Unit-III

Fluid Kinematics

Classification of fluid flow:- (i) Steady & Unsteady flow (ii) Uniform & Non-Uniform flow (iii) Laminar & Turbulent flow (iv) Rotational & ir-rotational flow (v) compressible & incompressible flow (vi) Ideal & real fluid flow.
(vii) 1D, 2D & 3D ~~continuity equations~~ flows,
(viii) vortex flows (ix) streamline (x.) Pathline
(xi) streakline & streamtube
(xii) 1D, 2D & 3D continuity equations in cartesian co-ordinates.

Fluid Kinematics:-

Kinematics is defined as that branch of science which deals with motion of particles without considering the forces causing the motion.

→ Kinematics involves the description of motion of fluids in terms of space-time relationship.

→ ~~A solid will move at same~~

→ When a solid body moves all its particles will also move in same velocity & same acceleration.

→ But in fluid, same particles of fluids does not move in same velocity & acceleration.

The fluid motion is described by two method.
1. Lagrangian Method & 2. Eulerian method.

Lagrangian Method:-
In this method, single fluid particle is considered & pursued throughout its course of motion and observation is made about the behaviour of this particle during its course of motion through space. Properties are function of time & ~~space~~.

ie, Only single particles velocity, acceleration, density etc., are described.

Eulerian Method:-
In this method, at a point velocity, acceleration, pressure, density etc., are described in flow field. Properties are function of time & space.

Out of these two methods Eulerian method is commonly adopted in fluid mechanics & the same is used in the following analysis.

Steady flow :-

Steady flow is defined as the fluid characteristics like velocity, pressure, density etc., at a point do not change with time.

$$\left(\frac{\partial v}{\partial t}\right) = 0 \; ; \; \left(\frac{\partial p}{\partial t}\right) = 0 \; ; \; \left(\frac{\partial \rho}{\partial t}\right) = 0 .$$

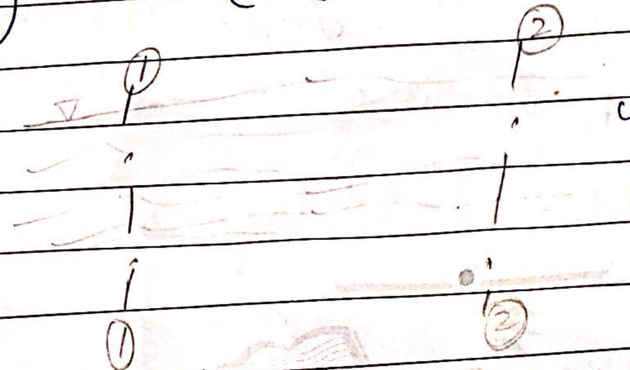


Fig:- flow through a pipe of variable diameter under constant pressure head ie, Reservoir or tank.

@ 10.AM $V_1 = 1$ m/s

@ 12 AM $V = 1$ m/s

@ 10 AM = $V_{2-2} = 3$ m/s

@ 12 AM $V_2 = 3$ m/s

Unsteady flow:-

Unsteady flow is defined as the fluid characteristics like velocity, pressure, density etc., at a point changes w.r.t time.

$$\left(\frac{\partial v}{\partial t}\right) \neq 0 \; ; \; \left(\frac{\partial P}{\partial t}\right) \neq 0 \; ; \; \left(\frac{\partial \rho}{\partial t}\right) \neq 0$$

In case of vector quantities such as velocity of flow even the change in direction of such quantities w.r.t at any point in the flowing fluid makes the flow unsteady.

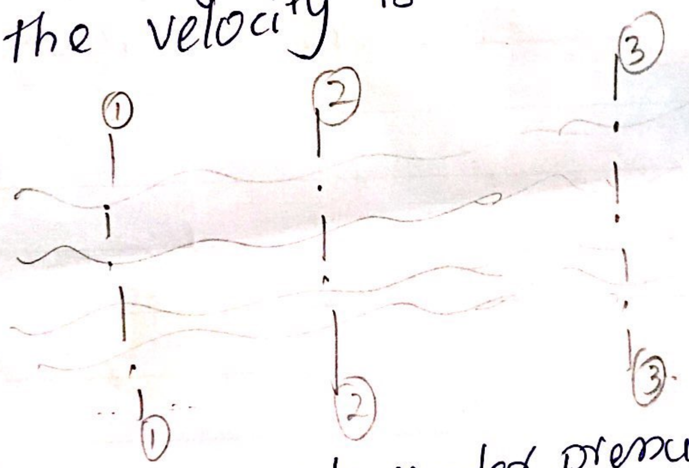
Unsteady flow example - Flow through a pipe of variable diameter under variable pressure due to an increasing/decreasing water level of the reservoir or opening or closure of a valve or stopping.

Uniform flow:-

It is defined as that type of flow in which velocity of flow of fluid does not change, both in magnitude & direction from point to point for any given instant of time, then the flow is said to be uniform.

$$\left(\frac{\partial V}{\partial S}\right)_{t=\text{constant}} = 0$$

i.e.) At a given time ~~the velocity~~ throughout the length the velocity is same or constant.



At 1:00 PM

at ①-①, V = 1 m/s

②-② , V = 1 m/s

③-③ , V = 1 m/s ..

Eg:- Flow of liquids under pressure through long pipelines of constant diameter is uniform flow.

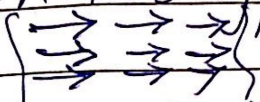
Non-Uniform flow:-

It is that type of flow in which the velocity at any given time changes wrt space.

$$\left(\frac{\partial v}{\partial s}\right)_{t=\text{constant}} \neq 0$$

Eg:- Flow of liquids under pressure through long pipelines of varying diameters is non-uniform flow.

Laminar flow:- A flow is said to be laminar when the various fluid particles move in layers with one layer of fluid sliding smoothly over an adjacent layer. This type of flow is called as Viscous flow.



Eg:- Flow of a very viscous fluid is treated as laminar flow.

Turbulent flow:- When the fluid particles move in an entirely haphazard or disorderly manner or zigzag way is known as "Turbulent flow".

→ Due to the movement of fluid particles in a zig-zag way, the eddies formation takes place which are responsible for high energy loss.

Eg:- Flow in natural streams, artifical channels, water supply

Compressible flow:- The flow of fluid in which the density of fluid changes from point to point is not constant for the fluid is known as "compressible flow". $\rho \neq$ constant.

→ All real fluid are compressible fluid flow.

Eg: (i) Flow of gases through nozzles & orifices
(ii) Aeroplanes moving at high altitude with high velocity

Incompressible flow:-

The flow in which the density is constant for the fluid flow.

$$\rho = \text{constant}.$$

Eg:- stream of water flowing at high speed from a garden hose pipe.

Rotational flow:-

A flow is said to be rotational flow if the fluid-particles while moving in the direction of flow rotate about their own axis is called as "Rotational flow".

Eg:- Liquid in the rotating tanks.

Irrotational flow:-

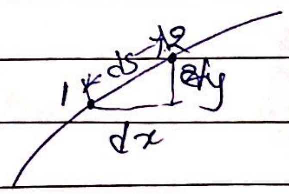
A flow is said to be irrotational if the fluid particles while moving in the direction of flow do not rotate about their own axis is known as "Irrotational flow".

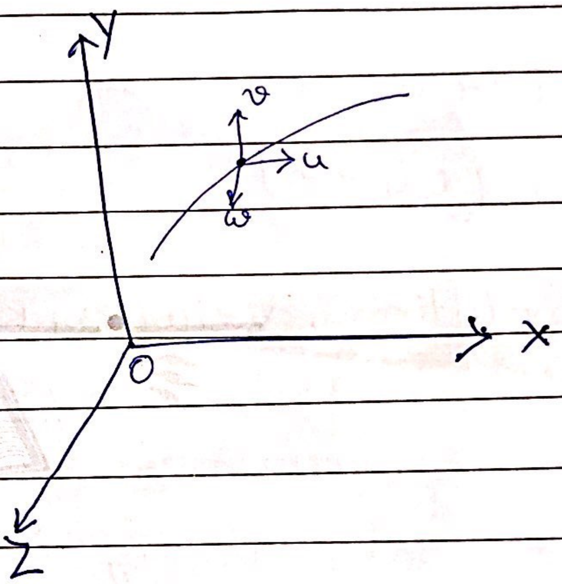
Eg:- Giantwheel.

Velocity of fluid particles –

Velocity is a function of space & time.

→ Velocity at any point of fluid mass is expressed as ratio between displacement of a fluid element along its path & the corresponding increment of time.





x, y & z are co-ordinates.

$\vec{V} = V(\vec{S}, t)$; $\vec{a} = a(\vec{S}, t)$

u, v & w are components of velocity x, y, z resp.

→ Since the point is fixed in space, the co-ordinates x, y, z & t are independent variables.

At a point if ds = Distance travelled by a fluid particle in time dt.

$$V = \frac{ds}{dt}$$

Velocity is a vector quantity.

→ Velocity at any point in the fluid can be resolved into 3 components, u, v & w along 3 mutually perpendicular directions x, y & z resp.

$V = f_1(x, y, z, t)$

$u = f_2(x, y, z, t)$

$v = f_3(x, y, z, t)$

$w = f_4(x, y, z, t)$

Vector notation, $V = iu + jv + kw$

i, j & k are unit vectors parallel to x, y, z axes resp.

One dimensional flow:- Eg:- Train

The various characteristics of flowing fluid such as velocity, pressure, density, temp etc., are function of time & one space co-ordinate only is known as 1-D flow.

steady flow:, $V = f(x)$

ie, $u = f(x)$

$v = 0$

$w = 0$.

The variation of velocities in other two mutually ⊥al directions is assumed negligible.

unsteady flow = $V = f(x, t)$.

⇒ In reality flow cannot be 1-D flow.

Two dimensional flow (2-D flow):-

The different characteristics of flowing fluid are function of time and two-space co-ordinates.
Steady flow.

$u = f_1(x,y)$; $v = f_2(x,y)$, $w = 0$

Three dimensional flow (3-D flow):-

The different characteristics of flowing fluid are function of time & three-space co-ordinates.
Steady flow - ~~$u=f_1(x,y)$, $v=f_2(x,y)$, $w=f_3(x,y)$~~

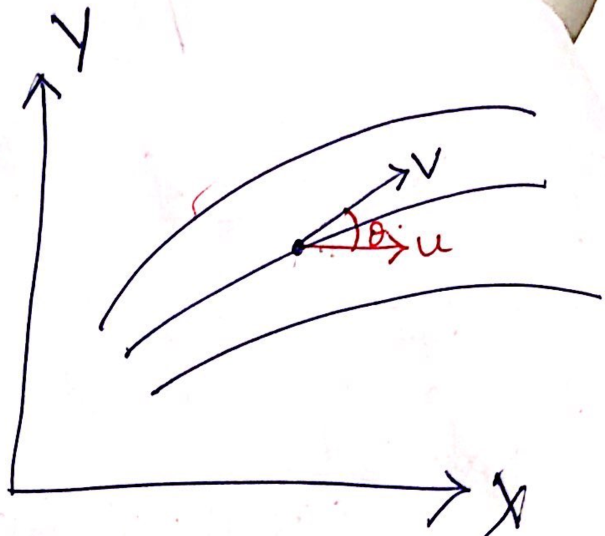
$u = f_1(x,y,z)$, $v = f_2(x,y,z)$ & $w = f_3(x,y,z)$.

~~Stream~~ Flow pattern:-

The flow pattern may be described by means of streamlines, stream-tubes, pathlines & streaklines.

Streamline:- A series of imaginary curves drawn in flow field such that at any point the tangent to the curve represent the velocity vector at that point at that instant is known as "streamline.

~~Streamtube~~ :-



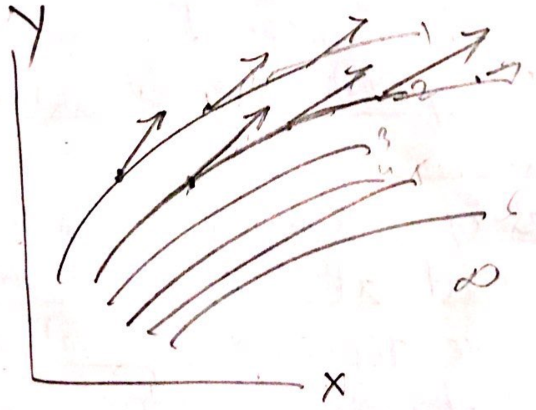
Slope :- $\frac{dy}{dx} = \frac{v}{u}$

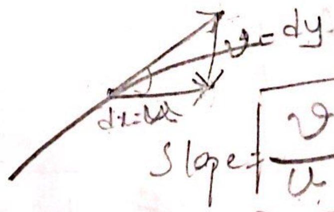
X & Y are directions
u & v are velocity components in x & y directions resp.

$$\frac{dx}{u} = \frac{dy}{v} \quad \text{or} \quad (v \cdot dx - u\,dy) = 0.$$

Differential equ^n of stream line in space for 2-D flow

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$ => Differential equ^n for 3-D flow.



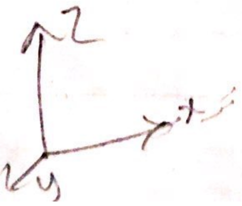


Slope $\boxed{\frac{v}{u} = \frac{dy}{dx}}$

$$\frac{v}{u} = \frac{dy}{dx}$$

$\boxed{\frac{dx}{dy}}$

$\boxed{\frac{v}{dy} = \frac{u}{dx}}$ — 2 I



$\boxed{v\,dx - u\,dy = 0}$

Pathline :- A pathline may be defined as the line traced by a single fluid particle as it moves over a period of time.

→ Pathline will show the direction of velocity of the same fluid particle at successive instants of time.

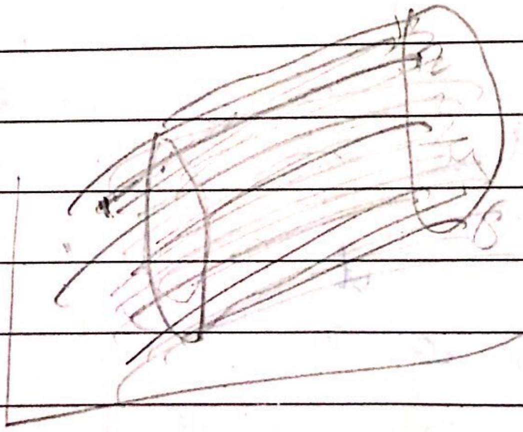
In steadyflow streamlines & pathlines are identical.

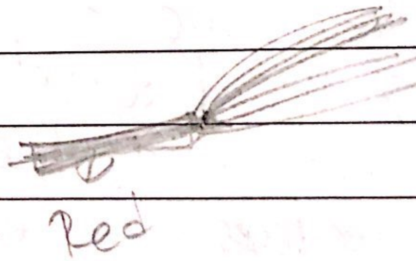
Streakline :- A streakline may be defined as the line that is traced by a fluid particle passing through a fixed point in a flow field.

In steadyflow streakline, streamline & pathline are identical.

Streamtube :- A streamtube is a tubular region of fluid surrounded by streamlines.

· A stream tube is a tube imagined to be formed by a group of streamlines passing through a small closed curve, which may or may not be circular

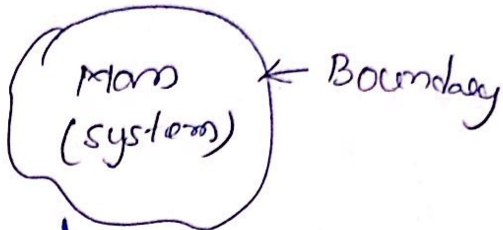




Fluid system :- (closed system)

A specific mass of fluid within the boundaries is defined by close surface.

i.e., Given mass within the boundary.



→ The shape of system & so the boundaries may change with time, as when fluid moves & deforms, so the system containing it also moves & deforms.

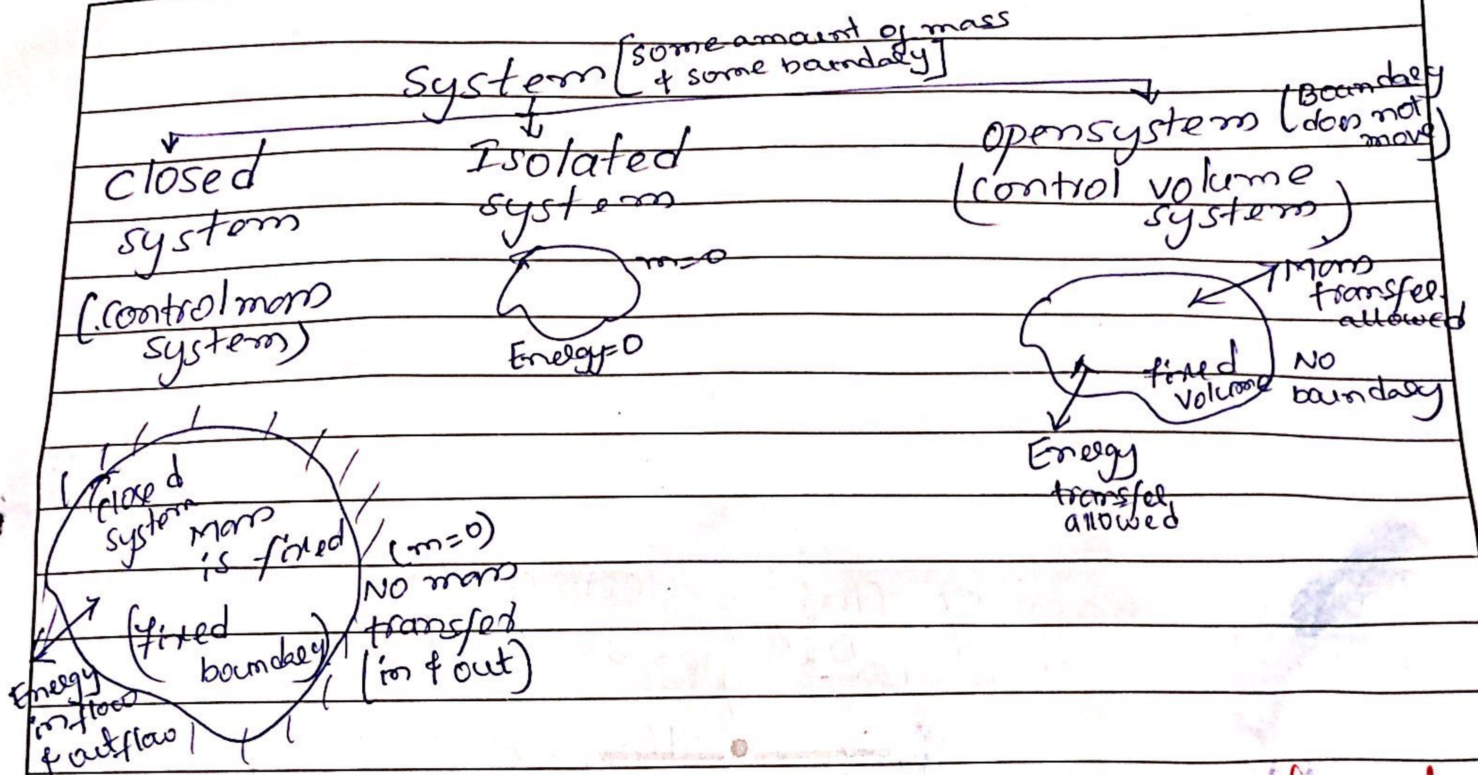
Control volume system :- (open system).

A fixed region in space, which does not move or change shape.
It is region in which fluid flow into & out.

Conservation of mass of fluid flow :-

Fluid is neither created nor destroyed within this region, it may be stated that rate of increase of fluid mass contained within the region must be equal to the difference b/w the rate at which the fluid mass enters the region & the rate at which fluid mass ~~enters the regi~~ leaves the region.

i.e., Rate of increase of fluid mass within the region = 0 [steady flow].



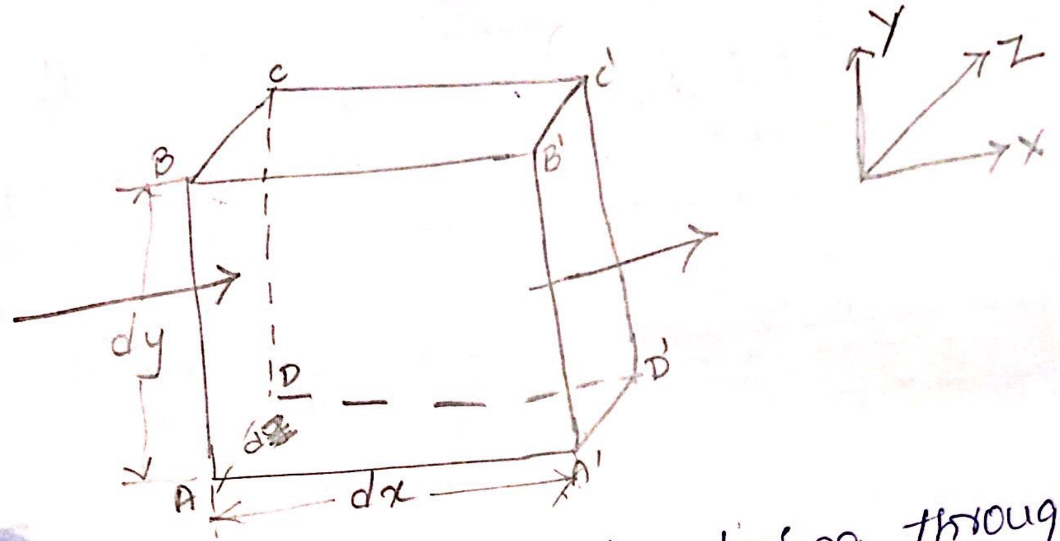
Continuity equation in cartesian co-ordinates:

Continuity equation is actually mathematical statement of principle of conservation of mass.

→ Mass is neither created nor destroyed in the fluid element within the fixed region.

Consider an elementary rectangular parallelopiped with sides of length dx, dy & dz in x, y & z directions

Let u, v, & w are the inlet velocity components in x, y & z directions resp.



Rate of mass of fluid entering through the left face in x-direction per second (YZ plane)

$= \rho \times \text{velocity in x-direction} \times \text{Area of ABCD}$

$= (\rho \times u \times dz \times dy)$

Rate of mass of fluid leaving through the rightface in x-direction persecond (YZ-plane)

$$= \left[\rho . u . dy . dz + \frac{\partial}{\partial x}(\rho . u dy . dz) . dx\right]$$

Net gain of mass in x-direction

$$= \left[\text{Mass of fluid entering} - \text{Mass of fluid leaving}\right]$$

$$= \left[\rho \cdot u \cdot \cancel{dy} \, dz\right] - \left[\rho \cdot u \cancel{\,} dy \cdot dz + \frac{\partial}{\partial x}(\rho \cdot u \cdot dy \cdot dz)\, dx\right]$$

$$= -\frac{\partial}{\partial x}(\rho u) \cdot dx \cdot dy \cdot dz$$

iii) Net gain of mass in y-direction

$$= -\frac{\partial}{\partial y}(\rho \cdot v) \cdot dx \cdot dy \cdot dz$$

Net gain of mass in z-direction

$$= -\frac{\partial}{\partial z}(\rho \cdot w) \cdot dx \cdot dy \cdot dz$$

Net gain of masses $= -\left[\frac{\partial}{\partial x}(\rho u) \cdot dx \cdot dy \cdot dz + \frac{\partial}{\partial y}(\rho \cdot v)\, dx \cdot dy \cdot dz + \frac{\partial}{\partial z}(\rho \cdot w) \cdot dx \cdot dy \cdot dz\right]$

$$= -\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] \cdot dx \cdot dy \cdot dz$$

1

→ Since Mass is neither created nor destroyed in the fluid element within the control volume,

Net increase of mass per unit time in fluid element = Rate of increase of mass of fluid in the element.

Mass of fluid in parallelopiped $= (\rho \cdot dx \cdot dy \cdot dz)$

its rate of increase of mass ~~of fluid~~ per unit time $= \frac{\partial}{\partial t}(\rho \cdot dx \cdot dy \cdot dz)$ — (2)

Equating (1) & (2) equations. We get

$$-\left[\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}\right]\cdot dx\cdot \cancel{dy}\cdot dz=\frac{\partial}{\partial t}\left(\rho\cdot dx\cdot \cancel{dy}\cdot dz\right)$$

$$-\left[\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}\right]=\frac{\partial\rho}{\partial t}$$

$$\boxed{\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0.}$$

⇓

General form of continuity equation in cartesian co-ordinates

This equation is applicable to

(i) steady & Unsteady flow

(ii) Uniform & Non-Uniform flow

(iii) compressible & Incompressible flow.

Apply the general form of continuity equation for Steady flow $\frac{\partial\rho}{\partial t}=0$

then $$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0.$$

) Incompressible flow, ρ = constant.
apply for G.E.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \Leftarrow \text{3D flow}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Leftarrow \text{2D flow.}$$

$$\frac{\partial u}{\partial x} = 0 \Leftarrow \text{1D flow.}$$