UNIT-I – (a) CURVES OF MAXIMUM BM & SF

In the case of rolling load or moving loads, the Bending moment and shear force at a section of the girder changes its position as the load moves. ~~from~~ Therefore the problem is (taken) two-fold:

(i) To determine the load positions for maximum BM or shear force for a given section of a girder and to compute its value.

(ii) To determine the load positions so as to cause absolute maximum bending moment or shear for anywhere on the girder.

→ Different train of loads :-

1. Single concentrated (point) load.
2. Uniformly distributed load longer than the span
3. Uniformly distributed load shorter than the span
4. Two concentrated (point loads) @ a specified distance between them.
5. Multiple concentrated loads (train of wheel loads).

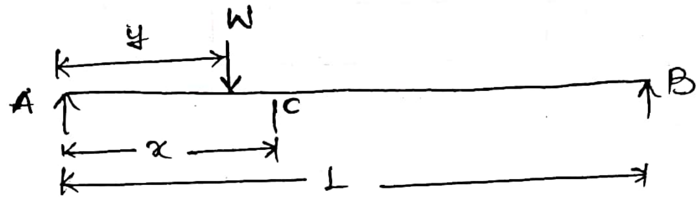
* SINGLE CONCENTRATED LOAD :-

Let us consider a single concentrated load 'W' travelling or rolling or moving along a simply supported beam or girder AB, of span 'L' from left to right.

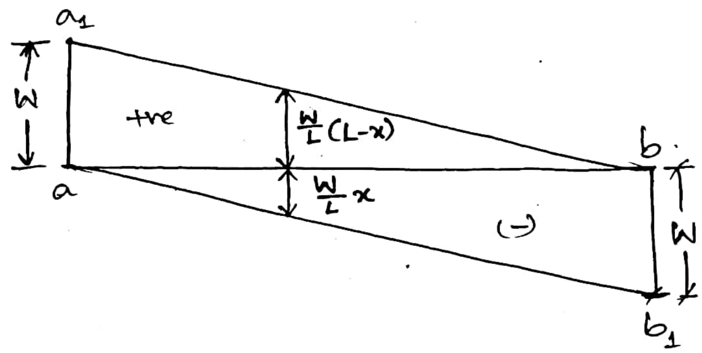
(a) Maximum shear force diagrams :-

Consider a point 'C', at a distance 'x' from left support A. Let the distance of load 'W' by 'y' from A. For any load position, the reaction

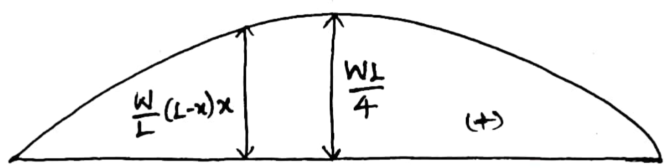
$\boxed{R_B = \frac{Wy}{L}}$ $\boxed{R_A = \frac{W}{L}(L-y)}$



→ (a) Load diagram



→ (b) Maximum Shear force diagram



→ (c) Maximum Bending moment diagram.

case (i) Load in AC $(y < x)$

Let the load 'W' be in AC, such that $y < x$ then,

$$F_x @ C = -R_B = -\frac{Wy}{L} \quad \text{— (1)} \Rightarrow F_x \uparrow \text{ as } y \uparrow$$

when $y = x$,

$$\boxed{F_{max} = -Wx/L} \quad \text{— (2)}$$

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When the load is on the section 'c', the maximum value of shear force is found.

For different values of 'x', the maximum shear force will be given by equation (a) & will vary linearly with 'x'.

Thus at $x=0$, $\Rightarrow F_{max} = 0$

$$x = L \Rightarrow F_{max} = \frac{-W*L}{L} = \boxed{-W = F_{max.max}}$$

• The absolute maximum negative shear force, therefore occurs at the right hand support $= -W$.

<u>case (ii) Load in CB ($y > x$)</u>

When the load moves further i.e $y > x$

$$F_x @ C = +R_A = + \frac{W(L-y)}{L} \quad \text{——} \quad (2)$$

Thus the shear force sign changes immediately when the load crosses the section.

$\Rightarrow$ Maximum SF would evidently be @ $y = x$

$$F_{max} = + \frac{W(L-x)}{L} \quad \text{——} \quad (b)$$

$$\Rightarrow \text{at } x=0, \quad F_{max}(+ve) = + \frac{WL}{L} = \boxed{+W = F_{max.max}}$$

$$x = L, \quad F_{max}(+ve) = + \frac{W(L-L)}{L} = 0.$$

The absolute maximum +ve S.F, therefore occurs at the left hand support, its value being +W.

(b) <u>Maximum bending moment diagram</u> :-

For a simply supported beam with a downward loading bend causing (concavity to the upper side) sagging $\Rightarrow$ Positive BM for all the sections of the beam.

case (ii) Load in AC ($y < x$)

$$M_x = +R_B (L - x)$$

$$\boxed{M_x = + \frac{W \cdot y}{L} (L - x)} \quad — \ ③$$

$\Downarrow$

M_x increases with increase in 'y'.

$$\Rightarrow y = x \Rightarrow M_{max} = + \frac{Wx}{L}(L - x) \quad — \ ⓒ$$

case (iii) Load in CB ($y > x$)

$$M_x = +R_A \cdot x$$

$$\boxed{M_x = + \frac{W(L-y)}{L} \cdot x} \quad — \ ④$$

$\Downarrow$

M_x increases as 'y' decreases

$\Rightarrow y = x,$

$$M_{max} = + \frac{W(L-x)}{L} \cdot x \quad — \ ⓓ.$$

Therefore the maximum bending moment at a section occurs when the load is on the section itself.
For different values of 'x' equation ⓒ or ⓓ vary parabolically.
For absolute BM_{max},

$$\frac{dM_{max}}{dx} = 0$$

$$\frac{WLx}{L} - \frac{Wx^2}{L}$$

$$W - \frac{2Wx}{L}$$

$$\therefore + \frac{W}{L}(L - 2x) = 0$$

$$\frac{W(L-2x)}{L}$$

$\Rightarrow \boxed{x = \frac{L}{2}} \Rightarrow$ The absolute maximum bending moment occurs @ centre of the span $\Rightarrow \boxed{M_{max.max} = + \frac{WL}{4}}$

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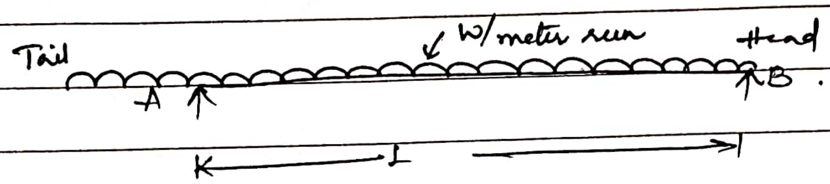
UNIFORMLY DISTRIBUTED LOAD LONGER THAN THE SPAN :-

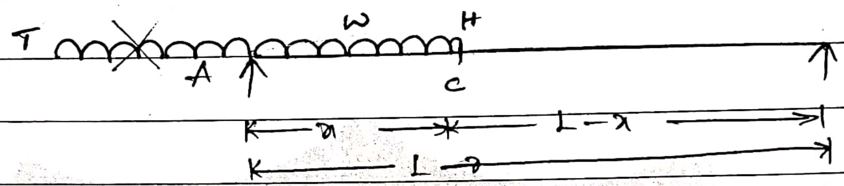


SF max @ supports

BM max @ centre.

(i) Maximum SF at :-

(a) Max. −ve SF



It occurs when the head of UDL just touches 'C'.

Taking moment about A,

$$R_B \times L - w.x.\frac{x}{2} + R_A \times 0 = 0.$$

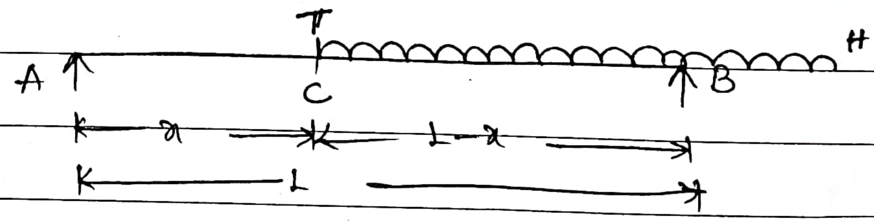
$$\boxed{R_B = \frac{wx^2}{2L}}$$

$F_x = -R_B \Rightarrow \boxed{F_x = -\frac{wx^2}{2L}}$ → Parabolic in nature & live for $x=0$ & $x=L$

$x=0$, $\boxed{F_0 = 0}$ — (1)

$x=L$, $\boxed{F_L = -\frac{wL}{2}}$ — (2)

(b) Max. +ve SF :-



It occurs when the tail of the UDL touches the section 'c'

Taking moment about B,

$$R_A \times L - \frac{w(L-x)^2}{2} + R_B \times 0 = 0$$

$$\boxed{R_A = \frac{w(L-x)^2}{2L}}$$

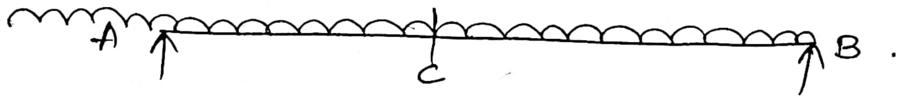
$F_x = +R_A$

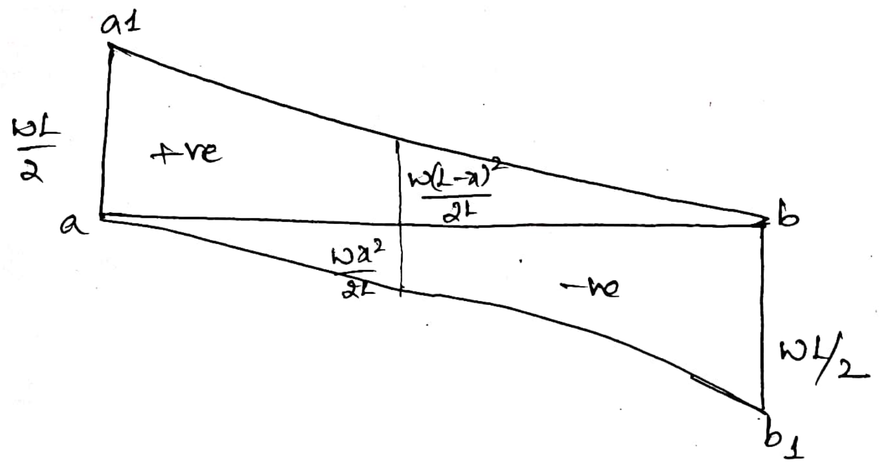
$$\Rightarrow \boxed{F_x = +\frac{w(L-x)^2}{2L}}$$ → Parabolic in nature and is true for $x=0$ & $x=L$

When,

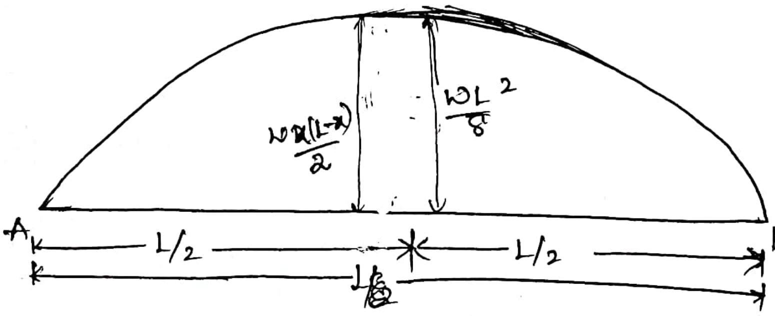
$x=0$, $\boxed{F_a = +\frac{wL}{2}}$ — (3)

$x=L$, $\boxed{F_L = 0}$ → (4).



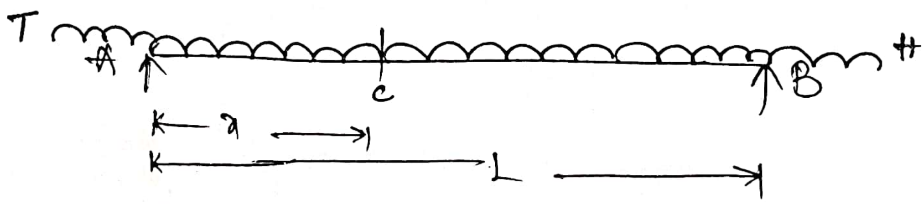


→ SFD



→ BMD.

(ii) Maximum BMD :- Occurs when the span is fully loaded.



Taking moment about x,

$$M_x = R_B \times (L-x) - \frac{w(L-x)^2}{2}.$$

$$M_x = \frac{wL(L-x)}{2} - \frac{w(L-x)^2}{2}$$

$$\boxed{M_x = \frac{w(L-x).x}{2}}$$

Max. when $x = L/2$

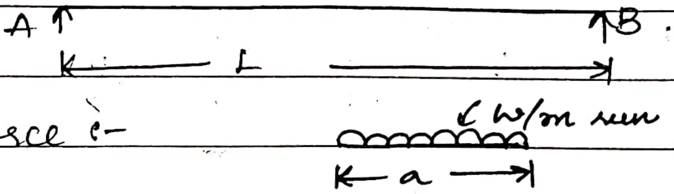
$$\boxed{M_{max} = \frac{w(L-L/2)\,L/2}{2}}$$

$$\Downarrow$$

$$\boxed{M_{max} = \frac{wL^2}{8}} \quad \text{—} (3).$$

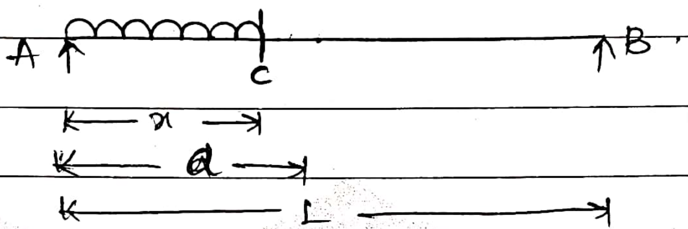
UNIFORMLY DISTRIBUTED LOAD SHORTER THAN THE SPAN OF GIRDER :-

(a) MAXIMUM SF DIAGRAM :-



I) Maximum negative shear force :-

(i) $x < a$



Taking moment about A,

$$R_B \times L - \frac{wx^2}{2} + R_A \times 0 = 0$$

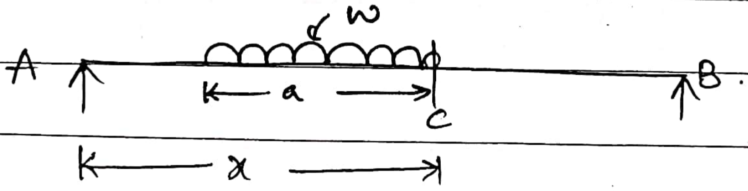
$$\boxed{R_B = \frac{wx^2}{2L}}$$

$F_x = -R_B \Rightarrow \boxed{F_x = -\frac{wx^2}{2L}}$ → Parabolic & true for values for $x = 0$ to $x = a$

$\Rightarrow x = 0 \Rightarrow \boxed{F_x = 0}$ — (1)

$x = a \Rightarrow \boxed{F_x = -\frac{wa^2}{2L}}$ — (2)

(ii) $x > a$



Taking moment about A,

$$R_B \times L - wa\left(x - \frac{a}{2}\right) + R_A \times 0 = 0$$

$$\boxed{R_B = \frac{wa}{L}\left(x - \frac{a}{2}\right)}$$

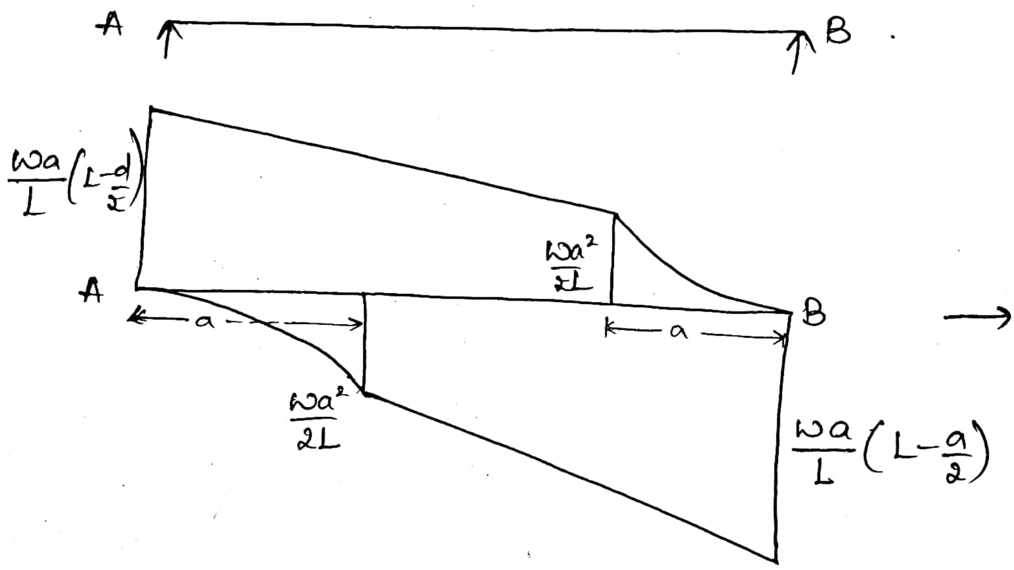
$F_x = -R_B$

$$\boxed{F_x = -\frac{wa}{L}\left(x - \frac{a}{2}\right)} \longrightarrow$$ This is linear and true for $x = a$ to $x = L$

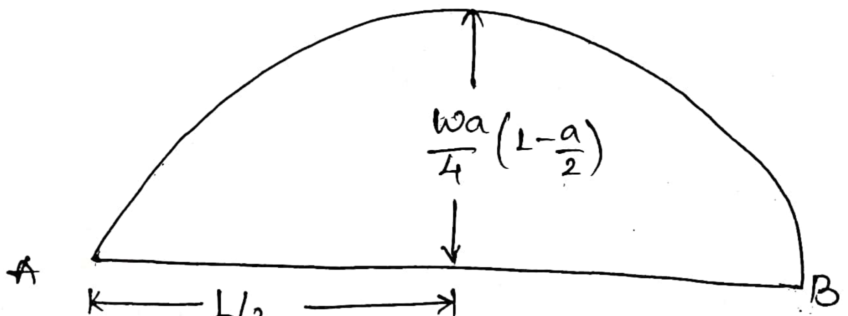
$x = a \Rightarrow \boxed{F_x = -\frac{wa^2}{2L}} \longrightarrow (2)$

$x = L \Rightarrow \boxed{F_x = -\frac{wa}{L}\left(L - \frac{a}{2}\right)} \longrightarrow (3)$

From equations (1), (2), (3), (4) (5) & (6)



→ SFD diagram



→ BM diagram

fig:- Max. SFD & BMD for SSB with UDL < span length

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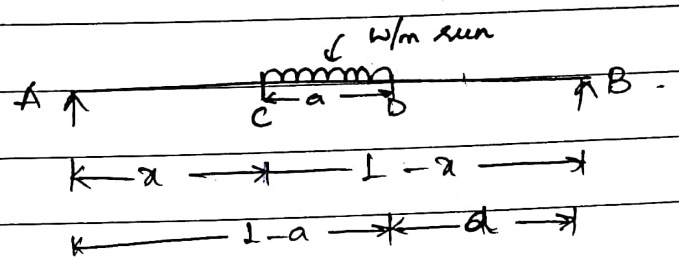
2) Maximum positive shearforce (diagram):

i) $x < L-a$



Taking moment about B,

$$R_A \times L - wa\left(L - x - \frac{a}{2}\right) + R_B \times 0 = 0.$$

$$\boxed{R_A = \frac{wa}{L}\left(L - x - \frac{a}{2}\right)}$$

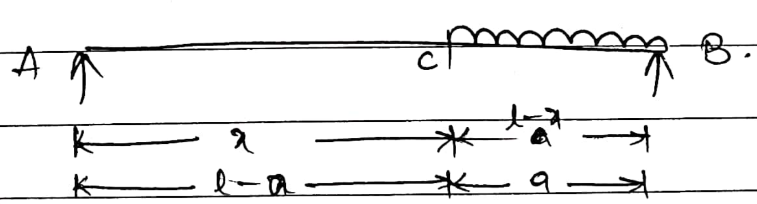
$F_x = +R_A$

$$\boxed{F_x = \frac{wa}{L}\left(L - x - \frac{a}{2}\right)}$$ → Linear & true for $x=0$ to $x=L-a$

when $x=0$, $\boxed{F_x = \frac{wa}{L}\left(L - \frac{a}{2}\right)}$ — (4)

$x = L-a$, $\boxed{F_x = \frac{wa^2}{2L}}$ — (5).

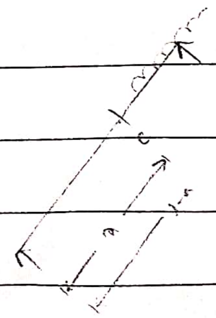
ii) $x > L-a$



Taking moment about B,

$$R_A \times L - wa\frac{(L-x)^2}{2} = 0.$$

$$\boxed{R_A = \frac{w(L-x)^2}{2L}}$$



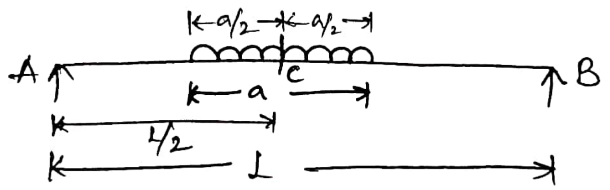
$F_x = +R_A$

$$\boxed{F_x = +\frac{w(L-x)^2}{2L}}$$ → Parabola & true for $x = L-a$, $x = L$

$x = L-a \Rightarrow \boxed{F_x = \frac{wa^2}{2L}}$ → (5)

$x = L \Rightarrow \boxed{F_x = 0.}$ → (6)

(b) Maximum BM :-



Occurs when UDL @ centre a/2 @ L/2

$$M_C = R_B \times \frac{L}{2} - \frac{wa}{2} \cdot \frac{a}{4}$$

$$\left(R_B = \frac{wa}{2}\right)$$

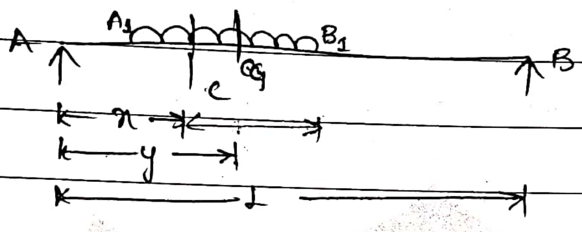
$$\boxed{M_C = \frac{wa}{4}\left(L - \frac{a}{2}\right)}$$ //

when not at centre,

AC,

BM @ C $\Rightarrow M_x = +R_B(L-x)$

For maximum BM at the section 'c' the load is to be arranged that its CG is at a distance 'y' from A,



In this condition, $R_B = \frac{way}{L}$

distance $CB_1 = \left(y - x + \frac{a}{2}\right)$

$M_x = +R_B(L-x) - \frac{w(CB_1)^2}{2}$

$$\Rightarrow +\frac{way}{L}(L-x) - \frac{w}{2}\left(y-x+\frac{a}{2}\right)^2$$

differentiate M_x w.r.t. y and equate it to zero,

$$\frac{dM_x}{dy} = 0 \Rightarrow +\frac{wa(L-x)}{y} - w\left(y-x+\frac{a}{2}\right)$$

$$\Rightarrow \frac{\overset{A_1B_1}{a}}{\underset{AB}{L}}\underbrace{(L-x)}_{CB} = \underbrace{\left(y-x+\frac{a}{2}\right)}_{CB_1} \Rightarrow \boxed{\frac{A_1C}{CB_1} = \frac{AC}{CB}}$$

$$y \Rightarrow \frac{a}{L}(L-x) + x - \frac{a}{2}.$$

a

$$M_{max} = \frac{wa}{L}(L-x)\times\left(\frac{a}{L}(L-x)+x-\frac{a}{2}\right) - \frac{w}{2}\left(\frac{a}{L}(L-x)\right)^2$$

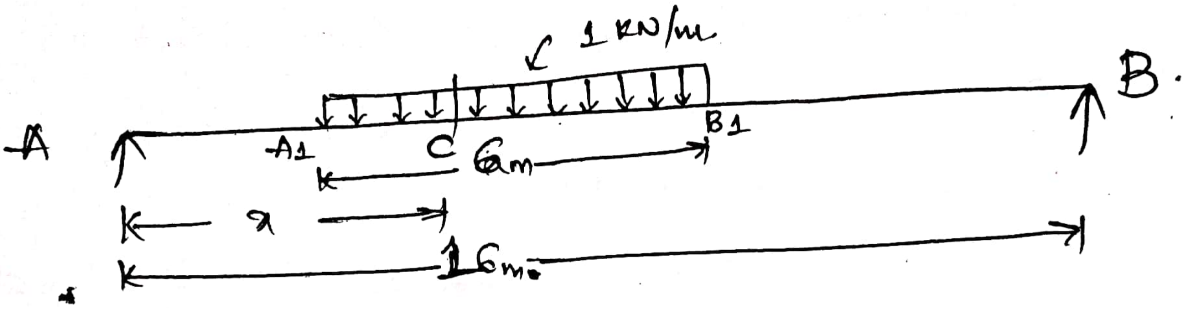
$$\boxed{\Rightarrow +\frac{wax}{L}(L-x)\left(1-\frac{a}{2L}\right)}$$

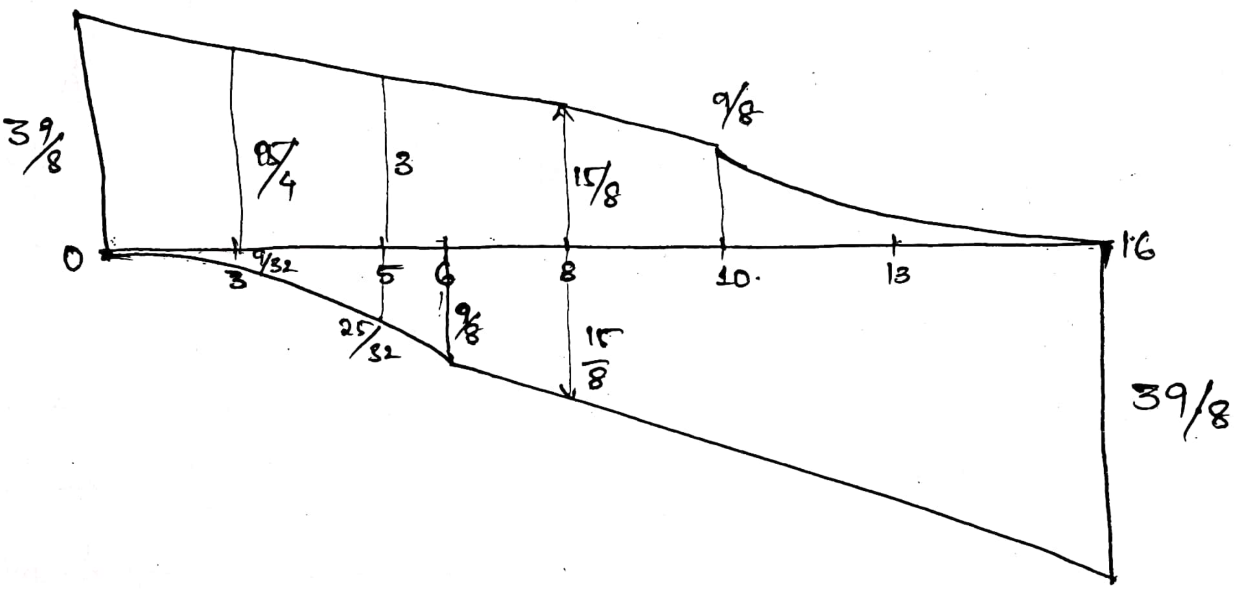
$$\left(R_B = \frac{wa}{2}\right)$$

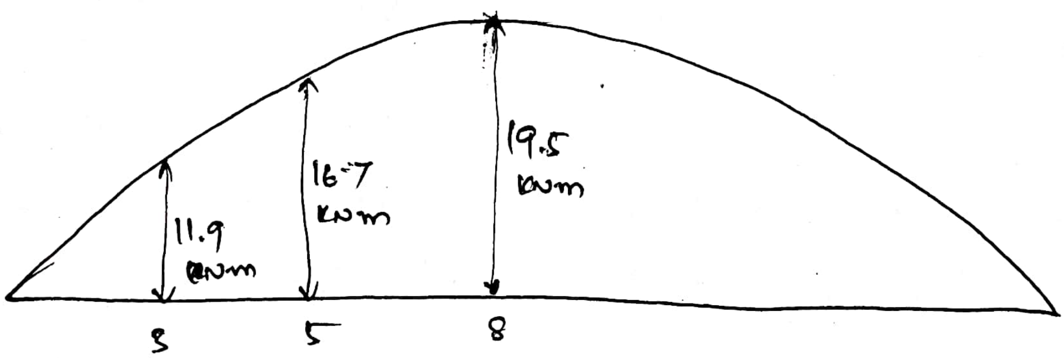
$$\boxed{M_c = \frac{wa}{4}\left(l - \frac{a}{2}\right)}$$

Prob:

→ A uniformly distributed load of 1 kN per meter run, 6 m long crosses a girder of 16 m span. Construct the maximum SF & BM diagrams and calculate the values at sections, 3m, 5m and 8m from the left hand support.


1 KN/m
B.
A
A1
C
6m
B1
x
16m


39/8
0
3
5
6
8
10.
13
16
15/8
9/8
9/32
25/32
39/8


11.9 KNm
16.7 KNm
19.5 KNm
3
5
8

Given data,

UDL = 1 KN/meter run

Length of UDL, a = 6 m

Span ⇒ L = 16 m

x = 3m ; x = 5m; x = 8 m.

(a) Max. -ve SF

from eqn. (1) $F_{max} \Rightarrow -\frac{wx^2}{2L}$

Case (i) upto x = 6m (x = 0 to x = a).

$$F_{max} = -\frac{1 \times x^2}{2 \times 16} = \frac{x^2}{32} \text{ KN.}$$

For, x = 0m, $F_0 = 0$

x = 3m, $F_3 = -\frac{9}{32}$ KN

x = 5m; $F_5 = -\frac{25}{32}$ KN

(x = a) ⇒ x = 6m; $F_6 = -\frac{36}{32} = -\frac{9}{8}$ KN.

Parabolic.

Case (ii) x > a ⇒ x > 6m

$$F_x = -\frac{wa}{L}\left(x - \frac{a}{2}\right)$$

$$\Rightarrow -\frac{1 \times 6}{16}\left(x - \frac{6}{2}\right) \Rightarrow -\frac{3}{8}(x-3)$$

At x = 6, $F_6 = -\frac{3}{8}(6-3) \Rightarrow \frac{-9}{8}$ KN.

x = 8, $F_8 = -\frac{3}{8}(8-3) \Rightarrow -\frac{15}{8}$ KN

x = 16, $F_{16} = F_{max\,max} \Rightarrow -\frac{3}{8}(16-3) \Rightarrow -\frac{39}{8}$ KN.

Linear

(b) Maximum +ve Shear force:-

(i) x between 0 to (L-a) ⇒ 16 - 6 ⇒ 10 m.

$F_{max} \Rightarrow +R_A \Rightarrow \frac{Wa}{L}\left(L - x - \frac{a}{2}\right) \rightarrow$ linear.

$$\Rightarrow \frac{1 \times 6}{16}\left(16 - x - \frac{6}{2}\right)$$

$$\Rightarrow +\frac{3}{8}(13 - x)$$

At $x = 0$, $F_0 = +\frac{3}{8}(13 - 0) = \underline{+\frac{39}{8}\ kN}$

$x = 3$, $F_3 = +\frac{3}{8}(13 - 3) = \underline{+\frac{15}{4}\ kN}$

$x = 5m$, $F_5 \Rightarrow +\frac{3}{8}(13 - 5) = \underline{+3\ kN}$

$x = 8m$, $F_8 = +\frac{3}{8}(13 - 8) = \underline{+\frac{15}{8}\ kN}$

$x = 10m$, $F_{10} = +\frac{3}{8}(13 - 10) = \underline{+\frac{9}{8}\ kN}$.

} linear.

(iii) x between 10 m to 16 m.

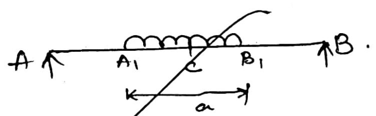
$$F_{max} = +\frac{w(L - x)^2}{2L} \Rightarrow \frac{1\,(16 - x)^2}{2 \times 16} \Rightarrow \frac{(16 - x)^2}{32}$$

At $x = 10m$, $F_{10} = +\frac{1}{32}(16 - 10)^2 \Rightarrow \underline{\frac{9}{8}}$

$x = 16m$, $\underline{F_{16} = 0.}$

} parabolic

(c) Maximum Bending moment :-



$$\frac{A_1C}{CB_1} = \frac{AC}{CB}$$

$\left.\begin{array}{l} AC = x \\ CB = L - x \end{array}\right\} \Rightarrow \frac{A_1C}{CB_1} = \frac{x}{L - x}$

$$\frac{A_1C + CB_1}{CB_1} = \frac{\cancel{x} + L - \cancel{x}}{L - x}$$

Maximum BM

$$M_{ax} = +\frac{Wax}{L}(L-x)\left(1-\frac{a}{2L}\right)$$

$x=3 \Rightarrow M_3 = 11.9$ kN-m

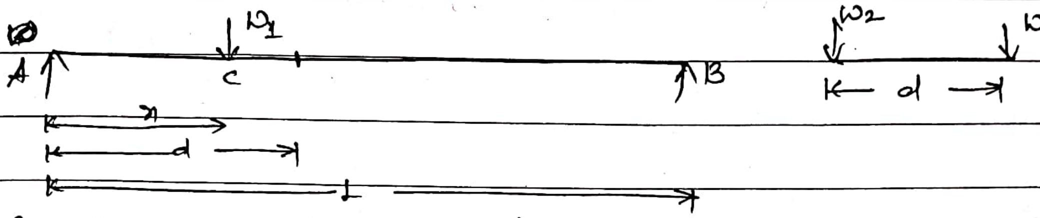
$x=5 \Rightarrow M_5 = 16.7$ kN-m

$x=8 \Rightarrow M_8 = 19.5$ kN-m $\leftarrow M_{max.\,max}$.

Section x	max. -ve SF	max +SF	max BM.
3m	$-9/32$ kN	$+15/4$ kN	+11.9 kN-m
5m	$-25/32$ kN	+3 kN	+16.7 kN-m
8m	$-15/8$ kN	$+15/8$ kN	+19.5 kN-m

(4) TWO POINT LOAD WITH A FIXED DISTANCE BETWEEN THEM :-

Let us consider two point loads, W_1 & W_2 at a fixed distance 'd' apart, moving from left to right with W_1 leading and $W_1 < W_2$.



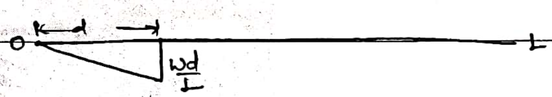
(a) Maximum negative shear force :-

(i) $x < d$ ⇒ occurs when W_1 is at x,

Taking moment about A,

$$R_B \times l - W_1 x + R_A \times 0 = 0$$

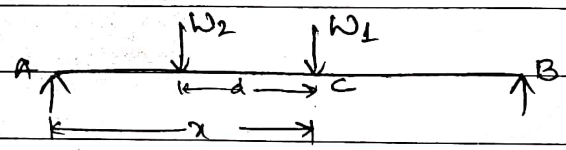
$$R_B = \frac{W_1 x}{L}$$



$$\boxed{F_x = -\frac{W_1 x}{L}} \quad \text{— (1)}$$

(ii) $x > d$

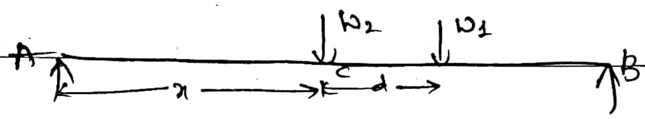
ⓐ when W_1 is at section,



$$R_B = \frac{W_1 x + W_2 (x-d)}{L}$$

$$F_1x = -R_B \Rightarrow \boxed{F_1 x = -\frac{[W_1 x + W_2 (x-d)]}{L}} \quad \text{— (2)}$$

ⓑ when W_2 is at section,
and (W_2 just left & W_1 right)



$$R_B = \frac{[W_2 x + W_1 (x+d)]}{L}$$

$$F_2 x = -R_B + W_1 \Rightarrow \boxed{F_2 x = -\frac{[W_2 x + W_1 (x+d)]}{L} + W_1} \quad \text{— (3)}$$

Any of the equations ② & ③ should give the maximum negative shear force depending on the relative x & d values.

ⓒ $(L-x) < d$ ⇒ W_2 @ C, W_1 off the girder, $F_{0_1 x} = -\frac{W_2 x}{L}$

$x = d$ to $x = L$,

eqn ② > eqn ③ if

${}^1F_{max} > {}^2F_{max}$.

$$\Rightarrow \frac{W_1 x + W_2(x-d)}{L} > \frac{W_2 x + W_1(x+d)}{L} - W_1$$

$$\Rightarrow (W_1 + W_2)d < W_1 L$$

$$\Rightarrow \boxed{d < \frac{W_1 L}{W_1 + W_2}} \quad \text{— (I).}$$

equation ② gives maximum -ve SF (as standard case when leading load reaches the section).

Beyond this eqn ③ gives max. i.e $d > \frac{W_1 L}{W_1 + W_2}$.

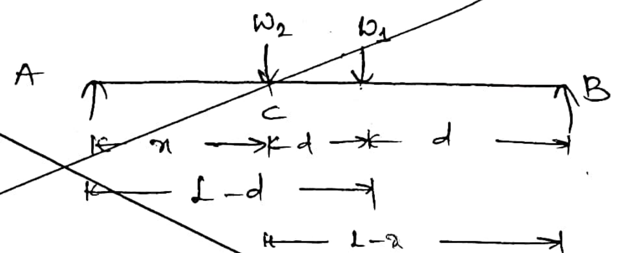
$x = d \Rightarrow \boxed{F_{2x} = -\frac{W_1 d}{L}}$

$x = L \Rightarrow F_{1x} = F_{max} = -\left[W_1 + \frac{W_2(L-d)}{L}\right]$

(b) Maximum positive B shearforce:-

(i) $x < L-d$.

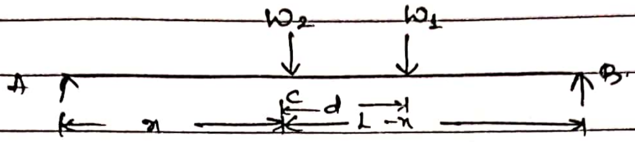
(ii) W_2 at the section,



$$R_A = \frac{W_2(L-x) + W_1(L-x-d)}{L}$$

$$\boxed{+F_{2x} \Rightarrow + \frac{W_2(L-x) + W_1(L-x-d)}{L}} \rightarrow (4).$$

(b) Maximum +ve SF



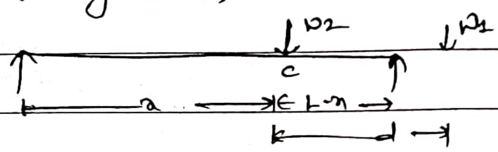
(i) $(L-x) > d$, W_2 & W_1 on the span & W_2 @ C,

$F_{2max} = +R_A$

$$\boxed{F_{2max} = + \frac{[W_2(L-x) + W_1(L-x-d)]}{L}} \quad — (4)$$

$\rightarrow x=0$ to $x=L-d$

(ii) $(L-x) < d$, W_2 @ C & W_1 off the girder,



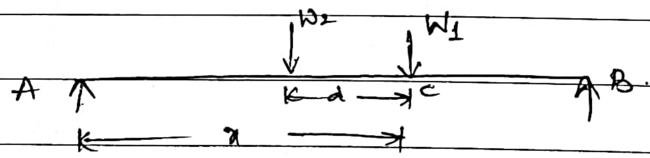
$$F_{2max} = +R_A = \boxed{+\frac{W_2(L-x)}{L}}$$

(5). $\rightarrow x=L-d$ to $x=L$

$$\boxed{F_{2max} = + \left(W_2 + \frac{W_1(L-d)}{L}\right)} \Rightarrow @ \ x=0$$

$F_{2max} \Rightarrow @ \ x = L-d \Rightarrow \boxed{F_{2max} = +\frac{W_2 d}{L}}$

(c) Maximum BMD.



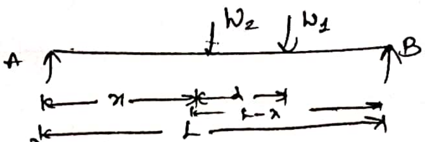
(i) W_1 at the section,

$$R_B = \frac{W_1 x + W_2(x-d)}{L}$$

BM at x, $M_x = R_B(L-x)$

$$\boxed{M_{1x} \Rightarrow \frac{W_1 x + W_2(x-d)}{L}(L-x)} \quad — (I)$$

(ii) W_2 at C,



$M_x = +R_A \cdot x$

$\Rightarrow R_A = \dfrac{W_2(L-x) + W_1(L-x-d)}{L}$

$$\Rightarrow \boxed{M_{2x} = \frac{W_2(L-x) + W_1(L-x-d)}{L} \cdot x} \quad \text{—— (II)}$$

Now, $M_{1x} > M_{2x}$ if,

$$\frac{W_1 x + W_2(x-d)}{L}(L-x) > \frac{W_1(L-x-d) + W_2(L-x)}{L} \cdot x$$

$$x > \frac{W_2 L}{W_1 + W_2}$$

$\Rightarrow$ For, $x < \dfrac{W_2 L}{W_1+W_2} \Rightarrow M_{2x}$ is max.

$x > \dfrac{W_2 L}{W_1+W_2} \Rightarrow M_{1x}$ max.

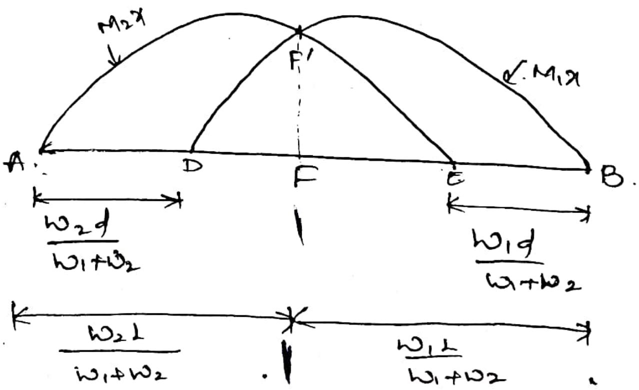
Now,

$M_{1x} = 0 \Rightarrow$ @ $x = \dfrac{W_2 d}{W_1+W_2}$ & $x = L$

$M_{2x} \Rightarrow 0$ @ $x = 0$ & $(L-x) = \dfrac{W_1 d}{W_1+W_2}$

$M_{1x} = M_{2x}$ @ F'

$$\boxed{x = \frac{W_2 L}{W_1 + W_2}}$$



F' divides AB with ratio $W_1 : W_2$

$$\boxed{x = \frac{AE}{2} = \frac{1}{2}\left(L - \frac{W_1 d}{W_1+W_2}\right)}$$

$$\Rightarrow \boxed{M'_{02x} = R_B(L-x) = \frac{W_2 x}{L}(L-x)} \text{ —— (III)}$$

↳ $x = L-d$ to $x = L$

$x < d$

$$\boxed{M_{01x} = \frac{W_1 x}{L}(L-x)}$$

↳ (IV) $x = 0$ to $x = d$

Problem:- Two point loads of 4 kN & 6 kN spaced 6m apart cross a girder of 16m span, the 4 kN load leading from left to right. Construct the maximum SF & BM diagrams stating the absolute maximum values.

Sol: Given data, $W_1 = 4\ kN$

$W_2 = 6\ kN$

$d = 6m$

$L = 16m$

$$\frac{W_1 L}{W_1 + W_2} = \frac{4 \times 16}{4+6} = 6.4$$

$6 < 6.4 \Rightarrow$ Standard case.

(a) Max. -ve SF

From $x = 0$ to $x = 6m \Rightarrow$ Max. -ve SF occurs due to leading load $W_1 = 4\ kN$.

$$F_x = -\frac{W_1 x}{L} = -\frac{4x}{16} \Rightarrow -\frac{x}{4}\ kN$$

$$F_6 = -\frac{6}{4} = -1.5\ kN$$

From $x > 6m$

$$F_1 x = -\frac{W_1 x + W_2(x-d)}{L} \Rightarrow -\left(\frac{5x-18}{18}\right)$$

At $x = 16 = F_{max} = -\frac{(5 \times 16) - 18}{8} \Rightarrow \boxed{-\frac{31}{4}\ kN}$

(b) Max +ve SF

$x = 0$ and $x = L - d = 16 - 6 = 10m$, Max. +ve SF occurs due to W_2 (6 kN) at the section and W_1 ahead of it.

$$F_{2max} = \frac{W_2(L-x) + W_1(L-x-d)}{L}$$

$x = 0$, $F_A = F_{max\,max} \Rightarrow \boxed{+8.5\ kN}$

$x = 10$, $\boxed{F_{10} = +9/4\ kN}$

For all the sections between $x = 10m$ to $x = 16m$, W_1 will be ... the girder when W_2 @ section,

$$F_{02}\,max = \frac{W_2(L-x)}{L}$$

$x = 10$, $\boxed{F_D = +\frac{9}{4}}$ → check

$x = 16$, $F_B = 0$:

(c) Maximum bending moment :-

eqn. (I) $\Rightarrow M_1 x = \frac{W_1 x + W_2(x-d)}{L}(L-x)$

$$\Rightarrow + (10x - 36)\left(1 - \frac{x}{16}\right)$$

$M_1 x = 0$ @ $x = \frac{W_2 d}{W_1 + W_2} \Rightarrow 3.6m$ and $x = 16m$

Thus, $DB = 16 - x \Rightarrow 12.4m$.

M^1_{max} will occur @ $\boxed{x = 3.6 + \frac{DB}{2} \Rightarrow 9.8m.}$

$$M^1_{max} = + (10 \times 9.8 - 36)\left(1 - \frac{9.8}{16}\right)$$

$$\boxed{M^1_{max} = 24.25\ KN\text{-}m.}$$

eqn. (II). $\Rightarrow M^2_x = \frac{W_2(L-x) + W_1(L-x-d)}{L} \cdot x$

$$\Rightarrow + \frac{x}{16}[136 - 10x].$$

This is at

$x = 0$ to ~~...~~ $L - x = \frac{W_1 d}{W_1 + W_2}$

$x = 13.6m.$

Now, $AE = 16 - 2.4 \Rightarrow 13.6m.$

$\Rightarrow M^2_{max}$ occur at $\boxed{x = \frac{AE}{2} = 6.8m.}$

$$\boxed{M^2_{max} = +28.8\ KN\text{-}m.}$$

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$$\frac{W_2 L}{W_1+W_2}$$

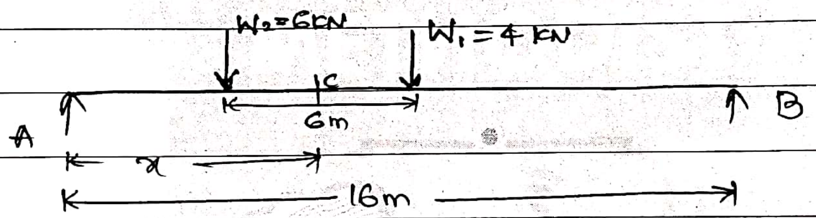
$$\frac{6\times16}{6+4} \Rightarrow 9.6\,m.$$

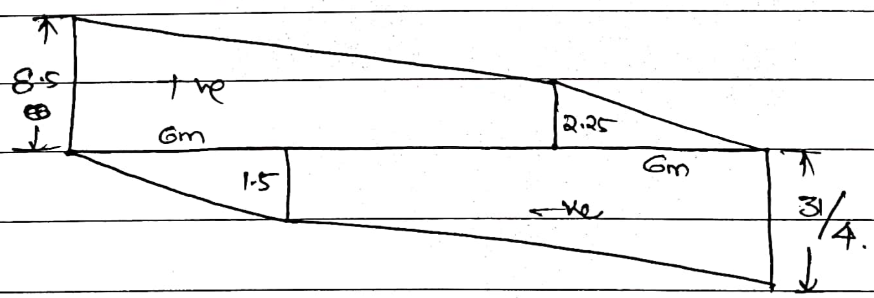
$x < 9.6\,m \Rightarrow M^2_{max} > M^1_x$.

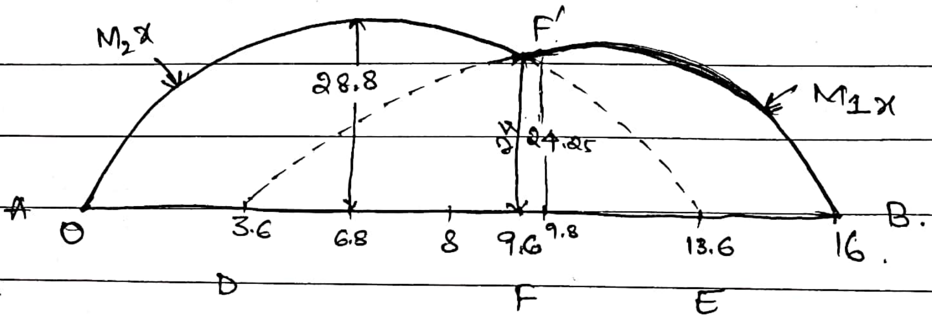
$x > 9.6\,m \Rightarrow M_1\,max > M_2\,x$.

at $x = 9.6$,

$$\left.\begin{aligned} M_2\,max &= 24\ kN\text{-}m \\ M_1\,max &= 24\ kN\text{-}m \end{aligned}\right\}\ \underline{\text{Check}}$$







Problem:- $W_1 = 4\,kN$

$W_2 = 6\,kN$

$d = 6\,m$

$L = 12\,m.$

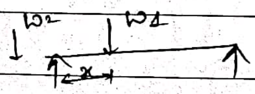
Sol: $\frac{W_1 L}{W_1+W_2} \Rightarrow \frac{4\times12}{4+6} = 4.8\,m.$

$d = 6 > \frac{W_1 L}{W_1+W_2}$

Thus the case is not standard one, SFD will not be similar as previous case.

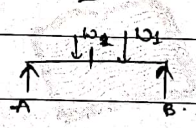
(a) Maximum -ve SF

(i) $x=0$ to $x=6$



$${}^1F_x = \frac{-W_1 x}{L} = -\frac{4x}{12} = -\frac{x}{3}\ kN \quad —①$$

(ii) $x=0$ to $x=6$,



$${}^2F_x = -R_B + W_1$$

$$\Rightarrow -\frac{W_2 x}{L} - \frac{W_1(x+d)}{L} + W_1$$

$$\Rightarrow -\left(\frac{5}{6}x - 2\right)\ kN \quad —②.$$

If

$${}^2F_x > {}^1F_x \Rightarrow \frac{5}{6}x - 2 > \frac{x}{3}$$

$$5x - 12 > 2x.$$

$$3x > 12$$

$$x > 4.$$

Thus for $x=0$ to $x=4 \rightarrow {}^1F_x \Rightarrow F_0 = 0$

$F_4 = -\frac{4}{3}\,kN$ ✓

$x=4$ to $x=6 \Rightarrow {}^2F_x \Rightarrow F_4 = \frac{5\times4}{6} - 2 \Rightarrow -\frac{4}{3}\,kN$ ✓ Check.

$F_6 = -3\,kN.$

(iii) $x = d$ to $x = L = 12$
$= 6.$

$$^1F_x = \frac{-W_1 x + W_2(x-d)}{L} \Rightarrow -\frac{5}{6}(x-3.6). \rightarrow (3).$$

(iv) $x=6$ to $x=12$

$$^{02}F_{max} = \frac{-W_2 x}{L} = -\frac{x}{2}. \quad (4)$$

if (4) > (3)

$$\frac{x}{2} > \frac{5}{6}(x-3.6)$$
$$x < 9.$$

Thus $x=6$ to $x=9 \Rightarrow F_{02}\ max \Rightarrow F_6 = -3$ kN.

$F_9 = -\frac{9}{2}$ kN $\Rightarrow -4.5$ kN ✓

$x=9$ to $x=12 \Rightarrow F_{1x} \Rightarrow F_9 = -4.5$ kN ✓ Check

$\Rightarrow F_{12} = -7$ kN

(6) Maximum +ve S.F

(i)
$$F_{2max} = \frac{W_2(L-x) + W_1(L-x-d)}{L}$$
$$= \left(8 - \frac{5}{6}x\right)$$

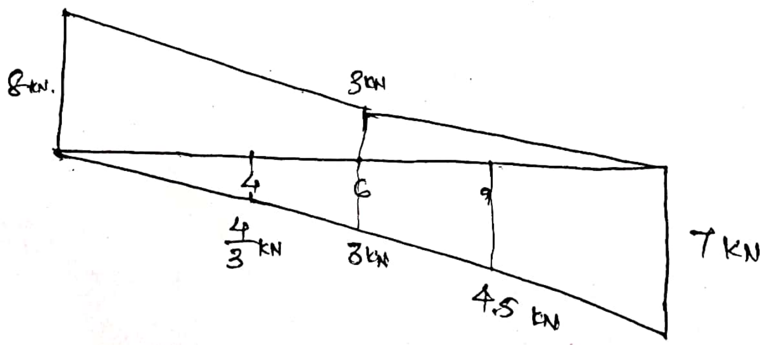
$x=0$ to $x=L-d=6$ m

at $x=0 \Rightarrow F_{max\,max} = 8$ kN.

$x=6 \Rightarrow +(8-5) = +3$ kN.

(ii) $F_{02\,max} = \frac{W_2(L-x)}{L} \Rightarrow x=6 \Rightarrow F_6 = +3$ kN

$F_{12} = 0.$



© Maximum BM

(i) $M_{2x} \Rightarrow \frac{x}{L}[W_1(L-x-d)+W_2(L-x)]$

$\downarrow$

$x=0 \quad 5(L-x) = \frac{W_1 d}{W_1+W_2} \Rightarrow \frac{\cancel{4}^{2}\times 6}{\cancel{10}_5}$

$x = 12 - \frac{12}{5} \Rightarrow \frac{48}{5} = 9.6\,m.$

M^2_{max} will be @ $x = \frac{9.6}{2} = 4.8\,m$

$^2M_{max} \Rightarrow +19.2$ KN-m

(ii) $M_1 x = \frac{[W_1 x + W_2(x-d)]}{L} L-x \Rightarrow \left(\frac{5}{6}x - 3\right)(12-x).$

$x = \frac{W_2 d}{W_1+W_2} \Rightarrow 3.6\,m$ to $x=12.$

$\Rightarrow M^1_x$ is max @ $x = 3.6 + \frac{12-3.6}{2} \Rightarrow 7.8\,m.$

$^1M_{max} = +14.7$ KN-m

F $\Rightarrow$ $^1M_{max} = M^2_{max}$

$x = \frac{W_2 L}{W_1+W_2} \Rightarrow 7.2\,m.$

M^1_{max} +14.4 KN.
$M^2_{max} = +14.4$ KN } check.

(iii) $M^{02}_{max} = \frac{W_2 x(L-x)}{L} \Rightarrow +x(6-0.5x) \Rightarrow M^{02}_{max} = \frac{L}{2} \Rightarrow +18$ KN-m

$M^2_{max} = M^{02}_{max}$

$\cancel{x}\left(6-\frac{5}{6}x\right) = \cancel{x}(6-0.5x)$

$x=6.$

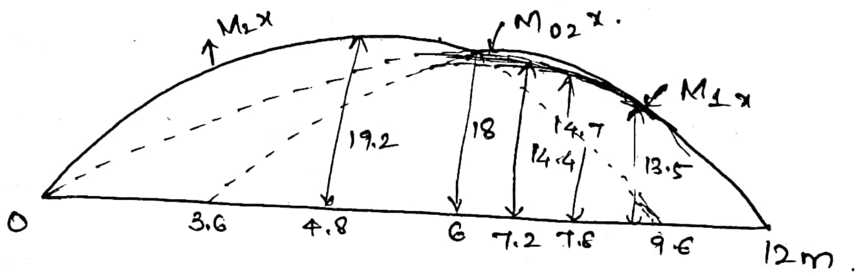
$\Rightarrow {^2M_6} = {^{02}M_6} = +18$ KN-m.

$$M_x^{02} = M_x^{1}$$

$$\left(\frac{5}{6}x - 8\right)(12 - x) = x(6 - 0.5x)$$

$$x = 9\ \text{m}.$$

$$M_9^{02} = M_9^{1} \Rightarrow +9(6 - 0.5 \times 9) = +13.5\ \text{kN-m}$$



6r

d

x 7 L-d

Q:8 Two point loads W_1 & W_2 ($W_2 > W_1$) spaced at a distance 'd' travel from left to right across a simply supported girder, with W_1 leading. Prove that the limiting span below which the greatest bending moment anywhere in the girder will occur when the load W_1 has gone off the girder, is equal to $\left(1 \pm \sqrt{\frac{W_2}{W_1+W_2}}\right)d$.

Sol Maximum value of BM is given by M_2x i.e when W_2 is at the section and W_1 is ahead ($\because W_2 > W_1$).

M_2x is maximum at,

$$x = \frac{1}{2}\left(L - \frac{W_1 d}{W_1+W_2}\right).$$

Substituting x in M_2x, we get,

$$M_2x = \frac{x}{L}\left[W_1(L-x-d) + W_2(L-x)\right]$$

$$\Rightarrow \frac{x}{L}\left[W_1(L-x) - W_1 d + W_2(L-x)\right]$$

$$\Rightarrow \frac{x}{L}\left[(W_1+W_2)(L-x) - W_1 d\right]$$

$$\Rightarrow \frac{1}{2L}\left\{L - \frac{W_1 d}{W_1+W_2}\right\}\left[\{W_1+W_2\}\left\{L - \frac{1}{2}\left(L - \frac{W_1 d}{W_1+W_2}\right)\right\} - W_1 d\right]$$

$$\Rightarrow \frac{1}{4L(W_1+W_2)}\left[\{(W_1+W_2)L - W_1 d\}\{(W_1+W_2)L - W_1 d\}\right]$$

$$\Rightarrow \frac{\left[(W_1+W_2)L - W_1 d\right]^2}{4L(W_1+W_2)} \quad \text{--- (1)}$$

Also when W_1 is off the girder, the max. BM under at the section is given by eqn. ~~(2)~~, $M_{02}x$.

$$M_{02}x = \frac{W_2 x(L-x)}{L} \quad @ \quad x = \frac{L}{2}.$$

$$M_{02}x \Rightarrow \frac{W_2 \cancel{L}}{\cancel{L}\, 2}\left(L - \frac{L}{2}\right)$$

$$\Rightarrow \frac{W_2 L}{4} \quad \text{— (2)}.$$

To have greatest BM governed by $M_{02}x$, we have

$$\frac{W_2 L}{\cancel{4}} > \frac{[(W_1+W_2)L - W_1 d]^2}{\cancel{4}L(W_1+W_2)}$$

$$W_2 L^2 (W_1+W_2) > (W_1+W_2)^2 L^2 + W_1^2 d^2 - 2W_1 d L (W_1+W_2)$$

$$L^2 (W_1+W_2)\left[(W_1+W_2) - W_2\right] + W_1^2 d^2 - 2W_1 d L (W_1+W_2) < 0$$

$$L^2 (W_1+W_2) - 2dL(W_1+W_2) + W_1 d^2 < 0$$

$$L < \left[1 \pm \sqrt{1 - \frac{W_1}{W_1+W_2}}\right] d$$

$$L < \left[1 \pm \sqrt{\frac{W_2}{W_1+W_2}}\right] d$$

Solve if $W_1 = 4$ kN, $W_2 = 6$ kN, $d = 6$, $L = 10$.

$L_{lim} \Rightarrow 1.35$ m or 10.65 m

$10 < 10.65 \Rightarrow M_{02}$ gives max.

Equivalent uniformly distributed load:

A given system of loading crossing or rolling across a girder can always be replaced by a uniformly distributed load, longer than the span, such that bending moment & Shear force, due to this equivalent static load, everywhere is atleast equal to that caused by the actual system of moving loads. Such a static load is known as 'Equivalent uniformly distributed load (E.U.D.L). This E.U.D.L will be different for B.M and S.F.

E.U.D.L for cases, (B.M).

ⓐ Single pt. load

ⓑ U.D.L Shorter than the span

ⓒ Two point loads W_1 & W_2 at a distance 'd' apart.

ⓐ E.U.D.L for single point load :-

$$M_{max} = + \frac{W.x}{L}(L-x) \quad \text{— (1)}$$

If w' is E.U.D.L over the whole span, B.M at a section 'C' will be given by,

$$M = + \frac{w'L}{2}.x - \frac{w'x^2}{2} \Rightarrow + \frac{w'x}{2}(L-x) \quad \text{— (2)}.$$

(1) = (2), $\frac{w'x}{2}(L-x) = \frac{W.x}{L}(L-x)$

$$\boxed{w' = \frac{2W}{L}}$$

@ centre, $\frac{w'L^2}{8} = \frac{WL}{4} \Rightarrow \boxed{w' = \frac{2W}{L}}$

(b) E.U.D.L for UDL shorter than the span :-

UDL @ centre, of the span,

$$+\frac{wa}{4}\left(L-\frac{a}{2}\right) = \frac{w'L^2}{8}$$

$$\Rightarrow \boxed{w' = \frac{2wa}{L^2}\left(L-\frac{a}{2}\right)} \rightarrow a = \text{Length of the UDL.}$$

(c) E.U.D.L for the 2 pt. loads at a fixed distance apart :-

The E.U.D.L for this must be in such a way that the B.M.D ~~...~~ ~~...~~ completely envelops M_{2x}, M_{02x}, M_{1x} dia. This is possible when the tangent of BM curve due to EUDL at the support is equal to the greater of the tangents to M_x^2 & M_x^1 (M_x^{02}) at their corresponding ends.

Eg: M_{2x}, $\Rightarrow y = +\frac{x}{16}(136-10x)$

$$\frac{dy}{dx} @ x=0 \Rightarrow 8.5$$

$$M_{1x} = (10x-36)\left(1-\frac{x}{16}\right) \Rightarrow (12.25x - 0.625x^2 - 36).$$

$$\frac{dy}{dx} (@ x=16 \Rightarrow -7.75$$

inclination of tangent is anti clockwise.

Greater tangent $\Rightarrow 8.5$.

$$M_x = y = \frac{w'x(L-x)L}{2L} \Rightarrow \frac{w'x(16-x)}{2}.$$

$$\frac{dy}{dx} @ x=0 \Rightarrow 8w'$$

$$8w' = 8.5 \Rightarrow w' = 1.06 \text{ kN/m}.$$

$$\text{Max BM} = \frac{w'L^2}{8} \Rightarrow \frac{8.5}{8} \times \frac{16 \times 16}{8} \Rightarrow 34 \text{ kN-m}. > 28.4 \text{ kN-m} @ \text{Eq: (2)}.$$

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Combined dead and moving loads SF diagrams: (Focal length)

Let a girder AB, SS over a span L, carry a uniformly distributed dead load w/unit length. Also due to certain system of moving loads, let w'' be the EUDL, based on shear considerations.

fig: (a) shows SFD due to dead load.

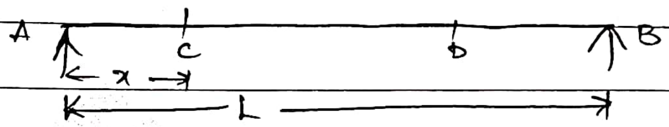
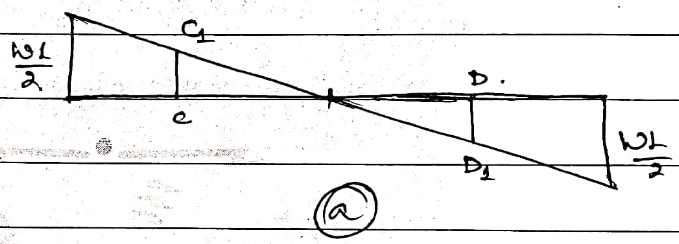


fig (b) shows SFD due to EUDL.

At any distance x, SF due to EUDL is given by,



$$F_x(-ve) = \frac{w''x^2}{2L}$$

$$F_x(+ve)\ \frac{w''(L-x)^2}{2L}$$

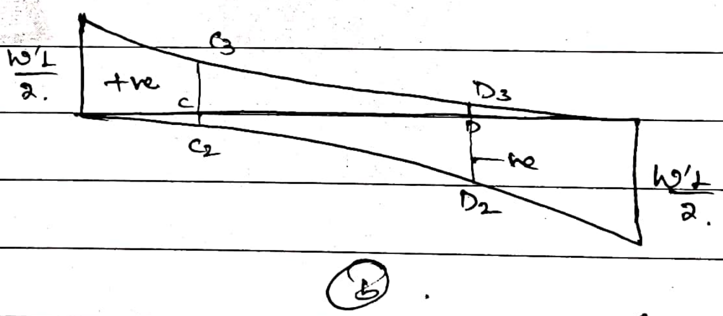
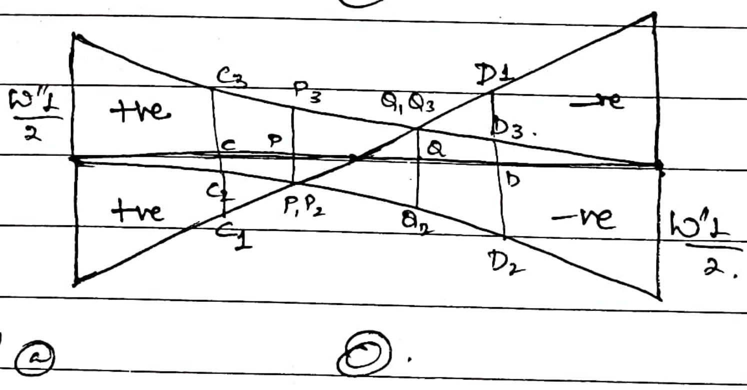


fig (c) indicates the combined SFD obtained after superimposing two (a) & (b) diagrams.



combining the +ve SF of (a) with -ve SF of (b) we get final shear = ordinate C_1C_2.

+ve SF of (a) with +ve SF of b ⇒ final shear = ordinate C_1C_3.

⇒ Final shear at any point is given by vertical intercepts between dead load SF & curves of EUDL.

from fig :-

At C, $SF = C_1C_2$ & C_1C_3 (both +ve)

At P, $SF = P_1P_2^{(0)}$ & P_1P_3 (+ve).

Q, $SF = Q_1Q_2$ (−ve), Q_1Q_3 (0).

D, $SF = D_1D_2$ & D_1D_3 (−ve both).

→ For all the sections to the left of P, the final SF is always positive.

→ For all the sections to the right of Q, the final SF is always negative.

→ For all the sections between P & Q, the final SF is both +ve & −ve. (i.e, SF changes the sign as the load moves over the portion PQ only). Such a portion of the girder, over which the final SF changes the sign, is called focal length.

(Q) Calculate focal length of girder of 16m span carrying a dead load of 3 KN/m & EUDL of 6 KN/m for shear.

Sol:- F_d ⇒ SF due to dead load any section.

F_L ⇒ SF due to EUDL at any section

$$F_d = +\frac{WL}{2} - wx \Rightarrow 24 + 3x \quad -① .$$

$$F_L = -\frac{wx^2}{2L} \Rightarrow -\frac{w''x^2}{2L} \Rightarrow -\frac{3x^2}{16} .$$

At P, $F_d + F_L(-ve) = 0 \Rightarrow 24 + 3x - \frac{3x^2}{16} = 0$.

$x = 5.85\,m. \Rightarrow AP = 5.85$

$BQ = 5.85$

Focal length $= AB - 2AP \Rightarrow 16 - (2 \times 5.85) \Rightarrow \underline{4.3m}$