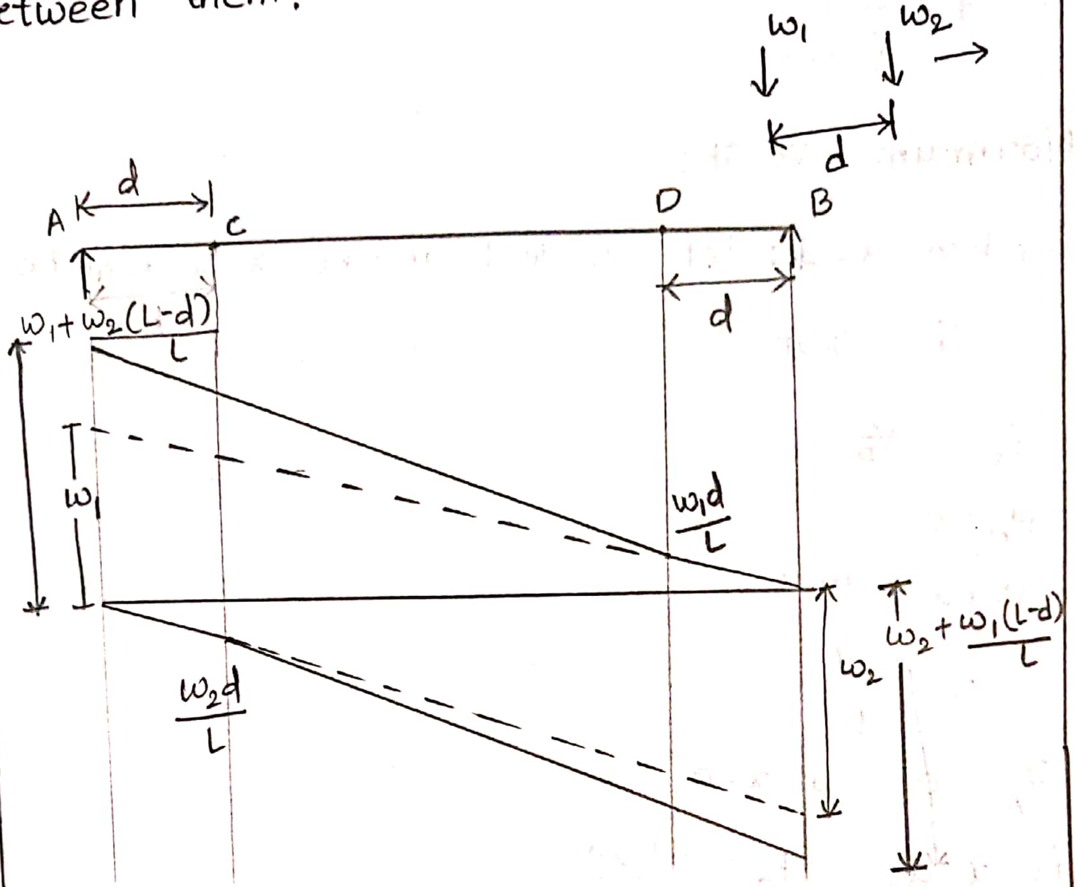
at $x=0$, $BM=0$

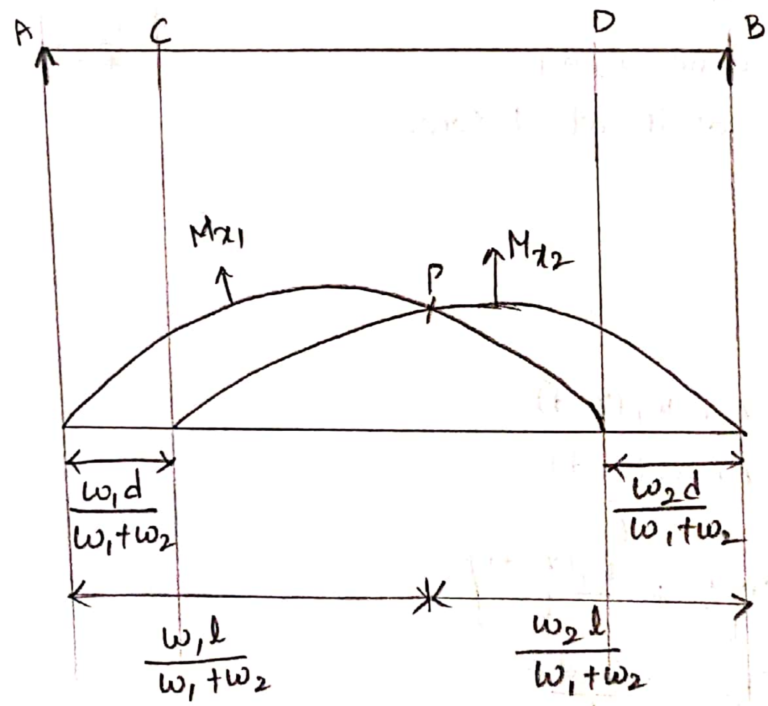
$x=l$, $BM=0$

$x=\frac{l}{2}$, $BM = \frac{wl_1(2l-l_1)}{2l^2} \times \frac{L}{2} \times \frac{L}{2}$

$$= \frac{wl_1(2l-l_1)}{8}$$

4. Two point loads at a fixed distance between them:



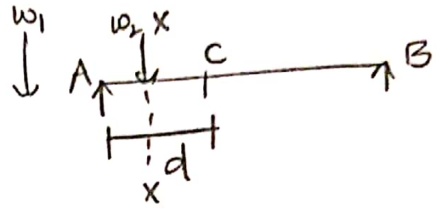
Pre conditions:

i) $w_2 < w_1$

ii) $d < \frac{w_2 L}{w_1 + w_2}$

Maximum -ve SF:

i) when $x < d$: let $w_2$ just reaches $x$, $w_1$ will be off the span.




$SF_x = -V_B$

$\Sigma M_A = 0$

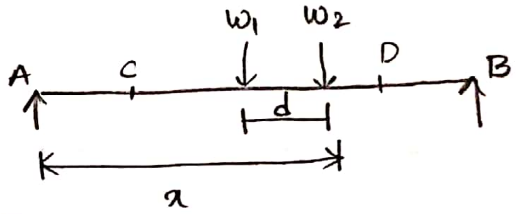
$V_B \times l = w_2 x$

$V_B = \frac{w_2 x}{L}$

$SF_A = 0$ at $x = 0$

if $x = d$, $SF_C = \frac{-w_2 d}{l}$

ii) when $x > d$:




when $w_2$ reaches $x$, $w_1$ will be left of it at distance 'd'.

$SF_x = -V_B$

$\Sigma M_A = 0$

$V_B \times l = w_2 x + w_1 (x - d)$

$V_B = \frac{w_2 x}{L} + \frac{w_1 (x-d)}{L}$

$SF_x = -\left[\frac{w_2 x}{L} + \frac{w_1 (x-d)}{L}\right]$

at $x = d$, $SF_C = \frac{-w_2 d}{L}$

at $x = l$ $SF_B = -\left[w_2 + \frac{w_1 (L-d)}{L}\right]$

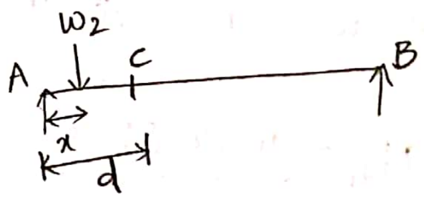
Maximum +ve SF = $\frac{w_1(L-x)}{L} + \frac{w_2(L-x-d)}{L}$.

Maximum Bending Moment:

let $M_{x_1}$ and $M_{x_2}$ be the curves of maximum Bending Moment when $w_1$ and $w_2$ reaches the Section respectively.

i) when $x<d$.

In this case $w_1$ will be off the span,

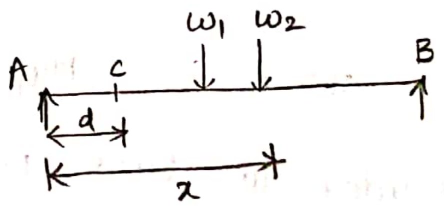



$$M_{x_2} = \frac{w_2 x(L-x)}{L}$$

when $x=0$, $M_{x_2}=0$

$x=d$ $\quad M_{x_2} = \frac{w_2 d(L-d)}{l}$

ii) when $x>d$




$$M_{x_2} = V_b(L-x)$$

$\Sigma M_A = 0$

$V_B \times l = w_2 x + w_1(x-d)$

$$M_{x_2} = \left[\frac{w_2 x}{L} + \frac{w_1(x-d)}{L}\right](L-x)$$

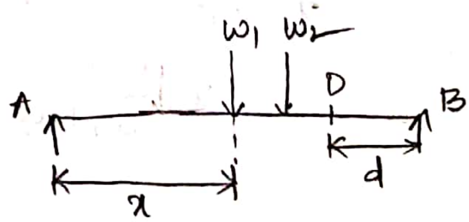
valid from $x=d$ to $x=l$

at $x=d$ $\quad M_{x_2} = \frac{w_2 d}{L}(L-d)$

at $x=l$ $\quad M_{x_2} = 0$.

$M_{x_1}$

when $x<(L-d)$




$$M_{x_1} = V_A x$$

for

$\Sigma M_B = 0$

$$w_1(L-x) + w_2(L-(x+d)) = V_A \times l$$

$$V_A = \frac{w_1(L-x) + w_2(L-x-d)}{l}$$

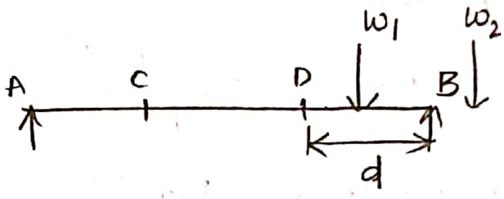
$$M_{x_1} = \frac{[w_1(L-x) + w_2(L-x-d)]}{l}$$

valid from $x=0$ to $L-d$

case - ii

when $x > L-d$

$w_2$ will be off the span and $w_1$ will be on span




$$M_{x_1} = \frac{w_1(x)(L-x)}{l}$$

at $x = L-d$ $\quad M_{x_1} = \frac{w_1 d(L-d)}{l}$

$x = l$ $\quad M_{x_1} = 0.$

Inorder to find point of contraflexure for $M_{x_2}$ curve,

$$M_{x_2} = 0$$

$$\frac{w_2 x + w_1(x-d)}{l} \times l - x = 0$$

$$w_2 x = -w_1(x-d)$$

$$x(w_1+w_2) = d \times w_1$$

$$x = \frac{w_1 d}{w_1+w_2}$$

inorder to find point of contra-flexure for curve $M_{x_1}$

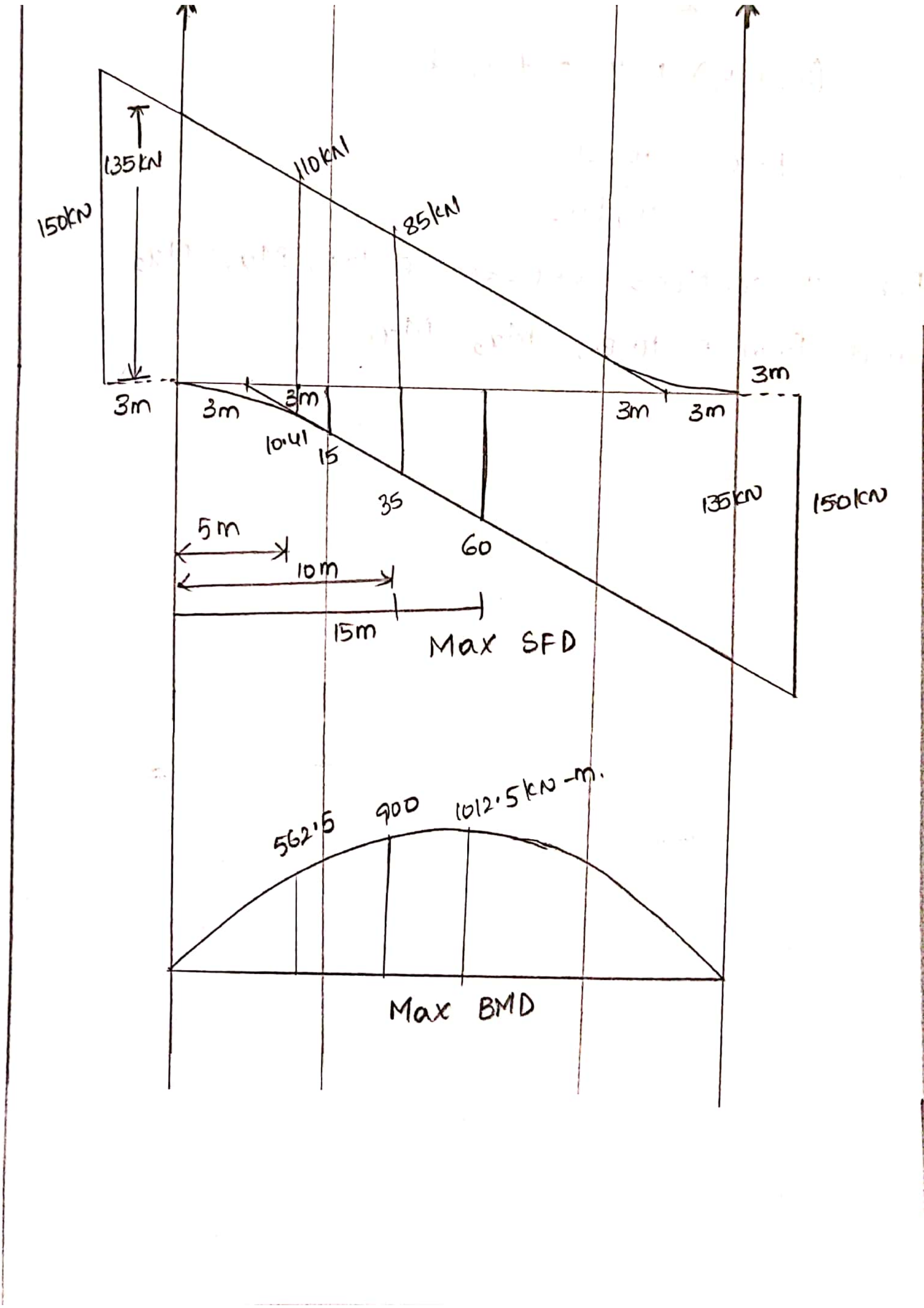
$M_{x_1} = 0$

$$\frac{[w_1(L-x) + w_2(L-x-d)] \times x}{L} = 0$$

$$(w_1 + w_2)\ L - x = + w_2 d$$

$$L - x = \frac{w_2 d}{w_1 + w_2}$$

For all sections between A to P, $M_{x_1} > M_{x_2}$ and from P to B, $M_{x_2} > M_{x_1}$.


150kN
135kN
110kN
85kN
3m
3m
3m
3m
3m
3m
10.41
15
35
60
135kN
150kN
5m
10m
15m
Max SFD
562.5
900
1012.5kN-m.
Max BMD

-ve SF:

SF Valid from $x = 0$ to $l_1$ is $\frac{wx^2}{2L}$

$x = 0$ to $6$

at $x = 5m$, $SF = -\frac{25 \times 25}{2 \times 30} = 10.41 kN$

at $x = 6m$, $SF = -\frac{25 \times 36}{2 \times 30} = 15 kN$

at $x > l_1$ $SF_x = \frac{-wl_1(x - \frac{l_1}{2})}{l}$

at $x = 10m$, $SF_x = -\frac{25 \times 6 (10-3)}{30} = -35 kN$

at $x = 15m$, $SF_x = \frac{-25 \times 6 (15-3)}{30} = -60 kN$

at $x = 30m$, $SF_x = \frac{-25 \times 6 (30-3)}{30} = -135 kN$

+ve SF:

at $x = 5m$ from left $\Rightarrow$ $x = 25m$ from right

$SF = \frac{wl_1(x - \frac{l_1}{2})}{L}$

$= \frac{25 \times 6 (25 - 3)}{30} = 110 kN$

at $x = 20m$, from left $\Rightarrow$ $x = 10m$ leftom right

$SF = \frac{25 \times 6 \times (10-3)}{30} = 85 kN.$

BM: $\frac{wl_1(2L - L_1)}{2L^2} \times x \times (L - x)$

at $x = 5m$, $\frac{25 \times 6 \times (2 \times 30 - 6)}{2 \times 30^2} \times 5 \times (30-5) = 562.$

$x = 10m$, $BM = 900 kN\text{-}m$

$x = 15m$, $BM = 1012.5 kN\text{-}m$

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