FLEXIBILITY MATRIX METHOD AND STIFFNESS MATRIX METHOD

→ Degree of static indeterminacy :-

Statically indeterminate structures are those structures which cannot be analysed with the help of equations of static equilibrium alone. These structures are also called as 'hyperstatic structures'.

→ The no. of unknowns are greater than the number of independent equations derived from static equations of equilibriums. ∴ Additional equations are required to determine the unknowns.

The number of these additional eqn's necessary for the solution of the structure is known as degree of static indeterminacy. (or) Degree of redundancy of the structure.

→ It is denoted as 'D_s' → Total degree of static indeterminacy.

Indeterminacies are

① Degree of External indeterminacy → D_{se}

② Degree of internal indeterminacy → D_{si}.

$$\therefore \boxed{D_s = D_{se} + D_{si}}$$

→ Degree of external indeterminacy :- It is related to the support - system of the structure.

Equ. eqn's ⇒ Space frame

$$\Sigma F_x = \Sigma F_y = \Sigma F_z = 0$$

$$\Sigma M_x = \Sigma M_y = \Sigma M_z = 0.$$

plane frame, $\Sigma F_x = \Sigma F_y = 0$

$\Sigma M_z = 0$

Independent equations for spaced frame are '6' & plane frame are '3'.

If the number of independent reaction components are more than that of the independent equilibrium eqns, then the structure is externally indeterminate.

Let 'r' be the external reaction components.

$\boxed{D_{se} = r - 6} \rightarrow$ Space frame

$\boxed{D_{se} = r - 3} \rightarrow$ Plane frame.

→ Degree of internal indeterminacy :-

ⓐ Pin-jointed frames :- It is statically determinate internally if it has just the min. no. of members (m') required to preserve its geometry. If not then it called statically indeterminate. i.e no. of members are more.

If. $j \rightarrow$ joints in a pin jointed frame,

$m' = 2j - 3 \rightarrow$ plane frame

$m' = 3j - 6 \rightarrow$ space frame

If $m > m' \rightarrow$ statically indeterminate. → over stiff.

$m < m' \rightarrow$ Unstable or deficient.

$m = m' \rightarrow$ Statically determinate (internally).

(actual no. of members)

he degree of internal indeterminacy, D_{si}

$$\boxed{D_{si} = m - (3j - 3)} \rightarrow \text{plane frame}$$

$$\boxed{D_{si} = m - (6j - 6)} \rightarrow \text{Space frame}$$

) Rigid jointed frames: A rigid jointed frame is statically determinate internally if its members form an open configuration resembling a tree like structure. Open configuration implies no closed cells. If the structure does not have an open configuration, it is statically indeterminate internally.

A statically indeterminate structure can be made statically determinate by making necessary cuts so that the open configuration can be obtained.

For a plane frame → at each cut three reactions components are released (i.e Two forces & 1 couple).

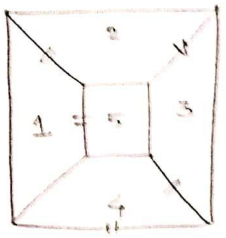
For a space frame → at each cut Six reactions components are released (i.e 3 forces & 3 couples).

$$\boxed{D_{si} = 3c} \rightarrow \text{plane frame}$$

$$\boxed{D_{si} = 6c} \rightarrow \text{Space frame.}$$

where, c → no. of cuts required for obtaining open conf.

Eg:



Generally,

$D_s = (m+r) - 2j \rightarrow$ plane
$\quad = (m+r) - 3j \rightarrow$ space } pin.

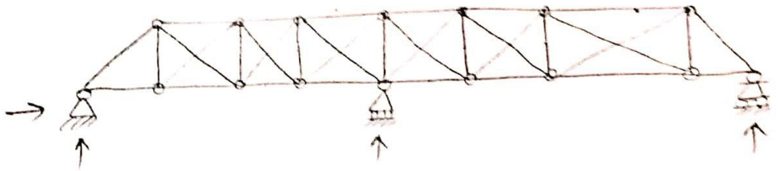
$D_s = (3m+r) - 3j \rightarrow$ plane
$\quad = (6m+r) - 6j \rightarrow$ space } Rigid.

Problems:

Determine the degree of static indeterminacy of the pin jointed plane frame shown in the figure.

Sol: Total no. of independent reaction components,

$r = 2+1+1 = 4$



$D_{se} = r - 3 \Rightarrow 4 - 3 = 1$ — ①

No. of joints, $j = 16$

Actual members $\Rightarrow m = 35$

min. mem, $m' = 2j - 3 \Rightarrow (2 \times 16) - 3 \Rightarrow 29$

$D_{si} = 35 - 29 = 6$ — ②

$D_s = D_{se} + D_{si}$

$\quad = 1 + 6 = 7$ //

(or)

$D_s = (m+r) - 2j$

$\quad = (35+4) - 2 \times 16$

$\quad = 7$ //

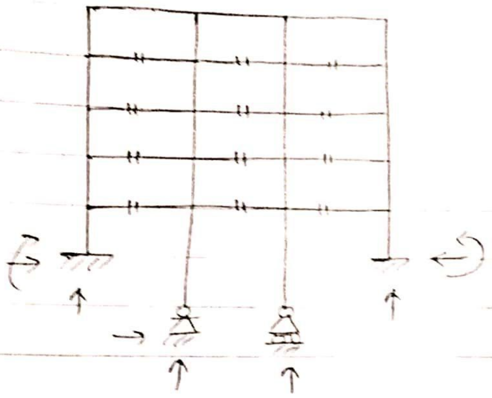
Q2) Determine the degree of static indeterminacy of the rigid jointed plane frame shown in the fig.

Sol:-

$r = 3+2+1+3$

$\Rightarrow 9$

$D_{se} = 9-3 = 6$



No. of cuts required

$c = 12$

$D_{si} = 3\times12$

$= 36$

$D_s = D_{se} + D_{si}$

$= 6+36 = \underline{\underline{42}}$

(or) $m = 55$, $r = 9$, $j = 24$

$D_s = (3m+r) - 3j$

$= ((3\times55)+9) - 3\times24$

$= \underline{\underline{42}}$

* Degree of Kinematic indeterminacy :-

A structure is said to be kinematically indeterminate if the displacement components of its joints cannot be determined by compatibility equations alone.

→ To evaluate the displacements components at the joints of these structure, it is necessary to consider the equations of static equilibrium.

Additional equations based on equilibrium must b written in order to obtain sufficient no. of eqn. to determine displacement components.

The number of these additional eqns. required for determination of independent displacement components is known as degree of kinematic indeterminacy or Degree of freedom of the structure.

Pin jointed frames:- It has 2 independent displacement components as it can move in any 2 orthogonal directions in a plane

$\boxed{D_k = 2j - e}$ → plane frame

where, j → joints

e = no. of compatibility eqns

$\boxed{D_K = 3j - e}$ → space frame

e → no. of constraints imposed by the support cond.

$\Rightarrow \boxed{D_K = 2j - r}$

Rigid jointed frames:- It has two linear movements and one rotation ∴ 3 displacement comp.

$\Rightarrow \boxed{D_K = 3j - e}$ → plane.

$\Rightarrow \boxed{D_K = 6j - e}$ → space

But the formula cannot be used directly coz,
the

① axial deformation of column
② Compatibility of the structure

} are neglected in the eqn.

↳ This is called inextensibility - column & beam.

$$D_K = 3j - (r+m) \rightarrow \text{Plane}$$

$$D_K = 6j - (r+m) \rightarrow \text{space}$$

FLEXIBILITY & STIFFNESS MATRICES

→ Flexibility and stiffness (converse of flexibility) are important properties which characterize the response of a structure by means of force - displacement relationships.

→ Flexibility of a structure is defined as the displacement caused by a unit force.

→ Stiffness of a structure is defined as the force required for unit displacement.

$$\text{Flexibility} = \frac{\Delta_1}{P_1} = \delta_{11}$$

$$\text{Stiffness} = \frac{P_1}{\Delta_1} = k_{11}$$

P → force

Δ ⇒ displacement

FUNDAMENTAL ASSUMPTIONS:-

The fundamental assumptions in the analysis of the structures is the linearity of the structural response. It follows that the internal stresses and the resulting displacements increase in proportion to the external forces. The response of a structure can be linear, only if the principle of superposition is valid.

According to this principle, the total response of a

structure on account of the combined action of any two systems of external forces (say P_1 & P_2) is equal to the sum of the response due to the two systems of force acting separately.
Say Δ be the displacement,

$$\boxed{\Delta_{1+2} = \Delta_{2+1} = \Delta_1 + \Delta_2}$$

where,

Δ_{1+2} → Total displacement due to the combined action of P_1 & P_2 applied in sequence of P_1 & P_2

Δ_{2+1} → Total displacement due to the combined action of P_1 & P_2 applied in sequence of P_2 & P_1

Δ_1 → displacement due to the action P_1 alone

Δ_2 → displacement due to the action P_2 alone.

→ The structural response is linear and the principle of superposition holds if the following assumptions are satisfeed:-

1. The structure is in a condition of static equilibrium.
2. The material ~~follows~~ is homogenous, isotropic and elastic. & also follows Hooke's law.
3. The supports are unyielding. If supports are yielding, ~~kepp~~ the structural response is not linear. & the principle of superposition is not valid.
4. The displacements are small.
5. There is no self-straining of the structure. i.e the internal force in every member of the structure is zero when no external load acts on the structure.

FLEXIBILITY MATRIX

Consider a structure which satisfies the basic fundamental assumptions. Let the system of forces $P_1, P_2 \ldots P_n$ act on the structure. The word "forces" has been generalized which include couples and reaction components. The system of forces $P_1, P_2 \ldots P_n$ may include all or some of the forces acting on the structure. Let the system of forces, produces displacements $\Delta_1, \Delta_2 \ldots \Delta_n$ at co-ordinates $1, 2 \ldots n$.

∴ Using principle of superposition,

$$\Delta_1 = \delta_{11}P_1 + \delta_{12}P_2 + \ldots + \delta_{1j}P_j + \ldots + \delta_{1n}P_n$$
$$\Delta_2 = \delta_{21}P_1 + \delta_{22}P_2 + \ldots + \delta_{2j}P_j + \ldots + \delta_{2n}P_n$$
$$\vdots$$
$$\Delta_i = \delta_{i1}P_1 + \delta_{i2}P_2 + \ldots + \delta_{ij}P_j + \ldots + \delta_{in}P_n$$
$$\vdots$$
$$\Delta_n = \delta_{n1}P_1 + \delta_{n2}P_2 + \ldots + \delta_{nj}P_j + \ldots + \delta_{nn}P_n$$

where,

δ_{ij} → displacement at co-ordinate 'i' due to the unit force at co-ordinate 'j'.

∴ $\delta_{i1}P_1$ → displacement at co-ordinate 'i' due to P_1.

This force-displacement relationship is expressed in the matrix form,

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_i \\ \vdots \\ \Delta_n \end{bmatrix} = \begin{bmatrix} \delta_{11} & \delta_{12} & \ldots & \delta_{1j} & \ldots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \ldots & \delta_{2j} & \ldots & \delta_{2n} \\ \vdots \\ \Delta_{i1} & \delta_{i2} & \ldots & \delta_{ij} & \ldots & \delta_{in} \\ \vdots \\ \delta_{n1} & \delta_{n2} & \ldots & \delta_{nj} & \ldots & \delta_{nn} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_j \\ \vdots \\ P_n \end{bmatrix} \quad — ①$$

In the compact form.

$$\boxed{[\Delta] = [\delta][P]} \quad \text{—} \, ②$$

where

$[\Delta] \rightarrow$ a column matrix of order $n \times 1$, known as displacement matrix.

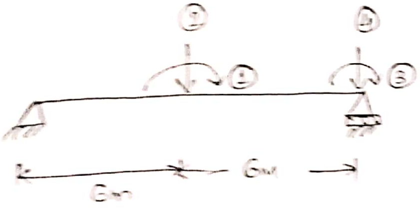
$[P] \rightarrow$ a column matrix of order $n \times 1$, known as force matrix

$[\delta] \rightarrow$ a square matrix of order n, known as flexibility matrix

Note: To generate the j^{th} column of the flexibility matrix, a unit force should be applied at co-ordinate j and the displacements at all the co-ordinates are determined.

∴ In order to [illegible] the flexibility matrix, a unit force should be applied successively at coordinates 1, 2, 3 .... n and the displacements at all coordinates are computed.

Eg:-



Sol: Applying unit force @ co-ordinate ①

$$f_{11} = \frac{\overset{P}{M}L}{12EI} \Rightarrow \frac{12 \times 1}{12EI} = \frac{1}{EI}$$

$$f_{21} = 0$$

$\delta_{31} \Rightarrow \frac{-PL}{24EI} \Rightarrow \frac{-12}{24EI} \Rightarrow \frac{-0.5}{EI}$

$\delta_{41} = 0.$

Now unit force @ coordinate ②

$\delta_{12} = 0.$

$\delta_{22} = \frac{PL^3}{48EI} \Rightarrow \frac{36}{EI}$

$\delta_{32} = \frac{-PL^2}{16EI} \Rightarrow \frac{-9}{EI}$

$\delta_{42} = 0.$

@ coordinate ③

$\delta_{13} = \frac{-PL}{24EI} = \frac{-12}{24EI} = \frac{-0.5}{EI}$

$\delta_{23} = \frac{-PL^2}{16EI} = \frac{-9}{EI}$

$\delta_{33} = \frac{PL}{3EI} = \frac{4}{EI}$

$\delta_{43} = 0.$

@ coordinate ④.

$\delta_{14} = \delta_{24} = \delta_{34} = \delta_{44} = 0.$

$$\therefore [\delta] = \frac{1}{EI}\begin{bmatrix} 1 & 0 & -0.5 & 0 \\ 0 & 36 & -9 & 0 \\ -0.5 & -9 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} //.$$

* STIFFNESS MATRIX

$P_1, P_2, P_3, \ldots P_n$ → system of forces

$\Delta_1, \Delta_2, \Delta_3 \ldots \Delta_n$ → displacements produced due to applied system of forces.

$1, 2, 3, \ldots n$ → respective co-ordinates

If a unit displacement is given at coordinate 'j' without any displacement at other coordinates, the forces req. at co-ordinates $1, 2, 3 \ldots n$ may be represented by – $K_{1j}, K_{2j} \ldots K_{nj}$ resp. $[\Delta_j = 1$ & $\Delta_i \, (j \neq i) = 0]$

In other words $k_{1j}, k_{2j} \ldots k_{nj}$ are the forces required at co-ordinate $1, 2, \ldots n$ resp in order to to produce a unit displacement at co-ordinate j & zero displacement at all other co-ordinates.

∴ k_{ij} → force at co-ordinate 'i' due to a unit disp. at co-ordinate 'j' only.

$$\therefore \; P_i = K_{i1}\Delta_1 + K_{i2}\Delta_2 + \ldots + K_{in}\Delta_n$$

(or)

$$\left.\begin{array}{l} P_1 = K_{11}\Delta_1 + K_{12}\Delta_2 + \ldots + K_{1j}\Delta_j + \ldots + K_{1n}\Delta_n \\ P_2 = K_{21}\Delta_1 + K_{22}\Delta_2 + \ldots + K_{2j}\Delta_j + \ldots + K_{2n}\Delta_n \\ \vdots \\ P_i = K_{i1}\Delta_1 + K_{i2}\Delta_2 + \ldots + K_{ij}\Delta_j + \ldots + K_{in}\Delta_n \\ \vdots \\ P_n = K_{n1}\Delta_1 + K_{n2}\Delta_2 + \ldots + K_{nj}\Delta_j + \ldots + K_{nn}\Delta_n. \end{array}\right\} \text{— Eq ③}$$

as a matrix,

$$\begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_i \\ \\ P_n \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \cdots & K_{1j} & K_{1n} \\ K_{21} & K_{22} \cdots & K_{2j} & K_{2n} \\ \vdots & & & \\ K_{i1} & K_{i2} \cdots & K_{ij} \cdots & K_{in} \\ \vdots & & & \\ K_{n1} & K_{n2} \cdots & K_{nj} & K_{nn} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_j \\ \vdots \\ \Delta_n \end{bmatrix}$$

in a compacted form,

$$\boxed{[P] = [K][\Delta]} \quad — ④$$

where,

$[K] \Rightarrow$ Stiffness matrix of order 'n' [Square matrix].

In order to generate the j^{th} column of the stiffness matrix, a unit displacement must be given at co-ordinate 'j' without any displacement at other coordinates and the forces required at all the co-ordinates are determined.

∴ In order to develop stiffness matrix, unit displacement should be given successively at co-ordinates 1, 2 .... n and forces at all the coordinates are calculated.

FLEXIBILITY MATRIX METHOD/ FORCE /COMPATIBILITY METHOD :-

Procedure :

→ The degree of static indeterminacy is determined & the redundants are identified.

→ The redundants are assigned with coordinates. (Say $P_1, P_2, P_3 \ldots P_n$ are the redundants at coordinates, $1, 2, 3 \ldots n$ resp.).

→ Remove all the redundants to obtain released - structure that is stable. This structure is called 'Released structure' (or) 'basic determinate structure'.

→ Determine flexibility matrix w.r.t chosen coordinates

→ Find $[\Delta_L]$ displacements at the coordinates due to applied loads acting on released structure.

→ Using compatibility eqns find the unknown redundant forces.

Compatibility condition :- It is based on principle of super-position ⇒ "The net displacement at any point in a statically determinate structure is the sum of the displacements in the released structure due to applied loads & the redundants.

$$\text{In-det} \leftarrow [\Delta] = [\Delta_L] + [\Delta_R] \rightarrow \text{det} \longrightarrow ⑥.$$

$$\left.\begin{aligned} \Delta_1 &= \Delta_{1L} + \Delta_{1R} \\ \Delta_2 &= \Delta_{2L} + \Delta_{2R} \\ &\vdots \\ \Delta_j &= \Delta_{jL} + \Delta_{jR} \\ &\vdots \\ \Delta_n &= \Delta_{nL} + \Delta_{nR} \end{aligned}\right\} \longrightarrow ⑤.$$

where,

Δ_j → displacement at coordinate 'j' in the statically indeterminate structure

Δ_{jL} → displacement at coordinate 'j' in the released structure due to applied loads

Δ_{jR} → displacement at coordinate 'j' in the released structure due to the redundants.

Now,

Sub. eqn. ① in eqn. ⑤.

$$\Delta_1 = \Delta_{1L} + \delta_{11}P_1 + \delta_{12}P_2 + \dots + \delta_{1n}P_n$$

$$\vdots$$

$$\Delta_n = \Delta_{nL} + \delta_{n1}P_1 + \delta_{n2}P_2 + \dots + \delta_{nn}P_n$$

$$\Rightarrow \quad [\Delta] = [\Delta_L] + [\delta][P]$$

$$\boxed{[P] = [\delta]^{-1}\{[\Delta] - [\Delta_L]\}} \text{ —— (I).}$$

If net displacement, $[\Delta] = 0$ then,

$$\boxed{[P] = -[\delta]^{-1}[\Delta_L]}$$

[illegible] METHOD / DISPLACEMENT METHOD / EQUILIBRIUM METHOD

[illegible]

- The degree of kinematic indeterminacy (degree of freedom) of the structure is determined and coordinates of each are assigned to independent displacement.
- In general, the displacement components at supports and joints are treated as independent displacement comp.
- 1, 2, ... n are the coordinates assigned to the independent displacement components $\Delta_1, \Delta_2 \ldots \Delta_n$ resp.
- Lock all the supports & joints to obtain a restrained structure ⇒ no displacement possible at coordinates.
- Let,

$P_1', P_2' \ldots P_n'$ = forces required at coordinates 1, 2, ... n in the restrained structure [illegible] $\Delta_1, \Delta_2 \ldots \Delta_n$ [illegible]

- [illegible] the supports & joints be released permitting displacements $\Delta_1, \Delta_2 \ldots \Delta_n$ at the coordinates. These displacements require forces $P_{1\Delta}, P_{2\Delta} \ldots P_{n\Delta}$ at resp. coordinates.
- If $P_1, P_2 \ldots P_n$ are the external forces at coordinates 1, 2, ... n then the conditions of equilibrium of the structure may be expressed by the eqn.

$$\left.\begin{aligned} P_1 &= P_1' + P_{1\Delta} \\ P_2 &= P_2' + P_{2\Delta} \\ &\vdots \\ P_n &= P_n' + P_{n\Delta} \end{aligned}\right\} \quad \text{—} ①$$

$$[P] = [P'] + [P_\Delta] \quad \text{—} ②$$

Sub. eqn. ⑤ in eqn. ⑦

$$P_1 = P_1' + K_{11}\Delta_1 + K_{12}\Delta_2 + \quad + K_{1n}\Delta_n$$

$$\vdots$$

$$P_n = P_n' + K_{n1}\Delta_1 + K_{n2}\Delta_2 + \dots \quad + K_{nn}\Delta_n$$

$$\Rightarrow \quad [P] = [P'] + [K][\Delta]$$

$$\Rightarrow \quad [\Delta] = [K]^{-1}\{[P] - [P']\} \quad \text{—} \quad (II)$$

If the external forces act only @ coordinates,

$$P_1' = P_2' = \dots = P_n' = 0$$

$$\boxed{[\Delta] = [K]^{-1}[P]}$$

If there are no external forces @ coordinates,

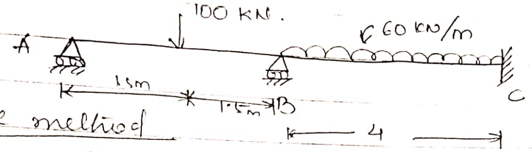
$$P_1 = P_2 = \quad = P_n = 0$$

$$\Rightarrow \quad \boxed{[\Delta] = -[K]^{-1}[P']}$$

Problem:

1) Analyse the continuous beam shown in fig. using - Force method & displacement method.

Sol.



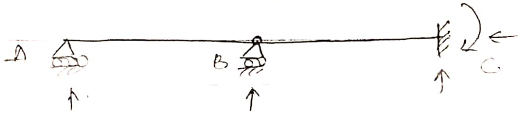
ⓐ Force method

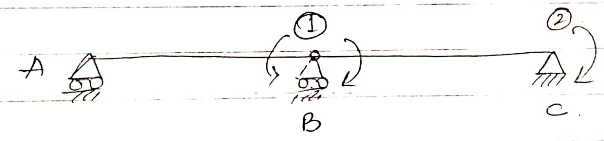
→ $D_S = r - 3$

$= 5 - 3$

$= 02$

→ Let basic determinate structure be obtained by releasing the BM at B and C





Flexibility matrix,

$\delta_{11} \Rightarrow \left(\frac{ML}{3EI}\right)_{BA+BC}$

⇒ Here M = 1

$\Rightarrow \frac{4}{3EI} + \frac{3}{3EI} \Rightarrow \frac{7}{3EI}$

$\delta_{21} = -\frac{ML}{6EI} \Rightarrow -\frac{4}{6EI} = \frac{-2}{3EI}$

$$\delta_{12} = -\frac{2}{3EI}$$

$$\delta_{22} = \frac{4}{3EI}$$

∴

$$[\delta] = \frac{1}{3EI}\begin{bmatrix} 7 & -2 \\ -2 & 4 \end{bmatrix} \longrightarrow ⓐ$$

For spans BA & BC

$$\Delta_{11} = \frac{Pl^2}{16EI} + \frac{Pl^3}{24EI}$$

$$\Rightarrow \frac{100\times3^2}{16EI} + \frac{60\times4^3}{24EI}$$

$$\Rightarrow \frac{216.25}{EI}$$

$$\left.\Delta_{21} \Rightarrow -\frac{60\times4^3}{24EI} \Rightarrow \frac{-160}{EI}\right\} \longrightarrow ⓑ$$

For continuity ⇒ $\Delta_1 - \Delta_2 = 0$.

Sub.

$$[P] = -[\delta]^{-1}[\Delta_1]$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = -3EI\begin{bmatrix} 7 & -2 \\ -2 & 4 \end{bmatrix}^{-1}\begin{bmatrix} \frac{216.25}{EI} \\ \frac{-160}{EI} \end{bmatrix}$$

$\Rightarrow$ $P_1 = -68$ kN-m

$P_2 = 86$ kN.m.

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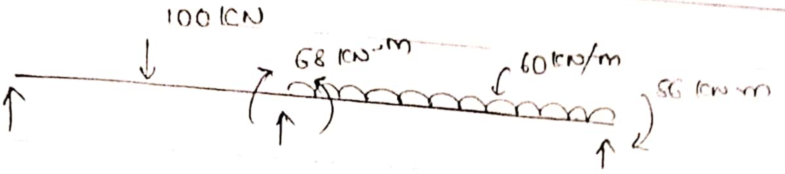
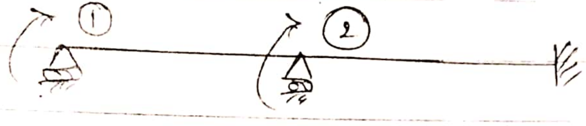


(ii) Displacement method:-

Dof ⇒ 2.



$$K_{11} \Rightarrow \frac{4EI}{L} \Rightarrow \frac{4EI}{3}$$

$$K_{21} = \frac{2EI}{L} \Rightarrow \frac{2EI}{3}$$

$$K_{12} \Rightarrow \frac{2EI}{L} \Rightarrow \frac{2EI}{3}.$$

$$K_{22} \Rightarrow \frac{4EI}{3} + \frac{4EI}{4} \Rightarrow \frac{7EI}{3}.$$

$$[K] = \frac{EI}{3}\begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix}$$

No external loads @ Co-ordinate.

$$P_1 = P_2 = 0.$$

$$P_1' = -\frac{100\times 3}{8} \Rightarrow -37.5 \text{ kN-m} \longrightarrow M_{FAB}$$

$$P_2' \Rightarrow \frac{100\times 3}{8} - \frac{60\times 4^2}{12} \Rightarrow -42.5 \text{ kNm.} \longrightarrow M_{FBC}$$

$\begin{bmatrix}\theta_B\\ \theta_C\end{bmatrix} = \frac{1}{EI}\begin{bmatrix}[illegible]\end{bmatrix}^{-1}\begin{bmatrix}[illegible]\end{bmatrix}$

$\theta_B = \frac{[illegible]}{EI}$

$\theta_C = \frac{11.9}{EI}$

$M_{AB} = M_{FAB} + \frac{2EI}{L}(2\times\theta_A + \theta_B)$

$\Rightarrow [illegible] + \frac{2EI}{8}\left(2\times\frac{[illegible]}{EI} + \frac{[illegible]}{EI}\right)$

$\Rightarrow 0$

$M_{BA} = [illegible] + \frac{2EI}{8}\left(2\times\frac{11.9}{EI} + \frac{[illegible]}{EI}\right)$

$\Rightarrow 68$ kN m

$M_{BC} = -80 + \frac{2EI}{4}\left(2\times\frac{11.9}{EI}\right) \Rightarrow [illegible]$ kN m

$M_{CB} = 80 + \frac{2EI}{4}\left(\frac{11.9}{EI}\right) \Rightarrow 86$ kN m

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[illegible]

Support reactions:

$R_A + R_{B_1} = 100$

$R_A \times 8 - 100\times1.6 + 68 = 0$

$\Rightarrow R_A = 27.33$ kN

$R_{B_1} = [illegible]$

$R_{B_2} + R_C = 60\times4 = 240$ kN

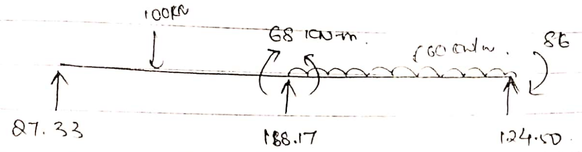
$R_{B_2}\times4 - \frac{60\times4\times4}{2} - 68 + 86 = 0$

$R_{B_2} \Rightarrow [illegible]$ 115.5 kN

$R_A = 27.33$ kN.

$R_B = 72.67 + 115.5 = 118.17$ kN

$R_C = 124.50$ kN



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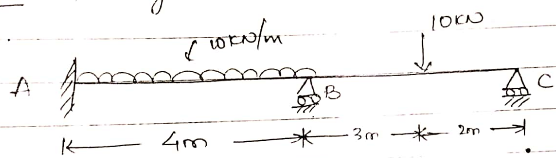
22. Analyse the given beam:

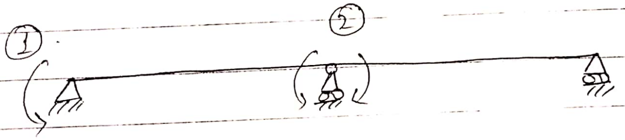


Sol: Force method

$D_S = r - 3$

$\Rightarrow 5 - 3 = 2$ (∴ Release two redundants).

Let the moments @ A & B be released.



Flexibility matrix,

unit force @ ① $\Rightarrow \delta_{11} = \frac{ML}{3EI} \Rightarrow \frac{4}{3EI}$

$\delta_{21} = \frac{ML}{6EI} = \frac{4}{6EI} = \frac{2}{3EI}$

unit force @ ②

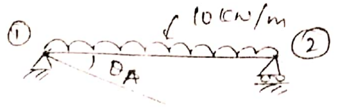
$$\delta_{12} = \frac{ML}{6EI} = \frac{4}{6EI} = \frac{2}{3EI}$$

$$\delta_{22} = \left(\frac{ML}{3EI}\right)_{BA} + \left(\frac{ML}{3EI}\right)_{BC} \Rightarrow \frac{4}{3EI} + \frac{5}{3EI} \Rightarrow \frac{9}{3EI}$$

$$[\delta] = \frac{1}{3EI}\begin{bmatrix} 4 & 2 \\ 2 & 9 \end{bmatrix}$$

$$[\delta]^{-1} = \frac{3EI}{32}\begin{bmatrix} 9 & -2 \\ -2 & 4 \end{bmatrix}$$

Now, $[\Delta_1]$



$$\Delta_{11} = -\frac{PL^3}{24EI}$$

$$\Rightarrow -\frac{10 \times 4^3}{24EI} = -\frac{26.66}{EI}$$

$$\Delta_{21} = -\frac{PL^3}{24EI} - ~~100~~ \frac{Pab(2L-a)}{6LEI}$$

$$\Rightarrow -\frac{40.67}{EI}$$

$$[P] = -[\delta]^{-1}[\Delta_L]$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = -\frac{3EI}{32}\begin{bmatrix} 9 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -26.66/EI \\ -40.67/EI \end{bmatrix}$$

$P_1 \Rightarrow 14.87$ kNm

$P_2 = 10.52$ kN-m.

placement method

$K_{11} \Rightarrow \frac{4EI}{4} + \frac{4EI}{5} \Rightarrow \frac{9EI}{5}$

$K_{21} \Rightarrow \frac{2EI}{5}$

$K_{12} \Rightarrow \frac{2EI}{5}$

$K_{22} = \frac{4EI}{5}$

$[K] = \frac{EI}{5}\begin{bmatrix} 9 & 2 \\ 2 & 4 \end{bmatrix} \Rightarrow K^{-1} = \frac{5}{32EI}\begin{bmatrix} 4 & -2 \\ -2 & 9 \end{bmatrix}$

No external loads @ coordinates,

$P_1, P_2 = 0$

$P_2' = -\frac{10 \times 4}{8} \Rightarrow -5 \text{ kNm}$

$P_1' = -\frac{10 \times 5^2}{12} + \frac{10 \times 4}{8} \Rightarrow -15.83 \text{ kNm}$

Now,

$[\Delta] = -[K]^{-1}[P']$

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \frac{-5}{22EI} \cdot \begin{bmatrix} 4 & -2 \\ -2 & 9 \end{bmatrix} \begin{bmatrix} -15.83 \\ -5 \end{bmatrix}$$

$$= \frac{-05}{32EI} \begin{bmatrix} -53.32 \\ -13.34 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{8.33}{EI} \\ 2.06/EI \end{bmatrix}$$

$$M_{AB} \Rightarrow MF_{BA} + \frac{2EI}{L} \left(2 \times \Delta_2 + \Delta_1 \right)$$

$$\Rightarrow 15.83 + \frac{\cancel{2}EI}{\cancel{4}_2} \left(2 \times \frac{2.08}{EI} + \right)$$

$$\Rightarrow 15.83 + \frac{\cancel{EI}}{2} \left(\frac{12.49}{\cancel{EI}} \right)$$

$$\Rightarrow 22.07$$

Page No. [illegible]

Analyse the continuous beam ABC, fixed at end A, supported on rollers at B & C, as shown in figure, if the beam undergoes settlement of supports B & C by $\frac{500}{EI}$ & $\frac{300}{EI}$ resp. EI is constant throughout. Determine reaction at supports and moments at supports.

Redundancy = 2

$\delta_{11} \Rightarrow \frac{8}{3EI}$

$\delta_{21} = \delta_{12} = \frac{4}{3EI}$

$\delta_{22} = \frac{8}{3EI} + \frac{8}{3EI} \Rightarrow \frac{16}{3EI}$

$[\delta] = \frac{1}{3EI}\begin{bmatrix} 8 & 4 \\ 4 & 16 \end{bmatrix}$

$[\Delta_L] \Rightarrow \begin{bmatrix} \Delta_{11} \\ \Delta_{21} \end{bmatrix}$

$\Delta_{11} = \frac{Wl^2}{16EI} \Rightarrow \frac{100 \times 8^2}{16EI} \Rightarrow -\frac{400}{EI}$

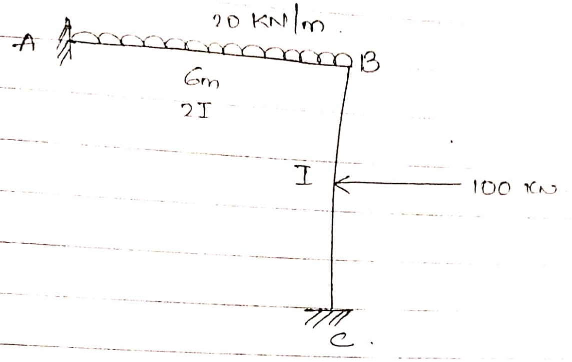
$\Delta_{21} = \frac{60 \times 8^2}{16EI} + \frac{100 \times 8^2}{16EI} \Rightarrow -\frac{640}{EI}$

Additional disp. due to settlement of supports,

$$\Delta_{1s} = \frac{-300}{8EI}$$

$$\Delta_{2s} = \frac{300}{8EI} + \frac{(300-200)}{8EI} \Rightarrow + \frac{50}{EI}$$

Analyse the given rigid jointed frame (Force method).

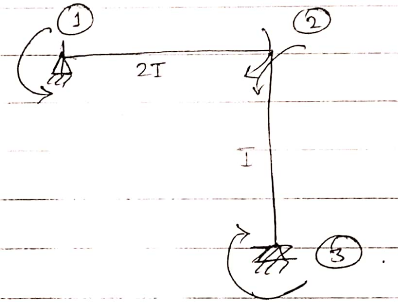


Sol

$D_s = r - 3 \Rightarrow 6 - 3 = 3$ (or) $D_s = (3m + r) - 3j \Rightarrow 3.$

∴ Releasing three redundants say M_A, M_B & M_C.

Flexibility matrix,



$M = 1$ @ ∈ ①

$$\delta_{11} \Rightarrow \left(\frac{ML}{3EI}\right)_{AB}$$

$$\Rightarrow \frac{6}{3 \times 2EI} \Rightarrow \frac{1}{EI}$$

$$\delta_{21} = \left(\frac{ML}{6EI}\right)_{AB} = \frac{6}{6 \times 2EI} \Rightarrow \frac{1}{2EI} \Rightarrow \frac{0.5}{EI}$$

$$\delta_{31} = 0.$$

$M = 1$ @ ②

$$\delta_{12} = \left(\frac{ML}{6EI}\right)_{BA} = \frac{6}{6 \times 2EI} \Rightarrow \frac{1}{2EI} \Rightarrow \frac{0.5}{EI}$$

$$\delta_{22} = \left(\frac{ML}{3EI}\right)_{BA} + \left(\frac{ML}{3EI}\right)_{BC} \Rightarrow \frac{6}{6EI} + \frac{4}{3EI} \Rightarrow \frac{2.33}{EI}$$

$$\delta_{32} = \frac{ML}{6EI} \Rightarrow \frac{4}{6EI} \Rightarrow \frac{0.67}{EI}$$

$M = 1$ @ ③

$\delta_{13} = 0$

$\delta_{23} = \left(\frac{mL}{6EI}\right)_{CB} \Rightarrow \frac{4}{6EI} \Rightarrow \frac{0.67}{EI}$

$\delta_{33} = \frac{4}{3EI} \Rightarrow \frac{1.33}{EI}$

$$[\delta] = \frac{1}{EI}\begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 2.33 & 0.67 \\ 0 & 0.67 & 1.33 \end{bmatrix}$$

$$[\delta]^{-1} = \frac{EI}{2.33}\begin{bmatrix} 2.67 & -0.67 & 0.33 \\ -0.67 & 1.33 & -0.67 \\ 0.33 & -0.67 & 2.08 \end{bmatrix}$$

$$\Rightarrow \cancel{EI}\begin{bmatrix} \cancel{1.434} & \cancel{-0.286} & \cancel{0.1445} \\ \cancel{-0.286} & \cancel{0.5788} & \end{bmatrix}$$

Displacement matrix ($[\Delta_L]$) due to applied loads:

$\Delta_{1L} = -\frac{wL^3}{24EI} \Rightarrow -\frac{20 \times 4^3}{24E \times 2I} \Rightarrow -\frac{90}{EI}$

$\Delta_{2L} = -\frac{wL^3}{24EI} - \frac{WL^2}{16EI} = -\frac{190}{EI}$

$\Delta_{3L} = -\frac{100 \times 4^2}{16EI} \Rightarrow -\frac{100}{EI}$

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \cancel{-\frac{EI}{2.33}}\begin{bmatrix} 62.8 \\ 54.3 \\ 47.8 \end{bmatrix} \begin{matrix} \text{kN-m} \\ \text{kN-m} \\ \text{kN-m.} \end{matrix}$$

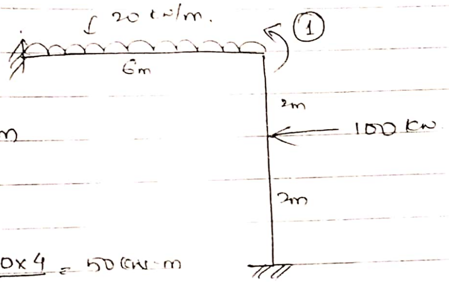
Displacement method:

$D_k = 3j - (m + r)$

$= 9 - (2 + 6)$

$= 9 - 8 = 1$

Let the frame undergo independent rotation at B.



B $M_{FAB} = \frac{wL^2}{12} \Rightarrow 60\ KN\text{-}m$

$M_{FBA} = -60\ kN\text{-}m.$

$M_{FBC} = \frac{wL}{8} \Rightarrow \frac{100 \times 4}{8} = 50\ kN\text{-}m$

$M_{FCB} = -50\ kN\ m.$

∴ Rotation @ B ⇒ $-60 + 50 = -10\ kN\text{-}m.$

$P_1' = -10\ kN\text{-}m.$

$[K] \Rightarrow \left(\frac{4EI}{L}\right)_{BA} + \left(\frac{4EI}{L}\right)_{BC}$

$\Rightarrow \frac{4 \times \cancel{2}EI}{\cancel{6}\,3} + \frac{\cancel{4}EI}{\cancel{4}}$

$\Rightarrow \frac{4EI}{3} + EI \Rightarrow \frac{7EI}{3}$

$K^{\dagger} \Rightarrow \frac{3\cancel{6}}{7EI}$

$[\Delta] = -k^{+}[P']$

$\Rightarrow \quad -\left(\frac{3}{7EI} \times -10\right)$

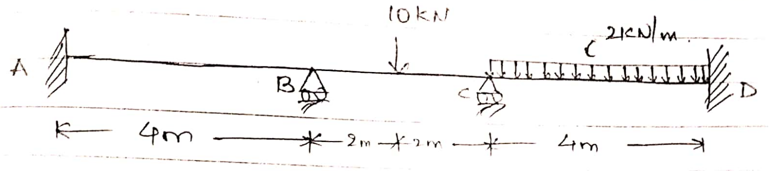
$[\Delta] \Rightarrow \quad \frac{30}{7EI}$ ↻

$\theta_B = \frac{4.25}{EI}$

$M_{AB} = 60 + \frac{2\cancel{EI}\times 2\cancel{I}}{6}\left[0 + \frac{30}{7\cancel{EI}}\right]$

$\Rightarrow$ 62.8 KN.m.

Direct Element Approach

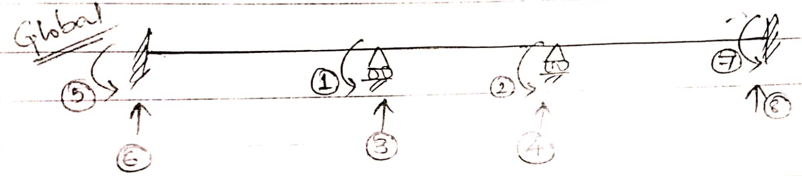


EI → constant.

Step 1

→ Numbering the joints showing displacements i.e possible displacements (degrees of freedom) first and the member the constrained displacements.

Global

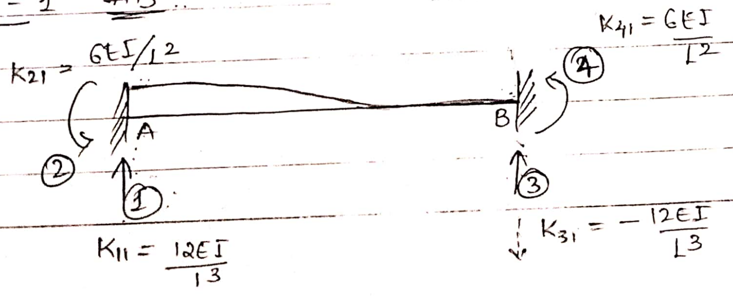


In the given co-ordinates, the continuous beam is divided into three beam elements with two 'dof' (one rotational and one translation) for each end of the members. For the given beam 'known' displacements u_3, u_4, u_5, u_6, u_7 & $u_8 = 0$. (for support conditions)

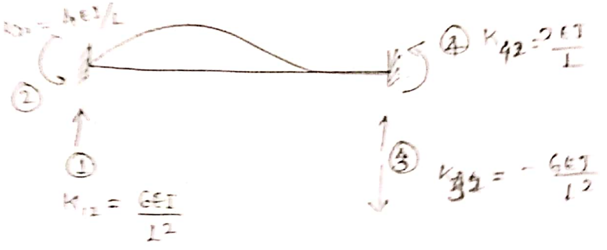
Step: 2 Local co-ordinates and stiffness matrix.

member - 1 AB.

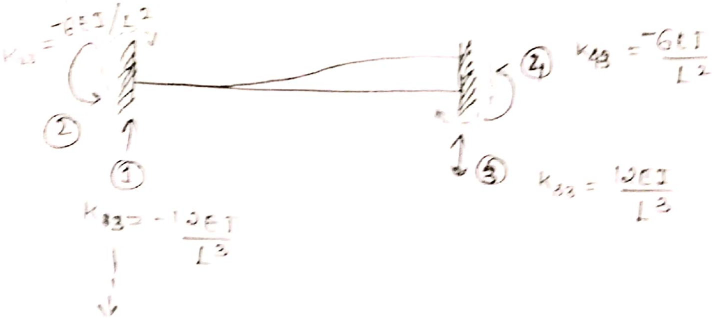
$\Delta_A = 1$



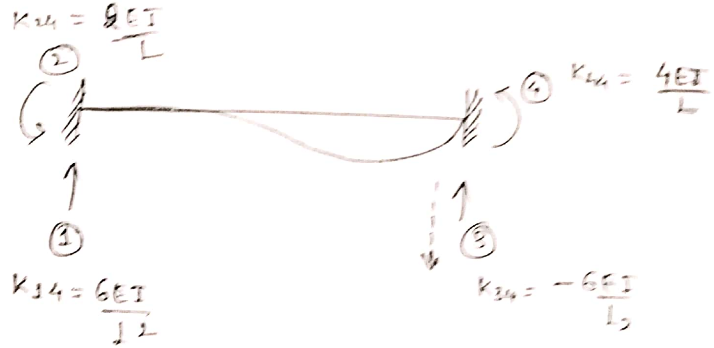
$\theta_A = 1$



$\Delta_B = 1$



$\theta_B = 1$



Note:

↑ , ↶ ⇒ Considered co-ordinates

⇣ , ⇢ → Behaviour of the beam due to applied coordinates direction

$$[K] = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & \frac{-6EI}{L^2} & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L^2} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

The same matrix can be considered for all elet member as they all are same in considered coordinates.

$\therefore$ For member A-B $\Rightarrow$ $L = 4$

$$[K_{AB}] = EI \begin{array}{c} \begin{array}{cccc} 6 & 5 & 3 & 1 \end{array} \\ \left[\begin{array}{cccc} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.0 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.0 \end{array}\right] \end{array} \begin{array}{c} \\ 6 \\ 5 \\ 3 \\ 1 \end{array}$$

$$K_{BC} = EI \begin{array}{c} \begin{array}{cccc} 3 & 1 & 4 & 2 \end{array} \\ \left[\begin{array}{cccc} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.0 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.0 \end{array}\right] \end{array} \begin{array}{c} \\ 3 \\ 1 \\ 4 \\ 2 \end{array}$$

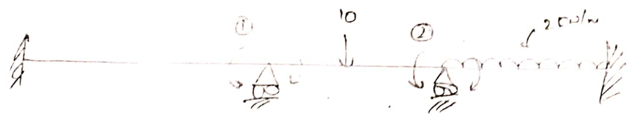
$$K_{CD} = EI \begin{array}{c} \begin{array}{cccc} 4 & 2 & 8 & 7 \end{array} \\ \left[\begin{array}{cccc} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.0 \end{array}\right] \end{array} \begin{array}{c} \\ 4 \\ 2 \\ 8 \\ 7 \end{array}$$

Step 3

Assemble the global stiffness matrix :-

$$K = EI \begin{array}{cccccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \\ 2 & 0.5 & 0 & -0.375 & 0.5 & 0.375 & 0 & 0 & 1 \\ 0.5 & 2.0 & 0.375 & 0 & 0 & 0 & 0.5 & -0.375 & 2 \\ 0 & 0.375 & 0.375 & -0.1875 & -0.375 & -0.1875 & 0 & 0 & 3 \\ -0.375 & 0 & -0.1875 & 0.375 & 0 & 0 & 0.375 & -0.1875 & 4 \\ 0.5 & 0 & -0.375 & 0 & 1 & 0.375 & 0 & 0 & 5 \\ 0.375 & 0 & -0.1875 & 0 & 0.375 & 0.1875 & 0 & 0 & 6 \\ 0 & 0.5 & 0 & 0.375 & 0 & 0 & 1 & -0.375 & 7 \\ 0 & -0.375 & 0 & -0.1875 & 0 & 0 & -0.375 & 0.1875 & 8 \end{array}$$

Step-4 → Equivalent load vector



$$@\ B = \frac{WL}{8} \Rightarrow \frac{10 \times 4}{8} \Rightarrow +5 \text{ kN-m}$$

$$@\ C \Rightarrow -\frac{WL}{8} + \frac{WL^2}{12} \Rightarrow -\frac{\overset{5}{\cancel{10}} \times 4}{\cancel{8}_2} + \frac{\cancel{2} \times 4^2}{\cancel{12}\,6}$$

$$\Rightarrow -5 + 2.67 \Rightarrow -2.33 \text{ kN-m}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -2.33 \end{bmatrix}$$

Step 5 ⇒ Solution of equilibrium equations.

$$[P] = [K][P].$$

$$\begin{bmatrix} +5 \\ -2.33 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = EI \begin{bmatrix} 2 & 0.5 & 0 & -0.375 & 0.5 & 0.375 & 0 & 0 \\ 0.5 & 2.0 & 0.375 & 0 & 0 & 0 & 0.5 & -0.375 \\ 0 & 0.375 & 0.375 & -0.1575 & -0.375 & -0.1875 & 0 & 0 \\ -0.375 & 0 & -0.1875 & 0.375 & 0 & 0 & 0.375 & -0.1875 \\ 0.5 & 0 & -0.375 & 0 & 1 & 0.375 & 0 & 0 \\ 0.375 & 0 & -0.1875 & 0 & 0.375 & 0.1875 & 0 & 0 \\ 0 & 0.5 & 0 & 0.375 & 0 & 0 & 1 & -0.375 \\ 0 & -0.375 & 0 & -0.1875 & 0 & 0 & -0.375 & 0.1875 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{bmatrix}$$

unknowns u_1 & u_2

$$\Rightarrow \begin{bmatrix} 5 \\ -2.33 \end{bmatrix} = EI \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\Rightarrow \boxed{\begin{aligned} u_1 &= 2.997/EI \\ u_2 &= -1.909/EI \end{aligned}}$$

Step-6: Unknown joint loads:

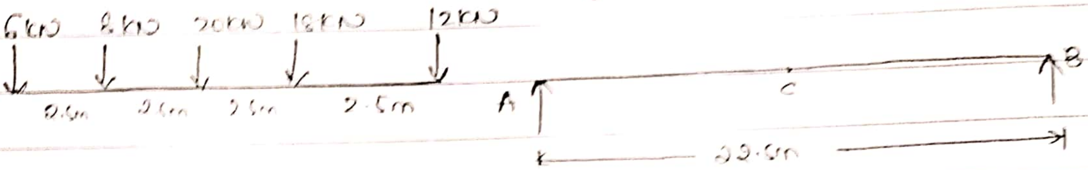
$$\begin{bmatrix} P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = \cancel{EI} \begin{bmatrix} 0 & 0.375 \\ -0.375 & 0 \\ 0.5 & 0 \\ 0.375 & 0 \\ 0 & 0.5 \\ 0 & -0.375 \end{bmatrix}_{6\times 2} \begin{bmatrix} 2.997 \\ -1.909 \end{bmatrix}_{2\times 1} \frac{1}{\cancel{EI}} = \begin{bmatrix} -0.715 \\ -1.123 \\ +1.4985 \\ 1.123 \\ -0.954 \\ 0.715 \end{bmatrix} \begin{matrix} \text{KN} \\ \text{KN} \\ \text{KN-m} \\ \text{KN} \\ \text{KN-m} \\ \text{KN.} \end{matrix}$$

Step: 7 Actual reactions at supports:

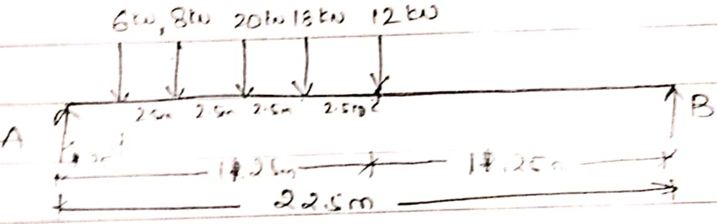
$$\begin{bmatrix} R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \end{bmatrix} = \begin{bmatrix} P_3^F \\ P_4^F \\ P_5^F \\ P_6^F \\ P_7^F \\ P_8^F \end{bmatrix} + \begin{bmatrix} P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \\ 0 \\ 0 \\ +2.67 \\ 4 \end{bmatrix} + \begin{bmatrix} -0.715 \\ -1.123 \\ -1.4985 \\ 1.123 \\ -0.754 \\ 0.715 \end{bmatrix} = \begin{bmatrix} 4.285 \\ 7.877 \\ -1.498 \\ 1.123 \\ 1.916 \\ 4.715 \end{bmatrix}$$

/

(Q) A train of wheel loads as shown in figure cross a simply supported beam of span 22.5m Find the max. (+ve) & (-ve) SF at centre of the span. Also find absolute max. BM anywhere in the span.



Sol:- ⓐ Max. -ve SF : When the leading load is at the (centre) section, max. -ve SF occurs.



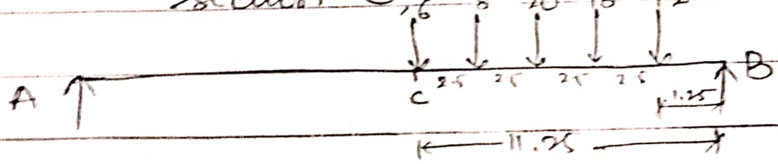
Taking moment about A,

$$R_B \times 22.5 = (12 \times 11.25) + (18 \times 8.75) + (20 \times 6.25) + (8 \times 3.75) + (6 \times 1.25)$$

$$\boxed{R_B = 20.22 \text{ kN}}$$

Max. -ve SF = $\boxed{F_x(-ve) = -20.22 \text{ kN}}$

ⓑ Max. -ve SF : Occurs when the end load is at the section 'C',



Taking moment about B,

$R_A \times$ [illegible] = ([illegible]) + ([illegible]) + [illegible]

[illegible] + [illegible] + ([illegible])

$\boxed{R_A = [illegible]}$

Max BM at C = $\boxed{R_A \times [illegible] = [illegible]}$

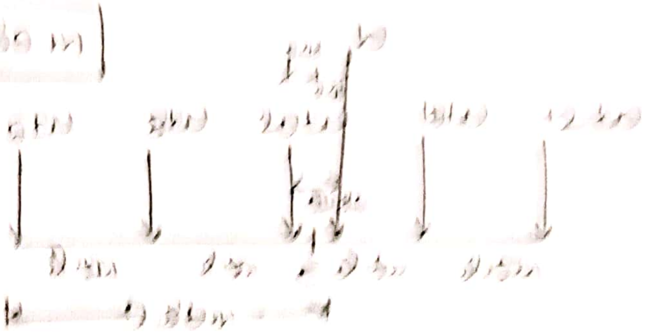
(ii) Absolute max Bending moment.

Centre of gravity of load system, $\bar{x}$

$\bar{x} = \frac{\Sigma wx}{W}$

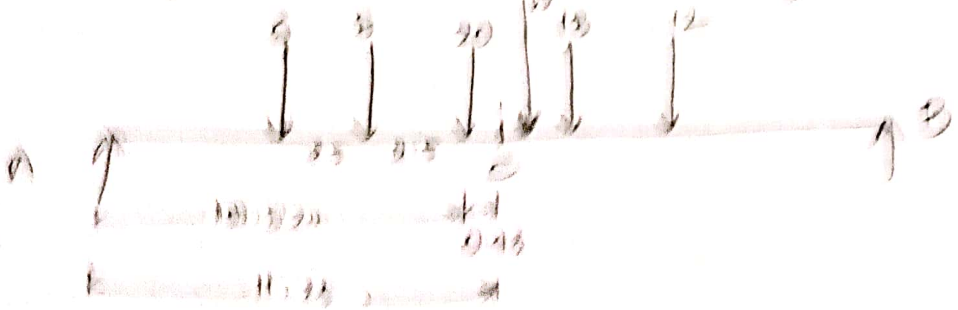
$= \frac{(8\times 0) + (15\times 2.5) + (20\times 5) + (15\times 7.5) + (12\times 10)}{8+15+20+15+12}$

$\boxed{\bar{x} = [illegible]\ m}$



considering 20 kN,

Max BM occurs when 'C' is midway between CG & 20 kN (5 [illegible])



Taking moment about A,

$$R_B \times 22.5 = (6 \times 5.82) + (8 \times 8.32) + (20 \times 10.82) + (18 \times 13.32) + (12 \times 15.82)$$

$$R_B = 33.22 \text{ KN.}$$

$$M_{max} \text{ (at 20 KN)} \Rightarrow R_B \times (11.25 + 0.43) - (12 \times 5) - (18 \times 2.5)$$

$$\boxed{M_{max} \Rightarrow 283 \text{ KN-m}}$$

(or) Taking moment about B,

$$R_A \times 22.5 \Rightarrow (6 \times 16.68) + (8 \times 14.18) + (20 \times 11.68) + (18 \times 9.18) + (12 \times 6.68)$$

$$R_A = 30.78 \text{ KN.}$$

$$M_{max} \text{ (at 20 KN)} \Rightarrow (R_A \times 10.82) - (8 \times 2.5) - (6 \times 5)$$

$$\Rightarrow 283 \text{ KN-m}$$

INFLUENCE LINES

Def:

"An Influence line for any given point or section of a structure is a curve whose ordinates represent to scale the variation of a function, such as SF, BM, deflection etc at the point or section as unit load moves across the structure."

(or)

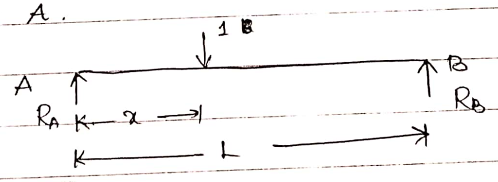
"An Influence line is a graph or curve of a response function when a unit load crosses the span of a beam from left to right."

where response functions are

(i) Reaction at support (R)
(ii) Shear force at a section (F)
(iii) Bending moment at a section (M).

IID for support reactions :-

Let us consider a beam/girder AB of span 'L'. Let a unit load be at a distance 'x' from the support A.



$\boxed{R_A + R_B = 1}$

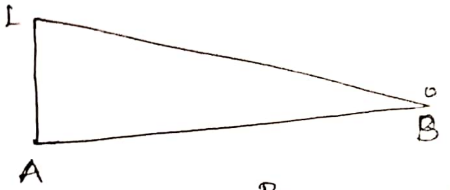
Taking moment about A,

$$R_B \times L - 1 \times x + R_A \times 0 = 0 \Rightarrow \boxed{R_B = \frac{x}{L}}$$

$\Rightarrow \boxed{R_A = 1 - \frac{x}{L}}$

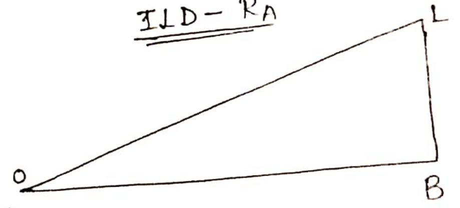
These reactions are linear functions and are true for $x=0$ to $x=L$

when $x=0$, $R_A = L$
when $x=L$, $R_A = 0$



ILD – R_A

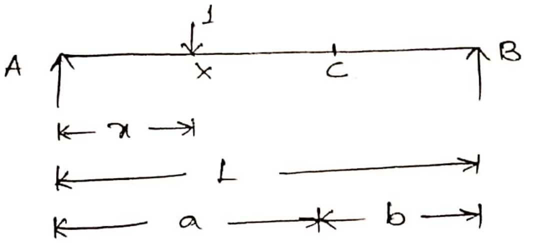
$R_B = \frac{x}{L}$, when $x=0$, $R_B = 0$
$x=L$, $R_B = L$



ILD – R_B

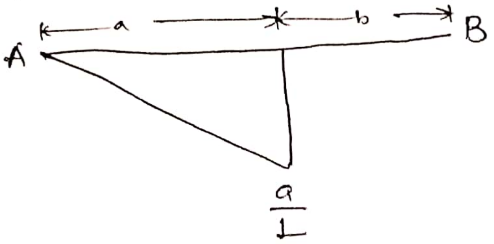
ILD for shear force at a section (say C) :-

Consider a section 'x' at a distance 'x' from the support A.



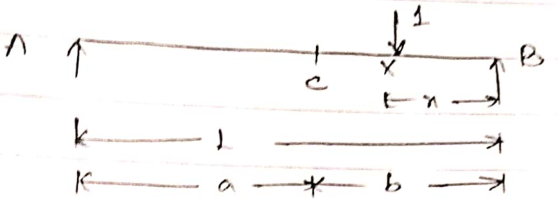
$R_B = \frac{x}{L}$
$R_A = 1 - \frac{x}{L}$
Shear force at C

$$F_C = -R_B$$
$$= -\frac{x}{L} \quad \text{(linear)}$$



when $x=0$, $F_C = 0$
$x=a$, $\boxed{F_C = -\frac{a}{L}}$

consider a section 'x' in the span of BC at a distance 'x' from the support B.



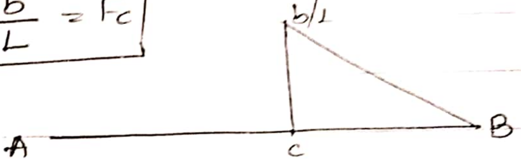
$F_C = R_A$

$= 1 - \frac{x}{L}$

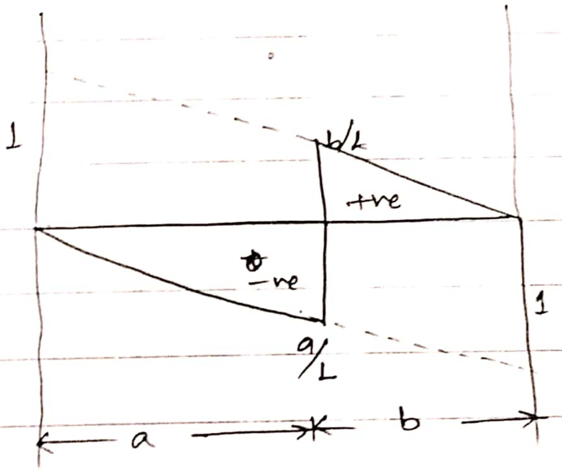
when $x = a$, $F_C = 1 - \frac{a}{L}$

, $F_C = \frac{L-a}{L} \rightarrow \boxed{\frac{b}{L} = F_C}$

$x = L$, $F_C = 0$



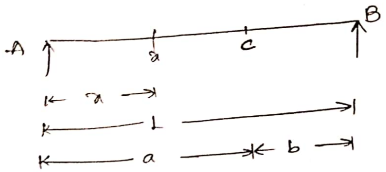
combined ILD



(b) * ILD for Bending moment (@ a section C)

Let section be at a distance x- from A.

$R_B = \frac{x}{L}$; $R_A = 1 - \frac{x}{L}$



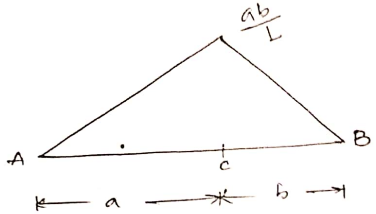
Taking moment at C,

$$M_C = R_B \times b$$
$$= \frac{x}{L} \times b$$

$$M_C = \frac{xb}{L}$$

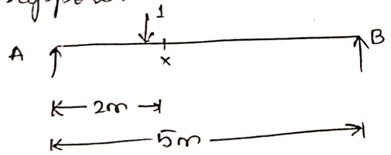
@ C, $x = a$

$$M_C = \frac{ab}{L}$$



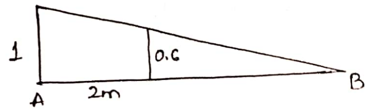
Eg: A simply supported beam AB has a span of 5m. Draw the ILD for R_A, R_B, V_x & M_x for a section at 2m from left hand support.

Sol:-



ILD for Support reactions :-

$$R_A = 1 - \frac{x}{L}$$

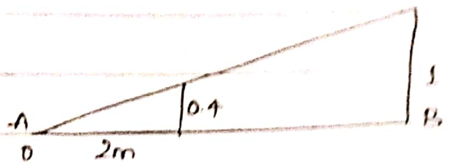


$x = 0$, $R_A = 1$ units

$x = 2$, $R_A = 1 - \frac{2}{5} \Rightarrow 0.6$ units

$x = 5$, $R_A = 0$

$R_B = \frac{x}{L}$



$x=0$, $R_B=0$

$x=2$, $R_B = 0.4$ units

$x=5$, $R_B = 1$ unit

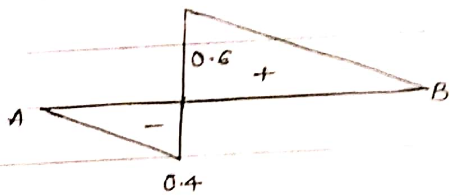
→ ILD for shear force :-

$F_c = -R_B$ for AX.

B $F_c = -\frac{x}{L} \Rightarrow -\frac{a}{L}$

@ $F_c \Rightarrow x = 2 \Rightarrow P_c = -\frac{2}{5} = 0.4$ unit

for XB, $F_c = \frac{b}{L} \Rightarrow \frac{3}{5} \Rightarrow 0.6$ unit

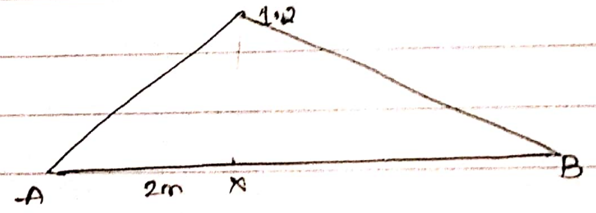


→ ILD for BM

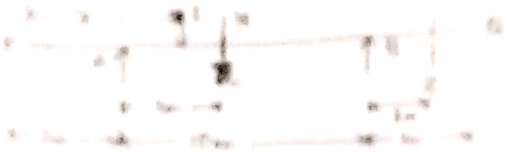
$M_c = \frac{ab}{L} \Rightarrow \frac{2\times3}{5} \Rightarrow 1.2$ units

~~(OR)~~

~~M_x for AX $\Rightarrow R_B \times (l-x) \Rightarrow \frac{x}{5}(5-x)$~~



⑤ Draw influence lines for R_A, R_B, V_D, M_D, V_E and M_E for an overhanging beam shown in figure



b. Support reactions :-

ⓐ $R_A \Rightarrow$ Taking moment about B,

$1 \times (12 - x) = R_A \times 10$

$$\boxed{R_A = \frac{12-x}{10}}$$

@ $x = 0 \Rightarrow R_A = 1.2$ units ; $x = 5 \Rightarrow R_B = 0.7$ units

$x = 12 \Rightarrow R_A = 0$

$x = 14 \Rightarrow R_A = \frac{12-14}{10} \Rightarrow -0.2$ units.

ⓑ $R_B \Rightarrow$ Taking moment about A,

$1 \times (x-2) = R_B \times 10.$

$$\boxed{R_B = \frac{x-2}{10}}$$

@ $x = 0 \Rightarrow R_B = -0.2$ units

$x = 2m \Rightarrow R_B = 0$

$x = 5m \Rightarrow R_B = 0.3$ units

$x = 14 \Rightarrow R_B = 1.2$ units.

(ii) ILD for shear force @ x (V_x) :-

Rx, PX , $V_x = -R_B$

$V_x = -\left(\frac{x-2}{10}\right) \Rightarrow$ @ $x = 0$, $V_x = 0.2$

@ $x = 2$, $V_x = 0$

@ $x = 5$, $V_x = -0.3$

For x 8, $V_x = R_A \Rightarrow \frac{12-x}{10} \Rightarrow$

@ $x = 5m = 0.7$ units

@ $x = 12 \Rightarrow 0$ units

@ $x = 14 \Rightarrow -0.2$ units.

(iii) ILD for bending moment (M_X) :-

AX, $M_X = R_B \times 7 \Rightarrow \frac{7(x-2)}{10}$

@ $x=0$, $M_X = -1.4$ units

@ $x=2$, $M_X = 0$

@ $x=5$, $M_X = 2.1$ units

for,

XB, $M_X = R_A \times 3 \Rightarrow \frac{3(12-x)}{10}$

@

$x = 5m$, $M_X = 2.1$ units

$x = 12$, $M_X = 0$

$x = 14$, $M_X = -0.6$ units.

(iv) ILD for V_Z :-

For any load position in portion ZY, $V_Z = 0$. When the load is at the section Z, $V_Z = 1$ and remains constant ~~[illegible]~~ ZD

(v) ILD for M_Z :- $M_Z = 0$ @ section

in ZD, it increases linearly.

When load is @ D, $M_Z = -1 \times 1$

$\Rightarrow -1$ units

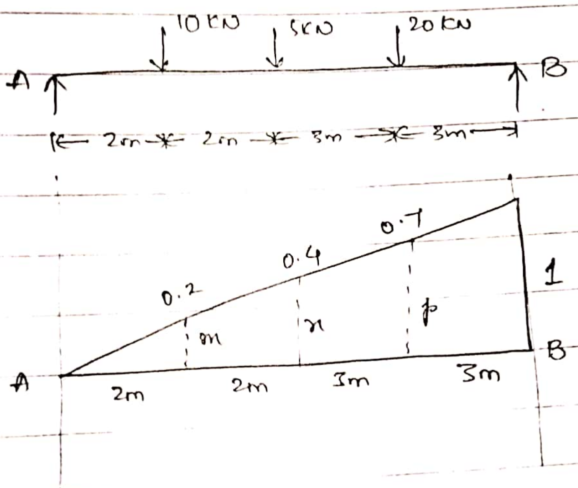
~~Application of ILD~~

~~Finding functions using ILD~~

~~Eg: 1 Find R_B using ILD :-~~

* Applications of ILD :-

1) Find R_B using ILD :-



using similar triangle properties,

$$\frac{m}{2} = \frac{1}{10} \Rightarrow m = 0.2 \text{ units}$$

$$\frac{n}{4} = \frac{1}{10} \Rightarrow n = 0.4 \text{ units}$$

$$\frac{p}{7} = \frac{1}{10} \Rightarrow p = 0.7 \text{ units.}$$

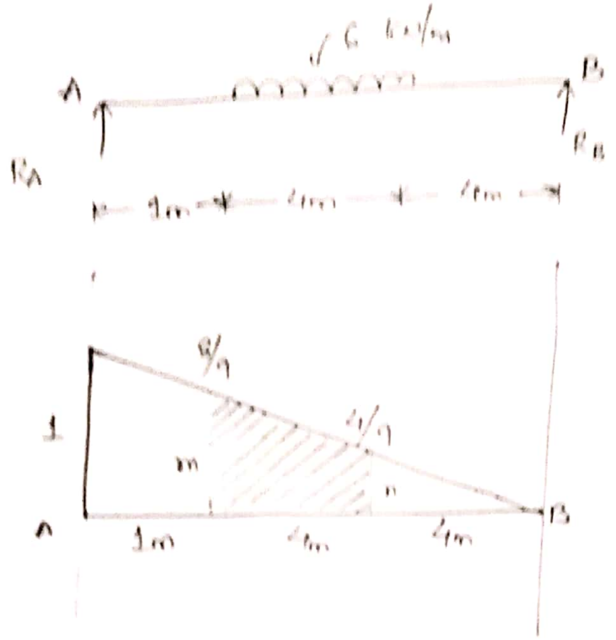
Reaction @ B $\Rightarrow$ $R_B = \Sigma wy$ → (y = ordinates of ILD)

$= 10 \times 0.2 + 5 \times 0.4 + 20 \times 0.7$

$\Rightarrow$ 18 kN.

$R_A \Rightarrow \underline{17 \text{ kN}}$

(2) Find Reaction @ A :



$\frac{m}{8} = \frac{1}{9} \Rightarrow m = \frac{8}{9}$

$\frac{n}{4} = \frac{1}{9} \Rightarrow n = \frac{4}{9}$

R_A = Load intensity × Area covered by loading

→ Trapezoid

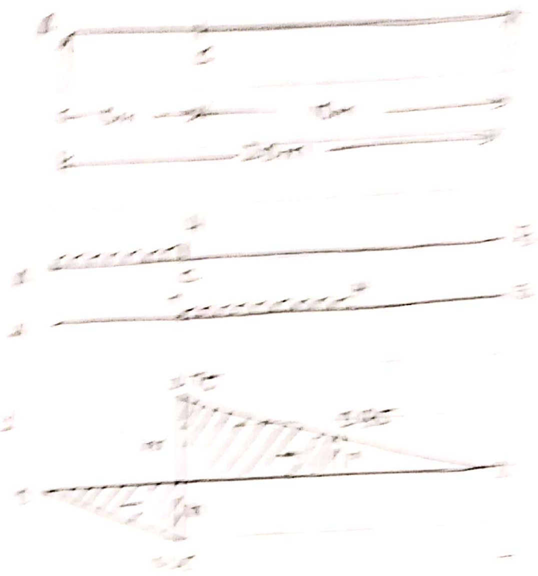
Area covered by loading = $\frac{1}{2} \times$ (Sum of parallel sides) × distance b/w parallel sides

$\Rightarrow R_A = 6 \times \frac{1}{2}\left(\frac{8}{9} + \frac{4}{9}\right) \times 4$

$\Rightarrow 16$ KN

$R_B = 8$ KN

Draw ILD for SF and BM @ 5m from left support for a SSB 20m long. Find Max SF and BM @ section due to an UDL (rolling load of length 5m and intensity 10 kN/m)



$\frac{m}{15} = \frac{1}{20}$

$m = 0.75$

$\frac{n}{5} = \frac{1}{20}$

$n = 0.25$

$\frac{P}{7} = \frac{1}{20}$

$P = 0.35$

(i) Max -ve SF

$F_c = 10 \times \text{Area}$

$\Rightarrow 10 \times \frac{1}{2} \times 5 \times 0.25 \Rightarrow -6.25\ \text{kN}$

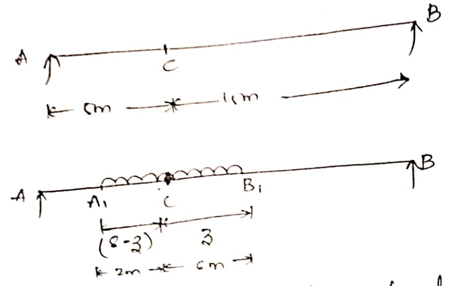
(ii) Max +ve SF

$F_c = 10 \times \text{Area}$

$\Rightarrow 10 \times \frac{1}{2} (0.75 + 0.35) \times 5$

$\Rightarrow 44\ \text{kN}$

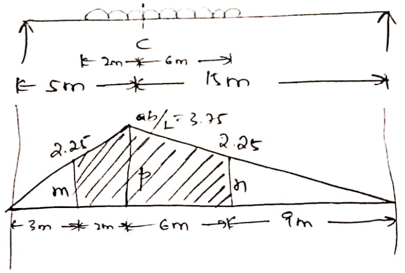
ⓑ BM @ K → ILD



Avg load on AC = Avg. load on BC

$$\frac{10 \times (8-z)}{5} = \frac{10z}{15}$$

$z = 6m$



$p = \frac{ab}{L} = \frac{5 \times 15}{20} = 3.75$

$\frac{n}{9} = \frac{3.75}{15}$

$n = 2.25$

& $\frac{m}{3} = \frac{3.75}{5} \Rightarrow m = 2.25$

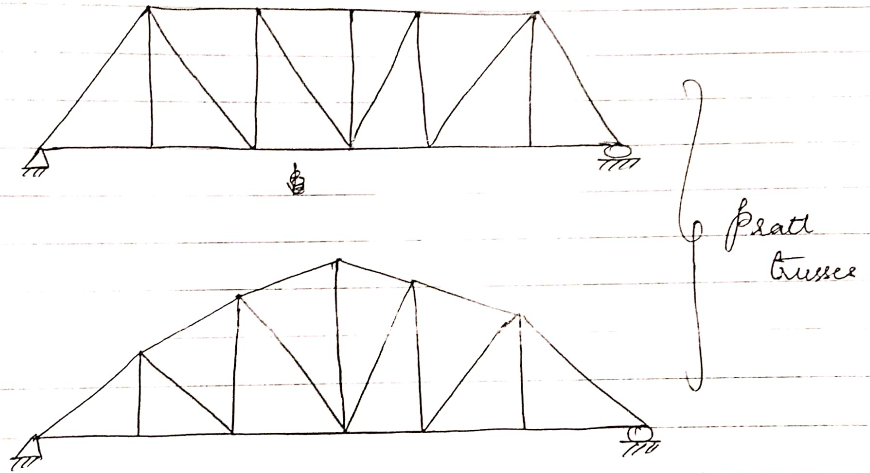
$M_{max} = 10 \times \text{Area}$

$\Rightarrow 10 \times \left[\frac{1}{2} (2.25 + 3.75)(2) + \frac{1}{2} (3.75 + 2.25) 6 \right]$

$M_{max} \Rightarrow 240 \text{ kN-m}$

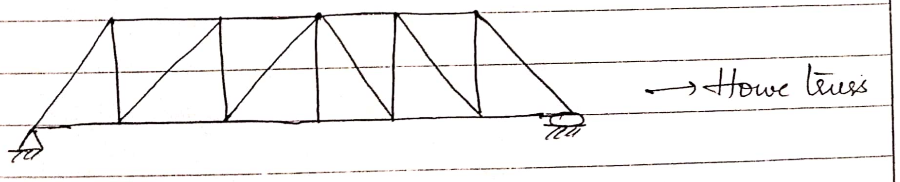
Types of Trusses :-

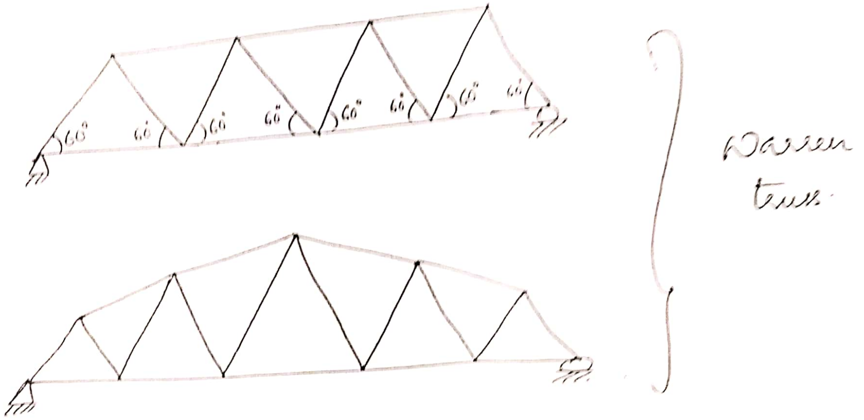
① Pratt truss with parallel chords.
② Pratt truss with inclined chords
③ Warren truss with parallel chords
④ Warren truss with inclined chords.



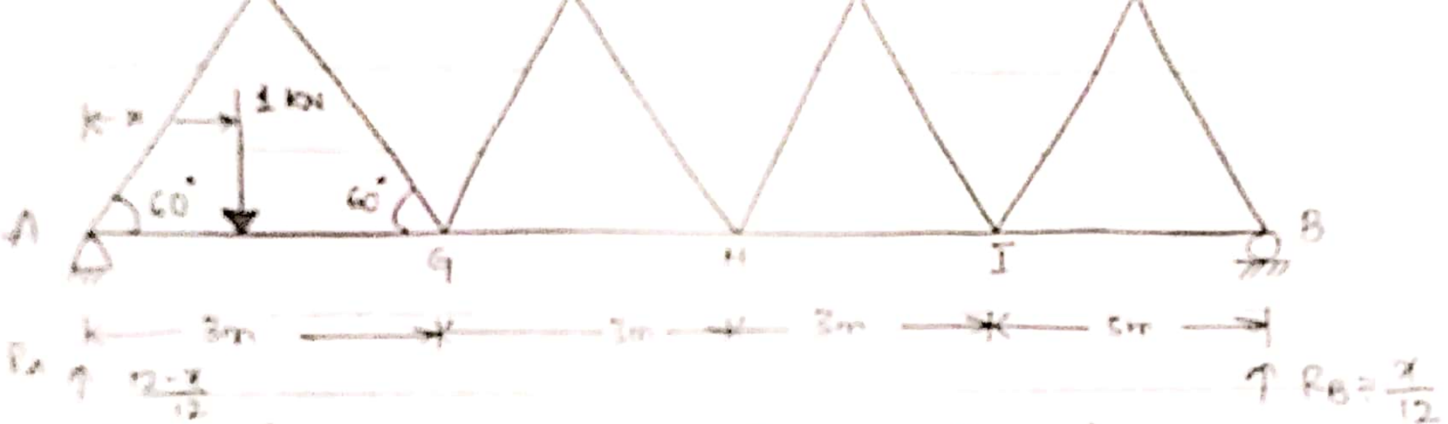
A pratt truss is one of the most common type of truss that includes vertical members and diagonals that slope down towards the centre.

→ Opposite of Pratt truss is Howe truss.





Warren truss comprises of diagonal members joined together to for inverted equilateral triangles together forming a framed-structure.



Draw ILD for member GH in the given truss.

Step 1 - Reactions in terms of 'x'.

$\Sigma M_A = 0$

$R_B \times 12 = 1 \times x \Rightarrow R_B = \frac{x}{12}$

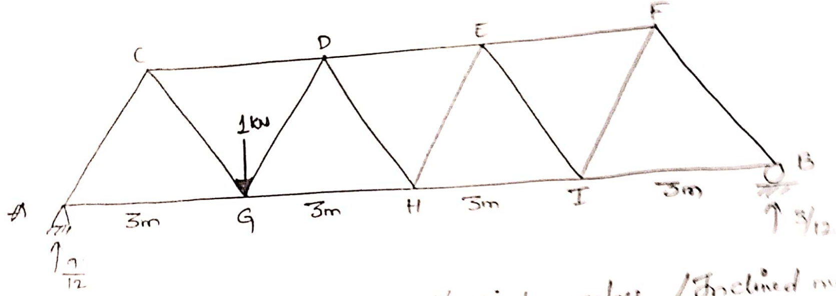
$\Rightarrow R_A = 1 - \frac{x}{12} \Rightarrow \frac{12-x}{12}$

Step 2 :- Placing the load at every joint.

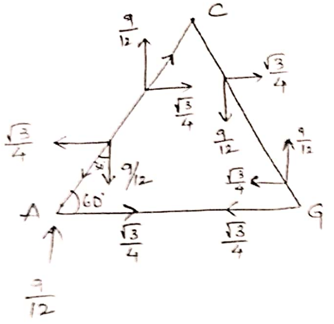
At joints A & B, when load is place the entire load is taken by the support itself. No load will be transferred to the members.

Let us place the load at pt. G. to find forces in GH

@ G ⇒ $x = 3 \Rightarrow R_A = \frac{9}{12}$; $R_B = \frac{3}{12}$



Note:- Vertical reactions ⇒ Vertical member / Inclined mem.
Horizontal reaction ⇒ Horizontal / Inclined mem.



→ To Balance all forces @ joint i.e Vertical & Horizontal.

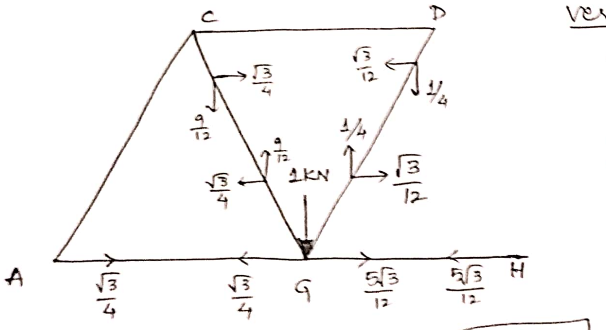
→ Sign convention:-

← • →
Tension (+ve)

→ • ←
Comp. (-ve)

Force in AC ⇒ x
⇒ $x\cos 30° = \frac{9}{12}$
$x = \frac{\sqrt{3}}{2}$

Horizontal.
$x\sin\theta \Rightarrow \frac{\sqrt{3}}{2}\sin 30$
$\Rightarrow \frac{\sqrt{3}}{4}$

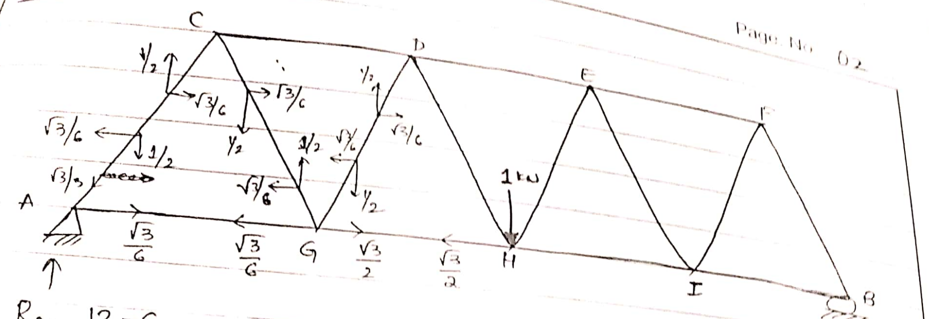


$-\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} + x + \frac{\sqrt{3}}{12} = 0 \Rightarrow \boxed{x = \frac{5\sqrt{3}}{12}}$

Vertical
$+\frac{9}{12} - 1 + x = 0$
$x = \frac{1}{4}$

$x\cos 30° = \frac{1}{4}$
⇒ [illegible]
⇒ [illegible]
x [illegible]

@ H ⇒ $x = 6m$

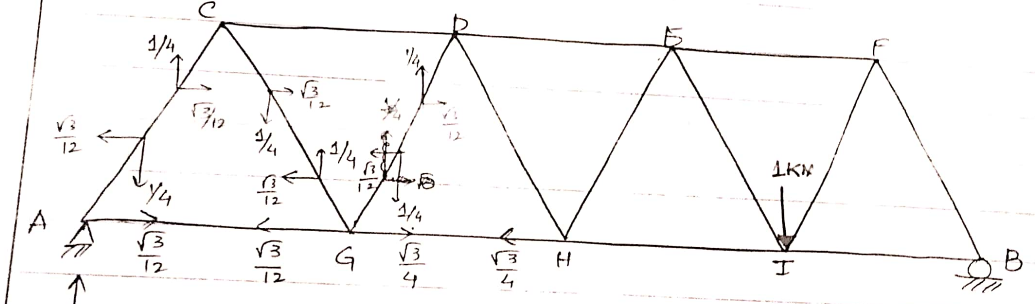


$R_A = \frac{12-6}{12}$

$\Rightarrow \frac{1}{2}$

$R_B = \frac{1}{12}$

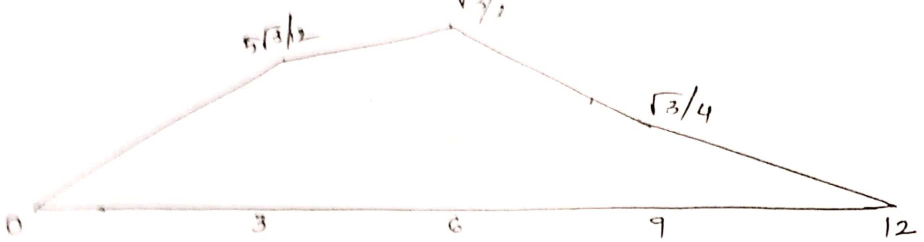
@ I ⇒ $x = 9m$.



$R_A = \frac{1}{4}$

$R_B = \frac{3}{4}$

x	Force in GH
0	0
3	$5\sqrt{3}/12$
6	$\sqrt{3}/2$
9	$\sqrt{3}/4$
12	0.


$5\sqrt{3}/12$
$\sqrt{3}/2$
$\sqrt{3}/4$
0
3
6
9
12