Unit - III [Sequencing :]

Introduction :-

The sequencing techniques deals with the problem of preparing optimal timetable for jobs, equipment, people, materials facilities and all other resources that are needed to support the production schedule.

→ Its objective is the minimization of the total elapsed time b/w the completion of first and last job in a particular order.

Priority Sequencing Rules :-

The following are the priority sequencing rules:-

① First Come first Serve (FCFS) :- Gives top priority to the waiting jobs that arrived earliest in the production system.

② Earliest due date :- Gives top priority to the waiting jobs whose due date is earliest. The objective of this rule is to reduce the lateness of the job.

③ Shortest processing time (SPT) :- Gives top priority to the waiting jobs whose operation time at the work centre is shortest. The objective of this rule is to reduce the waiting time in the queue.

④ Least slack :- Gives top priority to the job, which is having least slack.

Slack for job i = Earliest due date of job i - Processing time of job i.

⑤ Longest processing time :- Gives top priority to the waiting job whose processing time at the work centre is longest.

⑥ Preferred customer :- Gives top priority based on importance of the customer or order.

⑦ Last in first out (LIFO) :- when a m/c is dismantled for repair or overhaul, the parts are put back in LIFO system. Pipeline laying in water works, electrical wiring maintainance system will follow this model.

Job-shop and
Flow-shop Sequencing :-

→ The selection of appropriate order in which waiting jobs may be served is called sequencing. Here the jobs are different. But the sequence of operations performed on each job is the same. That is why these problems are called job shop and flow shop sequencing problems.

Analytical methods for solving cases.

Analytical methods have been developed for solving the following five cases:

① N jobs & 2 m/c's, A and B and all the jobs are processed in the order A-B.

② N jobs & 3 m/c's, A, B and C and all jobs are processed in the order A-B-C.

③ N jobs and M m/c's and all jobs are processed in the order $M_1, M_2, ---. M_m$.

④ Two jobs and M m/c's and each job are to be processed through the m/c's in the prescribed order.

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(3) Travelling Salesman problem.

Assumptions made in Sequencing problem :-

The following assumptions have beend made in sequencing problem:

(1) Only one operation is carried out on a machine at a particular time.

(2) Each operation once started on a job must be Completed

(3) Only one machine of each type is available.

(4) A job is processed as soon as possible, but only in the order specified.

(5) Processing times are independent of the order in which jobs are performed.

(6) The time required to transport jobs from one machine to another is negligible.

(7) All the jobs are available at the beginning.

Important terms used in sequence :-

(1) optimal Sequence :- It is the sequence which minimi-zes the total time of completing all jobs. There may be more than one optimal sequence.

(2) Total elapsed time :- It is minimum time of completing all jobs.

Gantt Chart :-

→ It is a tool used for both loading and scheduling of jobs on m/c's. The Gantt Chart is used to represent the scheduling of jobs on each m/c. The G chart was originated by Gantt and consists of a no. of rectangles. These rectangles are used to represent the work centre or m/c's for each job. The Gantt chart ~~cannot~~ must be updated periodically to account for new jobs.

Model 1:- Processing N jobs through two machines:-

This sequencing problem is completely described as follows:-

(i) Only two m/c's A & B are involved.
(ii) Each job is processed in order A – B.
(iii) The actual processing times are known.

A production system is shown below.

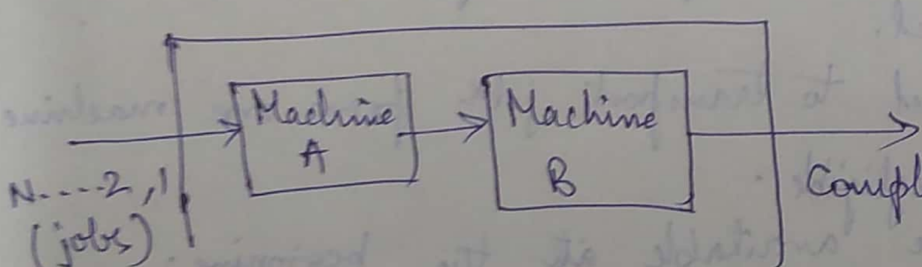

fig:- Production System consists of two m/c's & N jobs.

Johnson algorithm can be used to solve N jobs through 2 m/c's problem.

Johnson Algorithm:-

Steps in Johnson algorithm:-

The following are the steps in Johnson algorithm:-

1. Examine the processing times on machine A and machine B and find the smallest time. If there ~~are~~ is a tie select any one of the minimum values.

(2) If the minimum processing times occurs on machine A, do that job first. If the minimum processing time occurs on machine B, do that job last.

(3) Remove the job assigned and continue steps 1 and 2 until all the jobs are assigned.

(4) Determine the total elapsed time for the optimal sequence.

Problem:- A machine operator has to perform two operations turning and threading on a no. of different jobs. The time required to perform these operations for each job is known. Determine the order in which the jobs should be processed in order to minimize the total time required to complete all the jobs.

Job	Time for turning (hrs) (Machine A)	Time for threading (hrs) (Machine B)
1	3	8
2	12	10
3	5	9
4	2	6
5	9	3
6	11	1

sol:- There are six jobs and two m/c's.
Determining the optimal sequence using the Johnson algorithm
Steps 1-3: The smallest time is 1 hour to job 6. It is occurring on m/c B, therefore do the job last.

					6

Remove the job 6 from the table. The reduced table is:

Job	Turning Time (hrs) (Machine A)	Threading Time (hrs) (Machine B)
1	3	8
2	12	10
3	5	9
4	2	6
5	9	3

Now the smallest time is 2hrs for job 4. It is occuring on machine A, therefore do the job first.

4					6

Remove the job 4 from the table. The reduced table ~~below (Table 12.12)~~ is:

Job	Turning time (hrs) (Machine A)	Threading Time (hrs) (Machine B)
1	3	8
2	12	10
3	5	9
5	9	3

Now the smallest time is 3hrs. It is occuring on machine A on job 1 and also occuring on m/c B on job 5. Here there is a tie for minimum time 3. If there is tie, then select any job (ie, job 1 or job 5). If we select to job 5, minimum processing time occurs on machine B. Do the job 5 that.

4				5	6

Remove the job 5 from the above table. The reduced table is:

Job	Time for turning (hrs) (Machine A)	Time for threading (hrs) (Machine B)
1	3	8
2	12	10
3	5	9

Now the smallest time is 3 hrs for job 1, It is occuring on machine A, therefore do the job 1 first.

4	1			5	6

Remove the job 1 from the table. The reduced table is:

Job	Time for turning (hrs) (Machine A)	Time for threading (hrs) (Machine B)
2	12	10
3	5	9

Now the smallest time is 5 hrs for job 3. It is occuring on machine A, therefore do the job 3 first.

4	1	3		5	6

Remove the job 3 from the table. Now only job is 2. So, the optimal sequence is:

4	1	3	2	5	6

Step 4 : The total elapsed time is shown in below Table.

Job sequence	Turning operation Time in – Time out	Threading operation Time in – Time out
4	0–2	2–8
1	2–5	8–16
3	5–10	16–25
2	10–22	25–35
5	22–31	35–38
6	31–42	42–43

Minimum elapsed time = 43 hours.

Idle time for m/c A (turning operation) = 1 hr.

Idle time for m/c B (threading operation) = 6 hrs.

The Gantt chart is shown in below figure.

Machine A

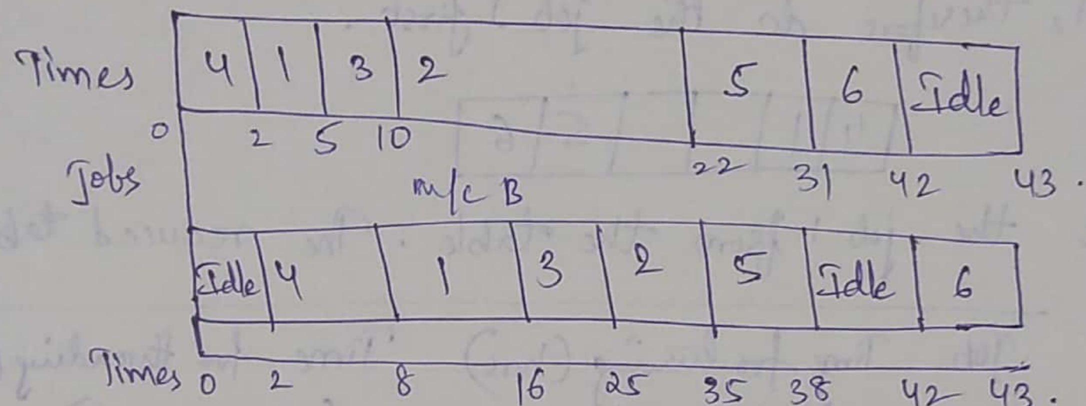

Fig :- Gantt chart.

Prob 2 :- A readymade garment manufacturer has to process seven items, through two stages of production, Cutting & sewing. The time taken for each of these items is given in the below table:

Find an order in which these items are to be processed through these stages so as to minimize the total processing time.

Item	1	2	3	4	5	6	7
Cutting Time	5	7	3	4	6	7	12
Sewing Time	2	6	7	5	9	5	8

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Model 2: Processing of N jobs through three machines

This sequencing problem is completely described as follows:

(i) Only three machines A, B and C are involved.

(ii) Each job is processed in the prescribed order A-B-C.

(iii) No passing of jobs is permitted.

(iv) The actual processing times are known.

Johnson method can be extended to cover the special cases where either one or both of the following conditions hold good:

(a) The minimum time on machine A $\geq$ maximum time on machine B.

(b) The minimum time on machine C $\geq$ maximum time on machine B.

If either one or both conditions are satisfied, then convert the N jobs through three machines problem into N jobs through two machines problem. Here G and H are two fictitious machines & their corresponding processing times are given by

$$G_i = A_i + B_i \quad i = 1, 2, 3 \ldots N.$$

$$H_i = B_i + C_i \quad i = 1, 2, 3, \ldots N:$$

Problem:- Find the sequence that minimizes the total time required in performing the following jobs on three machines in the order A-B-C as shown in table. Also find the total elapsed time.

m/c \ Job	1	2	3	4	5	6
A	8	3	7	2	5	1
B	3	4	5	2	1	6
C	8	7	6	9	10	9

Sol^n: Minimum of $A = 1$

Maximum of $B = 6$.

Minimum of $C = 6$.

The Condition (b) ie, the minimum time on m/c C $\geq$ maximum time on m/c B is satisfied. So, the Johnson rule is applicable to N jobs through three m/c's problem. Convert the N jobs through three m/c's problem into N jobs through two m/c's problem.

Table: N jobs three m/c's problem into N jobs two m/c's.

m/c \ Job	1	2	3	4	5	6
G = A+B	11	7	12	4	6	7
H = B+C	11	11	11	11	11	15

Determining the optimal sequence using the Johnson rule (this problem has 2 optimal sequences).

Optimal sol^n 1:

4	5	6	2	1	3

optimal sol^n 2:

4	5	2	6	1	3

The total elapsed time for optimal sequence is shown in below table :-

Elapsed time - Table :-

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Job Sequence	Machine A Time in – Time Out	Machine B Time in – Time Out	Machine C Time in – Time Out
4	0 – 2	2 – 4	4 – 13
5	2 – 7	7 – 8	13 – 23
6	7 – 8	8 – 14	23 – 32
2	8 – 11	14 – 18	32 – 39
1	11 – 19	19 – 22	39 – 47
3	19 – 26	26 – 31	47 – 53

Minimum elapsed time = 53 hrs.
Idle time for machine A = 27 hrs.
" " " " B = 32 hrs.
" " " " C = 4 hrs.

Prob. 9: Find the sequence for the following eight jobs as shown in table that will minimize the total elapsed time for the completion of all jobs. Each job is processed in the order of C – A – B. Calculate the idle time.

m/c \ Job	1	2	3	4	5	6	7	8
A	4	6	3	4	5	3	6	2
B	8	10	7	8	4	8	9	13
C	5	6	2	3	4	9	15	11

Sol^n:- Here the sequence of m/c's is C – A – B.

Minimum of C = 2
Maximum of A = 6
Minimum of B = 7

The condition i.e., the minimum time on m/c B ≥ max^m time on m/c A is satisfied.

So the Johnson rule is applicable to N jobs through three machines problem.

Converting the N jobs through three machines problem into N jobs through two machines problem.

Job / Machine	1	2	3	4	5	6	7	8
A	4	6	3	4	5	3	6	2
B	8	10	7	8	11	8	9	13
C	5	6	2	3	4	9	15	11

So

Job / M/C	1	2	3	4	5	6	7	8
G=C+A	9	12	5	7	9	12	21	13
H=A+B	12	16	10	12	16	11	15	15

The optimal sequence of jobs using the Johnson rule is:

3	4	1	5	2	8	7	6

The total elapsed time for optimal sequence is shown in table

Job Sequence	Machine C Time in-Time Out	Machine A Time in-Time Out	Machine B Time in-Time Out.
3	0-2	2-5	5-12
4	2-5	5-9	12-20
1	5-10	10-14	20-28
5	10-14	14-19	28-39
2	14-20	20-26	39-49
8	20-31	31-33	49-62
7	31-46	46-52	62-71
6	46-55	55-58	71-79.

Minimum elapsed time = 79 hrs.

Idle time for machine A = 46 hrs.

" " " " B = 5 hrs.

" " " " C = 24 hrs.

Model 3:- Two jobs through 'm' machines :-

This sequencing problem is described as follows:-

(i) only two jobs are to be performed.

(ii) Sequence of m/c's for each job is known.

(iii) Sequencing of the m/c's for each job may be same (or) different.

(iv) Processing times are known.

(V) Each machine can work only one job at a time.

The problem is to minimize the total elapsed time, i.e., the minimum time of completing both jobs. Here the objective is to determine the scheduling of jobs on each machine so that the time of completing both jobs is minimized. Using the graphical method solve the two jobs problem through M machines problem. The ~~general~~ graphical procedure is given below:

Graphical method :-

(1) Draw two axes at right angles to each other. Represent the processing times of job 1 along the horizontal axis &, processing times of job 2 along the vertical axis.

(2) Lay out the machine times for two jobs on corresponding axis in the given order of machines. Rectangular areas

represent that both jobs are competing the same machine at same time. Only one job is processed on each machine at a time. So passing through these rectangular areas is not permitted.

③ Determine the optimal path starting from the origin to the final point F.

(i) Line inclined to 45° to x-axis or y-axis represents that both jobs are processing.

(ii) Line parallel to x-axis represents that job 1 is processing and job 2 is idle.

(iii) Line parallel to y-axis represents that job 2 is processing and job 1 is idle.

④ Determine the total elapsed time from the graph using the following equation.

Total elapsed time = Processing time of job 1 + Idle time of job 1.

(or)

Processing time of job 2 + idle time of job 2.

Prob:- Using the graphical method, calculate the minimum time needed to process jobs 1 and 2 on five machines A, B, C, D and E, as shown in table ie., for each machine, find the job, which should be done first.

Job 1	Sequence	A	B	C	D	E	
	Time (hrs)	1	2	3	5	1	= 12
Job 2	Sequence	C	A	D	E	B	
	Time (hrs)	3	4	2	1	5	= 15

Sol^n:- Draw the graph using the procedure presented in the graphical method. The resultant graph is shown in fig.

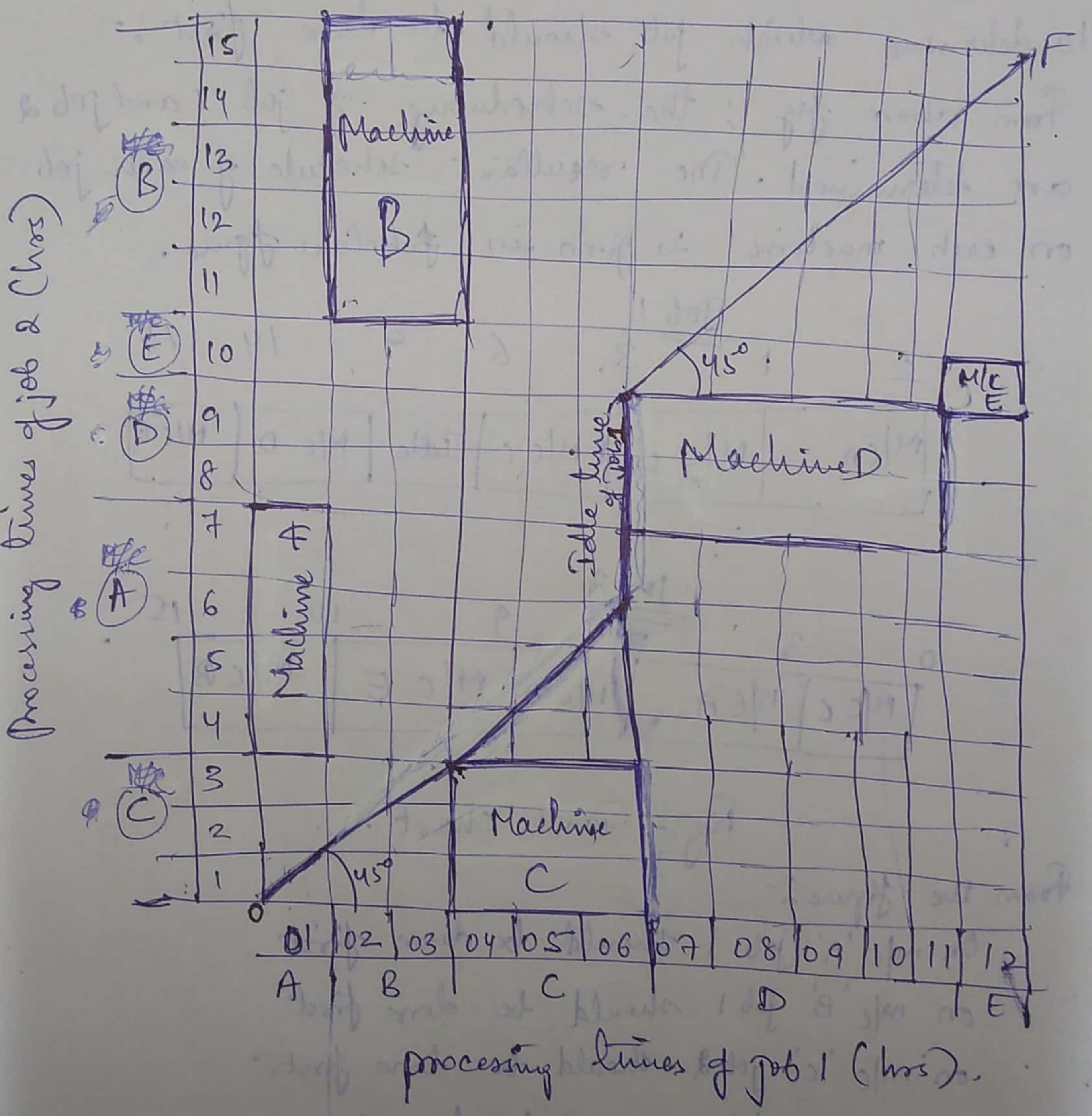


Fig:- Determining the optimal path.

From the graph:

Total elapsed time = Processing time of job 1 + Idle time of job 1

(or)

Processing time of Job 2 + Idle time of Job 2.

$= 12 + 3 = 15$ hrs.

(or)

$= 15 + 0 = 15$ hrs.

To determine which job should be done first :-

From above fig ; the scheduling of job 1 and job 2 are determined. The resultant schedule of each job on each machine is given in below figure.

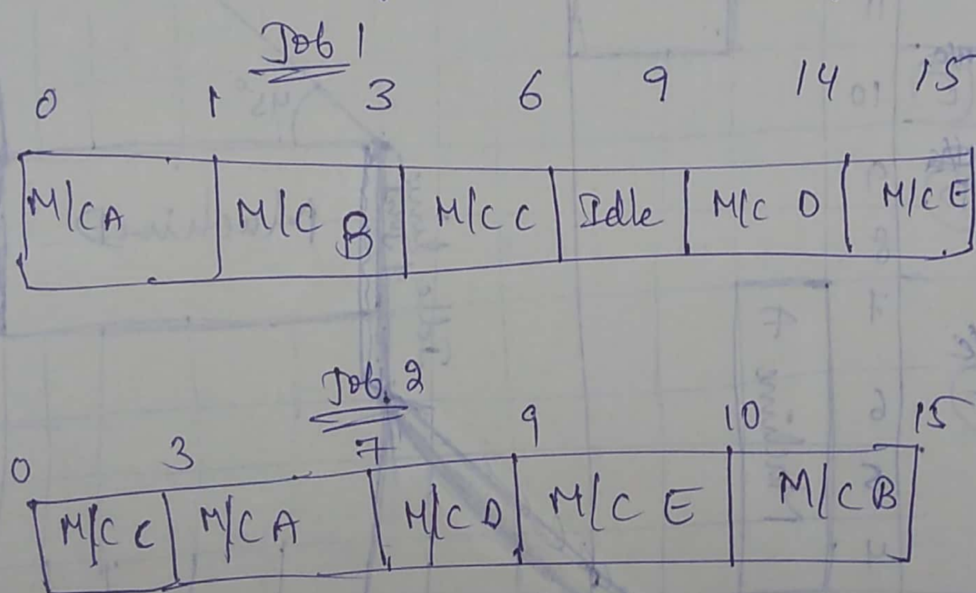


fig :- Gantt chart.

From the figure :-

On m/c 'A' job 1 should be done first

on m/c 'B' job 1 should be done first

on m/c 'C' job 2 should be done first

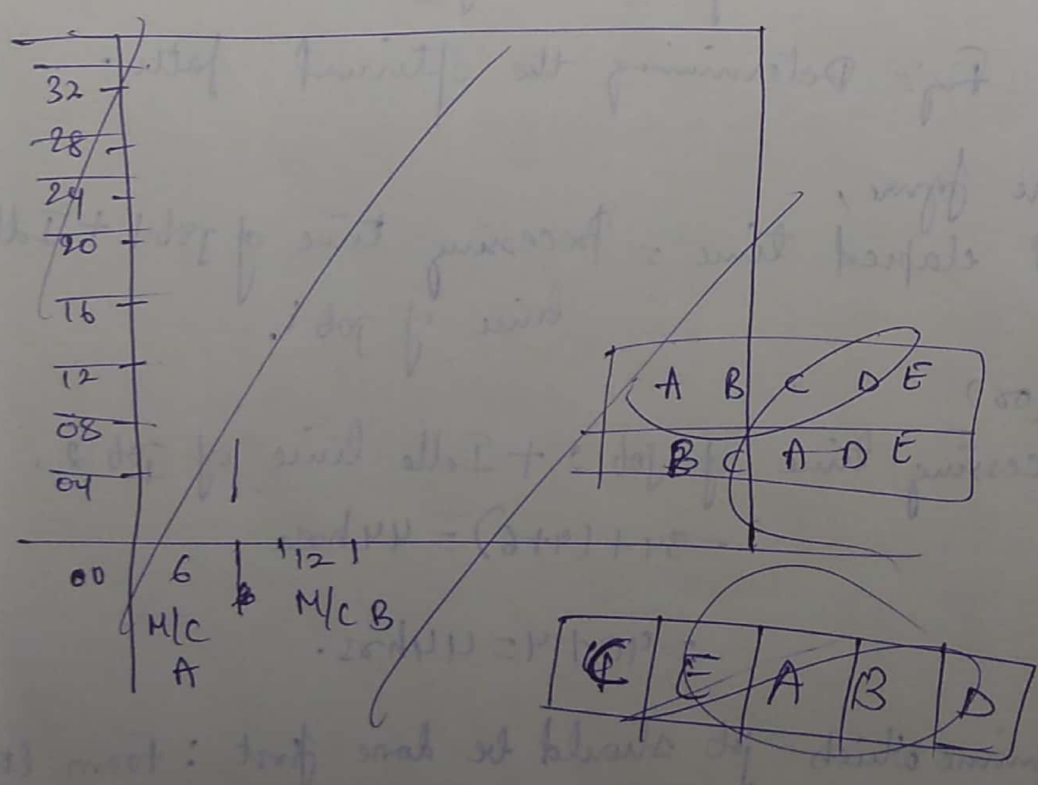
on m/c 'D' job 2 should be done first

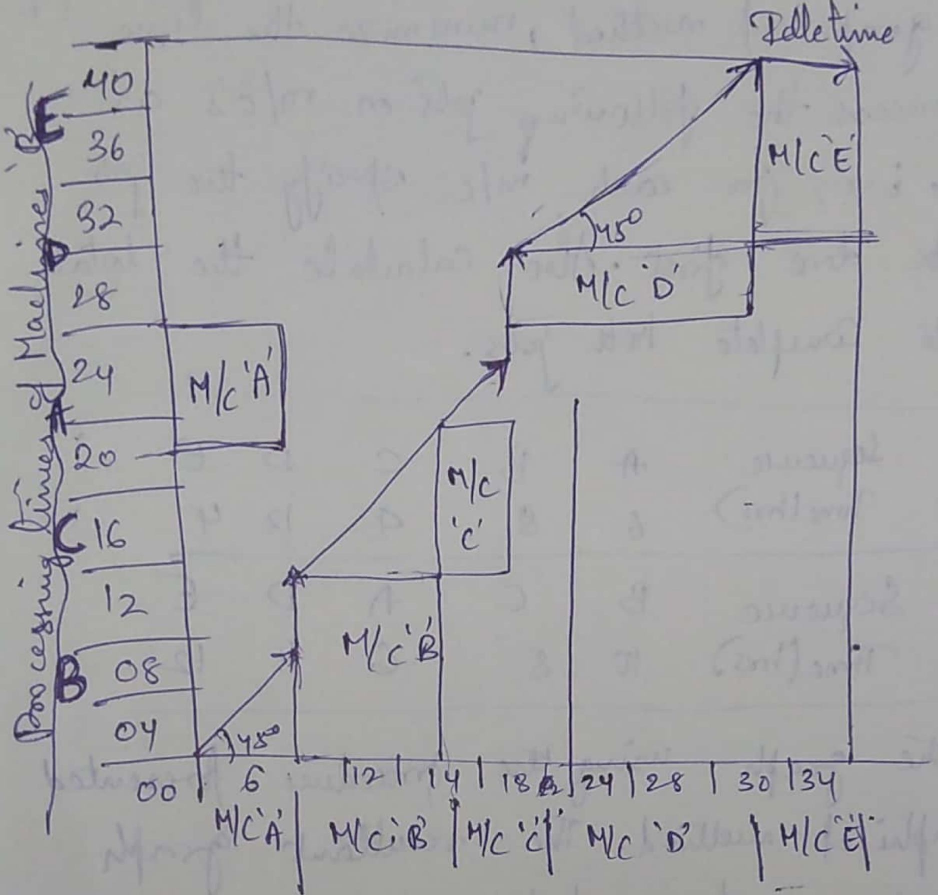
on m/c 'E' job 2 should be done first.

Prob:- Using the graphical method, minimize the time required to process the following jobs on m/c's as shown in table i.e., for each m/c specify the job which should be done first. Also calculate the total elapsed time to complete both jobs.

Job 1	Sequence	A	B	C	D	E
	Time (hrs)	6	8	4	12	4
Job 2	Sequence	B	C	A	D	E
	Time (hrs)	10	8	6	4	12

Soln:- Draw the graph using the procedure presented in the graphical method. The resultant graph is shown in ~~show~~ figure below.





Processing times of Job 1.

Fig:- Determining the optimal path.

From the figure,

Total elapsed time = Processing time of Job 1 + Idle time of job 1.

(or)

Processing time of job 2 + Idle time of Job 2.

$= 34 + (4+6) = 44$ hrs.

(or)

$= 40 + 4 = 44$ hrs.

To determine which job should be done first : From the above figure, the scheduling of job 1 and job 2 are determined. The resultant schedule of each job on each m/c is given in figure below.

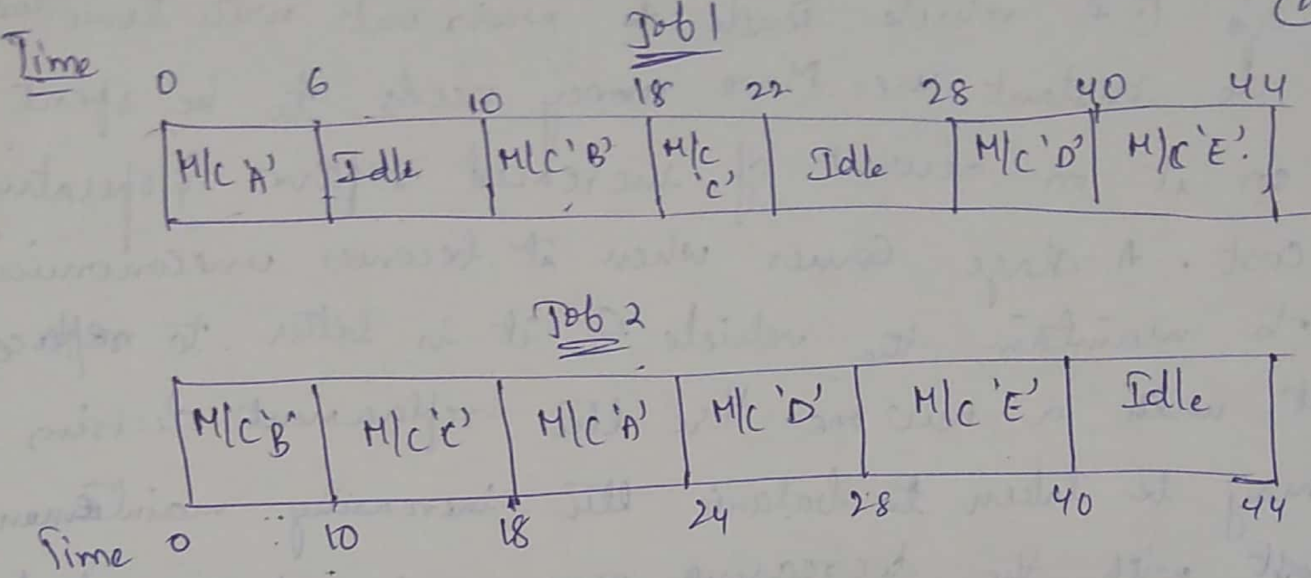


fig:- Gantt chart.

From the figure,

On m/c 'A' job 1 should be done first
On m/c 'B' job 2 should be done first.
On m/c 'C' job 2 should be done first
On m/c 'D' " '2' should be done first
On m/c 'E' job '2' should be done first

Replacement :-

Introduction :- The problem of replacement is felt when the job performing units such as men, machines, equipments, parts, etc., become less effective or useless due to either sudden, or gradual deterioration in their efficiency, failure (or) breakdown. By replacing them with new ones at frequent intervals, maintanance and other overhead costs can be reduced. However, such replacements would increase the need of Capital cost for new ones.

for eg:- ① A vehicle tends to wear out with time due to constant use. More money needs to be spent on it on account of increased repair & operating cost. A stage comes when it becomes uneconomical to maintain the vehicle & it is better to replace it with a new one. Here the replacement decision may be taken to balance the increasing maintenance cost with the decreasing money value of the vehicle, with the passing of time.

Thus, the basic problem in such situations is to formulate a replacement policy in order to determine an age (or period) at which the replacement of the given machinery/equipment is most economical, keeping in view all possible alternatives.

We shall discuss the replacement policies in the context of the following 3 types of replacement situations

i) Items such as m/c's, vehicles, types, etc., whose efficiency deteriorates with age due to constant use & which need increased operating & of maintenance costs. In such cases the deterioration level is predictable & is represented by ⓐ increased maintenance/operational cost, ⓑ its waste (or) scrap value & damage to item & safety risk.

(ii) Items such as light bulbs and tubes, electric motors, radio, television parts, etc., which do not give any indication of deterioration with time but fail all of a sudden and are rendered useless. Such cases require an anticipation of failures to specify the probability of failure for any future time period. With this probability distribution and the cost information, it is desired to formulate optimal replacement policy in order to balance the wasted life of an item, replaced before failure against the costs incurred when the items fail in service.

(iii) The existing working staff in an organization gradually reduces due to retirement, death, retrenchment, and other reasons.

Replacement of items whose efficiency deteriorates with time :-

→ When operational efficiency of an item deteriorates with time (gradual failure), it is economical to replace the same with a new one. For eg, the maintenance cost of a m/c increases with time & a stage is reached where it may not be economical to allow the m/c to continue in the system. Besides, there could be no. of alternative choices & one may like to compare the available alternatives on the basis of running costs (avg maintenance & operating costs) involved.

~~In this section~~.
→ we shall discuss various techniques for making such comparisions under different conditions. While making such Comparisions it is assumed that suitable expressions for running Costs are available.

Model 1:- Replacement policy for items whose running Costs increases with time and value of Money Remains Constant during a period.

Theorem 1:- The cost of maintenance of a m/c is given as a function increasing with time, whose scrap value is constant.

(a) If time is measured Continuously then the average annual cost will be minimized by replacing the m/c when the average cost to date becomes equal to the current maintence cost.

(b) If time is measured in discrete units, then the average annual cost will be minimized by replacing the machine when the next period's maintenance cost becomes greater than the current average cost.

Proof:- The aim here is to determine the optimal replacement age of a piece of equipment whose running cost increases with time & the value of money remains constant (i.e., value is not counted) during that period.

Let, C = capital (or) purchase cost of new equipment.
S = Scrap (or salvage) value of the equipment at the end of t years.

$R(t)$ = running cost of equipment for the year t

n = replacement age of the equipment.

(a) when time 't' is a continuous variable :- If the equipment is used for 't' years, then the total cost incurred over this period is given by :

TC = Capital (or purchase) cost − Scrap value at the end of 't' years + Running cost for 't' years.

$$= C - S + \int_0^n R(t)\,dt.$$

Therefore, the average cost per unit time incurred over the period of 'n' years is :

$$ATC_n = \frac{1}{n}\left\{C - S + \int_0^n R(t)\,dt\right\} \quad \text{——— (1)}$$

To obtain the optimal value 'n' for which ATC_n is minimum, differentiate ATC_n with respect to n, & set the first derivative equal to zero. That is, for minimum of ATC_n.

$$\frac{d}{dn}\{ATC_n\} = -\frac{1}{n^2}\{C - S\} + \frac{R(n)}{n} - \frac{1}{n^2}\int_0^n R(t)\,dt = 0$$

(or) $$R(n) = \frac{1}{n}\left\{C - S + \int_0^R R(t)\,dt\right\},\ n \neq 0 \quad \text{——— (2)}$$

$$R(n) = ATC_n.$$

Hence, the following replacement policy can be derived with the help of eqn (2)

Policy Replace the equipment when the average annual cost for 'n' years becomes equal to the current / annual running cost. That is :

$$R(n) = \frac{1}{n}\left\{C - S + \int_0^n R(t)\,dt\right\}.$$

(b) when time 't' is a discrete variable; The average Cost incurred over the period 'n' is given by:

$$ATC_n = \frac{1}{n}\left\{C - S + \sum_{t=0}^{n} R(t)\right\} \quad \text{— (3)}$$

If $C-S$ and $\sum_{t=0}^{n} R(t)$ are assumed to be monotonically decreasing and increasing, respectively, then there will exist a value of n for which ATC_n is minimum.

Thus, we shall have inequalities:

$$ATC_{n-1} > ATC_n < ATC_{n+1}$$

(or) $\quad ATC_{n-1} - ATC_n > 0.$

and $\quad ATC_{n+1} - ATC_n > 0.$

Eqⁿ (3) for period n+1, we get:

$$ATC_{n+1} = \frac{1}{n+1}\left\{C - S + \sum_{t=1}^{n+1} R(t)\right\}.$$

$$= \frac{1}{(n+1)}\left\{C - S + \sum_{t=1}^{n} R(t) + R(n+1)\right\}.$$

$$= \frac{n}{(n+1)} \frac{\left\{C - S \sum_{t=1}^{n} R(t)\right\}}{n} + \frac{R(n+1)}{n+1}.$$

$$= \frac{n}{n+1} \cdot ATC_n + \frac{R(n+1)}{n+1}.$$

Therefore,

$$ATC_{n+1} - ATC_n = \frac{n}{n+1} ATC_n + \frac{A(n+1)}{n+1} - ATC_n.$$

$$= \frac{R(n+1)}{n+1} + ATC_n\left[\frac{n}{n+1} - 1\right] = \frac{R(n+1)}{n+1} - \frac{ATC_n}{n+1}$$

Since, $ATC_{n-1} - ATC_n > 0$, we get

$$\frac{R(n+1)}{n+1} - \frac{ATC_n}{n+1} > 0.$$

$$R(n+1) - ATC_n > 0 \text{ (or) } R(n+1) > ATC_n.$$

Similarly, $ATC_{n-1} - ATC_n > 0$, implies that $R(n+1) < ATC_{n-1}$

This provides the following replacement policy.

Policy 1 If the running cost of next year, $R(n+1)$ is more than the average cost of n^{th} year, ATC_n, then it is economical to replace at the end of 'n' years. That is:

$$R(n+1) > \frac{1}{n}\left\{C - S + \sum_{t=0}^{n} R(t)\right\}.$$

Policy 2 If the present year's running cost is less than the previous year's average cost, ATC_{n-1}, then do not replace. That is:

$$R(n) < \frac{1}{n-1}\left\{C - S + \sum_{t=0}^{n-1} R(t)\right\}.$$

Model 1:- Money value is not counted:-

Eg:- A firm is considering the replacement of a machine, whose cost price is Rs. 12,200/- and its scrap value is Rs. 200, from experience the running (maintenance and operating) costs are found to be as follows:

Year:	1	2	3	4	5	6	7	8
Running cost (Rs):	200	500	800	1200	1800	2500	3200	4000

When should the machine be replaced?

Soln:- We are given the running cost, $R(n)$, the scrap value $S = Rs. 200/-$ & the cost of the machine, $C = Rs. 12200/-$

In order to determine the optimal time 'n' when the m/c should be replaced, we first calculate the average cost per year during the life of the machine, as shown in table below:

Year of Service	Running Cost (Rs) $(R(n))$	Cummulative Running Cost (Rs) $(\Sigma R(n))$	Depreciation Cost (Rs) $\boxed{C-S}$	Total cost (Rs) TC	Average Total Cost (Rs) ATC_n
(1)	(2)	(3)	(4)	(5)=(3)+(4)	(6)=(5)/(1)
1	200	200	12000	12200	12200
2	500	700	12000	12700	6350
3	800	1500	12000	13500	4500
4	1200	2700	12000	14700	3675
5	1800	4500	12000	16500	3300
6	2500	7000	12000	19000	$\boxed{3167}$
7	3200	10,200	12000	22,200	3171
8	4000	14,200	12000	26,200	3275

From the table; it may be noted that the average cost per year, ATC_n is minimum in the Sixth year (Rs. 3167). Also, the average cost in seventh year (Rs. 3171) is more than the cost in the sixth year. Hence, the machine should be replaced after every six years.

eg:- The data collected in running a machine & the cost of which is Rs. 60,000/- are given below:

Year:	1	2	3	4	5
Resale value (Rs):	42000	30000	20400	14400	9650
Cost of spares (Rs):	4000	4270	4880	5700	6800
Cost of labour (Rs):	14000	16000	18000	21000	25000.

Determine the optimum period for replacement of the machine.

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Sol:- The costs of spares and labour, together, determine the
running (operational (or) maintenance) cost. Thus, the running costs & the resale price of the machine in successive years are as follows:

Year:	1	2	3	4	5
Resale Value (Rs):	42000	30000	20400	14400	9650
Running Cost (Rs):	18000	20270	22800	26700	31800.

The calculations of average running cost per year during the life of the machine are shown in table below.

Year of Service n	Running Cost (Rs) $R(n)$	Cummulative Running Cost (Rs) $\Sigma R(n)$	Resale Value (Rs) S	Depreciation Cost (Rs) $C-S$	Total Cost (Rs) TC	Avg. Cost (Rs) ATC_n.
(1)	(2)	(3)	(4)	(5)=60000-(4)	(6)=(3)+(5)	(7)=(6)/(1)
1	18,000	18,000	42,000	18,000	36,000	36,000.00
2	20,270	38,270	30,000	30,000	68,270	34,135.00
3	22,880	61,150	20,400	39,600	1,00,750	33,583.33
4	26,700	87,850	14400	45,600	1,33,450	33,362.50
5	31,800	1,19,650	9,650	50,350	1,70,000	34000.00

The calculations in above table show that the avg. cost is the lowest during the fourth year. Hence, the m/c should be replaced after every four years, otherwise the average cost per year for running the m/c would start increasing.

eg. - M/c 'A' Costs Rs. 45000/- and its operating costs are estimated to be Rs. 1000/- for the first year increasing by Rs 10000/- per year in the second and subsequent years. Machine 'B' Costs Rs. 50000/- & operating costs are Rs. 2000 for the first year, increasing by Rs. 4000/- in the second and subsequent years. If at present we have a machine of type A, should we replace it with B? If so when? Assume that both M/c's have no resale value and their future costs are not discounted?

soln :- The calculations of average running cost per year during the life of machines A & B are shown in tables below respectively.

$S = 0$ (no resale value)

Year of Service n	Running Cost (Rs) $R(n)$	Cummulative Running Cost (Rs) $\Sigma R(n)$	Depreciation Cost (Rs) $C-S$	Total Cost (Rs) TC	Avg. Total Cost (Rs) ATC_n
(1)	(2)	(3)	(4)	(5)=(3)+(4)	(6)=(5)/(1)
1	1,000	1,000	45,000	46000	46,000
2	11,000	12,000	45,000	57,000	28,500
3	21,000	33,000	45,000	78,000	26,000
4	31,000	64,000	45,000	1,09,000	27,250
5	41,000	1,05,000	45,000	1,50,000	30,000
6	51000	1,56,000	45,000	2,01,000	33,500

From above table, it may be noted that the average running cost per year is lowest in the third year, i.e., Rs. 26000/-. Hence machine A should be replaced after every three years of service.

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the sum of all discounted costs is minimum.

eg:- Let the value of the money be assumed to be 10% per year and suppose that m/c A is replaced after every three years, whereas m/c B is replaced every six years. The yearly costs (in Rs) of both the m/c's are given below;

Year	1	2	3	4	5	6
M/C A	1000	200	400	1000	200	400
M/C B	1700	100	200	300	400	500

Determine which m/c should be purchased.

soln:- The discounted cost (present worth) at 10% per year for m/c A for three years & B for six years is given in table I & II respectively.

Table I

Year	Discounted cost at 10% Rate (Rs) cost	present worth
1	1000	$1000 \times \left(\frac{100}{100+10}\right)^0 = \boxed{1000}$
2	200	$200\left(\frac{100}{100+10}\right)^1 = 200 \times 0.9091$ $\boxed{=181.82}$
3	400	$400\left(\frac{100}{100+10}\right)^2 = 400 \times 0.8264$ $\boxed{=330.56}$

Discounted cost of m/c A -

Total Rs. 1512.38

Thus, the average yearly cost of m/c A is $\frac{1512.38}{3}$
$= 504.13$

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Years	Discounted Cost at 10% Rate (Rs)	
	Cost	Present worth
1	1700	$1700 \times \left(\frac{100}{100+10}\right)^0 = 1700.00$
2	100	$100 \times (100/110)^1 = 100 \times 0.9091 = 90.91$
3	200	$200 \times \left(\frac{100}{110}\right)^2 = 200 \times 0.8264 = 165.28$
4	300	$300 \times \left(\frac{100}{110}\right)^3 = 300 \times 0.7513 = 225.39$
5	400	$400 \times \left(\frac{100}{110}\right)^4 = 400 \times 0.6830 = 273.20$
6	500	$500 \times \left(\frac{100}{110}\right)^5 = 500 \times 0.6209 = 310.45$

Discounted Cost of M/C B.

Total Rs. 2765.23

Thus, the average yearly cost of m/c 'B' is $\frac{2765.23}{6}$

$\boxed{= 460.87}$

With the data, on average yearly cost of both the m/c's, the apparent advantage is in purchasing m/c B. But the periods for which the costs are considered are different. Therefore, let us first calculate the total present worth of m/c 'A' for six years.

Total present worth $= 1000 + 200 \times 0.9091 + 400 \times 0.8264 + 1000 \times 0.7513 + 200 \times 0.6830 + 400 \times 0.6209 = \text{Rs. } 2648.64.$

average yearly Cost of m/c 'A' for 6 yrs $= 2648.64/6 = 441.44$

This is less ~~than~~ than the total present worth of m/c B. Thus m/c A should be purchased.

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Year of Service n (1)	Running Cost (Rs) R(n) (2)	Cumm. Running Cost (Rs) ΣR(n) (3)	Depreciation Cost (Rs). C−S (4)	Total Cost (Rs) TC (5)=(3)+(4)	Average Cost (Rs), ATCn (6) = (5)/(1)
1	2000	2000	50000	52000	52000
2	6000	8000	50000	58000	29000
3	10000	18000	50000	68000	22667
4	14000	32000	50000	82000	20500
5	18000	50000	50000	100000	20000
6	22000	72000	50000	1,22000	20333

The per year avg. running cost of m/c B is lowest in the 5th year, i.e., Rs. 20000/- ∴ This cost is less than the lowest avg. running cost (Rs. 26,000) per year for m/c A. Hence, m/c A should be replaced by m/c B.

Now to find the time of replacement of m/c A by m/c B, the total cost of m/c A in the successive years is computed as follows:

Year	:	1	2	3	4
Total Cost incurred Rs	:	46,000 —	57,000−46,000 =11000	78,000−57,000 = 21000	1,09,000 −78,000 = 31,000

M/c A should be replaced by m/c B at the time (age) when m/c 'A''s running cost for the next year exceeds the lowest average running cost (Rs. 20,000) per year of m/c B.

calculations show that the running cost (Rs. 21000) of m/c A in the third year is more than the lowest average cost (Rs. 20000) of m/c B. Hence, m/c A should be replaced by m/c B after two years.

Model 2:- when money value is counted:

Value of money Criterion :- If the effect of the time-value of money is to be considered then replacement decision analysis must be based upon an equivalent annual cost. For eg, if the interest rate on Rs. 100 is 10% per year; then the value of Rs. 100 to be spent after one year will be Rs. 110/-. This is also called value of money. Also, the value of money that decreases with constant rate is known as its depreciation ratio or discounted factor. The discounted value is the amount of money required at the time of the policy decision in order to build up funds at compound interest large enough to pay the required cost when due. For eg, if the interest rate on Rs. 100 is 'r' percent per year, then the present value of Rs 100 to be spent after 'n' years will be:

$$\boxed{d = \left(\frac{100}{100+r}\right)^{n}}$$

where

d = discount rate (or) depreciation value. After having an idea of discounted cost, the objective should be to determine the critical age at which an item should be replaced so that

Eg:- A Company is considering the purchase of a new m/c at Rs. 15000/-. The economic life of the m/c is expected to be 8 years. The salvage value of the m/c at the end of the life will be Rs. 3000/-. The annual running cost is estimated to be Rs. 7000/-. (17)

(a) Assuming an interest rate of 5 %, determine the present worth of future costs of the proposed m/c.

(b) Compare the new m/c with the presently-owned m/c that has an annual operating cost of Rs. 5000/- & cost of repair Rs 1500/- in the second year, with an annual increase of Rs. 500 in the subsequent years of its life. (Note:- Salvage value = Estimated resale value)

Soln:- (a) New m/c.

(i) Purchase cost C = Rs. 15000

(ii) Present worth of annual operating cost

= 7000 x Pwf at 5% interest for 8 years

$= 7000 \times \left(\frac{100}{105}\right)^8 = 7000 \times 0.6768 = Rs.$ 4737.60,

(iii) Present worth of the salvage value

= 3000 x Pwf at 5% interest for 8 years

$= 3000 \times \left(\frac{100}{105}\right)^8 = 3000 \times 0.6768 = Rs\ 2030.40$

Thus, the present worth of total future costs for new m/c for 8 yrs, will be

$15000 + 4737.60 - 2030 = 17{,}707/-$

(b) Old m/c.

The calculations of the present worth of the old m/c are shown in table:

Yr. of Service	Operating Cost (Rs).	Repair Cost (Rs)	Total Operating & Repair Cost (Rs)	Pwf for single payment	Present worth (Rs).
1	5000	–	5000	0.9524	5000×0.9524 =4762.00
2	5000	1500	6500	0.9072	5895.50
3	5000	2000	7000	0.8638	6046.60
4.	5000	2500	7500	0.8227	6252.52
5	5000	3000	8000	0.7835	6268.00
6	5000	3500	8500	0.7462	6342.70
7	5000	4000	9000	0.7107	6396.30
8	5000	4500	9500	0.6778	6429.60
				Total Rs.	44,103.20

Since the present worth of old m/c as shown in above table is more than that of the new m/c, the new m/c should be purchased.

Replacement of Items that Fail Completely :-

The examples of this type of models are bulbs, resistors, transistors, etc.

The following assumptions are made in this model:

1. The system contains a large no. of identical items.
2. The individual replacement cost per item is greater than the group replacement cost per item.

There are two types of replacement policies:

1. Individual replacement policy.
2. Group replacement policy.

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Individual replacement policy:

Under this policy, an item is replaced as soon as it fails. The no. of items failed per unit time is determined by using mortality theorem. According to mortality theorem,

$$\text{No. of failures/unit time} = \frac{\text{Total no. of items}}{\text{Average age of the item}}.$$

Group Replacement policy:

Under this policy, items are replaced individually as and when they fail till optimum period T. After time T, all the items are replaced irrespective of the fact that items have failed or items have not failed. Here the problem is to find the optimum period T.

eg:- A Computer contains 10,000 resistors. When any resistor fails, it is replaced. The cost of replacing a resistor individually is ₹.10 only. If all the resistors are replaced at a time, the cost of resistor would be reduced to ₹3.50. The percent surviving by the end of month 't' is shown in below table.

Percent surviving by the end of month

Month(t)	0	1	2	3	4	5	6
percent surviving by the end of the month	100	97	90	70	30	15	0

What is the optimum replacement plan?

Sol^n :- The data are given in table below:

Data table

Month (t)	1	2	3	4	5	6
Percent surviving by the end of the month	97	90	70	30	15	0
Percent failing at the end of the month	3	10	30	70	85	100
Per cent failing in each month	3	7	20	40	15	15
Probability of failure in each month	0.03	0.07	0.2	0.4	0.15	0.15

Total no. of resistors in a computer = 10000

The cost of replacing single resistor = ₹. 10/-

For group replacement of the cost per resistor = ₹3.50/-

Individual replacement policy:-

Average age of the resistor $= (0.03 \times 1) + (0.07 \times 2) + (0.2 \times 3) + (0.4 \times 4) + (0.15 \times 5) + (0.15 \times 6)$

$= 4.02$ months.

Average no. of resistors failed in each month $= \frac{10000}{4.02}$

$= 2488$ resistors.

Individual replacement cost $= 2488 \times 10 =$ ₹. 24880/month.

Group replacement policy :-

No. of resistors failed in each month is :

1st month $= N_1 = 10000 \times 0.03 = 300$

2nd " $= N_2 = 10000 \times 0.07 + 300 \times 0.03 = 709$

3rd month $= N_3 = 10000 \times 0.2 + 300 \times 0.07 + 709 \times 0.03 = 2042$.

4th month $= N_4 = 10000 \times 0.4 + 300 \times 0.2 + 709 \times 0.07 + 2042 \times 0.03 = 4171$

5th month $= N_5 = 10000 \times 0.15 + 300 \times 0.4 + 709 \times 0.2 + 2042 \times 0.07 + 4171 \times 0.03 = 2030.$

6th month $= N_6 = 10000 \times 0.15 + 300 \times 0.15 + 709 \times 0.14 + 2042 \times 0.2 + 4171 \times 0.07 + 2030 \times 0.03$

$= 2590.$

To determine the optimum group replacement period:

The month at which all the resistor are to be replaced is shown in below table.

Table: Determination of the month at which all the resistors are to be replaced.

Month	No. of failures in each month	Cummulative no. of failures at the end of month	Individual replacement cost (₹)	Total cost = Individual replacement cost + Group replacement cost (₹)	Avg. cost = Total cost / Month (₹)
1	300	300	3000	35000+3000 = 38000	38000
2	709	1009	10090	45090	22,545
3	2042	3051	30510	65,510	21836
4	4171	7222	72220	107,220	26 805
5	2030	9252	92520	127,520	25504
6	2590	11,842	118420	153,420	25,570.

Note: group replacement cost = 10000 × 3.50 = 35000/-

Minimum group replacement cost/month = ₹. 21,836/-.

From above table, the optimum group replacement period is at the end of 3rd month. The group replacement cost per month is less than the individual replacement cost per month. So, decision is to follow group replacement policy.

i.e., $\underset{\text{(group replacement cost)}}{21,836/-} \leq \underset{\text{(Individual replacement cost)}}{24880/-}$

Eg:- The mortality rate shown in table below was observed for a certain type of light bulb.

Table :- Problem.

Week (t)	1	2	3	4	5
Percent failing at the end of week	10	25	50	80	100

There are ~~&~~ 1000 bulbs in use and it's costs ₹.10/- to replace an individual bulb, which has burnt out. If all the bulbs were replaced simultaneously, it ~~was~~ would cost ₹.4 per bulb. It is proposed to replace all the bulbs as and when they fail. At what intervals, all the bulbs should be replaced.] At what group replacement price per bulb would a policy of strictly group replacement become ~~for~~ preferable to the individual replacement policy?

Soln:- Number of bulbs = 1000

The cost of replacing single bulb = ₹.10/-

For group replacement the cost per bulb = ₹.4/-

The ~~g~~ data are given in below table.

Table :- Data

Week (t)	1	2	3	4	5
percent failing at the end of week	10	25	50	80	100
Number of bulbs failed in each week	10	15	25	30	20
Probability of failure in each week	0.1	0.15	0.25	0.3	0.2

Individual replacement policy :-

Average age of the bulb $= (0.1 \times 1) + (0.15 \times 2) + (0.25 \times 3) + (0.3 \times 4) + (0.2 \times 5)$
$= 3.35$ weeks.

Average no. of bulbs failed in each week $= \frac{1000}{3.35} = 299$ bulbs.

Individual replacement cost $= 299 \times 10 = 2990$ /week

Group replacement policy :-

No. of bulbs failed in each week is :-

1st week $= N_1 = 1000 \times 0.10 = 100$.

2nd week $= N_2 = 1000 \times 0.15 + 100 \times 0.1 = 160$.

3rd week $= N_3 = 1000 \times 0.25 + 100 \times 0.15 + 160 \times 0.1 = 281$.

4th week $= N_4 = 1000 \times 0.3 + 100 \times 0.25 + 160 \times 0.15 + 281 \times 0.1 = 377$

5th week $= N_5 = 1000 \times 0.2 + 100 \times 0.3 + 160 \times 0.25 + 281 \times 0.15$
$+ 377 \times 0.1 = 350$.

6th week $= N_6 = 1000 \times 0 + 100 \times 0.2 + 160 \times 0.3 + 281 \times 0.25 + 377 \times 0.15$
$+ 350 \times 0.1 = 230$.

The optimum group replacement period is shown in table below

Table: Determination of the optimum group replacement period.

Week	No. of bulbs failed in each week	Cumulative no. of bulbs failed	Individual replacement cost (₹)	Total Cost = Individual replacement Cost + group replacement Cost (₹)	Average Cost = Total cost / week (₹)
1	100	100	1000	5000	5000
2	160	260	2600	6600	3300
3	281	541	5410	9410	3136
4	377	918	9180	13180	3295
5	350	1268	12680	16680	3336

From the last table of previous Questⁿ, the optimum group replacement period is at the end of 3rd week.
The group replacement Cost per week is more than the

individual replacement cost per week. So, the optimum replacement policy is individual replacement policy.

Determining group replacement price per bulb at which group replacement is economical: In order that group replacement is economical then the average cost should be less than 2990. Then we have to reduce the group replacement cost per bulb.

$$\frac{1000x + 541 \times 10}{3} = 2990.$$

$$1000x + 5410 = 8970.$$

$$1000x = 3560.$$

$$x = ₹.356.$$

Group replacement cost / bulb is less than ₹.3.56 group replacement is preferred.