Unit-IV Theory of Games

Introduction :-

Game is defined as an activity b/w two or more persons involving activities by each person according to ~~the~~ a set of rules, at the end of which each person receives some benefit or suffers loss. For eg, bidding, advertisements, sports, elections, etc.

→The mathematical analysis of game is fundamentally based upon the minimax criterion of J. Von Neumann (Called upon the Father of Game Theory). Game theory is only based on this assumption. The primary contribution of the game theory is only the theoretical aspects rather than its application for solving the real problems.

Characteristics of Games :-

The following are the characteristics of games :-

① **Chance or strategy** :- If in a game activities are determined by skill, it is said to be a game of strategy; If the game is determined by chance, it is a game of chance. In general, a game may involve game of strategy as well as game of chance.

② **Number of persons** :- A game is called N persons game if the number of persons playing the game is N.

③ **Number of strategies** :- The no. of strategies for each player is finite or infinite.

④ **Payoff** :- A quantitative measure of satisfaction which a person gets at the end of each play is called payoff.

⑤ Two-person Zero sum games or rectangular games:- A game with only two players is called a two-person zero sum game, if the losses of one player are equivalent to the gain of the other, so that the sum of their net gains is zero.

⑥ Strategy:- There are two types of strategies:-

ⓐ Pure strategy:- If the player knows exactly what the other player is going to do, then the problem is pure strategy problem. Pure strategy games are called games with the saddle point.

ⓑ Mixed strategy problem:- If a player is guessing which activity the other player selects, a probablistic situation is obtained and the objective is to maximize the expected gain. Thus, the mixed strategy is selection among strategies with fixed probabilities.

⑦ Saddle point:- A saddle point of a payoff matrix is the position of such element in the payoff matrix which is minimum in its row & maximum in its column. A game may have more than one saddle point.

Rules for determining a saddle point:-

ⓐ Select the minimum element of each row of the payoff matrix and mark them by 'O'.

ⓑ Select the maximum element of each column of the payoff matrix and mark them by '□'.

ⓒ If there appears an element in the payoff matrix marked by 'O' & '□' both, the position of element is a saddle point of the payoff matrix.

Minimax (Maximin) Criterion of optimality

→ The minimax (maximin) criterion of the optimality states that if a player ~~limits~~ lists worst possible outcomes of all his potential strategies, he will choose that strategy which corresponds to the best of the worst outcomes. Such a strategy is called an optimal strategy. All the games are based on the minimax (maximin) criterion.

Models in Game Theory Problems:-

They are:

Model 1: Game with a saddle point (or) pure strategy problems.

Model 2: Game without a saddle point (or) mixed strategy problems.

ⓐ 2×2 matrix

ⓑ 2×n matrix

ⓒ m×2 matrix

ⓓ m×n matrix

Model 1:- Game with a Saddle Point (or) Pure Strategy Problems

The first step in any game theory problem is to find out a saddle point. If the problem has a saddle point, then it is a pure strategy problem.

Problem:- Solve the game matrix shown in table, which represents the payoff to the player A. Find optimal strategy.

		Player B		
		I	II	III
Player A	I	-3	-2	6
	II	2	0	2
	III	5	-2	-4

Sol^n:- The given matrix is a payoff matrix for the player A. So the numbers ~~of~~ in the payoff matrix are gains for the player A and losses for the player B.

The first step in any game theory problem is to find a saddle point. Determine the saddle point by encircling the minimum value of each row and putting the square around maximum value of each column. The resultant matrix is given ~~in~~ table below. For convenience, minimum values are shown by 'O' & maximum values by '□' in the table.

Payoff matrix for ~~the~~ player A.

Player A		Player B		
		I	II	III
	I	-③	-2	$\boxed{6}$
	II	2	$\boxed{⓪}$	2
	III	5	-2	-④

This game has a saddle point. So, the problem is a pure strategy problem.

ⓐ The best strategy for the player A is II.

ⓑ The best strategy for the player B is II.

ⓒ The value of game is 0.

Eg:- Solve the game whose payoff matrix for the player A is given in table.

		Player B		
		I	II	III
Player A	I	-2	15	-2
	II	-5	-6	-4
	III	-5	20	-8

Soln:- The given matrix is a payoff matrix for the player A. So, the numbers in the payoff matrix are gains for the player A and losses for the player B.

The first step in any game theory problem is to find a saddle point. Determine the saddle point by encircling the minimum value of each row and putting the square around maximum value of each column. The resultant matrix is given in table below. For convenience, minimum value are shown by 'O' and maximum values by '□' in the table.

payoff matrix for the player A.

	~~player B~~		
	~~I~~	~~II~~	~~III~~
~~I~~	~~-2~~	~~15~~	~~-2~~
~~II~~	~~-5~~	~~-6~~	~~-4~~
~~III~~	~~-5~~	~~+20~~	~~-8~~

		Player B		
		I	II	III
Player A	I	-2	15	-2
	II	-5	-6	-4
	III	-5	+20	-8

This game has two saddle points.
So the problem is pure strategy problem.

ⓐ The best strategy for the player A is I.

ⓑ The best strategy for the player B is I (or) III.

ⓒ The value of game is -2.

Q:- Find the range of values p & q, which will render (2,2) as a saddle point.

		Player B		
		I	II	III
Player A	I	2	4	5
	II	10	7	q
	III	4	p	6

Sol:- The given matrix represents gains for the player A & losses for the player B. Since the entry (2,2) is a saddle point, which is minimum in its row and maximum in its Column.

The resultant matrix is given in below table.

Payoff matrix for the player A

		Player B		
		I	II	III
Player A	I	2	4	5
	II	10	$\boxed{7}$	q
	III	4	p	6

ⓐ 7 is minimum of row, so $q \geq 7$.

ⓑ 7 is maximum of Column, So ~~q ≤ 7~~. $p \leq 7$.

Model 2:- Game without a saddle point (or) Mixed Strategy

Graphical Method:- It is used to solve the games whose payoff matrix has ① Two rows & n columns (2×n) ② m rows; 2 Column (m×2)

ⓐ **2xn matrix:-**

Algorithm for Solving 2xn matrix games.

* Draw two vertical axes 1 unit apart. The two lines are $x_1 = 0$, $x_1 = 1$ ('x_1' is the probability).

* Take the points of the first row in the payoff matrix on the vertical line $x_1 = 1$ & the points of the second row in the payoff matrix on the vertical line $x_1 = 0$.

* The point 'a_{1j}' on axis $x_1 = 1$ & is then joined to the point 'a_{2j}' on the axis $x_1 = 0$ to give a straight line. Draw 'n' straight lines for $j = 1, 2 \ldots n$ and determine the highest point of the lower envelope obtained. This will be the maximin point.

* The two or more ~~l~~ lines passing through the maximum point determines the required 2x2 payoff matrix. This in turn gives the optimum solution by making ~~the~~ use of analytical method.

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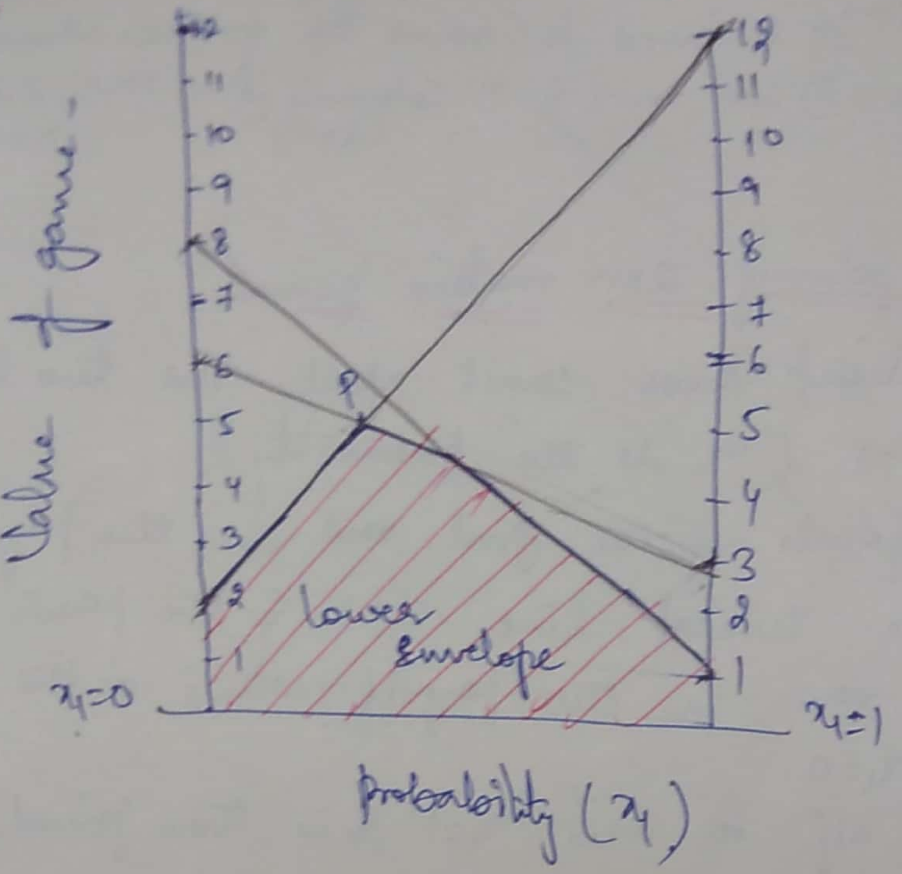
eg 1:- Solve by the graphical method the following game matrix.

$$\begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline A_1 & 1 & 3 & 12 \\ A_2 & 8 & 6 & 2 \end{array}$$

Soln:-



P= maximin point.

maximin pt = maximum of the minimum gains.

Eliminating (B_1) strategy ($\because$ it is not coinciding with point P).

the reduced payoff matrix is;

$$\begin{array}{c|cc} & B_2 & B_3 \\ \hline A_1 & 3 & 12 \\ A_2 & 6 & 2 \end{array}$$

optimal Strategy for $A = S_A = [x_1, x_2]$

" " $B = S_B = [y_2, y_3]$ ($\because$ 'y_1' $= 0$]

⑧ ⑤

$$x_1 = \frac{a_{22}-a_{21}}{(a_{11}+a_{22})-(a_{21}+a_{12})}, \quad x_2 = \frac{a_{11}-a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$$

$$y_2 = \frac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{21}+a_{12})} \; ; \quad y_3 = \frac{a_{11}-a_{21}}{(a_{11}+a_{22})+(a_{21}+a_{12})}$$

$$x_1 = \frac{2-6}{(3+2)-(6+12)} \; ; \quad x_2 = \frac{3-12}{(3+2)-(6+12)}$$

$$x_1 = \frac{-4}{+13} = \frac{4}{13} \qquad = \frac{-9}{-13} = 9/13.$$

$$y_2 = \frac{2-12}{(3+2)-(6+12)} \; ; \quad y_3 = \frac{3-6}{(3+2)-(6+12)}$$

$$= \frac{-10}{-13} = 10/13 \qquad = \frac{-3}{-13} = 3/13.$$

$$\therefore S_A = \left[\frac{4}{13}, \frac{9}{13}\right]; \quad S_B \left[0, \frac{10}{13}, \frac{3}{13}\right].$$

$$\text{Value of game} = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11}+a_{22})-(a_{21}+a_{12})} = \frac{6-72}{-13} = \frac{66}{13}.$$

$$\begin{array}{c|cc} & I & II \\ \hline I & 4 & 2 \\ II & -1 & 7 \\ III & 5 & 3 \end{array} \qquad \begin{array}{c|cc} & I & II \\ \hline I & 6 & 3 \\ II & 2 & 4 \\ III & 9 & 3 \end{array}$$

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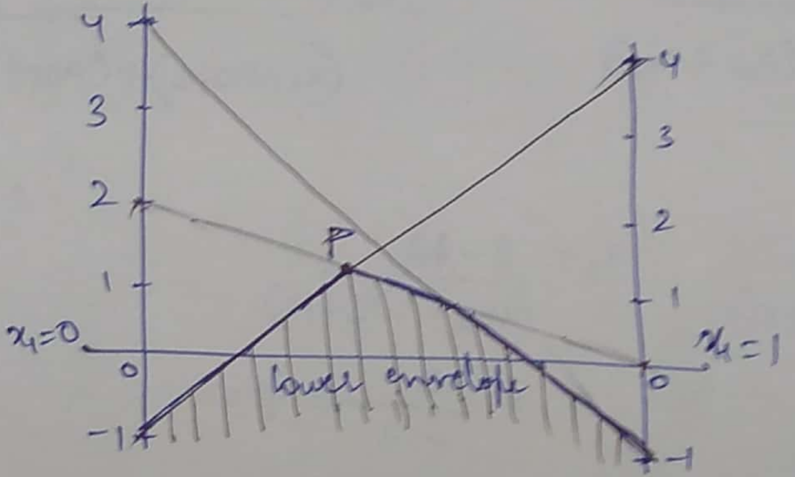
eg 2 :- Solve by the graphical method the following game.

$$\begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline A_1 & 4 & -1 & 0 \\ A_2 & -1 & 4 & 2 \end{array}$$

sol^n :-



P = maximin point.

Eliminating B_2 strategy :- ($\because$ it is not coinciding with point P.)

$$\begin{array}{c|cc} & B_1 & B_3 \\ \hline A_1 & 4 & 0 \\ A_2 & -1 & 2 \end{array}$$

$$V = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11}+a_{22}) - (a_{21}+a_{12})} = \frac{8-0}{6+1} = \frac{8}{7}.$$

$$S_A = \left[\frac{a_{22}-a_{21}}{(a_{11}+a_{22})-(a_{21}+a_{12})}, \frac{a_{11}-a_{12}}{(a_{11}+a_{22})-(a_{21}+a_{12})} \right] = \left[\frac{3}{7}, \frac{4}{7}\right]$$

$$S_B = \left[\frac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{21}+a_{12})}, 0, \frac{a_{11}-a_{21}}{(a_{21}+a_{22})-(a_{12}+a_{21})} \right] = \left[\frac{2}{7}, \frac{5}{7}\right]$$

Algorithm for solving mx2 matrix games.

- Draw two vertical axes 1 unit apart. The two lines are $x_1=0, x_1=1$
- Take the points of first row in the payoff matrix on the vertical line $x_1=1$ & the points of the second row in the payoff matrix on the vertical line $x_1=0$.
- The point a_{ij} on axis $x_1=1$ is then joined to the point a_{2j} on the axis $x_1=0$ to give a straight line. Draw 'n' straight lines for $j=1,2,---n$. and determine the lowest point of the upper envelope obtained. This will be the minimax point.
- The two or more lines passing through the minimax point determines the required 2x2 payoff matrix. This in turn gives the optimum solution by making use of analytical method.

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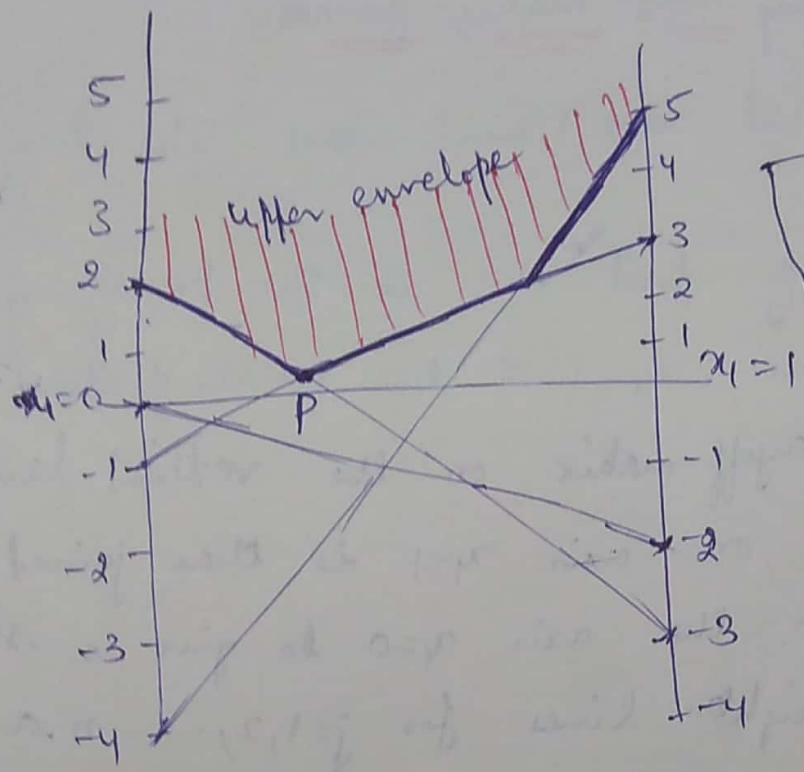
eg1:- Solve by the graphical method the following game.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{cc} B_1 & B_2 \\ \end{array}$$

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & -2 & 0 \\ A_2 & 3 & -1 \\ A_3 & -3 & 2 \\ A_4 & 5 & -4 \end{array}$$

Soln:- First we draw the graph for the given payoff matrix to Convert the given matrix to 2x2 form.



P = minimax point

= minimum out of the max losses.

eliminating A1, A4 [∵ these two are not ~~touching~~ (coinciding) with point P]

∴ we get a reduced 2×2 matrix;

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_2 & 3 & -1 \\ A_3 & -3 & 2 \end{array}$$

$$\text{strategy for } A = \left[0, \frac{a_{22} - a_{21}}{(a_{11}+a_{22})-(a_{21}+a_{12})}, \frac{a_{11}-a_{12}}{(a_{11}+a_{22})-(a_{21}+a_{12})}, 0\right]$$

$$= \left[0, \frac{(2+3)}{(3+2)-(-1-3)}, \frac{(3+1)}{(3+2)-(-1-3)}, 0\right]$$

$$S_A = \left[0, \frac{5}{9}, \frac{4}{9}, 0\right]$$

Strategy for $B = \left[\frac{a_{22} - a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}, \frac{a_{11} - a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})} \right]$

$$= \left[\frac{2+1}{9}, \frac{3+3}{9} \right]$$

$$S_B = \left[\frac{3}{9}, \frac{6}{9} \right]$$

Value of game $= \left[\frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11}+a_{22})-(a_{21}+a_{12})} \right]$

$$= \frac{6-3}{9} = \frac{3}{9} = \frac{1}{3}.$$

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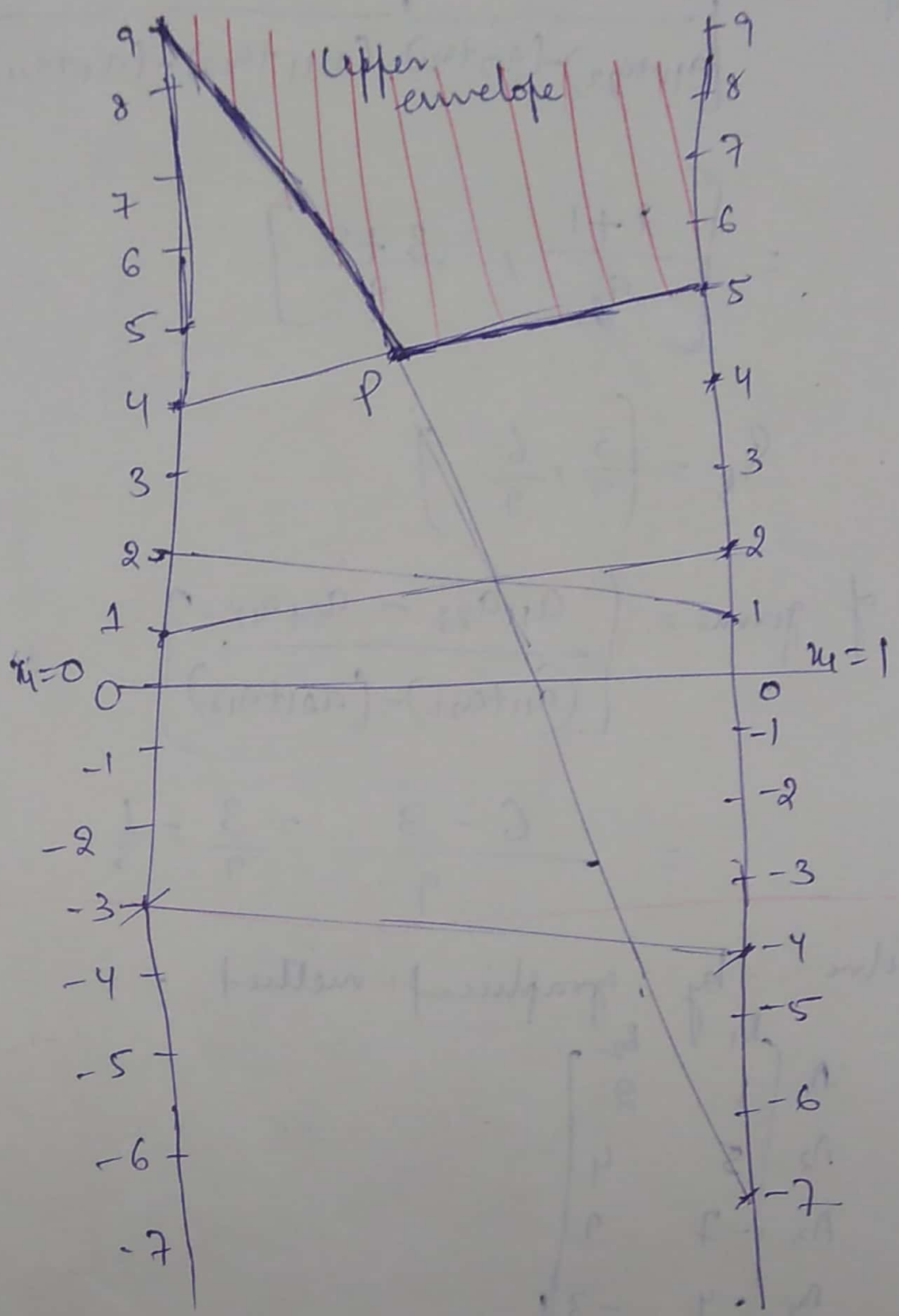
eg 2:- Solve by graphical method.

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & 1 & 2 \\ A_2 & 5 & 4 \\ A_3 & -7 & 9 \\ A_4 & -4 & -3 \\ A_5 & 2 & 1 \end{array}$$

Soln:- In the step1; we draw a graph to reduce the size of a matrix from (2×5) to (2×2).



P = mini max point.

= minimum of the maximum losses.

eliminating A1, A4, A5 ($\because$ not coinciding with point P),

we get the reduced 2x2 matrix as;

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_2 & 5 & 4 \\ A_3 & -7 & 9 \end{array}$$

Value of game = (v) =

$$V = \frac{a_{11}\,a_{22} + a_{21}\,a_{12}}{(a_{11}+a_{22})-(a_{21}+a_{12})} = \frac{45+28}{14+3} = \frac{73}{17}.$$

$$\text{Strategy of } A = \left[0, \frac{a_{22}-a_{21}}{(a_{11}+a_{22})-(a_{21}+a_{12})}, \frac{a_{11}-a_{12}}{(a_{11}+a_{22})-(a_{21}+a_{12})}, 0, 0\right]$$

$$S_A = \left[0, \frac{9+7}{17}, \frac{5-4}{17}, 0, 0\right] = \left[0, \frac{16}{17}, \frac{1}{17}, 0, 0\right]$$

$$\text{Strategy of } B = \left[\frac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{21}+a_{12})}, \frac{a_{11}-a_{21}}{(a_{11}+a_{22})-(a_{21}+a_{12})}\right]$$

$$S_B = \left(\frac{9-4}{17}, \frac{5+7}{17}\right) = \left[\frac{5}{17}, \frac{12}{17}\right]$$

Dominance Principle:-

Rules of Dominance ~~of~~ to reduce the size of matrix:-

The following are the rules of dominance to reduce the size of the matrix:

(1) If each element in one row is ≤ to the corresponding element in the other row, then the

row Corresponding to ≤ is inferior strategy. So remove the inferior strategy (the row which is having less values).

2. If each element in one Column is ≥ to the Corresponding element in the other Column then, the Column Corresponds to ≥ is the inferior strategy. So, remove the inferior strategy (the Column which is having more values).

3. A given strategy can be dominated if it is inferior to an average of two (or) more other pure strategies.

Game With Mixed Strategies

Consider the following payoff matrix with respect to player A and solve it optimally.

$$\begin{array}{c|cc|l} & \text{Player B} & & \text{Row Minima} \\ \hline \text{Player A} & 9 & 7 & \boxed{7} \to \text{Maximin} \\ & 5 & 11 & 5 \\ \hline \text{Column Maxima} \to & \boxed{9} & 11 & \\ & \text{Minimax} & & \end{array}$$

$$\text{Maximin} \neq \text{Minimax}$$

$$7 \neq 9.$$

∴ No saddle point exists

$$\begin{array}{c|cc|c} & & & \text{oddments} \\ \hline & 9 & 7 & 6 \\ & 5 & 11 & 2 \\ \hline \text{oddments} & 4 & 4 & \end{array}$$

Probability
$\Downarrow$

$$P_1 = \frac{6}{2+6} = \frac{3}{4}$$

$$P_2 = \frac{2}{2+6} = \frac{1}{4}$$

Probability $\Rightarrow q_1 = \frac{4}{4+4} \quad q_2 = \frac{4}{4+4}.$

$$q_1 = \frac{1}{2} \; ; \; q_2 = \frac{1}{2}$$

Value of game(v) $= \frac{(9\times 6)+(5\times 2)}{6+2} = \frac{54+10}{8} = 8.$

(or).

$V = \frac{(7\times 6)+(11\times 2)}{6+2} = 8$ (or) $V = \frac{(9\times 4)+(7\times 4)}{4+4} = 8$

(or) $V = \frac{(5\times 4)+(11\times 4)}{4+4} = 8.$

Hence, the strategies of player A = $\left(\frac{3}{4}, \frac{1}{4}\right)$

the Strategies of player B = $\left(\frac{1}{2}, \frac{1}{2}\right)$

Value of game = 8.

Solve the following game using Dominance Principle: (Pure strategy problem)

Player B.

		1	2	3	4	5	Row Minima.
	1	4	6	5	10	6	4
Player A	2	7	8	5	9	10	5
	3	8	9	11	10	9	⑧ Maximin
	4	6	4	10	6	4	4
Column Maxima		⑧ Minimax	9	11	10	10	

Minimax = Maximin = Saddle pt = 8.

V = 8.

Using Dominance principle;

Player B		1	2	3	4	5	
	~~1~~	~~4~~	~~6~~	~~5~~	~~10~~	~~6~~	㉛
Player A	2	7	8	5	9	10	㊴
	3	8	9	11	10	9	47
	~~4~~	~~6~~	~~4~~	~~10~~	~~6~~	~~4~~	㉚

Reduced matrix

Player B

		1	~~2~~	3	~~4~~	~~5~~
Player A	2	7	~~8~~	5	~~9~~	~~10~~
	3	8	~~9~~	11	~~10~~	~~9~~
		15	~~17~~	16	~~19~~	~~19~~

Reduced matrix, Player B

$$\begin{array}{c|cc|c} & 1 & 3 & \\ \hline \cancel{2} & \cancel{7} & \cancel{5} & ⑫ \\ 3 & ⑧ & \cancel{11} & ⑲ \end{array}$$

Player A

Player A's optimal strategy is '3'.
Player B's optimal strategy is '1'.

Value of game = 8.

Dominance Property with Mixed strategy Problem

Player B ~~Row Minima~~

$$\begin{array}{c|cccc|c} & 1 & 2 & 3 & 4 & \\ \hline 1 & 5 & -3 & 3 & 4 & 9 \\ 2 & -4 & 5 & 4 & 5 & 10 \\ \cancel{3} & \cancel{4} & \cancel{-4} & \cancel{-3} & \cancel{3} & 0 \end{array}$$

Player A

~~Column Maxima~~

Player B

$$\begin{array}{c|cccc} & 1 & 2 & 3 & \cancel{4} \\ \hline 1 & 5 & -3 & 3 & \cancel{4} \\ 2 & -4 & 5 & 4 & \cancel{5} \\ \hline & ① & ② & ⑦ & ⑨ \end{array}$$

Player A

Player B

$$\begin{array}{c|ccc} & 1 & 2 & \cancel{3} \\ \hline 1 & 5 & -3 & \cancel{3} \\ 2 & -4 & 5 & \cancel{4} \end{array}$$

Player A

$$\left.\begin{array}{l} \frac{5-3}{2} = 1 \\ \frac{-4+5}{2} = \frac{1}{2} \end{array}\right\}$$ average of first two columns

$$\begin{array}{cc|cc|c|l} & & \text{Player B} & & \text{oddments} & \text{Probability} \\ & & 1 & 2 & & \\ \hline \text{Player A} & 1 & 5 & -3 & 9 & P_1 = \frac{9}{9+8} = \frac{9}{17} \\ & 2 & -4 & 5 & 8 & P_2 = \frac{8}{9+8} = \frac{8}{17} \\ \hline \text{oddments} & & 8 & 9 & & \end{array}$$

Probability $q_1 = \frac{8}{8+9}$ $q_2 = \frac{9}{8+9}$

$q_1 = \frac{8}{17}$ $q_2 = \frac{9}{17}$

Value of game $= \frac{(5\times9)+(-4\times8)}{9+8} = \frac{45-32}{17} = \frac{13}{17}$.

Player A's optimal strategy $= \left[\frac{9}{17}, \frac{8}{17}\right]$

Player B's optimal strategy $= \left[\frac{8}{17}, \frac{9}{17}\right]$

Game with Saddle point

Eg: Find the optimum strategies of the players in the following game.

$$\begin{array}{cc|ccc|c} & & & \text{Player B} & & \text{Row Minima.} \\ & & 1 & 2 & 3 & \\ \hline & 1 & 25 & 20 & 35 & 20 \\ \text{Player A} & 2 & 50 & 45 & 55 & \boxed{45} \rightarrow \text{Maximin Value} \\ & 3 & 58 & 40 & 42 & 40 \\ \hline \text{Column Maxima} & & 58 & \underset{\text{Minimax Value.}}{\underset{\downarrow}{\boxed{45}}} & 55 & \end{array}$$

Minimax Value = Maximin Value.

45 = 45

Saddle point = 45. (2,2).

Value of game = 45.

Optimal strategy of the player A = $\overline{\underline{\text{II}}}$

Optimal strategy of the player B = $\overline{\underline{\text{II}}}$

~~Game~~

Unit VI Inventory.

Introduction:- The stock, which is kept in the stores for the smooth running of an organization, is called, inventory. Inventory may be 1 day stock or 15 days stock or 1 month stock or 1 year stock.

The inventory (or) stock of the goods may be kept in any of the following forms:-

① Raw material inventory.

Examples:

Factory	Raw material
Rice mill	Paddy
Oil refinery	Crude
Power plant	Coal
Cement plant	Lime stone
Automobile	steel.

② Semi-finished goods inventory: Inventory is kept in semi-finished form. Examples: Castings of pistons, Cylinders, etc.

③ Finished goods inventory: Inventory or stock is kept in finished goods form. Examples: Televisions, laptops, machine tools, cement bags, automobiles, ~~etc~~ etc.

④ Spareparts inventory: These spare parts are used when parts of a machine are worn out (or) failure of part in a machine.

Advantages of understocking:-

① Less space is required.

② Less Capital investment.

③ Less taxes

④ Less depreciation

⑤ Less damages.

Disadvantages of understocking:-

① More possibility of Company going out of stock.

② More no. of orders.

Advantages of overstocking:-

① Less possibility of Company going out of stock.

② Less no. of orders.

③ Price discounts when Company purchases large quantities

Disadvantages of overstocking:-

① More space is required.

② More Capital investment

③ More taxes

④ More depreciation

⑤ More damages.

Inventory decisions:-

There are two important decisions to be made regarding the inventory.

① Ordering quantity (or) batch quantity: Determining quantity to be ordered per each ~~other~~ order.

② Ordering Time: Time between the orders.

Costs involved in Inventory:

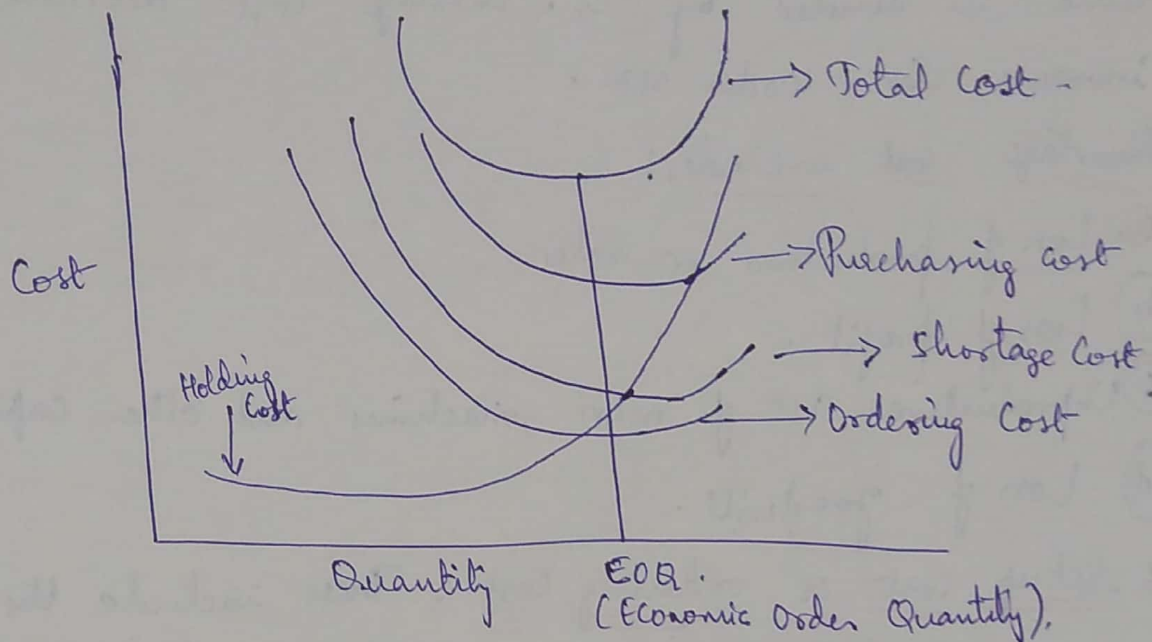


Fig:- Various cost Curves.

① Holding Cost (or) storage Cost (or) inventory Carrying cost: The cost associated with Carrying (or) holding the goods in stores. Holding cost increases with the increase of Cost. Holding cost per unit per unit time is denoted by C_1. Holding cost increases with increase of stock or inventory. Components of holding cost are:-

ⓐ Interest on invested capital.
ⓑ Cost of storage
ⓒ Taxes.
ⓓ Insurance.
ⓔ Depreciation Cost, etc.

up

② Shortage Cost (or) stock out cost: The penalty that is incurred as a result of shortage of stock. Production is stopped when raw material or spare parts are not available. The loss incurred due to the shortage of stock (or) spare part is called shortage cost. Shortage cost per unit per unit time is denoted by C_2. Shortage cost decreases with increase in order size.

Shortage cost includes:

ⓐ. Loss of production or sales.

ⓑ Loss of profit.

ⓒ Unproductive cost of men, machines and other capital resources.

ⓓ Loss of goodwill.

3. Setup cost or ordering cost :- These include the fixed cost associated with obtaining goods by placing an order. Ordering cost per order or setup cost per setup is denoted by C_3. Ordering cost decreases with increase in ordersize.

Ordering Cost includes:

ⓐ Cost of purchase

ⓑ Cost of stationery

ⓒ Transportation cost.

ⓓ Phone cost.

ⓔ Inspection costs

ⓕ Follow-up costs

Costs associated with setup of production:

ⓐ Rearrangement of machines.

ⓑ Cost of changing cutting tools, jigs, figures, etc.

ⓒ Unproductive cost of machines during changeover.

④ Purchasing Cost: If the supplier allows discount when the company purchases the quantities in large then purchasing

Cost is also a variable Cost. Purchasing cost decreases with increases in quantity ordered.

Total Cost of inventory:

Total Cost of inventory for time t = C = Purchasing cost + Holding cost + Ordering Cost + Shortage Cost.

Economic order quantity (EOQ): It is the quantity ordered per order so that total cost of inventory is minimum.

Classification of Inventory Models or Various Inventory Models:

① Deterministic Models: In deterministic models, demand is fixed and is known in advance.

Model 1: Uniform demand, instantaneous replenishment without shortage Cost. (simple EOQ model).

Assumptions made in simple EOQ model:

① Uniform demand

② Demand is known in advance.

③ Purchasing situation

④ Lead time is zero, ie, time between placing an order an replenishment of stock is zero.

⑤ Shortages are not allowed, so shortage cost per unit is ∞.

⑥ Purchasing price per unit is constant.

⑦ Holding cost is calculated based on average time spent by the material in the production system.

Notations used:

R = Demand rate

C_1 = Holding cost per unit per unit time.

C_2 = Shortage cost per unit per unit time.

C_3 = Ordering cost per order.

P = purchasing cost per ~~ood~~ unit

t = Time between production or purchasing orders.

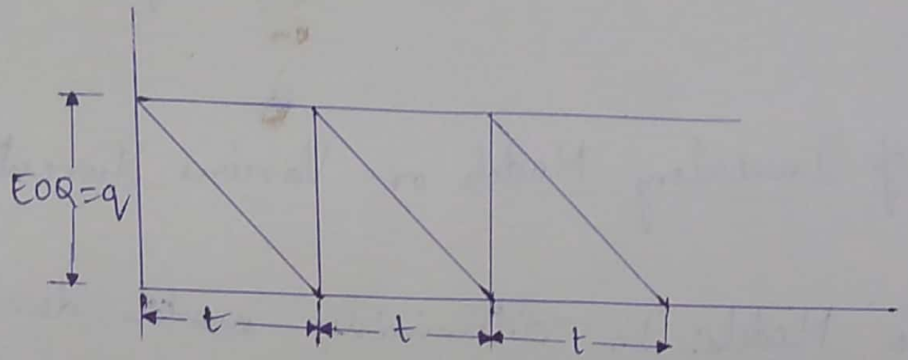


fig. Simple EOQ model 1.

In the simple EOQ model, there are two variable costs.

1. Holding Cost
2. Ordering Cost.

Description of model:

Let 'q' is the quantity ordered at the beginning of cycle t. There 'q' units are consumed in time 't' at the rate of R units.

So,

$$q = Rt$$

$$t = \frac{q}{R}.$$

Total inventory cost for time t = c = Holding Cost + Ordering Cost

$$= \left(\frac{q}{2}\right)C_1 t + C_3$$

Total inventory cost per unit time $= C = \dfrac{\frac{q}{2}C_1 t + C_3}{t}$

$$\Rightarrow C = \left(\frac{q}{2}\right)C_1 + C_3\left(\frac{1}{t}\right). \quad \text{——} ①$$

Substituting $t = \frac{q}{R}$ in eq^n ①

$$C = \left(\frac{q}{2}\right)C_1 + C_3\frac{R}{q} \quad \text{——} ②$$

$$\frac{dc}{dq} = \frac{C_1}{2} + C_3R\left(\frac{-1}{q^2}\right) = 0$$

$$C_3R\left(\frac{1}{q^2}\right) = \frac{C_1}{2}.$$

$$EOQ = q = \sqrt{\frac{2C_3R}{C_1}}$$

Substituting 'q' in eq^n ②

$$c = \frac{\sqrt{\frac{2C_3R}{C_1}}}{2}C_1 + C_3R\sqrt{\frac{C_1}{2C_3R}}.$$

Total Cost of inventory $= C = \sqrt{2C_1C_3R}$.

Total Cost of inventory including purchasing cost $= C$

$$C = \sqrt{2C_1C_3R} + PR.$$

Cycle time $= t = \frac{q}{R}$; No. of orders $= n = \frac{R}{q}$.

Q:- A particular item has demand of 9000 units per year. The cost of procurement is ₹100 and the holding cost per unit is ₹. 2.40 per year. The replacement is instantaneous and no shortages are allowed. Determine:

i) The economic lot size.

ii) The number of orders per year.

iii) The time b/w orders.

iv) The total cost per year, if the cost of one unit is ₹1.

Solⁿ:- R = Demand rate = 9000 units / year.

C_1 = Holding cost per unit per unit time = ₹. 2.40/unit/year

C_2 = Shortage cost per unit per ~~year~~ time = ∞.

C_3 = ordering cost per order = ₹100.

P = purchasing cost per unit = ₹1.

t = Time b/w production orders.

i) $EOQ = q = \sqrt{\frac{2C_3R}{C_1}}$

$EOQ = q = \sqrt{\frac{2\times 100\times 9000}{2.40}}$

EOQ = 866 units/year.

ii) $q = Rt$.

Cycle time(t) = Time b/w orders $= \frac{q}{R} = \frac{866}{9000} = 0.0962$ year

iii) Number of orders(n) $= \frac{R}{q} = \frac{9000}{866} = 10.4$ orders/year.

iv) Total cost of inventory including purchasing cost = C =

$= \sqrt{2C_1C_3R} + PR$.

Total cost of inventory $= \sqrt{2\times 2.4\times 100\times 9000} + 9000\times 1$ = ₹.11,080/year.

Eg:- A manufacturer has to supply 12000 units of a product ⑤ per year to the customer. The demand is fixed and the shortage cost is assumed to be infinite. The inventory holding cost is ₹ 0.20 per unit per ~~month~~ year and the setup cost per run is ₹.350. Determine :

i) The optimum run size.

ii) Optimum Scheduling period.

iii) Minimum total variable yearly cost.

Solⁿ:- R = Demand rate = 12000 units/year = 1000 units/month.

C_1 = Holding Cost per unit per ~~tim~~ unit time = ₹ 0.2/unit/month

C_2 = Shortage cost per unit per unit time = ∞.

C_3 = Ordering Cost per order = ₹.350.

i) $EOQ = q = \sqrt{\frac{2C_3R}{C_1}}$

$EOQ = q = \sqrt{\frac{2\times 350\times 1000}{0.2}}$ = 1870 units/run,

ii) Cycle time = $t = \frac{q}{R} = \frac{1870}{1000} =$ 1.87 months.

iii) Total cost of inventory = $C = \sqrt{2C_1C_3R}$.

Purchasing Cost is not given

Total cost of inventory excluding purchasing cost = C =

$= \sqrt{2\times 0.2\times 350\times 1000}$

= ₹374.1/month

= ₹ 4490/year.

Eg:- A stockist has to supply 400units of product every Monday to his customers. He gets the product at ₹ 50 per unit from the manufacturer. The cost of ordering and transportation from the manufacturer is ₹ 75 per order. The cost of carrying inventory is 7.5% per unit per year of the cost of the product. Find:

(i) The economic lot size.

(ii) The total optimal cost

Soln:- R = Demand rate = 400 units/week = 400 × 52 = 20,800 units/year

C_1 = Holding cost per unit per unit time = 0.075 × 50
= ₹ 3.75/unit/year.

C_3 = Ordering cost per order = ₹ 75.

P = Purchasing cost per unit = ₹ 50.

(i) $EOQ = q = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 75 \times (400 \times 52)}{3.75}}$ = 912 units/order.

(ii) Total cost of inventory including purchasing cost = C

$= \sqrt{2C_1C_3R} + PR$

$= \sqrt{2 \times 3.75 \times 75 \times 20800} + 20800 \times 50$

= ₹.10,43,420/year = ₹.86,951/month

= ₹ 20,065/~~month~~ week.

Model 2 :- Models with Price Breaks

So far we have described models in which purchasing cost is constant. However, many enterprises offer discounts for large purchases. Such discounts are typically referred ~~as~~ as quantity discounts or price breaks. In these models purchasing price /unit is also a variable cost.

Notations used:

R = Demand rate ; C_3 = Ordering Cost per unit - per unit time

C_1 = Holding Cost per unit per unit time

P = Purchasing Cost per unit

t = Time between production runs or purchasing orders.

I = Interest in percentage on one rupee capital invested /unit time.

In this EOQ model, there are three variables.

① Holding Cost

② Ordering Cost

③ Purchasing Cost

$$EOQ = q = \sqrt{\frac{2C_3R}{PI}}$$

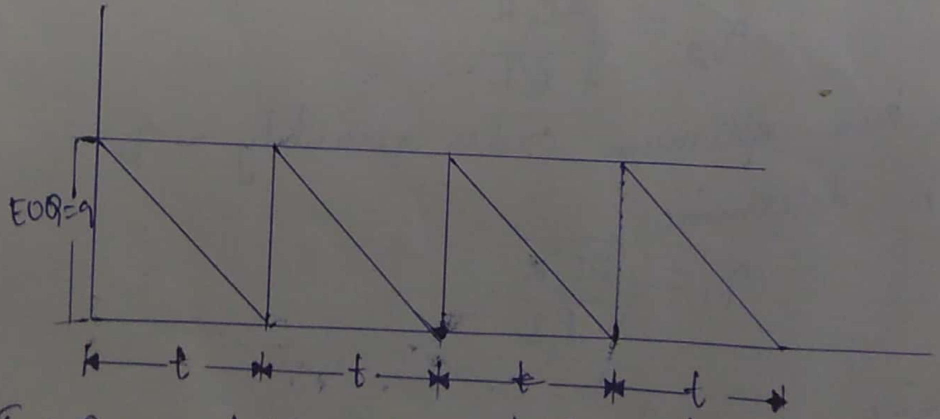


Fig:- Representation of simple price breaks model.

Total Cost Curves (C) for different prices are given in figure below. So Economic order quantity is determined by using search procedure presented in detailed below.

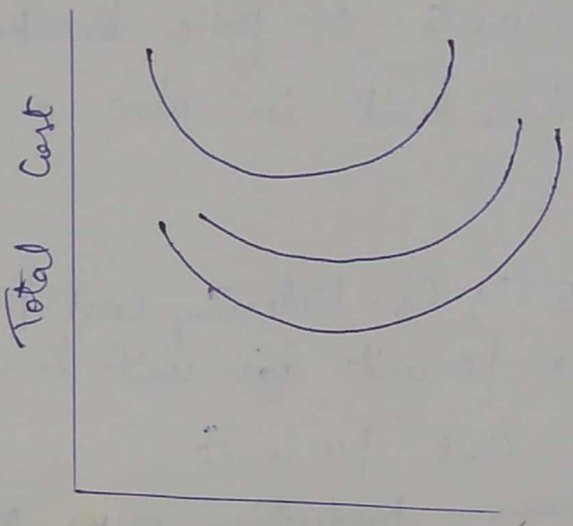


Fig.- Total Cost Curves for different purchasing prices per unit

Model with single price break:

Price/unit	Quantity purchased
P_1	$1 \leq q < b$
P_2	$q \geq b$

Steps to determine optimal order quantity:

Step 1: Determine

$$q_2 = \sqrt{\frac{2C_3R}{P_2I}}.$$

If $q_2 \geq b$ then optimum order quantity is q_2.

If $q_2 < b$ determine

$$q_1 = \sqrt{\frac{2C_3R}{P_1I}}.$$

Also determine,

⑦

$$c(q_1) = P_1R + \frac{C_3R}{q_1} + \frac{C_3I}{2} + \frac{P_1q_1I}{2}.$$

$$c(b) = P_2R + \frac{C_3R}{b} + \frac{C_3I}{2} + \frac{P_2bI}{2}.$$

whichever be the minimum cost, ie, optimal ordered quantity.

Eg:- find the optimum order quantity for a product for which the price breaks are as follows:

Quantity	Unit Cost (₹)
$0 \le q < 500$	10
$q \geq 500$	9.25.

The monthly demand for a product is 200 units. The cost of storage is 2% of unit cost per month and the cost of ordering is ₹350.

Soln:- R = Demand rate = 200 units/month.

$I = 0.02$ /unit/month.

C_3 = ordering cost per order = ₹350.

P_1 = ₹10/unit

P_2 = ₹.9.25/unit.

Steps to determine optimal order quantity:

Step 1: Determine

$$q_2 = \sqrt{\frac{2C_3R}{P_2I}} = \sqrt{\frac{2\times 350\times 200}{9.25\times 0.02}}$$

$q_2 = 870$ units

$q_2 \geq 500$

So optimum order quantity is q_2.

optimum order quantity $= q_2 = 870$ units.

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eg:- find the optimum order quantity for a product for which the price breaks are as follows:

Quantity	Unit Cost (₹)
$0 \le q < 500$	10
$q \ge 500$	9.25

The monthly demand for a product is 200 units. The Cost of storage is 2% of unit Cost per month and the Cost of ordering is ₹ 100.

Sol:- R = Demand rate = 200 units/month.

$I = 0.02$/unit/month

C_3 = Ordering Cost per order = ₹ 100.

P_1 = ₹ 10/unit

P_2 = ₹ 9.25/unit.

Steps to determine optimal order quantity:

Step 1: Determine ; $q_2 = \sqrt{\frac{2C_3R}{P_2I}} = \sqrt{\frac{2\times100\times200}{9.25\times0.02}}$

$q_2 = 465$ units

$q_2 < 500$, so goto step 2.

Step 2: $q_1 = \sqrt{\frac{2C_3R}{P_1I}} = \sqrt{\frac{2\times100\times200}{10\times0.02}}$.

$q_1 = 447$ units

$$C(q_1) = P_1R + \frac{C_3R}{q_1} + \frac{C_3I}{2} + \frac{P_1q_1I}{2}.$$

$$C(447) = 10\times200 + \frac{100\times200}{447} + \frac{100\times0.02}{2} + \frac{10\times447\times0.02}{2}$$

$$= ₹.\ 2{,}090.42/\text{month}$$

$$C(b_2) = P_2 R + \frac{C_3 R}{b_2} + \frac{C_3 I}{2} + \frac{P_2 b_2 I}{2}.$$

$$C(500) = 9.25 \times 200 + \frac{100 \times 200}{500} + \frac{100 \times 0.02}{2} + \frac{9.25 \times 500 \times 0.02}{2}$$

$$= ₹\ 1,937/\text{month}.$$

$$C(500) \le C(447).$$

So optimum order quantity is 500 units.

eg:- A Company uses annually 24,000 units of raw materials which Cost ₹1.25 per unit. Placing each ~~other~~ order costs ₹22.50 and the Carrying Cost is 5.4% per year of the average inventory. Find the economic lot size and total inventory cost (including Cost of material). Should the Company accept the offer made by the supplier of discount of 5% on the Cost price on a single order of 24,000 units?

Soln:- R = Demand rate = 24,000 units/year.

$I = 0.054$/unit/year.

C_3 = Ordering cost per order = ₹22.50.

p = purchasing price per unit = ₹1.25.

Case 1:- $$EOQ = q = \sqrt{\frac{2C_3R}{PI}} = \sqrt{\frac{2 \times 22.50 \times 24000}{0.054 \times 1.25}}$$

$q = EOQ = 4000$ units.

Case 2:- If the Company purchases 24000 units in single order then purchasing price per unit is $0.95 \times 1.25 = ₹1.187$.

(i.e. 5% discount)

[P.T.O]

$$c(q) = P_1 R + \frac{C_3 R}{q_1} + \frac{C_3 I}{2} + \frac{P_1 q_1 I}{2}.$$

$$C(4000) = 1.25 \times 24000 + \frac{22.5 \times 24000}{4000} + \frac{22.5 \times 0.054}{2} + \frac{1.25 \times 4000 \times 0.054}{2}$$

$$= ₹\,30{,}270\ /\text{year}.$$

$$C(b) = P_2 R + \frac{C_3 R}{b} + \frac{C_3 I}{2} + \frac{P_2 b I}{2}.$$

$$C(24000) = 1.18 \times 24000 + \frac{22.5 \times 24000}{4000} + \frac{22.5 \times 0.054}{2} + \frac{1.18 \times 24000 \times 0.054}{2}$$

$$c(24000) = ₹.\,29{,}292\ /\text{year}$$

The Company can accept the single order of 24000 units.

Model with no. of price breaks (Multiple price breaks):-

price /unit	Quantity purchased
p_1	$1 \le q < b_1$
p_2	$b_1 \le q < b_2$
p_3	$q \ge b_2$.

Steps to determine optimal order quantity:

Step 1: Determine

$$q_3 = \sqrt{\frac{2C_3 R}{P_3 I}}.$$

If $q_3 \ge b_2$ then optimum order quantity is q_3.

If $q_3 < b_2$ then go to step2.

Step2 : Determine

$$q_2 = \sqrt{\frac{2C_3R}{P_2I}}.$$

If $q_2 \geq b_1$ then determine

$$c(q_2) = P_2R + \frac{C_3R}{q_2} + \frac{C_3I}{2} + \frac{P_2q_2I}{2}.$$

$$c(b_2) = P_2R + \frac{C_3R}{b_2} + \frac{C_3I}{2} + \frac{P_2b_2I}{2},$$

If $q_2 < b_1$ then go to step3.

Step 3:- $$q_1 = \sqrt{\frac{2C_3R}{P_1I}}.$$

and also determine

$$C(q_1) = P_1R + \frac{C_3R}{q_1} + \frac{C_3I}{2} + \frac{P_1q_1I}{2}$$

$$c(b_1) = P_2R + \frac{C_3R}{b_1} + \frac{C_3I}{2} + \frac{P_2b_1I}{2}$$

$$c(b_2) = P_2R + \frac{C_3R}{b_2} + \frac{C_3I}{2} + \frac{P_2b_2I}{2}$$

whichever be the minimum Cost, ie, optimal order quantity.

eg:- find the optimal order quantity for which the price breaks are as follows:

Quantity	Unit Cost (₹)
$0 \leq q < 500$	10
$500 \leq q < 750$	9.25
$q \geq 750$	8.75

The monthly demand for a product is 200 units and the cost of storage is 2% of the unit cost per month and the Cost of ordering is ₹350.

Solⁿ:- R = Demand rate = 200 units/month

I = 0.02/unit/month

C_3 = Ordering cost per order = ₹350.

P_1 = ₹10/unit

P_2 = ₹9.25/unit

P_3 = ₹8.75/unit

Steps to determine optimal ordered quantity:

$$q_3 = \sqrt{\frac{2C_3R}{P_3I}} = \sqrt{\frac{2\times350\times200}{8.75\times0.02}} = 894 \text{ units}$$

$q_3 \geq 750$ then optimum order quantity is q_3 ie, 894.

eg:- The annual demand for a product is 64,000 units. The cost per order is ₹10 and the estimated cost of carrying one unit stock for a year is 20%. The normal price of the product is ₹10 per unit. However, the supplier offers a quantity discount of 2% on order of atleast 1000units at a time and a discount of 5%, if the order is atleast 5000 units. Suggest the most economical purchase quantity per order.

Solⁿ:- R = Demand rate = 64000 units/year.

I = 0.2/unit/year

C_3 = ordering Cost per order = ₹10.

P_1 = ₹10/unit.

P_2 = ₹9.8/unit

P_3 = ₹9.5/unit.

Quantity	Unit Cost (₹)
$0 \leq q < 1000$	10
$1000 \leq q < 5000$	9.8
$q \geq 5000$	9.5

Steps to determine optimal order quantity:

$$q_3 = \sqrt{\frac{2C_3R}{P_3I}} = \sqrt{\frac{2\times10\times64000}{9.5\times0.2}}$$

$q_3 = 820$ units

$q_3 < 5000$, go to step 2.

Step 2: $$q_2 = \sqrt{\frac{2C_3R}{P_2I}} = \sqrt{\frac{2\times10\times64000}{9.8\times0.2}}$$

$q_2 = 808$ units

$q_2 < 1000$, go to step 3.

Step 3: $$q_1 = \sqrt{\frac{2C_3R}{P_1I}} = \sqrt{\frac{2\times10\times64000}{10\times0.2}}$$

$q_1 = 800$ units

$$C(q_1) = P_1R + \frac{C_3R}{q_1} + \frac{C_3I}{2} + \frac{P_1q_1I}{2}.$$

$$C(800) = 10\times64000 + \frac{10\times64000}{800} + \frac{10\times0.2}{2} + \frac{10\times800\times0.2}{2}$$

$= ₹6,41,601.$

$$C(b_1) = P_2R + \frac{C_3R}{b_1} + \frac{C_3I}{2} + \frac{P_2b_1I}{2}.$$

$$C(1000) = 9.8\times64000 + \frac{10\times64000}{1000} + \frac{10\times0.2}{2} + \frac{9.8\times1000\times0.2}{2}$$

$= ₹.6,28,821.$

$$C(b_2) = P_2 R + \frac{C_3 R}{b_2} + \frac{C_3 I}{2} + \frac{P_2 b_2 I}{2}$$

$$C(5000) = 9.5 \times 64000 + \frac{10 \times 64000}{5000} + \frac{10 \times 0.2}{2} + \frac{9.5 \times 5000 \times 0.2}{2}$$

$$C(5000) = ₹.6{,}12{,}879.$$

Optimal order quantity = 5000 units.

Eg:- The annual demand for a product is 500 units. The cost of storage per unit per year is 10% of unit cost. The ordering cost is ₹ 180 for each ~~other~~ order. The unit cost depends upon the quantity ordered. The range of quantity ordered and the unit cost price are as follows:

Range of quantity ordered	Unit Cost (₹)
$0 \le q < 500$	25
$500 \le q < 1500$	24.80
$1500 \le q < 3000$	24.60
$q \ge 3000$	24.40.

Find the optimal order quantity.

Sol^n:- R = Demand rate = 500 units /year

$I = 0.1$/unit /year.

C_3 = Ordering Cost per order = ₹ 180.

P_1 = ₹ 25/unit

P_2 = ₹ 24.80/unit

P_3 = ₹ 24.60 /unit

P_4 = ₹.24.40/unit

Steps to determine optimal order quantity:

Step 1:

$$q_4 = \sqrt{\frac{2C_3R}{P_4 I}} = \sqrt{\frac{2\times180\times500}{0.1\times24.4}}$$

$q_4 = 271$ units

$q_4 < 3000$, go to step 2

Step 2:

$$q_3 = \sqrt{\frac{2C_3R}{P_3 I}} = \sqrt{\frac{2\times180\times500}{0.1\times24.6}}.$$

$q_3 = 270.5$ units

$q_3 < 1500$, go to step 3.

Step 3:

$$q_2 = \sqrt{\frac{2C_3R}{P_2 I}} = \sqrt{\frac{2\times180\times500}{0.1\times24.8}}$$

$q_2 = 269$ units

$q_2 < 500$, go to step 4.

Step 4:

$$q_1 = \sqrt{\frac{2C_3R}{P_1 I}}$$

$$q_1 = \sqrt{\frac{2\times180\times500}{0.1\times25}} = 268 \text{ units}$$

$$C(268) = 25\times500 + \frac{180\times500}{268} + \frac{180\times0.1}{2} + \frac{268\times25\times0.1}{2}$$

$= ₹\ 13,179$

$$C(500) = 24.8\times500 + \frac{180\times500}{500} + \frac{180\times0.1}{2} + \frac{500\times24.8\times0.1}{2}$$

$= ₹\ 13,209.$

$$C(1500) = 24.6\times500 + \frac{180\times500}{1500} + \frac{180\times0.1}{2} + \frac{1500\times24.6\times0.1}{2}$$

$= ₹\ 14,214.$

$$C(3000) = 24.4 \times 500 + \frac{180 \times 500}{3000} + \frac{180 \times 0.1}{2} + \frac{3000 \times 24.4 \times 0.1}{2}$$

$$= ₹.15,899.$$

Optimal order quantity = 268 units.

Probablistic Models or Stochastic Models:

→ In all previous models, it is assumed that the demand is fixed and known. In probabilistic models future demand is not known exactly, but probability distribution of demand is known.

There are two types of demands:

① **Discrete demand**: Discrete demand occurs in discrete units. Demand occurs only in integers. Discrete probability demand distribution functions are assumed.

Examples: Demand for two wheelers, news papers, etc.

② **Continuous demand**: Demand occurs in continuous units. Continuous probability demand distribution functions are assumed.

Examples: Demand for cake, oil, ~~etc~~ vegetables, etc.

Model 1:- Instantaneous Demand (Discrete Demand) No setup cost.

In this model, it is assumed that demand occurs once and only once in a period, t and there is no setup cost.

Notations Used: q = Stock at the beginning of the cycle (or) optimum order quantity.

C_1 = Holding cost/unit/unit time.

C_2 = Shortage cost / unit / unit time.

$p(d)$ = Probability that the demand is d.

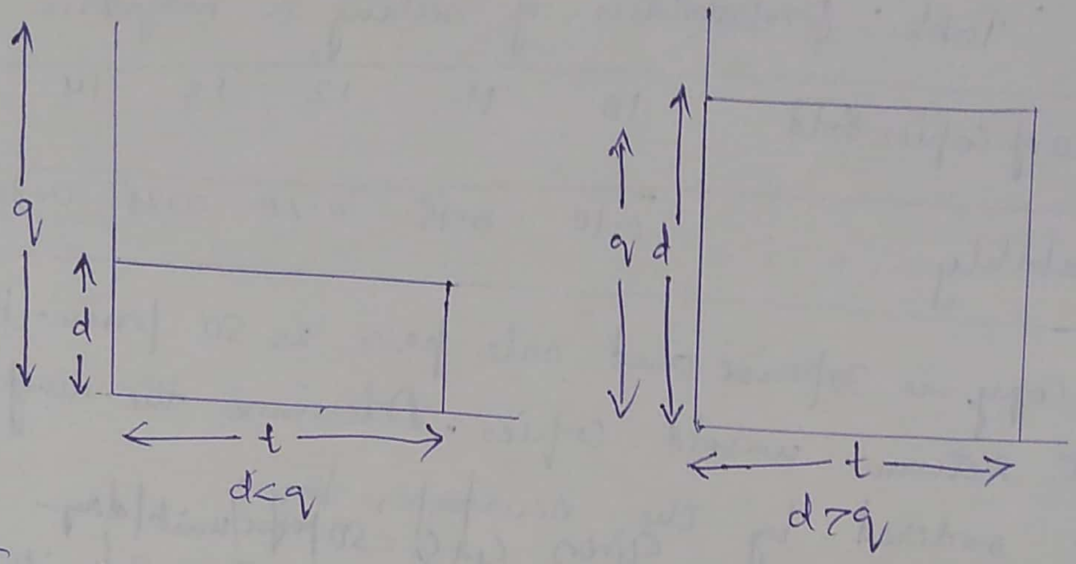

Fig: Graphical representation of instantaneous demand in time t

There are three possibilities of demand in period t:

① $d<q$ then there is holding cost for the $(q-d)$ units in period t.

② $d=q$ then there is no holding or shortage cost for period t.

③ $d>q$ then there is shortage cost for $(d-q)$ units in the period t.

'q' is optimum order quantity, if the following condition is satisfied.

$$\boxed{\sum_{0}^{q-1} p(d) < \frac{C_2}{C_1+C_2} < \sum_{0}^{q} p(d)}$$

where,

$\sum_{0}^{q-1} p(d)$ = Cummulative probability that demand $\leq q-1$ units.

$\sum_{0}^{q} p(d)$ = Cummulative probability that demand $\leq q$ units.

Eg:- A Newspaper boy has following probabilities of selling a magazine.

Table: Probabilities of selling a magazine

The no.of copies sold	10	11	12	13	14
Probability	0.10	0.15	0.20	0.25	0.30

Cost of copy is 30 paise and sale price is 50 paise. He cannot return unsold copies. Determine the no.of copies to be ordered by the newspaper boy

Given $C_1+C_2 = 50$ paise/unit/day

Sol^n:- Holding Cost (C_1) = 30 paise/unit/day = ₹ 0.3 /unit/day

Shortage Cost (C_2) = 20 paise/unit/day = ₹ 0.2 /unit/day.

$$\frac{C_2}{C_1+C_2} = \frac{20}{20+30} = 0.4$$

Table: To determine Cummulative probability

The no.of Copies sold (d)	10	11	12	13	14
probability (p(d))	0.10	0.15	0.20	0.25	0.30
Cumm. probability $\Sigma(p(d))$	0.10	0.25	0.45	0.70	1.00

'q' is optimum order quantity, if the following Condition is satisfied.

$$\sum_0^{q-1} p(d) < \frac{C_2}{C_1+C_2} < \sum_0^{q} p(d).$$

$$\sum_0^{12-1} p(d) < \frac{20}{20+30} < \sum_0^{12} p(d).$$

$$0.25 < 0.4 < 0.45$$

So optimal order quantity = q = 12 news papers.

eg :- The probability distribution of monthly sales of certain item is shown in the table below. The cost/ of carrying inventory is ₹ 30 per unit per month and the cost of unit shortage is ₹ 70 per month. Determine the optimum stock level which minimizes the total expected cost.

Table: The probability distribution of monthly sales of certain item

Monthly Sales	0	1	2	3	4	5	6
probability	0·01	0·06	0·25	0·35	0·20	0·03	0·1

Solⁿ :- Holding cost = C_1 = ₹ 30/unit/month.

Shortage Cost = C_2 = ₹ 70/unit/month

$$\frac{C_2}{C_1+C_2} = \frac{70}{70+30} = 0\cdot7$$

Table: To determine Cummulative probability

Monthly sales(d)	0	1	2	3	4	5	6
Probability p(d)	0·01	0·06	0·25	0·35	0·20	0·03	0·1
Cummulative probability $\Sigma p(d)$	0·01	0·07	0·32	0·67	0·87	0·9	1·0

q is optimum order quantity, if the following condition is satisfied.

$$\sum_{0}^{q-1} p(d) < \frac{C_2}{C_1+C_2} < \sum_{0}^{q} p(d)$$

$$\sum_{0}^{4-1} p(d) < 0\cdot7 < \sum_{0}^{4} p(d)$$

$$0\cdot67 < 0\cdot7 < 0\cdot87$$

So optimal order quantity = q = 4 units.

eg:- Some of the spare parts of a ship cost ₹ 1,00,000 each. These spare parts can only be ordered together with the ship. If not ordered at the time when the ship is constructed, these parts cannot be available on need. Suppose that a loss of ₹ 1,00,00,000 is suffered for each spare part that is needed when none is available in the stock. Further, suppose that, probabilities that spare parts will be needed as replacement during the life term of the class of the ship is given in table below.

Table: Probabilities that spare parts will be needed as replacement during the life term.

Spare parts	0	1	2	3	4	5 or more
probability	0.9488	0.0400	0.0100	0.0010	0.0002	0.00

How many spare parts should be procured?

Solⁿ:- Holding cost = ₹ 1,00,000 /unit

Shortage Cost = ₹ 1,00,00,000/ unit

$$\frac{C_2}{C_1+C_2} = \frac{1,00,00,000}{1,00,00,000 + 1,00,000} = 0.99009.$$

Table: Determining Cummulative probability.

Spare parts (d)	0	1	2	3	4	5 or mor
Probability $p(d)$	0.9488	0.0400	0.0100	0.0010	0.0002	0.00
Cumm. Probability $\Sigma p(d)$	0.9488	0.9888	0.9988	0.9998	1.00	1.00

q is optimum order quantity, if the following condition is satisfied.

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$$\therefore \sum_{0}^{q-1} p(d) < \frac{C_2}{C_1+C_2} < \sum_{0}^{q} p(d)$$

$$0.9888 < 0.990009 < 0.9988$$

So optimal order quantity $= q = 2$ units.

* Instantaneous Demand (continuous Demand) No setup Cost :

In this model, it is assumed that demand occurs once and only once in a period t and there is no setup cost. The demand function is assumed to be continuous.

Notations used :

q = Stock at the beginning of the cycle or optimum order quantity.

C_1 = Holding cost /unit /unit time.

C_2 = Shortage cost /unit /unit time

$f(x)$ = Probability that the demand is x.

'q' is the optimal order quantity, if the following condition is satisfied.

$$\int_{0}^{q} f(x)\, d(x) = \frac{C_2}{C_1+C_2}. \quad ; \quad 0 \leq x \leq q$$

eg :- An icecream company sells one of its types of icecream by the weight. If the product is not sold on the day it is prepared, it can be sold at a loss of 50 paise per kg. But there is unlimited market for one day old ice cream. On the other hand, the company makes a profit of ₹ 3.20 on every kg of icecream sold on the day it is prepared. Past daily distribution with

$$f(x) = 0.02 - 0.0002x, \cancel{0 \le x \le 100}$$

Determine optimum order quantity. $0 \le x \le 100$

Sol^n:- Holding Cost $= C_1 =$ ₹ 0.5/unit/day.

Shortage Cost $= C_2 =$ ₹ 3.20/unit/day.

probability distribution function of demand x.

$$f(x) = 0.02 - 0.0002x, \quad 0 \le x \le 100$$

'q' be the optimal order quantity, if the following Condition is satisfied.

$$\int_0^q f(x)\, d(x) = \frac{C_2}{C_1 + C_2}$$

$$\int_0^q (0.02 - 0.0002x)\, d(x) = \frac{3.2}{0.5 + 3.2}$$

$$\left[0.02x - \frac{0.0002x^2}{2}\right]_0^q = 0.864.$$

substituting values of q and zero.

$$\left[0.02q - \frac{0.0002q^2}{2}\right] = 0.864.$$

solving the above equation,

optimal order quantity $= q = 63.2$ kg.

Eg:- A baking company sells one of its types of cakes by weight. It makes a profit of 95 paise a pound on every pound of cake sold on the day it is baked. It disposes all cakes not sold on the day they are baked at a loss of 15 paise a pound. If the demand is known to be rectangular b/w 3000 and 4000 pounds; determine the optimum amount to be baked.

Solⁿ:- Holding cost = ₹ 0.15/pound/day. ⑮

Shortage cost = ₹ 0.95/pound/day.

probability distribution function

$$f(x) = \frac{1}{4000-3000} = \frac{1}{1000}$$

$$\int_0^q f(x)\,d(x) = \frac{C_2}{C_1+C_2}$$

$$\int_{3000}^{q} \left(\frac{1}{1000}\right) d(x) = \frac{0.95}{0.95+0.15} = 0.8636.$$

$$\left(\frac{1}{1000}\right) |x|_{3000}^{q} = 0.8636.$$

$$q - 3000 = 863.6.$$

optimal order quantity = ~~Q~~ q = 3863.6 units.

eg:- The probability density of demand of a certain item during a week to be $f(x) = 0.1,\ 0 \le x \le 10$. The demand is assumed to occur with a uniform pattern over the week. The unit carrying cost of the item in inventory is ₹ 2 per week and unit shortage cost ₹ 8/week. Determine optimal order level of the inventory.

solⁿ:- Holding cost = C_1 = ₹ 2/unit/week.

Shortage cost = C_2 = ₹ 8/unit/week.

probability distribution function

$$f(x) = 0.1, \quad 0 \le x \le 10$$

$$\int_0^q f(x)\,d(x) = \frac{C_2}{C_1+C_2}.$$

$$\int_0^q 0.1\,d(x) = \frac{8}{10} = 0.8$$

$$0.1\,|x|_0^q = 0.8$$

$$0.1q = 0.8$$

Optimal order quantity $= q = 8$ units.