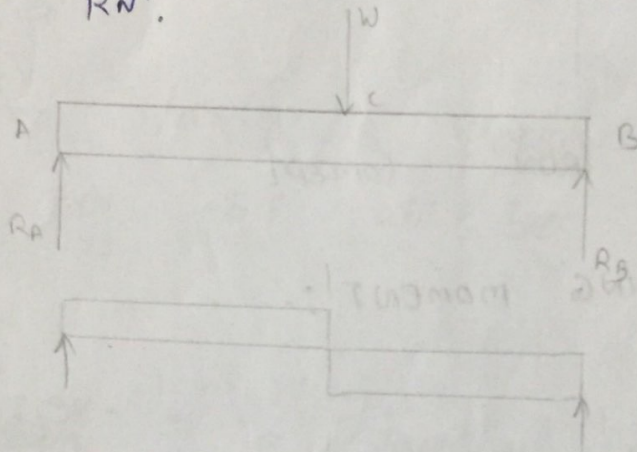


SHEAR FORCE AND BENDING MOMENT

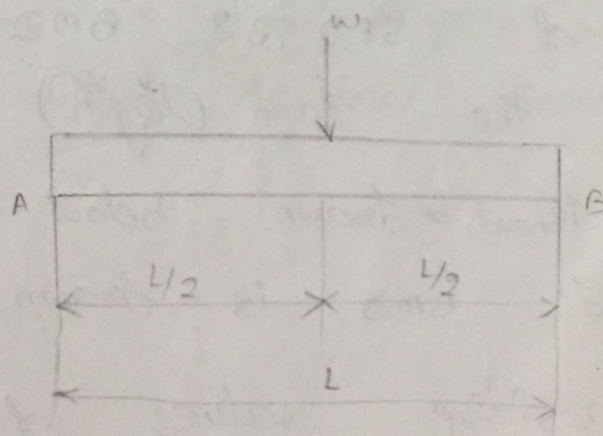
SHEAR FORCE (SF) :-

The algebraic sum of forces at any section of a beam either on left hand side or right hand side of the section is known as shear force. It is expressed in 'N' (or) 'kN'.

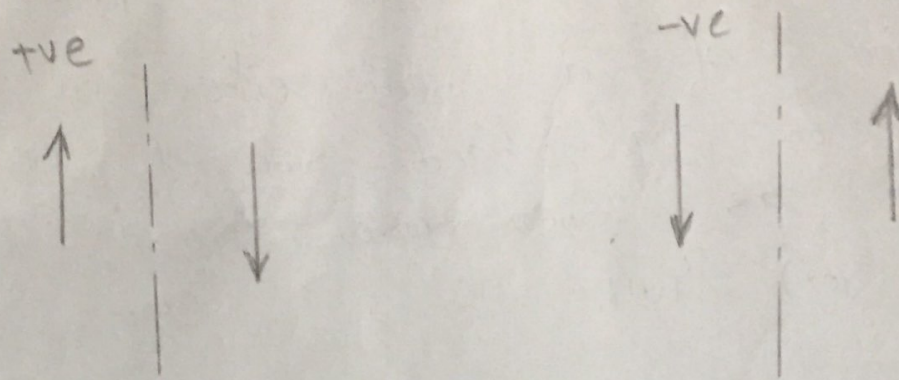


BENDING MOMENT :-

It is defined as the algebraic sum of moments of forces acting on the left side or right side of the section. It is expressed in 'N-m' (or) 'kN-m'.

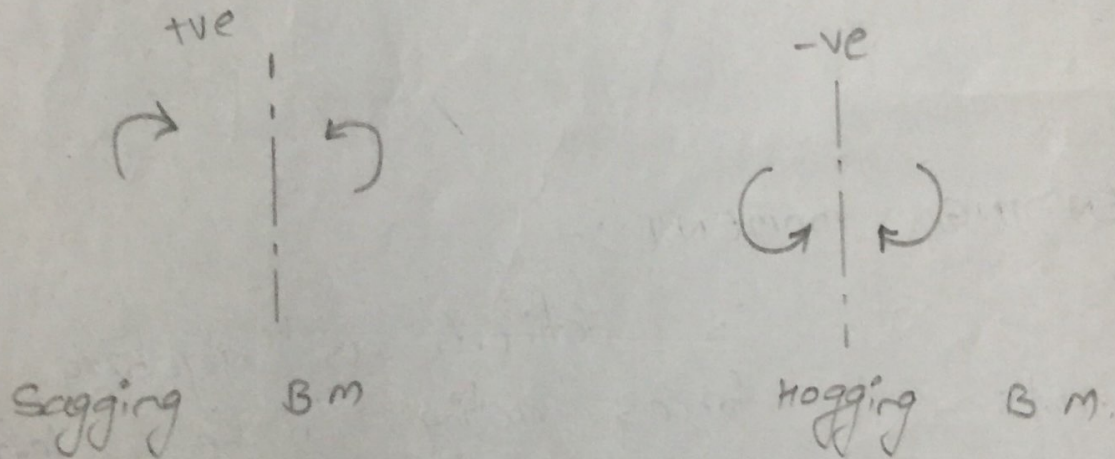


SIGN CONVENTIONS FOR SHEAR FORCE AND BENDING MOMENT.



LUP AND RUN Concept.

FOR BENDING MOMENT :-



IMPORTANT POINTS FOR DRAWING SFD & BMD :-

- Length of SFD & BMD must be equal to the span (length) of the beam.
- SFD should be drawn below the loaded beam and BMD is drawn below the SFD.
- The positive values of shear force, bending moments are plotted above the

base line, and negative values below the base line.

- For simply supported beam, B.M is zero at the supports.
- For cantilever beam, B.M will be zero at free end.
- If no load is present between two points then S.F will be constant.

BEAM :-

A beam is a structural member, it is subjected to a system of external forces at right angles to its axis.

- Any section of the beam experiences shearing off by the load and bending by the moment.
- Basically the longitudinal dimension (i.e length) of a beam is too large when compared with its cross-sectional dimensions (i.e $b \times d$).

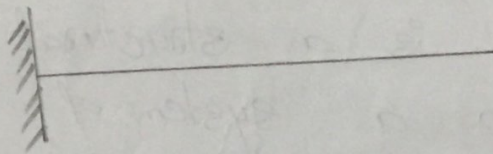
TYPES OF BEAMS :-

The various types of beams are

- 1) Cantilever beam
- 2) Simply supported beam.
- 3) overhanging beam
- 4) Fixed beam.
- 5) Continuous beam.

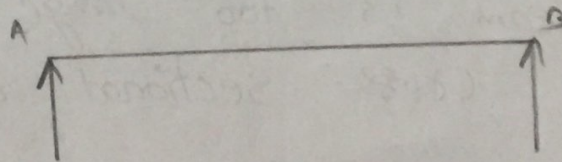
1) CANTILEVER BEAM :-

A beam which is fixed at one end and free at the other end, is known as cantilever beam.



2) SIMPLY SUPPORTED BEAM :-

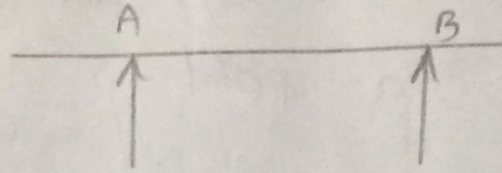
A beam supported or resting freely on the supports at its both ends, is known as simply supported beam.



3) OVERHANGING BEAM :-

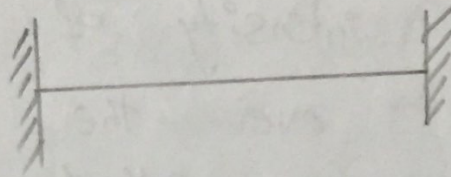
if the end portion of a beam is extended beyond the support, such

beam is known as overhanging beam.



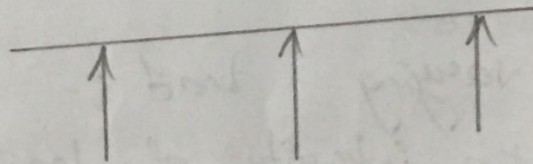
4) FIXED BEAM:-

A beam whose both ends are fixed or built in walls. is known as fixed beams.



5) CONTINUOUS BEAMS:-

A beam which is provided more than two supports is known as continuous beam.



TYPES OF LOADS:-

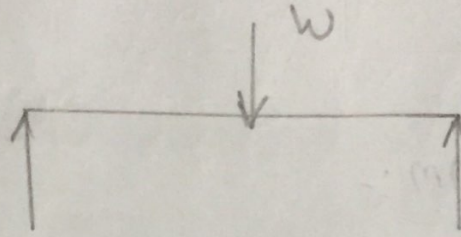
The various types of loads acting on

a beam are

- 1) Concentrated (or) point load.
- 2) Uniformly distributed load (UDL)
- 3) Uniformly varying load (UVL)

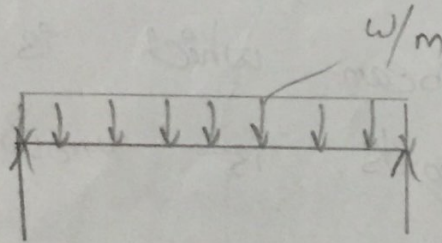
1) Concentrated (or) point load :-

A point load is one which is considered to act at a point.



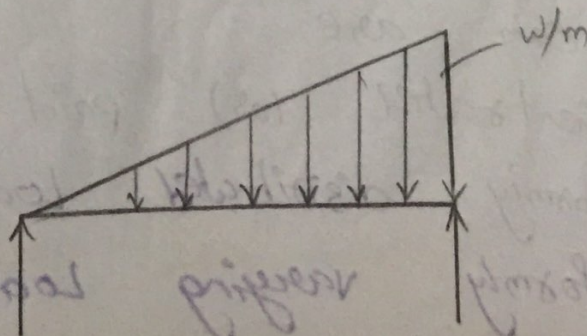
2) Uniformly distributed load :-

If the intensity of load is uniform (i.e) constant over the entire length of the beam it is called u.d.l. It is expressed in ' N/m '.

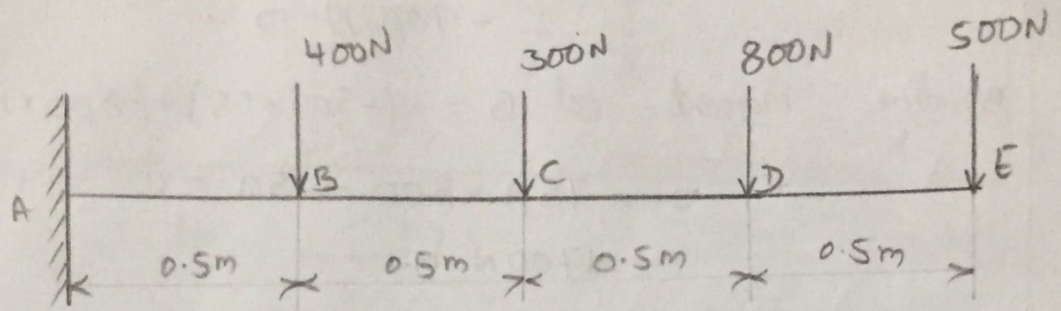


3) Uniformly varying load :-

If the intensity of load is uniformly varying from one end to another end, it is called U.V.L. In which the load is zero at one end and increases uniformly to the other end.



Q) Draw the shear force and Bending moment diagrams for the Cantilever beam as shown in below fig



Sol Before drawing the shear force and Bending moment diagrams first we have to calculate shear forces acting at different point and as well as Bending moment at various points

Shear force Calculations:-

$$\text{Shear force at point E} = 500 \text{ N}$$

$$\begin{aligned} \text{Shear force at point D} &= 500 + 800 \text{ N} \\ &= 1300 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Shear force at point C} &= 500 + 800 + 300 \\ &= 1600 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Shear force at point B} &= 500 + 800 + 300 + 400 \\ &= 2000 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Shear force at point A} &= 500 + 800 + 300 + 400 + 0 \\ &= 2000 \text{ N} \end{aligned}$$

Bending moment Calculations:-

$$\text{Bending moment at E} = 0$$

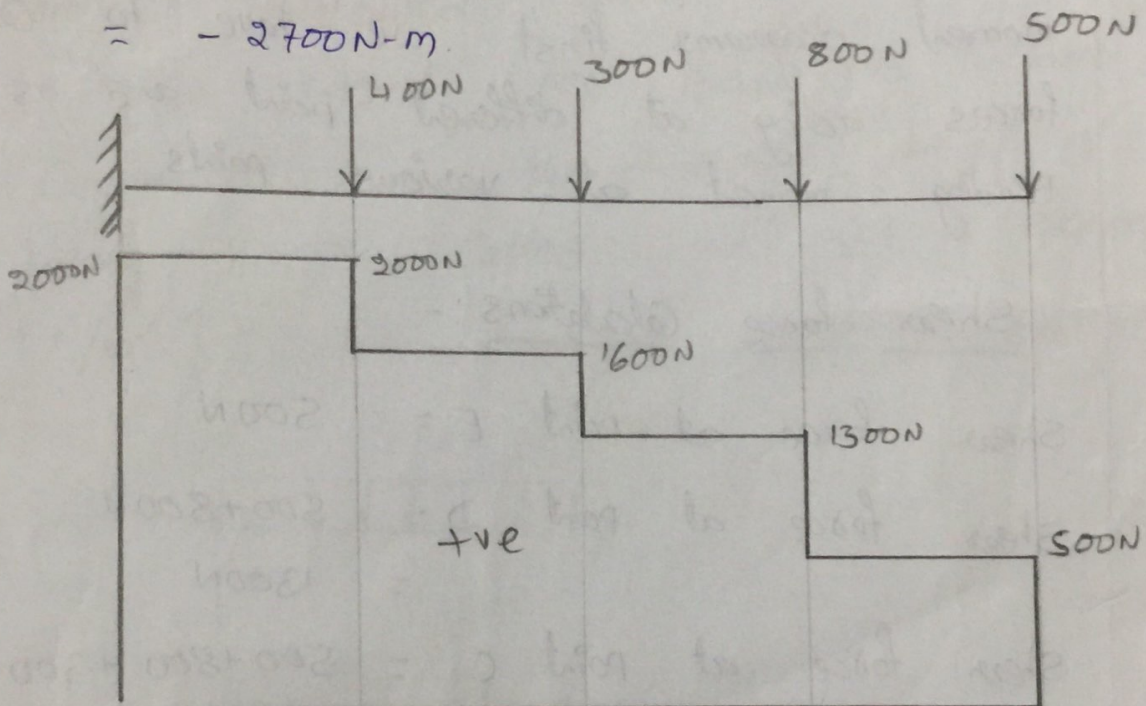
$$\begin{aligned} \text{Bending moment at D} &= (500 \times 0.5) + (800 \times 0) \\ &= -250 \text{ N-m} \end{aligned}$$

$$\begin{aligned}\text{Bending moment at C} &= -(500 \times 1) + (-800 \times 0.5) + (-300 \times 0) \\ &= -500 - 400 \\ &= -900 \text{ N-m}\end{aligned}$$

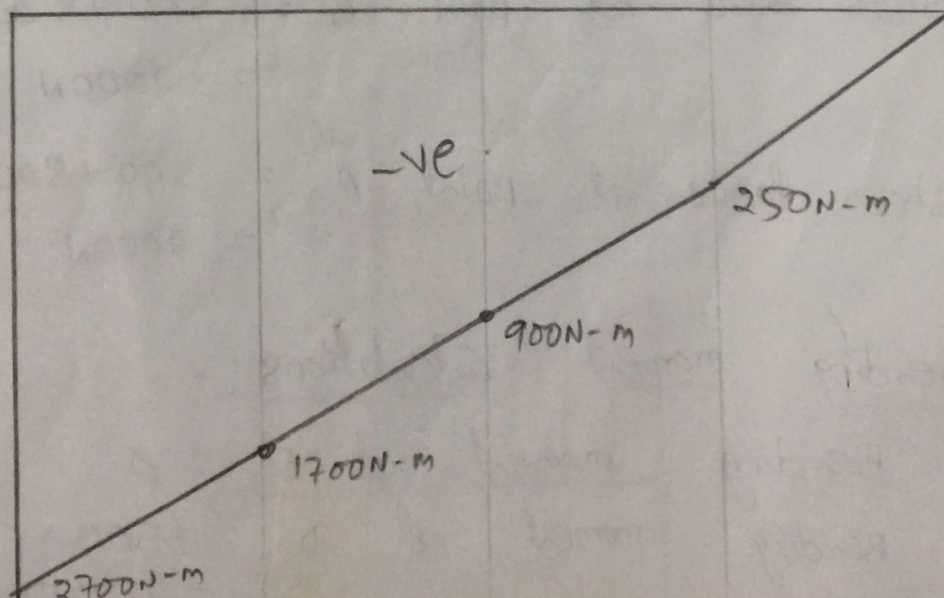
$$\begin{aligned}\text{Bending moment at B} &= (-500 \times 1.5) + (-800 \times 1) + (-300 \times 0.5) + (-400 \times 0) \\ &= -750 - 800 - 150 - 0 \\ &= -1700 \text{ N-m}\end{aligned}$$

$$\begin{aligned}\text{Bending moment at A} &= (-500 \times 2) + (-800 \times 1.5) + (-300 \times 1) + (-400 \times 0.5) \\ &= -1000 - 1200 - 300 - 200 \\ &= -2700 \text{ N-m}\end{aligned}$$

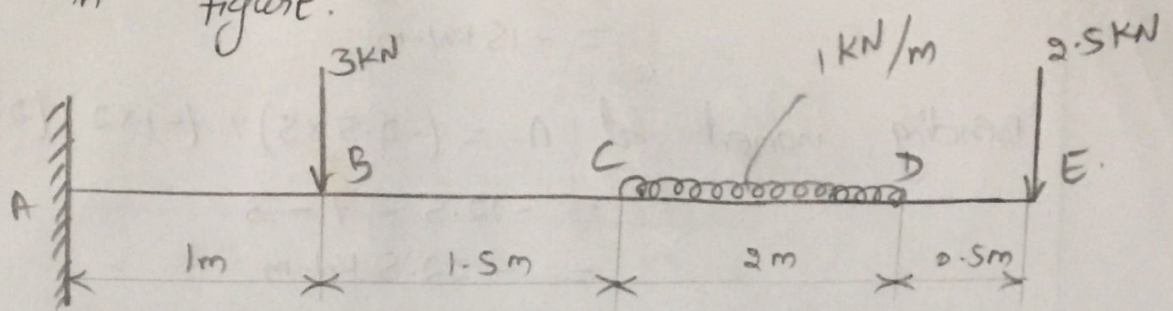
SFD



BMD



Draw the SFD & BMD for the beam as shown in figure.



Shear force calculations:-

$$\text{Shear force at E} = 2.5 \text{ kN}$$

$$\begin{aligned} \text{Shear force at D} &= 0 + 2.5 \text{ kN} \\ &= 2.5 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{Shear force at C} &= 2.5 + 0 + (1 \times 2) \\ &= 2.5 + 0 + 2 \\ &= 4.5 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{Shear force at B} &= 2.5 + (1 \times 2) + 3 \\ &= 2.5 + 2 + 3 \\ &= 7.5 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{Shear force at A} &= 2.5 + (1 \times 2) + 3 + 0 \\ &= 7.5 \text{ kN} \end{aligned}$$

Bending moment calculations:-

$$\text{Bending moment at E} = 0$$

$$\begin{aligned} \text{Bending moment at D} &= (-2.5 \times 0.5) \\ &= -1.25 \text{ kN-m} \end{aligned}$$

$$\begin{aligned} \text{Bending moment at C} &= (-2.5 \times 2.5) + (-1 \times 2) \times \left(\frac{2}{2}\right) \\ &= -6.25 - 2 \\ &= -8.25 \text{ kN-m} \end{aligned}$$

$$\text{Bending moment at B} = (-2.5 \times 4) + (-1 \times 2)(1.5 + \frac{2}{2})$$

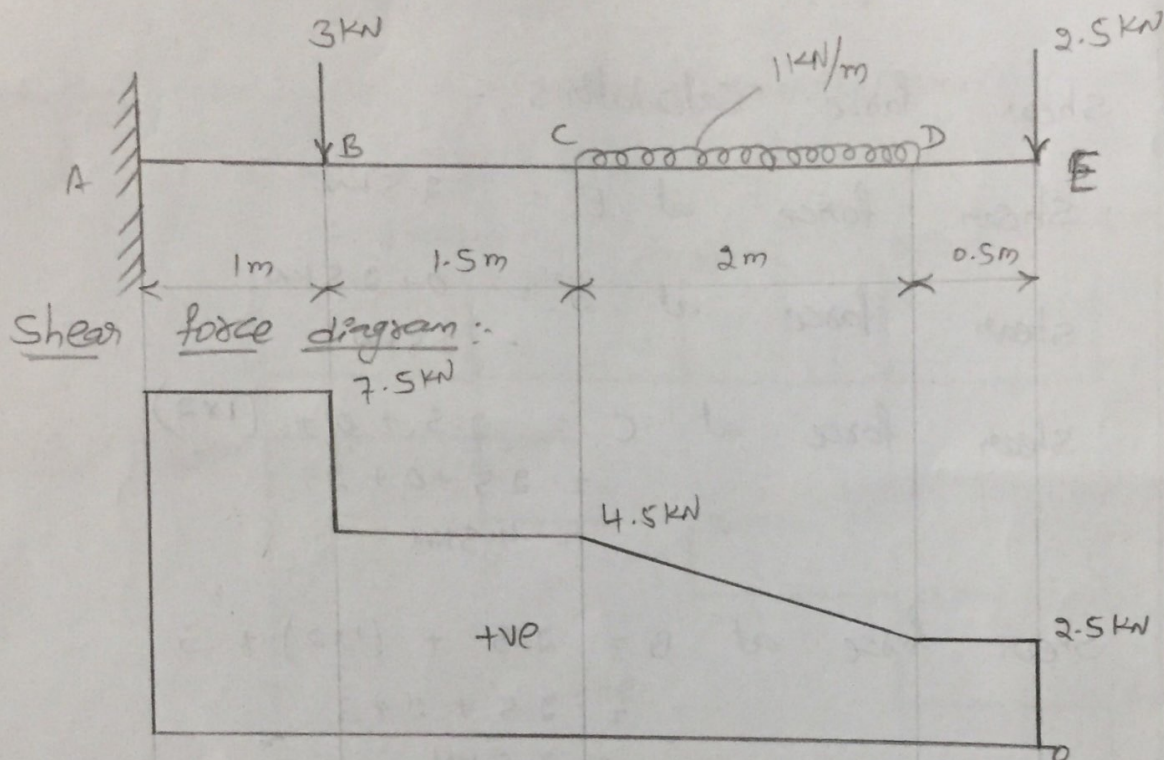
$$= -10 - 5$$

$$= -15 \text{ KN-m}$$

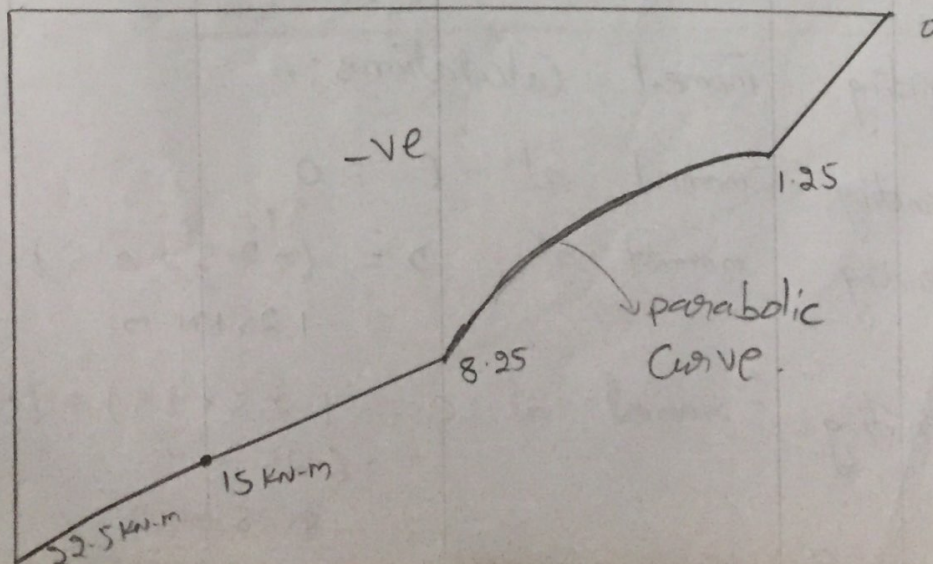
$$\text{Bending moment at A} = (-2.5 \times 5) + (-1 \times 2)(2.5 + \frac{2}{2}) + (-3 \times 1)$$

$$= -12.5 - 7 - 3$$

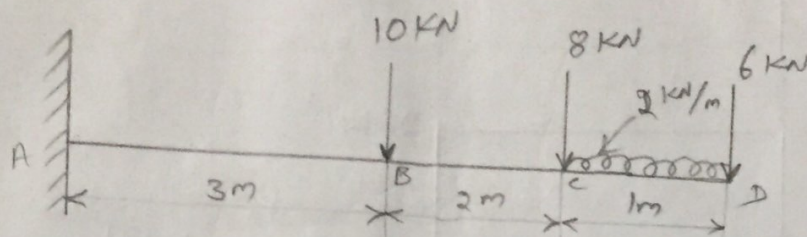
$$= -22.5 \text{ KN-m}$$



Bending moment diagram:-



Draw the SFD and BMD for the beam loaded as shown in below.



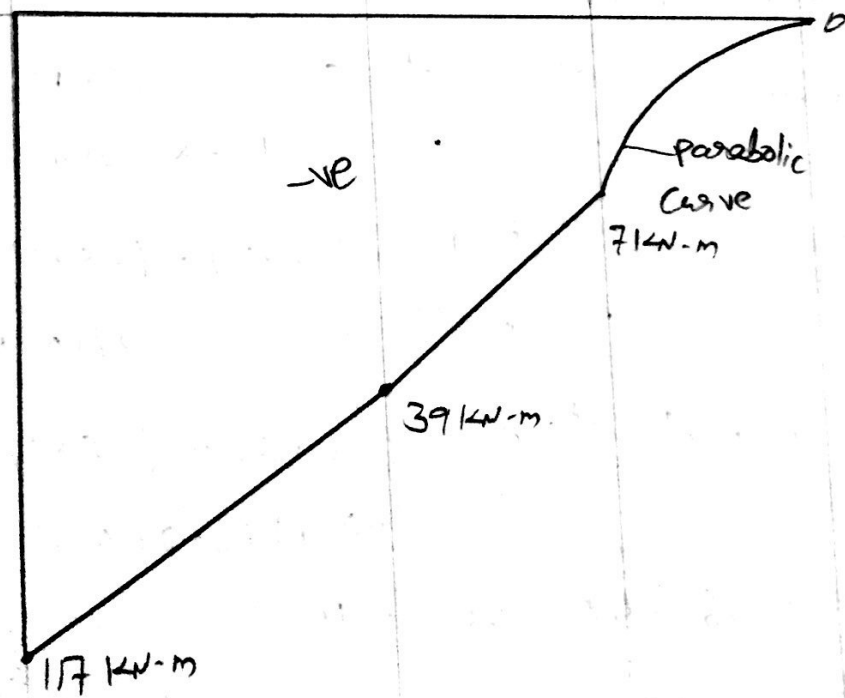
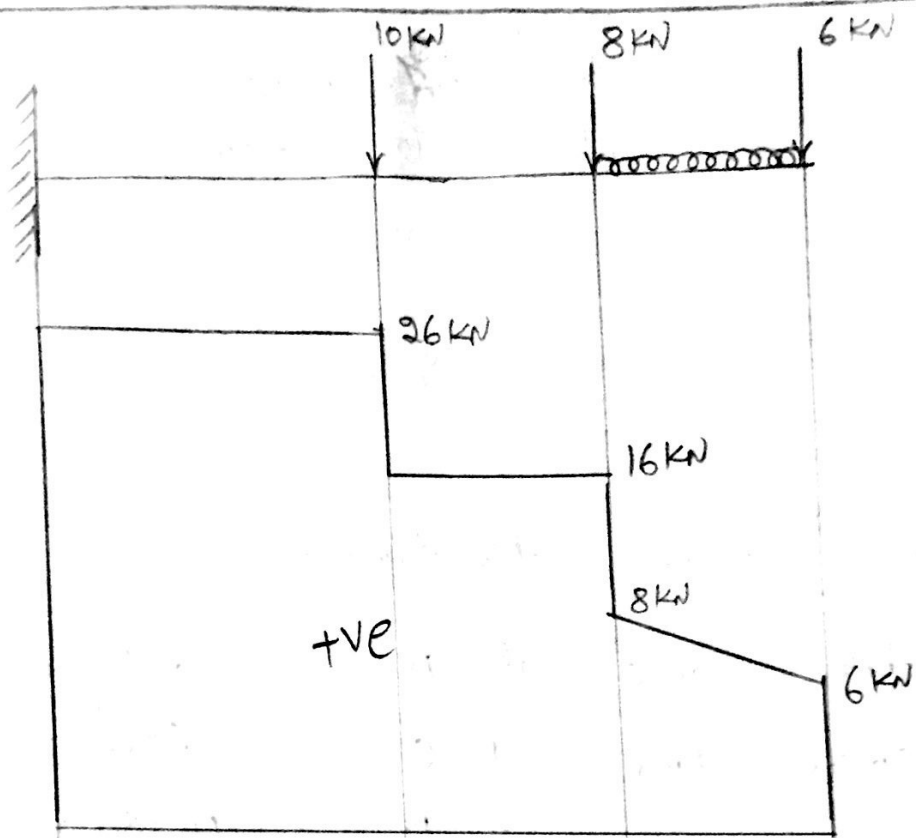
Shear Force Calculations:

$$\begin{aligned}
 \text{Shear force at D} &= 6 \text{ kN} \\
 \text{Shear force at C} &= 6 + 8 = 14 \text{ kN} \quad \text{with point load} \\
 \text{" " " B} &= (2 \times 1) + 6 + 8 + 10 = 26 \text{ kN} \\
 \text{Shear force at A} &= (2 \times 1) + 24 + 0 = 26 \text{ kN}
 \end{aligned}$$

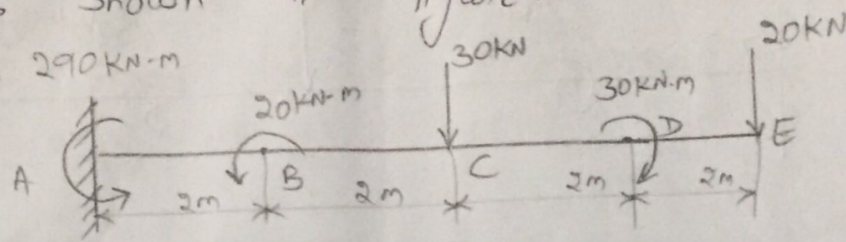
Notes: $6 + (2 \times 1) = 8 \text{ kN-m}$ without point load. $6 + 8 = 14 \text{ kN}$ with point load.

Bending moment Calculations:-

$$\begin{aligned}
 \text{Bending moment at D} &= 0 \\
 \text{Bending moment at C} &= -(6 \times 1) + \left(-\frac{(2 \times 1)}{2} \times \frac{1}{2} \right) \\
 &= -6 - 1 \Rightarrow -7 \text{ kN-m} \\
 \text{Bending moment at B} &= (-6 \times 3) + (-8 \times 2) + (-2 \times 1) \left(2 + \frac{1}{2} \right) \\
 &= -18 - 16 - 5 = -39 \text{ kN-m} \\
 \text{Bending moment at A} &= (-6 \times 6) + (-8 \times 5) + (-10 \times 3) \\
 &\quad + (-2 \times 1) (3 + 2 + 0.5) \\
 &= -36 - 40 - 30 - 11 \\
 &= -117 \text{ kN-m}
 \end{aligned}$$



Draw the SFD and BMD for cantilever beam loaded as shown in figure.



Shear force calculations:

$$\text{Shear force at point E} = 20 \text{ kN}$$

$$D = 20 + 0 = 20 \text{ kN}$$

$$C = 20 + 0 + 30 = 50 \text{ kN}$$

$$B = 50 \text{ kN}$$

$$A = 50 \text{ kN}$$

Bending moment calculations:

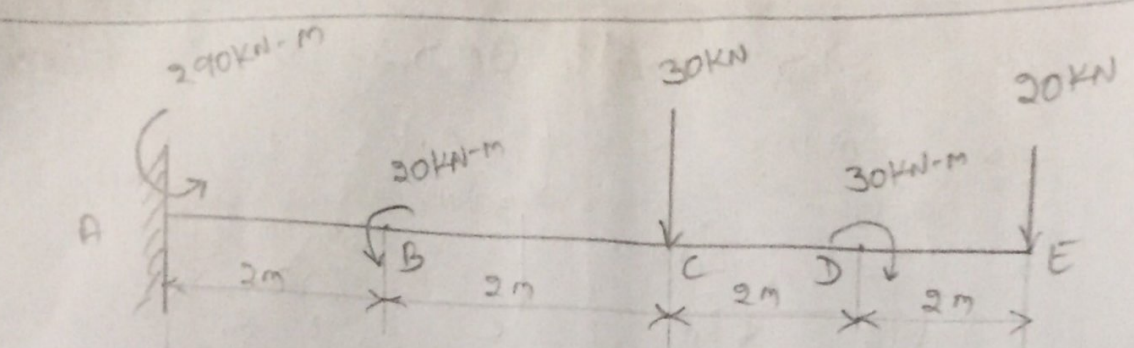
$$\text{Bending moment at point E} = 0$$

$$\begin{aligned} \text{" " " " D} &= -(20 \times 2) - 30 \\ &= -40 - 30 \\ &= -70 \text{ kN-m} \end{aligned}$$

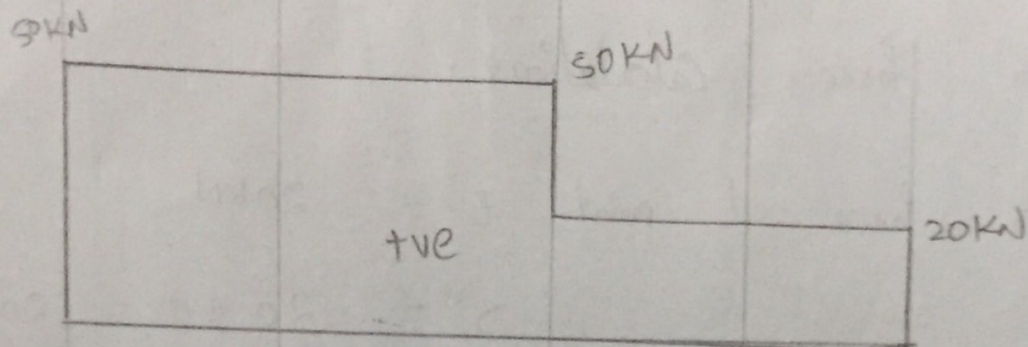
$$Bm_C = (-20 \times 4) - 30 \Rightarrow -80 - 30 \Rightarrow -110 \text{ kN-m}$$

$$\begin{aligned} Bm_B &= (-20 \times 6) - 30 - (30 \times 2) \Rightarrow -120 - 30 - 60 \\ &= -210 \text{ kN-m} \Rightarrow -210 + 20 \Rightarrow -190 \text{ kN-m} \end{aligned}$$

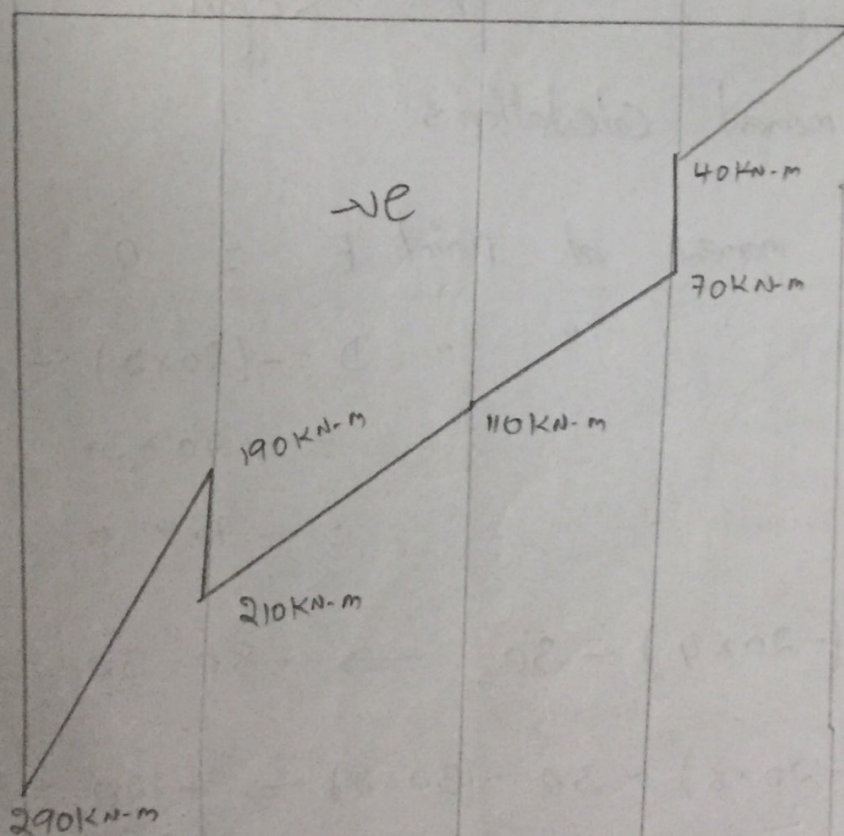
$$\begin{aligned} Bm_A &= (-20 \times 8) - 30 - (30 \times 4) + 20 + 290 \\ &= -160 - 30 - 120 + 20 + 290 \\ &= -310 + 20 + 290 \\ &= 0 \end{aligned}$$



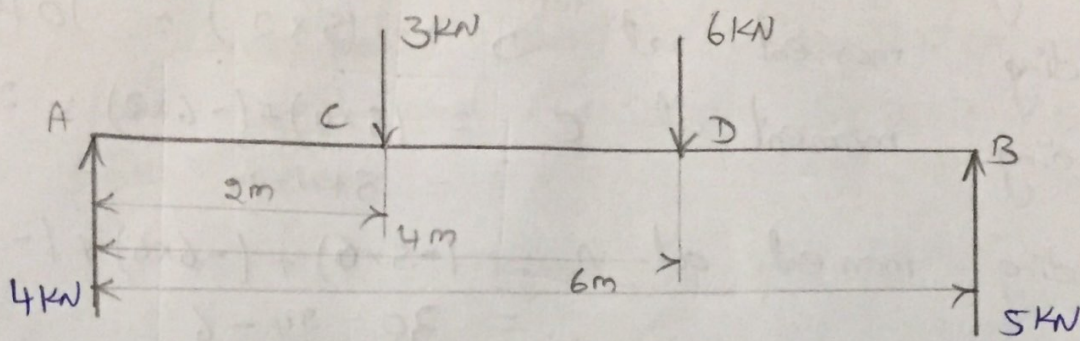
SFD



Bending moment diagram:



A Simply supported beam of length 6m carries a point load of 3kN & 6kN at a distance of 2m & 4m from the left end. Draw SFD & BMD.



Whenever the simply supported beam is given the we have to calculate the vertical reactions we know.

Sum of upward forces = sum of downward forces

$$R_A + R_B = 3 + 6$$

$$R_A + R_B = 9 \text{ kN} \quad \text{--- (1)}$$

we also know

sum of counter clockwise moments = sum of clockwise moments
Taking moments about A.

$$R_B \times 6 = 6 \times 4 + 3 \times 2$$

$$6R_B = 24 + 6$$

$$R_B = 30/6$$

$$R_B = 5 \text{ kN} \quad \text{--- (2)}$$

substitute R_B value in equation (1) to get R_A .

$$R_A = 9 - R_B \Rightarrow 9 - 5$$

$$R_A = 4 \text{ kN}$$

Shear force calculations:-

$$\text{Shear force at B} = -5 \text{ kN}$$

$$\text{Shear force at D} = -5 + 6 \Rightarrow 1 \text{ kN}$$

$$\text{Shear force at C} = -5 + 6 + 3 \Rightarrow 9 - 5 \Rightarrow 4 \text{ kN}$$

Shear force at A = $-5 + 6 + 3 - 4 = 9 - 9 = 0$

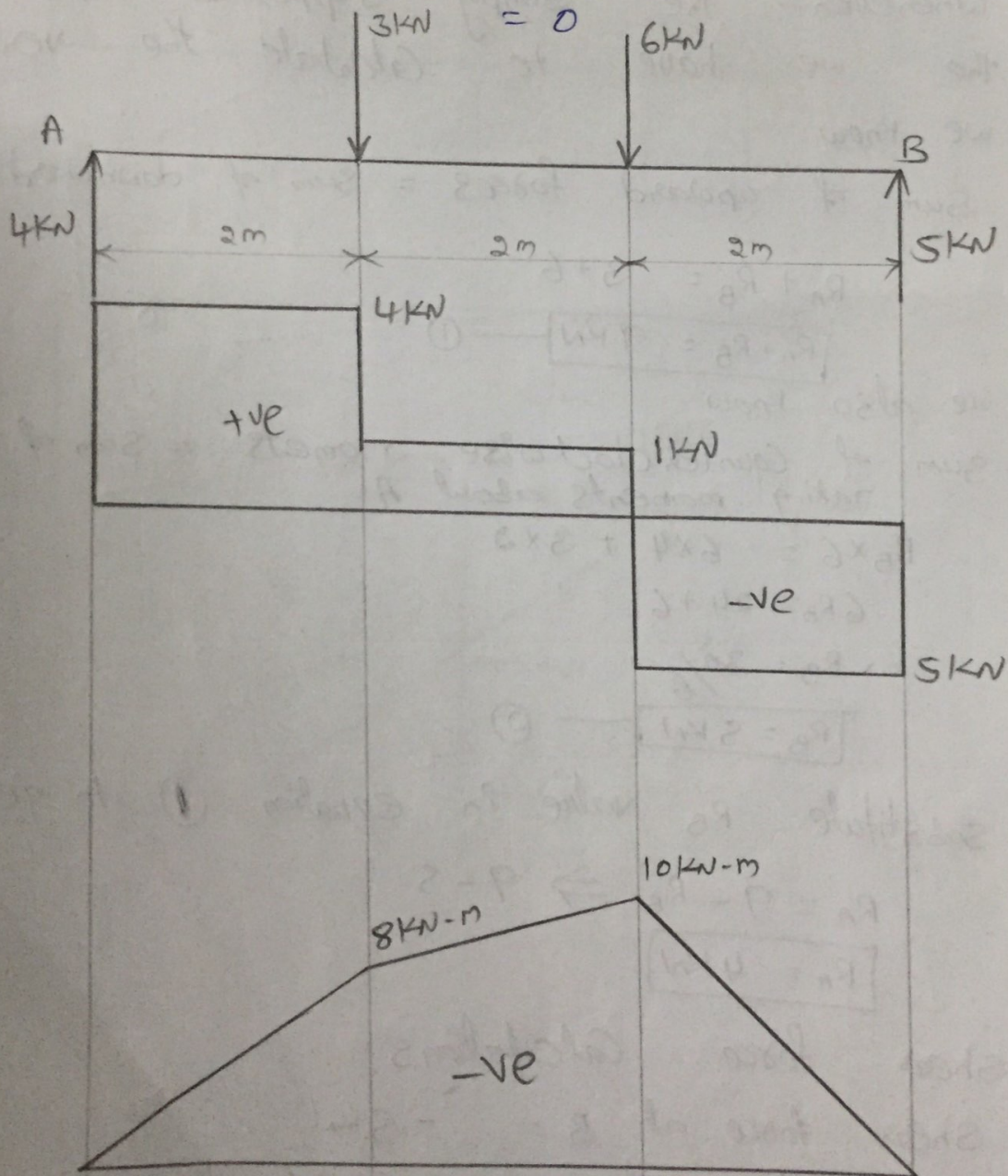
Bending moment calculations:-

Bending moment at B = 0

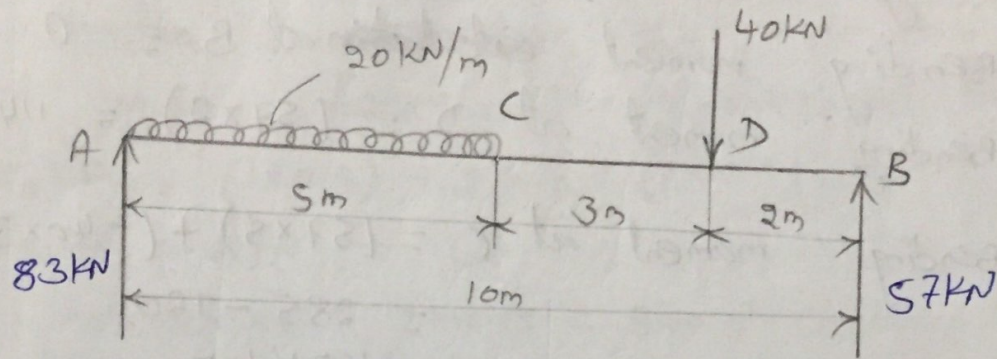
Bending moment at D = $(5 \times 2) = 10 \text{ kN-m}$

Bending moment at C = $(5 \times 4) + (-6 \times 2) = 20 - 12$
 $= 8 \text{ kN-m}$

Bending moment at A = $(5 \times 6) + (-6 \times 4) + (-3 \times 2)$
 $= 30 - 24 - 6$
 $= 30 - 30$
 $= 0$



A beam AB 10m long is simply supported at A and B. It carries a uniformly distributed load of 20 kN/m for a distance of 5m from the left end, and a concentrated load of 40 kN at a distance of 2m from right end B. Draw SFD & BMD for the given beam.



we know

Sum of upward forces = Sum of downward forces.

$$R_A + R_B = (20 \times 5) + 40$$

$$\boxed{R_A + R_B = 140 \text{ kN}} \quad \text{--- (1)}$$

Sum of Anticlockwise moments = Sum of clockwise moments
Taking moments about A.

$$R_B \times 10 = (40 \times 8) + (20 \times 5) \left(\frac{5}{2} \right)$$

$$10R_B = 320 + 250$$

$$R_B = 570/10$$

$$\boxed{R_B = 57 \text{ kN}} \quad \text{--- (2)}$$

Substitute R_B value in eqn (1) to get R_A value

$$R_A = 140 - R_B \Rightarrow 140 - 57$$

$$\boxed{R_A = 83 \text{ kN}}$$

Shear force calculations :-

$$\text{Shear force at B} = -57 \text{ kN}$$

$$\text{Shear force at D} = -57 + 40 \Rightarrow -17 \text{ kN}$$

$$\text{Shear force at C} = -57 + 40 \Rightarrow -17 \text{ kN}$$

$$\begin{aligned}
 \text{Shear force at A} &= -57 + 40 + (20 \times 5) \\
 &= -57 + 140 \\
 &= -83 \rightarrow \text{without point load.} \\
 &= -83 + 83 \\
 &= 0 \rightarrow \text{with point load.}
 \end{aligned}$$

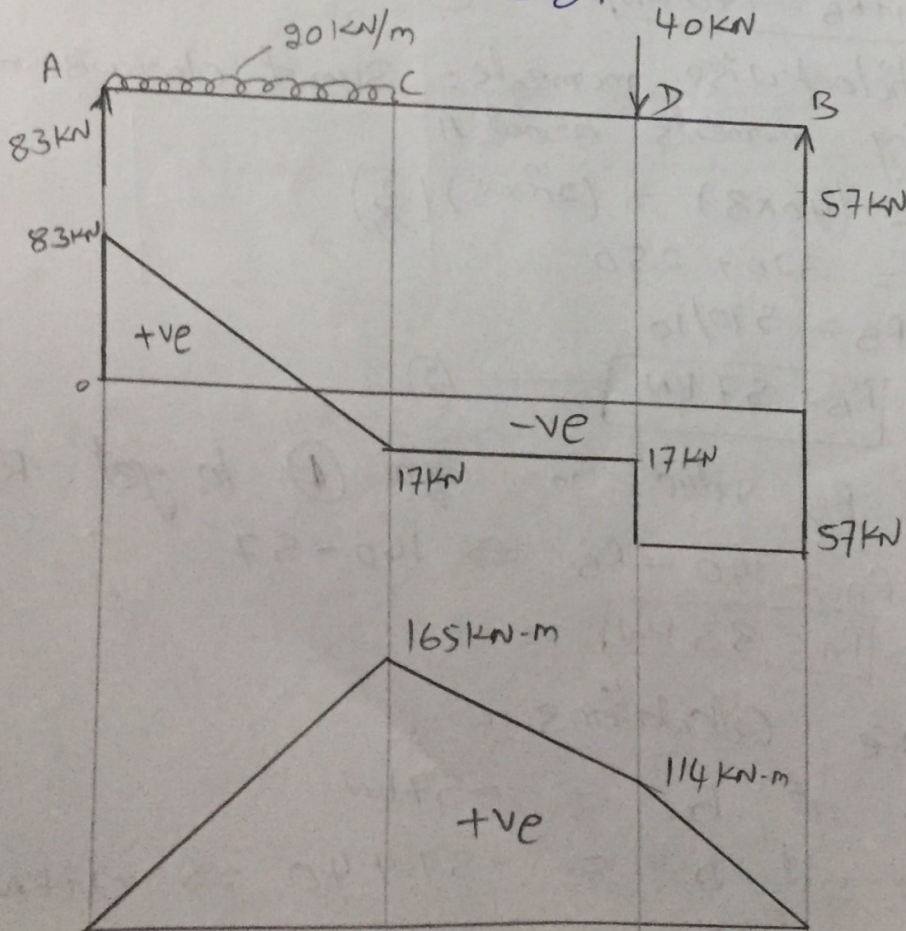
Bending moment calculations :-

$$\text{Bending moment } ~~\text{at B}~~ \text{ at B} = 0$$

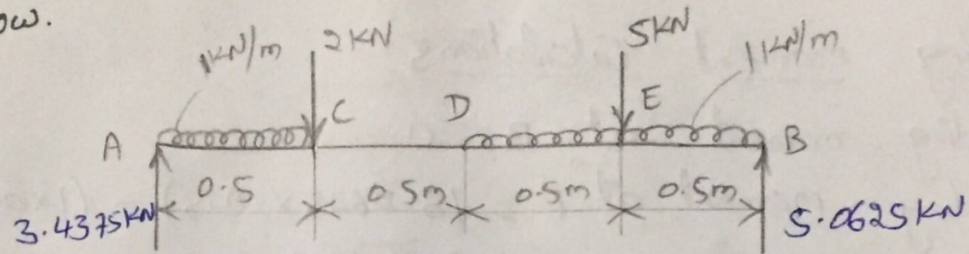
$$\text{Bending moment at D} = (57 \times 2) = 114 \text{ kN-m}$$

$$\begin{aligned}
 \text{Bending moment at C} &= (57 \times 5) + (-40 \times 3) \\
 &= 285 - 120 \\
 &= 165 \text{ kN-m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Bending moment at A} &= (57 \times 10) + (-40 \times 8) + (-20 \times 5) \times \frac{5}{2} \\
 &= 570 - 320 - 250 \\
 &= 570 - 570 \\
 &= 0
 \end{aligned}$$



Q) Draw the SFD & BMD for the beam as shown in below.



Sol For the simply supported beam first we have to calculate Reaction. for calculating this we know
Sum of upward forces = Sum of downward forces

$$R_A + R_B = (1 \times 0.5) + 2 + 5 + (1 \times 1.5)$$

$$R_A + R_B = 8.5 \text{ kN} \quad \text{--- (1)}$$

Sum of Anticlockwise moments = Sum of clockwise moments
Taking moments about A.

$$R_B \times 2 = 2 \times (0.5) + (5 \times 1.5) + (1 \times 0.5) \left(\frac{0.5}{2} \right) + (1 \times 1) \left(1 + \frac{1}{2} \right)$$

$$= 1 + 7.5 + 0.125 + 1.5$$

$$R_B = \frac{10.125}{2} = 5.0625 \text{ kN} \quad \text{--- (2)}$$

Substitute R_B value in Eqn (1).

$$R_A = 8.5 - 5.0625$$

$$R_A = 3.4375 \text{ kN}$$

Shear force Calculations :-

$$\text{Shear force at B} = -5.0625 \text{ kN}$$

$$\text{Shear force at E} = -5.0625 + (1 \times 0.5)$$

$$= -5.0625 + 0.5$$

$$= -4.5625 \text{ kN} \quad \text{--- without point load}$$

$$= -4.5625 + 5 = 0.4375 \text{ kN} \quad \text{--- with point load}$$

$$\text{Shear force at D} = -5.0625 + 5 + (1 \times 1)$$

$$= 0.9375 \text{ kN}$$

$$\text{Shear force at C} = -5.0625 + 5 + (1 \times 1) + 2$$

$$= 2.9375 \text{ kN}$$

$$\text{Shear force at A} = (-5.0625) + (1 \times 1) + 5 + 2 + (1 \times 0.5)$$

$$= -5.0625 + 8.5 = 3.4375 \text{ kN} \quad \text{--- without point load}$$

Shear force at A = $3.4375 - 3.4375$ — with point load.
 $= 0$

Bending moment Calculations:-

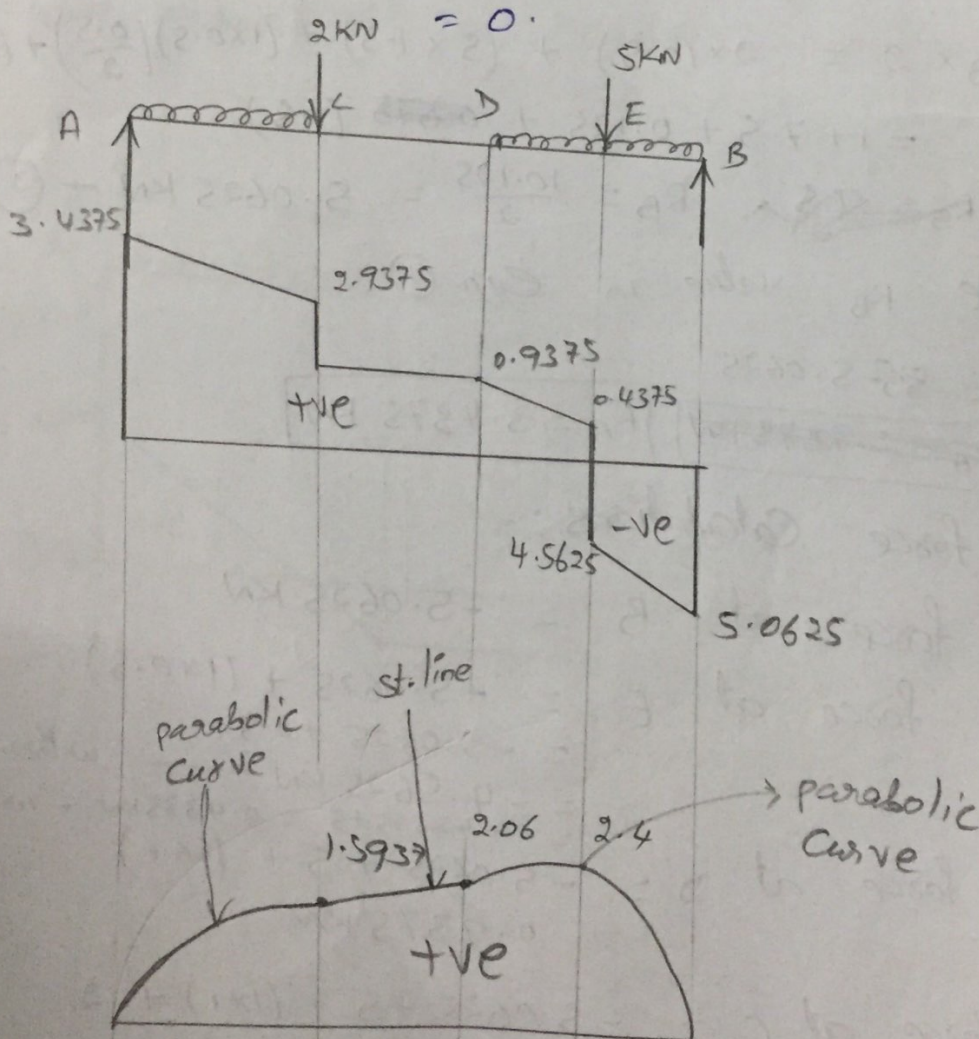
Bending moment at B = 0

$$\text{Bending moment at E} = (5.0625 \times 0.5) - (1 \times 0.5) \left(\frac{0.5}{2} \right) \\ = 2.406 \text{ kN-m}$$

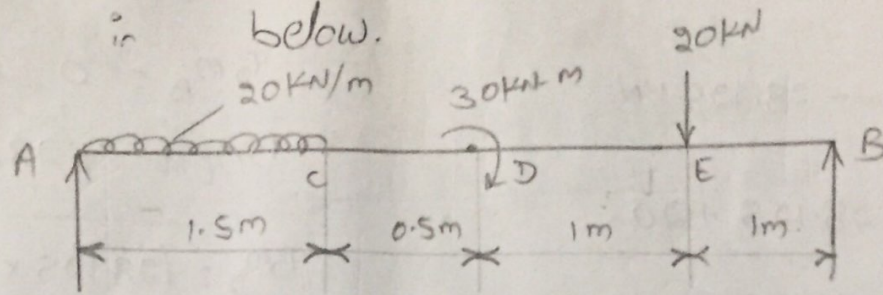
$$\text{Bending moment at D} = (5.0625 \times 1) - (5 \times 0.5) - (1 \times 1) \left(\frac{1}{2} \right) \\ = 2.0625 \text{ kN-m}$$

$$\text{Bending moment at C} = (5.0625 \times 1.5) - (5 \times 1) - (1 \times 1) \left(0.5 + \frac{1}{2} \right) \\ = 1.5937 \text{ kN-m}$$

$$\text{Bending moment at A} = (5.0625 \times 2) - (5 \times 1.5) - (2 \times 0.5) - (1 \times 1) \left(1 + \frac{1}{2} \right) \\ - (1 \times 0.5) \left(\frac{0.5}{2} \right)$$



Draw the SFD and BMD for the loaded beam as shown in below.



we know that whenever simply supported beam is given then we have to calculate Reactions at ends

we also know the equilibrium condition.

Sum of upward forces = sum of downward forces
here

$$R_A + R_B = 20 + (20 \times 1.5) \Rightarrow 20 + 30$$

$$\boxed{R_A + R_B = 50 \text{ kN}} \quad \text{--- (1)}$$

we have one more equilibrium condition.

Sum of Anticlockwise moments = sum of clockwise moments
here Taking moments about A.

$$R_B \times 4 = (20 \times 3) + (20 \times 1.5) \left(\frac{1.5}{2}\right) + 30$$

$$4R_B = 60 + (30) (0.75) + 30$$

$$R_B = \frac{112.5}{4} \Rightarrow \boxed{R_B = 28.125} \quad \text{--- (2)}$$

$$\boxed{R_B = 28.125 \text{ kN}} \quad \text{--- (2)}$$

substitute eqn (2) in eqn (1) to get R_A value.

$$R_A = 50 - 28.125$$

$$\boxed{R_A = 21.875 \text{ kN}}$$

$$\boxed{R_A = 21.875 \text{ kN}}$$

Shear force Calculations

$$SF_B = -28.125 \text{ kN}$$

$$SF_E = -28.125 + 20 \\ = -8.125 \text{ kN}$$

$$SF_D = -8.125 \text{ kN}$$

$$SF_C = -8.125 \text{ kN}$$

$$SF_A = -28.125 + 20 + (20 \times 1.5) \\ = -28.125 + 50 \\ = -21.875 + 21.875 \\ = 0.$$

Bending moment Calculations

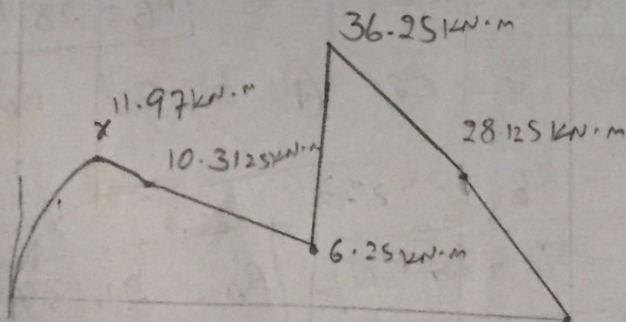
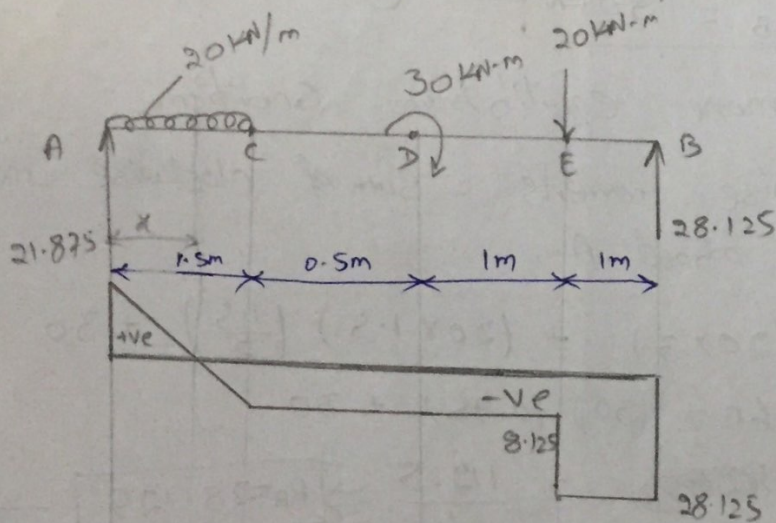
$$BM_B = 0$$

$$BM_E = +28.125 \times 1 = 28.125 \text{ kN-m}$$

$$BM_D = (28.125 \times 2) - 20 \times 1 \Rightarrow \cancel{48.125 - 20} \\ = \cancel{28.125 - 20} \\ = 56.25 - 20 \\ = 36.25 - 30 \\ = 6.25 \text{ kN-m}$$

$$BM_C = (28.125 \times 2.5) + (20 \times 1.5) - 30 \\ = 70.3125 - 30 - 30 \\ = 10.3125 \text{ kN-m}$$

$$BM_A = 0.$$



$$SF_x = 0$$

$$-21.875 + 20 \times x = 0$$

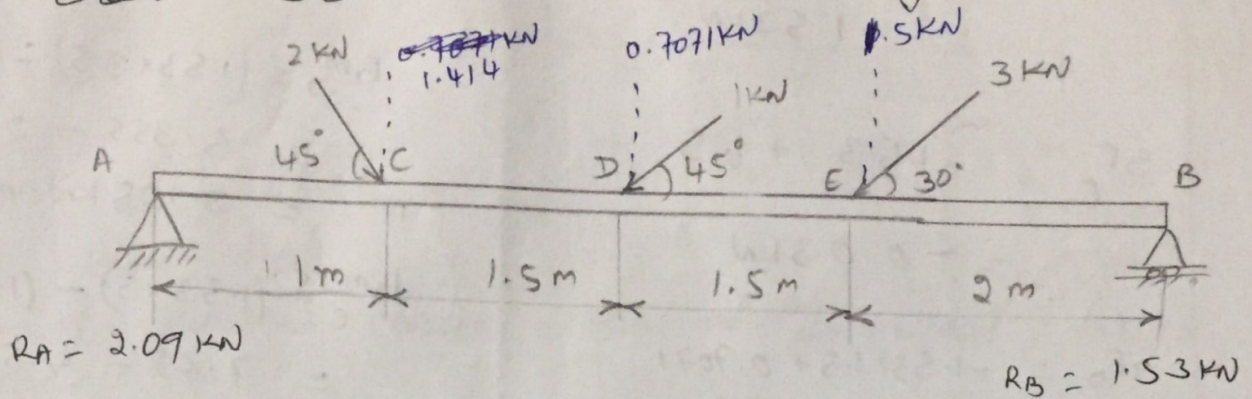
$$x = \frac{21.875}{20}$$

$$SF_x = 1.093 \text{ kN}$$

$$BM_x = R_A \times x - (20 \times x) \left(\frac{x}{2} \right)$$

$$= 21.875 \times 1.093 - (20 \times 1.093) \left(\frac{1.093}{2} \right) \\ = 11.97 \text{ kN-m}$$

A beam is loaded as shown in below fig find the reactions at A and B, and also draw shear force and bending moment diagram



The above fig shows simply supported beam carrying inclined point loads. First we have to resolve into vertical components.

we know

Sum of upward forces = sum of downward force

$$R_A + R_B = 2 \sin 45^\circ + 1 \sin 45^\circ + 3 \sin 30^\circ$$

$$= 1.4142 + 0.7071 + 1.5$$

$$R_A + R_B = \cancel{2.7113} \quad \text{--- (1) ---} \quad R_A + R_B = 3.6213 \text{ kN}$$

Sum of Anticlockwise moments = Sum of clockwise moments

$$(R_B \times 6) = (1.5 \times 4) + (0.7071 \times 3) + (\cancel{1.4142} \times 1)$$

$$6.5 R_B = 2.25 + 2.1213 + \cancel{1.4142} \quad 2.1213$$

$$R_B = \cancel{5.43} / 6.5$$

$$6 R_B = 6 + 1.7677 + 1.4142$$

$$R_B = \cancel{0.83 \text{ kN}} \quad \text{--- (2) ---}$$

$$R_B = \frac{9.1819}{6}$$

$$R_B = 1.53$$

Substitute eqn (2) value in (1)

$$R_A = \cancel{2.7113} \quad \cancel{0.83}$$

$$R_A = 3.6213 - 1.53$$

$$R_A = \cancel{2.0913 \text{ kN}}$$

$$R_A = 2.09 \text{ kN}$$

Shear force calculations

Bending moment calculations

shear force at point B

$$= -1.53 \text{ kN}$$

$$SF_E = -1.53 + 1.5$$

$$= -0.03 \text{ kN}$$

$$SF_D = -1.53 + 1.5 + 0.7071$$

$$= 0.6771 \text{ kN}$$

$$SF_C = -1.53 + 1.5 + 0.7071 + 1.414$$

$$= 2.09 \text{ kN}$$

$$SFA = 2.09 - 2.09 \Rightarrow 0$$

$$BM_B = 0$$

$$BM_E = (1.53 \times 2) = 3.06 \text{ kN-m}$$

$$BM_D = (1.53 \times 3.5) - (1.5 \times 1.5)$$

$$= 5.355 - 2.25$$

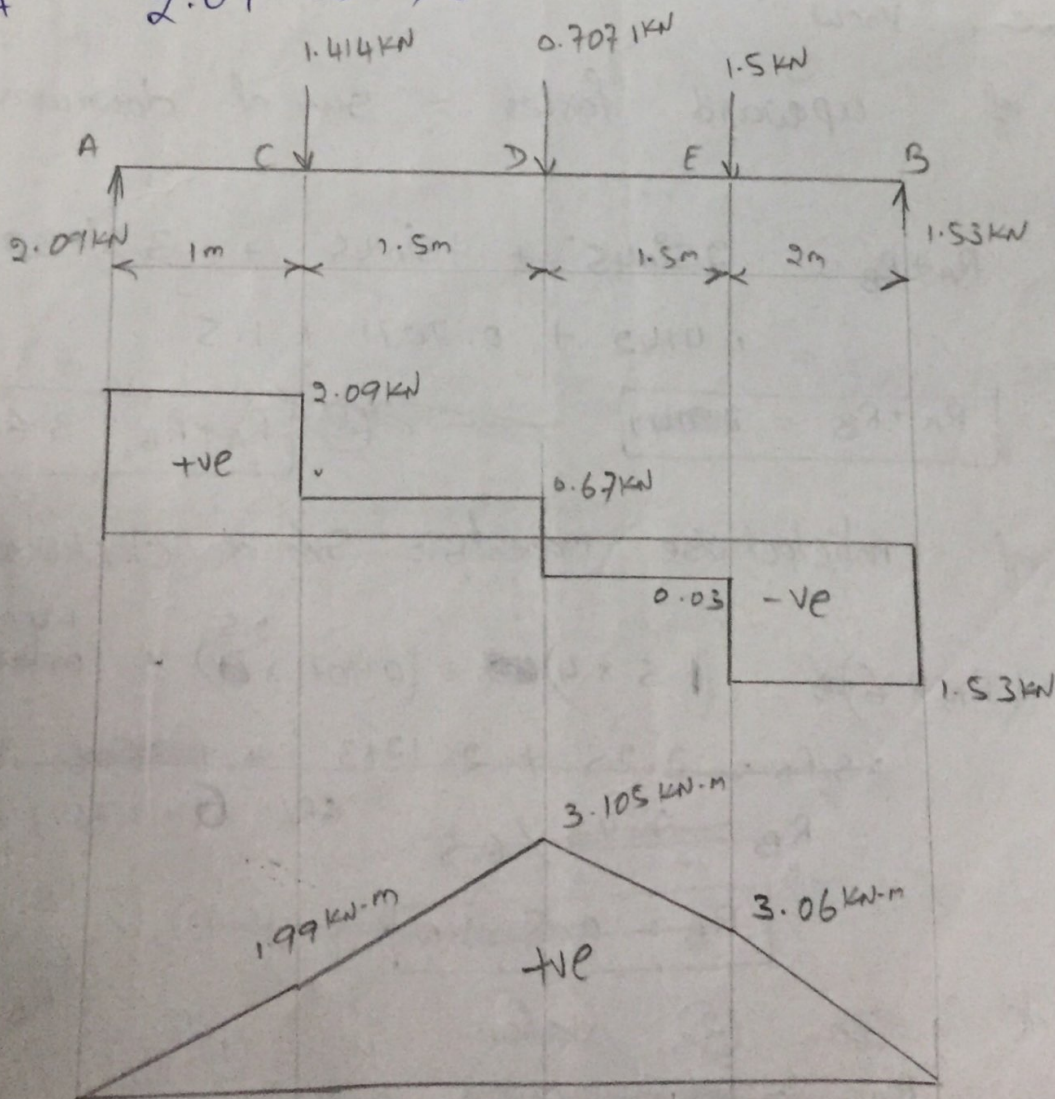
$$= 3.105 \text{ kN-m}$$

$$BM_C = (1.53 \times 5) - (1.5 \times 3) - (0.7071 \times 1.5)$$

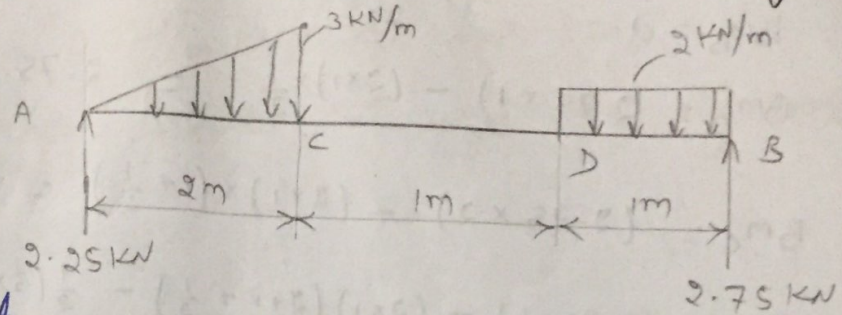
$$= 7.65 - 4.59 - 1.0606$$

$$= 1.99 \text{ kN-m}$$

$$BM_A = 0$$



Q) Draw the shear force and bending moment diagrams for the beam as shown in fig.



Sol we know that
sum of upward forces = sum of downward forces.

$$R_A + R_B = (2 \times 1) + \left(\frac{1}{2} \times 3 \times 2\right)$$

$$\boxed{R_A + R_B = 5 \text{ kN}} \quad \text{--- (1)}$$

we also know

sum of anticlockwise moments = sum of clockwise moments

$$R_B \times 4 = (2 \times 1) \left(3 + \frac{1}{2}\right) + \left(\frac{1}{2} \times 3 \times 2\right) \times \left(2 \times \frac{2}{3}\right)$$

$$4 R_B = 2 \times 3.5 + 3 \times \frac{4}{3}$$

$$R_B = 11/4$$

$$\boxed{R_B = 2.75 \text{ kN}} \quad \text{--- (2)}$$

Substitute eqn (2) value in eqn (1)

$$R_A = 5 - R_B \Rightarrow 5 - 2.75$$

$$\boxed{R_A = 2.25 \text{ kN}}$$

Shear force Calculations:-

$$SF_B = -2.75 \text{ kN}$$

$$SF_D = -2.75 + (2 \times 1) = -0.75 \text{ kN}$$

$$SF_C = -2.75 + (2 \times 1) = -0.75 \text{ kN}$$

$$\begin{aligned} SF_A &= -2.75 + (2 \times 1) + \frac{1}{2} (3 \times 2) = 2.25 \text{ kN} \\ &= 2.25 - 2.25 \\ &= 0 \text{ kN} \end{aligned}$$

Bending moment calculations:

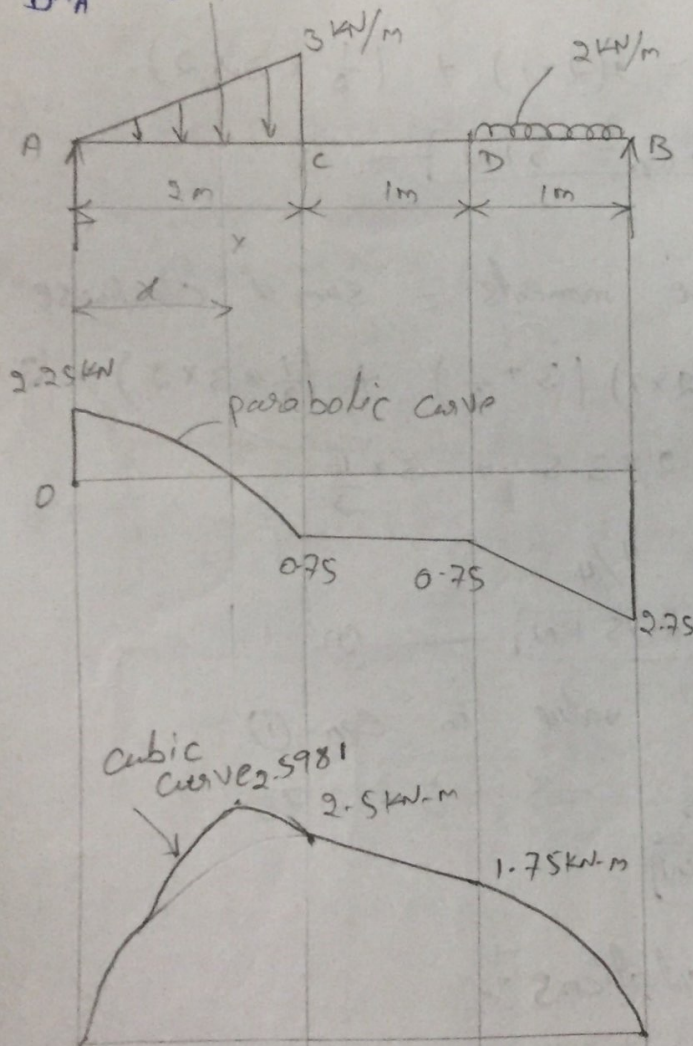
$$Bm_B = 0$$

$$Bm_D = (2.75 \times 1) - (2 \times 1) \times \frac{1}{2} = 2.75 - 1 = 1.75 \text{ kN-m}$$

$$Bm_C = (2.75 \times 2) - (2 \times 1) \times (1 + \frac{1}{2}) = 5.5 - 3 = 2.5 \text{ kN-m}$$

$$Bm_A = (2.75 \times 4) - (2 \times 1)(2 + 1 + \frac{1}{2}) - \frac{1}{2}(3 \times 3) \times \frac{2 \times 2}{3} \Rightarrow 11 - 7 - 4$$

$$Bm_A = 0$$



wherever the shear force is zero between two points then there will be maximum bending moment.

$$SF_x = 0$$

$$2.25 -$$

from similar Δ 's

$$\frac{AC}{AGC} = \frac{AE}{AF} \Rightarrow \frac{3}{3} = \frac{x}{EF}$$

$$EF = \frac{3}{2}x$$

$$2.25 - \frac{1}{2} \left(\frac{3}{2}x \times x \right)$$

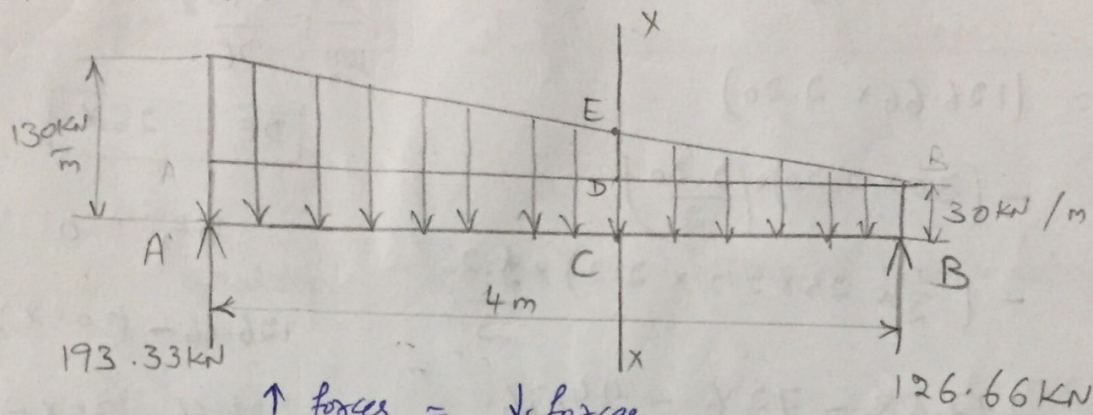
$$2.25 = \frac{3}{4}x^2$$

$$x^2 = \frac{2.25 \times 4}{3} = \frac{9}{3} = 3$$

$$x = 1.732 \text{ m}$$

$$\begin{aligned} Bm_x &= (2.25 \times 1.73) \times \frac{1}{2} \left(1.73 \times \frac{3}{2} \times 1.73 \right) \\ &= 3.8935 + (1.2944) \times \left(\frac{1.73}{3} \right) \\ &= 2.5981 \text{ kN-m} \end{aligned}$$

The intensity of loading on a simply supported beam of 4m span increases gradually from 30 kN/m run at one end to 130 kN/m run at the other. Draw S.F and B.M diagrams.



↑ forces = ↓ forces

$$R_A + R_B = (30 \times 4) + \left(\frac{1}{2} \times 4 \times 100 \right)$$

$$= 120 + 200$$

$$R_A + R_B = 320 \text{ kN} \quad \text{--- (1)}$$

Taking moments about A. (↺ = +)

$$R_B \times 4 = (30 \times 4) \times \left(\frac{4}{2} \right) + \left(\frac{1}{2} \times 4 \times 100 \right) \left(\frac{4}{3} \right)$$

$$= 240 + 266.66$$

$$= 506.66 / 4 =$$

$$R_B = 126.66 \text{ kN} \quad \text{--- (2)}$$

substitute R_B value in eqn (1)

$$R_A = 320 - 126.66$$

$$R_A = 193.33 \text{ kN}$$

Shear force calculation :-

$$SF_B = -126.66$$

$$SF_A = -126.66 + (30 \times 4) + \frac{1}{2} \times 4 \times 100$$

$$= -126.66 + 120 + 200$$

$$= -126.66 + 320$$

$$= 193.34 \text{ kN}$$

Bending moment calculations

$$BM_B = 0$$

$$BM_A = 0$$

$$BM_x = (126.66 \times 2.20)$$

$$- (30 \times 2.20) \times \left(\frac{2.20}{2} \right)$$

$$- \left(\frac{1}{2} \times 25 \times 2.2 \times 2.2 \right) \times \frac{2.2}{3}$$

$$= 277.68 - 72.6 - 44.36$$

$$= 277.68 - 116.96$$

$$= 160.71 \text{ kN-m}$$

From Similar Δ s

$$\frac{AB}{AF} = \frac{BD}{DF}$$

$$\frac{4}{\frac{100}{25}} = \frac{BD}{DE}$$

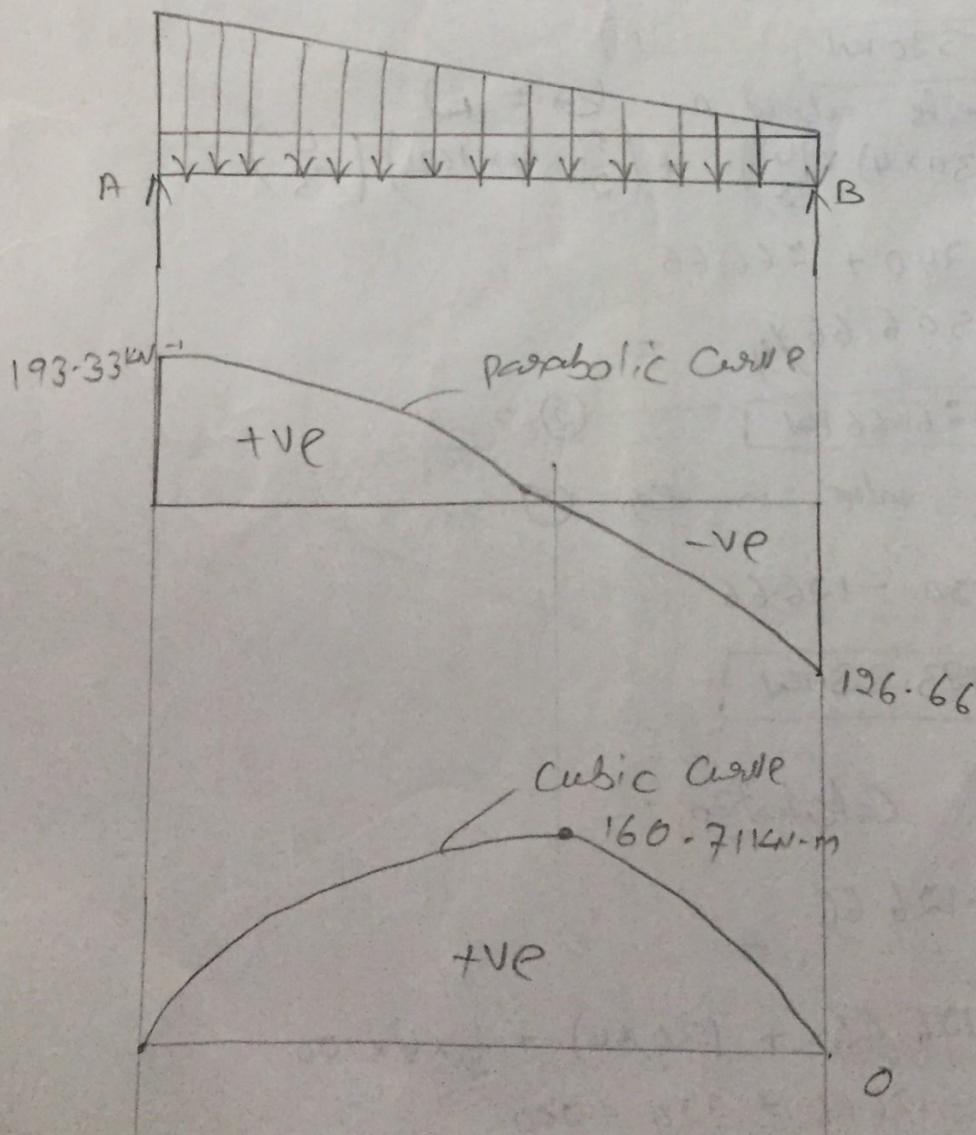
$$DE = 25x$$

$$SF_x = 0$$

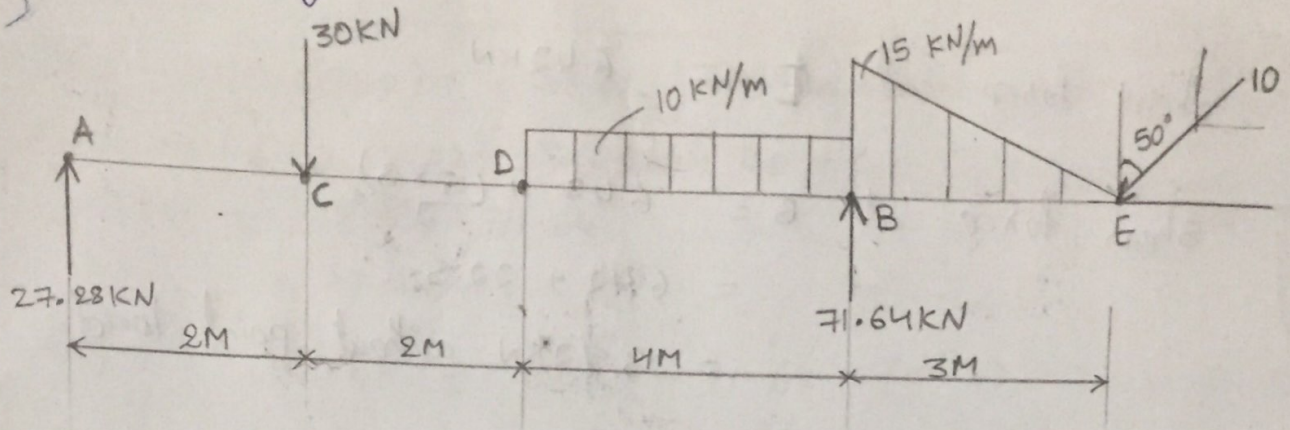
$$126.66 - (30 \times x) - \frac{1}{2} \times x$$

$$126.66 - 30x - 12.5x^2 = 0$$

$$x = 2.20 \text{ m}$$



Q) Draw SFD and BMD for the loaded beam as shown in below fig



sol we know
Sum of upward forces = sum of downward forces

$$R_A + R_B = 30 + (10 \times 4) + \frac{1}{2}(15 \times 3) + 10 \cos 50$$

$$= 30 + 40 + 22.5 + 6.42$$

$$\boxed{R_A + R_B = 94.92 \text{ kN}} \quad \text{--- (1)}$$

we also know

Sum of Anticlockwise moments = sum of clockwise moments

$$R_B \times 8 = (30 \times 2) + (10 \times 4) \left(2 + 2 + \frac{4}{2}\right) + \frac{1}{2}(15 \times 3) \left(2 + 2 + 4 + \frac{3}{3}\right) + (10 \cos 50 \times 11)$$

$$8 R_B = 573.20$$

$$\cancel{R_B = 71.65} \quad \boxed{R_B = 71.64 \text{ kN}}$$

Substitute R_B value in eqn (1) to get R_A

$$R_A = 94.92 - 71.65$$

$$\boxed{R_A = 23.28 \text{ kN}}$$

$$\text{Shear force at E} = 6.42 \text{ kN}$$

$$\begin{aligned} \text{Shear force at B} &= 6.42 + \frac{(15 \times 3)}{2} \\ &= 6.42 + 22.5 \\ &= 28.92 \text{ kN without point load.} \end{aligned}$$

$$\begin{aligned} \text{Shear force at B} &= 28.92 - 71.64 \\ &= -42.72 \text{ kN with point load.} \end{aligned}$$

$$\begin{aligned} \text{Shear force at point D} &= 6.42 + \left(\frac{1}{2} \times 15 \times 3\right) \\ &\quad + (10 \times 4) - 71.64 \\ &= -2.72 \text{ kN} \end{aligned}$$

$$\begin{aligned}
 SF_C &= (10 \cos 50) + (15 \times 3 \times \frac{1}{2}) - 71.64 + (10 \times 4) + 60 \sin 30 \\
 &= 6.42 + 22.5 - 71.64 + 40 + 30 \\
 &= 98.92 - 71.64 \\
 &= 27.28 \text{ kN}
 \end{aligned}$$

$$\begin{aligned}
 SF_A &= 6.42 + (\frac{1}{2} \times 15 \times 3) - 71.64 + (10 \times 4) + 30 - 27.28 \\
 &= 98.92 - 98.92 \\
 &= 0.
 \end{aligned}$$

Bending moment calculations :-

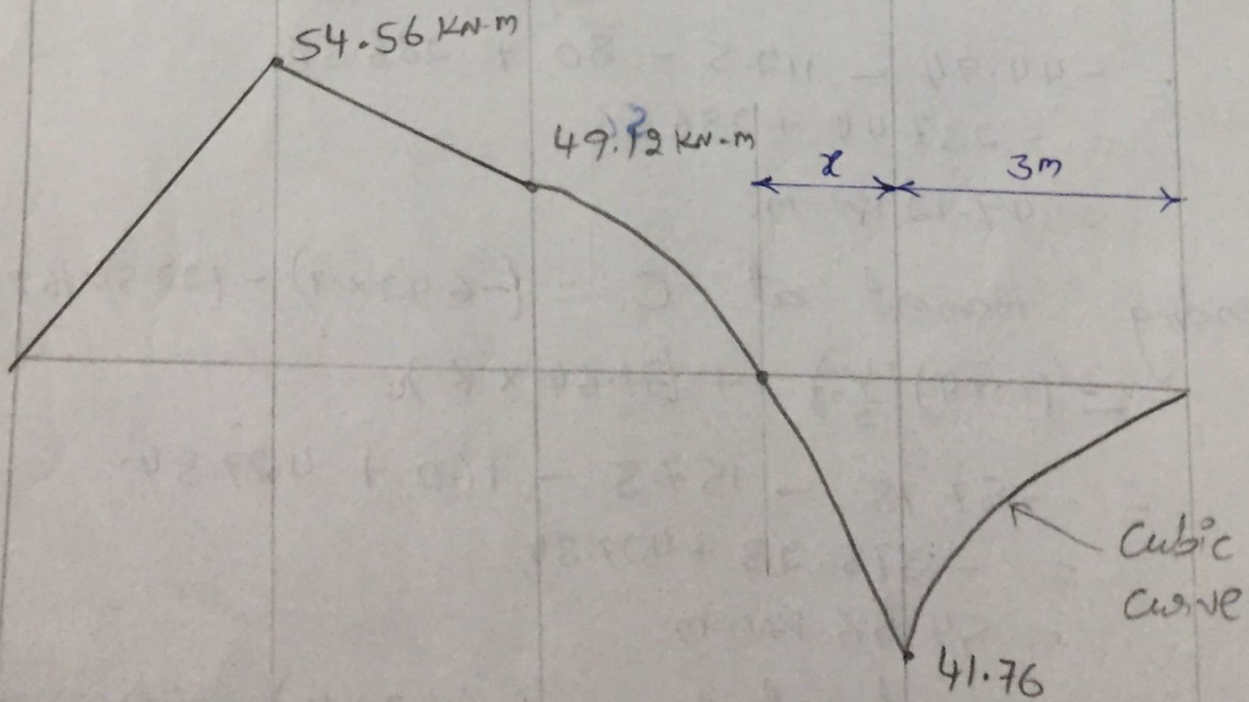
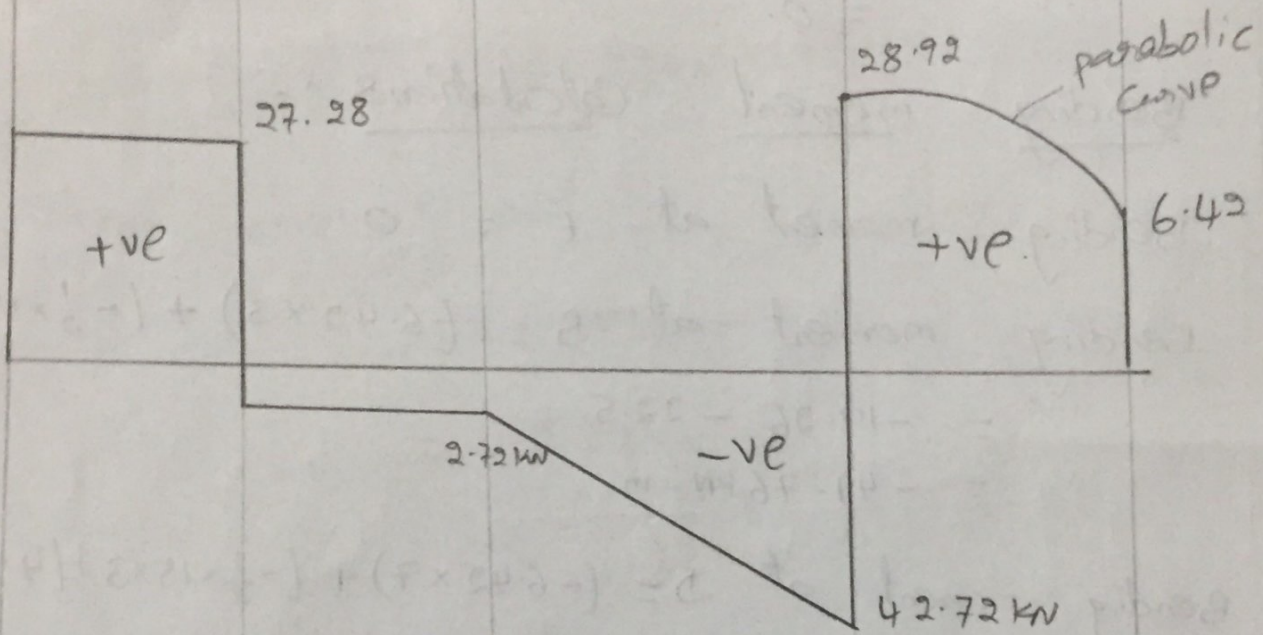
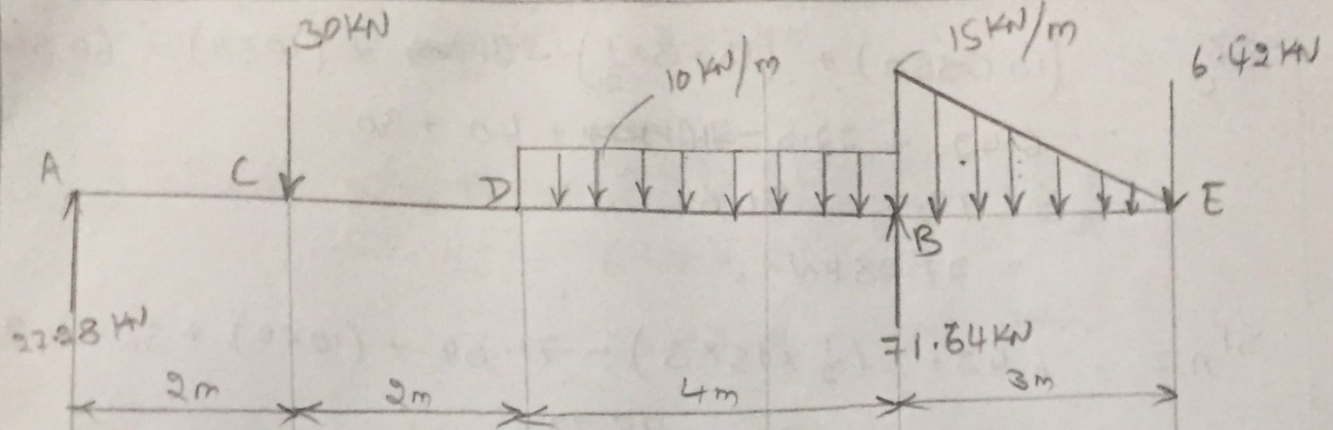
$$\text{Bending moment at E} = 0$$

$$\begin{aligned}
 \text{Bending moment at B} &= (-6.42 \times 3) + (-\frac{1}{2} \times 15 \times 3)(\frac{3}{2}) \\
 &= -19.26 - 22.5 \\
 &= -41.76 \text{ kN-m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Bending moment at D} &= (-6.42 \times 7) + (-\frac{1}{2} \times 15 \times 3)(4 + \frac{3}{3}) \\
 &\quad + (-10 \times 4)(\frac{4}{2}) + (71.64 \times 4) \\
 &= -44.94 - 112.5 - 80 + 286.56 \\
 &= -237.44 + 286.56 \\
 &= 49.12 \text{ kN-m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Bending moment at C} &= (-6.42 \times 9) - (22.5)(6 + \frac{3}{3}) \\
 &\quad - (10 \times 4)(\frac{4}{2} + 2) + (71.64 \times 6) \\
 &= -57.78 - 157.5 - 160 + 429.84 \\
 &= -375.28 + 429.84 \\
 &= 54.56 \text{ kN-m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Bending moment at A} &= (-6.42 \times 11) - (22.5)(8 + \frac{3}{3}) \\
 &\quad - (10 \times 4)(4 + \frac{4}{2}) + (71.64 \times 8) - (30 \times 2) \\
 &= -70.62 - 202.5 - 240 + 573.12 - 60
 \end{aligned}$$



In the above bending moment diagram we got the point of Contraflexure.

Now we have to locate the point of Contraflexure.

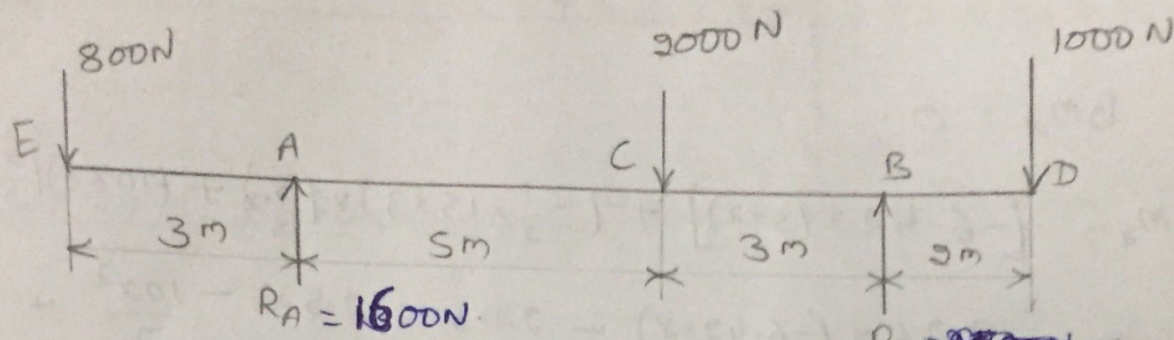
$$BM_x = 0$$

$$\begin{aligned} BM_x &= [-6.42 \times (3+x)] + \left(-\frac{1}{2} \times 15 \times 3\right) \times \left(\frac{3+x}{3}\right) + (10 \times x) \left(\frac{x}{2}\right) + (71.64 \times x) \\ &= -19.26 + (-6.42 \times x) - 22.5 - 22.5x - \frac{10x^2}{2} + 71.64x \\ &= -19.26 - 6.42x - 22.5 - 22.5x - 5x^2 + 71.64x \\ &= -5x^2 + 42.72x - 41.76 \end{aligned}$$

$$x = 1.125 \text{ m}$$

The point of Contraflexure is at a distance of $(3+x) \Rightarrow (3+1.125) = 4.125 \text{ m}$ from Right support.

Draw the shear force and bending moment diagram for the overhanging beam as shown in below.



we know

Sum of upward forces = sum of downward forces

$$R_A + R_B = 800 + 2000 + 1000$$

$$R_A + R_B = 3800 \text{ N} \quad \text{--- (1)}$$

we also know

Sum of anticlockwise moments = sum of clockwise moments

Taking moments about A.

$$(R_B \times 8) + (800 \times 3) = (2000 \times 5) + (1000 \times 10)$$

$$R_B = \frac{(10,000 + 10,000) + 2400}{8} = \frac{20,000 + 2400}{8} = 17600$$

$$R_B = 2200 \text{ N}$$

substitute R_B value in equation (1) to get R_A

$$R_A = 3800 - 2200$$

$$R_A = 1600 \text{ N}$$

~~sum of anticlockwise moments = sum of clockwise moments~~

SHEAR FORCE CALCULATIONS:-

Shear force at D = 1000 N

Shear force at B = 1000 - 2200

$$= -1000N = -1200N$$

$$\text{Shear force at C} = 1000 - 2200 + 2000 \\ = -800N$$

$$\text{Shear force at A} = 1000 - 2500 + 2000 - 1300 \\ = 3000 - 3800 \\ = -800$$

$$\text{Shear force at E} = 1000 - 2200 + 2000 - 1600 + 800 \\ = 0.$$

BENDING MOMENT CALCULATIONS:-

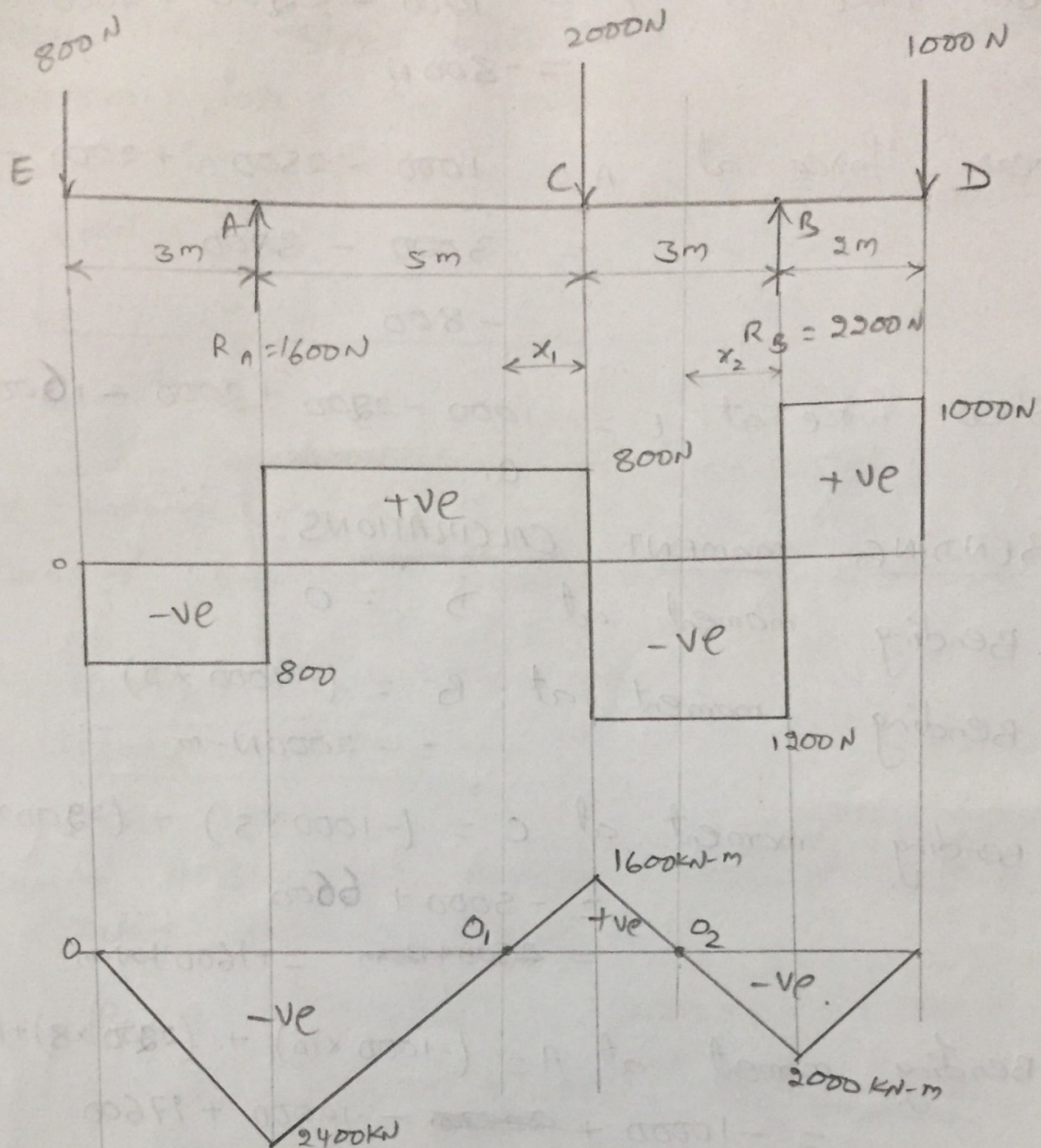
$$\text{Bending moment at D} = 0$$

$$\text{Bending moment at B} = (-1000 \times 2) \\ = -2000N-m$$

$$\text{Bending moment at C} = (-1000 \times 5) + (2200 \times 3) \\ = -5000 + 6600 \\ = \cancel{2500N-m} = +1600KN-m$$

$$\text{Bending moment at A} = (-1000 \times 10) + (2200 \times 8) + (-2000 \times 5) \\ = -10000 + \cancel{22000} - 10000 + 17600 \\ = -20000 + 17600 = -2400KN-m$$

$$\text{Bending moment at E} = (-1000 \times 13) + (2200 \times 11) + (-2000 \times 5) \\ + (1600 \times 3) \\ = -13000 + 24200 - 10000 + 4800 \\ = -29000 + 29000 \\ = 0.$$



Here in the above bending moment diagram we got two point of contraflexure. It is shown as O_1 & O_2 in the BMD.

$$\begin{aligned}
 B_{m_{O_1}} &= [1000 \times (5 + x_1)] + (2200)(3 + x_1) + (-2000 \times x_1) \\
 &= -5000 - 1000x_1 + 6600 + 2200x_1 - 2000x_1 \\
 0 &= 1600 - 800x_1 \\
 800x_1 &= 1600 \Rightarrow x_1 = \frac{1600}{800} \\
 \boxed{x_1 = 2 \text{ m}}
 \end{aligned}$$

$$BMO_2 = [-1000 \times (2+x_2)] + (2200 \times x_2)$$

$$= -2000 - 1000x_2 + 2200x_2$$

$$= -2000 + 1200x_2$$

$$2000 = 1200x_2$$

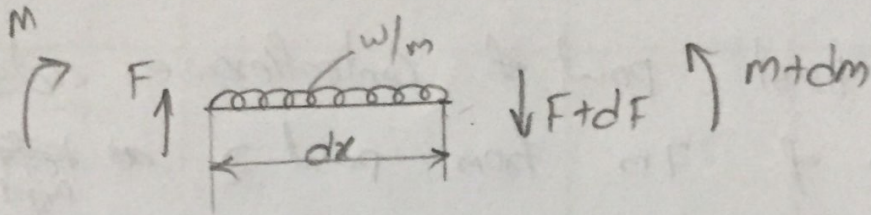
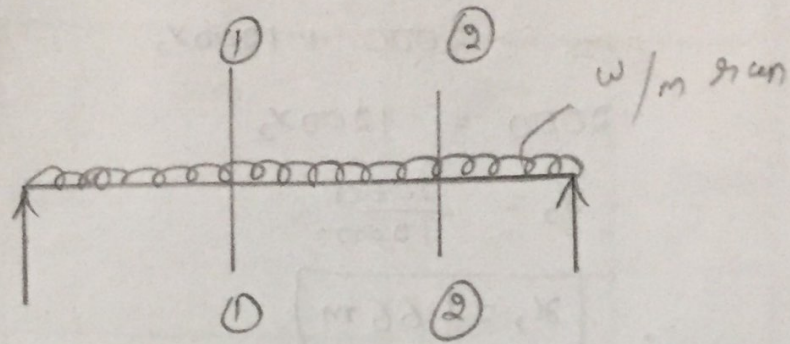
$$x_2 = \frac{2000}{1200}$$

$$x_2 = 1.66 \text{ m}$$

The first point of Contraflexure acts at a distance of 7m from point D ~~at left~~ ^{Right} side of Kelem.

The second point of Contraflexure acts at a distance of 3.66 from point D. Right side.

Relationship between Load, shear force and Bending moment :-



shear force.

$$F - w \cdot dx - (F + dF) = 0$$

$$\cancel{F} - w dx - \cancel{F} - dF = 0$$

$$\boxed{-w = \frac{dF}{dx}}$$

Bending moment :

$$M + F \cdot dx - w \cdot dx \cdot \frac{dx}{2} - (M + dm)$$

$$\cancel{M} + F dx - \frac{w dx^2}{2} - \cancel{M} - dm$$

neglecting the powers of smaller quantities

$$F dx - dm = 0$$

$$\boxed{F = \frac{dm}{dx}}$$

17

CHAPTER

THIN CYLINDERS AND SPHERES

17.1. INTRODUCTION

The vessels such as boilers, compressed air receivers etc., are of cylindrical and spherical forms. These vessels are generally used for storing fluids (liquid or gas) under pressure. The walls of such vessels are thin as compared to their diameters. If the thickness of the wall of the cylindrical vessel is less than $\frac{1}{15}$ to $\frac{1}{20}$ of its internal diameter, the cylindrical vessel is known as a *thin cylinder*. In case of thin cylinders, the stress distribution is assumed uniform over the thickness of the wall.

17.2. THIN CYLINDRICAL VESSEL SUBJECTED TO INTERNAL PRESSURE

Fig. 17.1 shows a thin cylindrical vessel in which a fluid under pressure is stored.

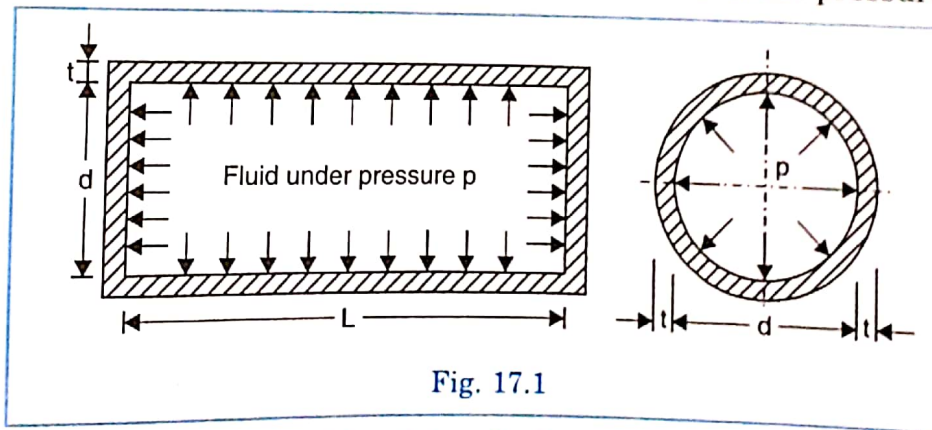


Fig. 17.1

- Let d = Internal diameter of the thin cylinder
 t = Thickness of the wall of the cylinder
 p = Internal pressure of the fluid
 L = Length of the cylinder.

On account of the internal pressure p , the cylindrical vessel may fail by splitting up in any one of the two ways as shown in Fig. 17.2 (a) and 17.2 (b).

The forces, due to pressure of the fluid acting vertically upwards and downwards on the thin cylinder, tend to burst the cylinder as shown in Fig. 17.2 (a).

The forces, due to pressure of the fluid, acting at the ends of the thin cylinder, tend to burst the thin cylinder as shown in Fig. 17.2 (b).

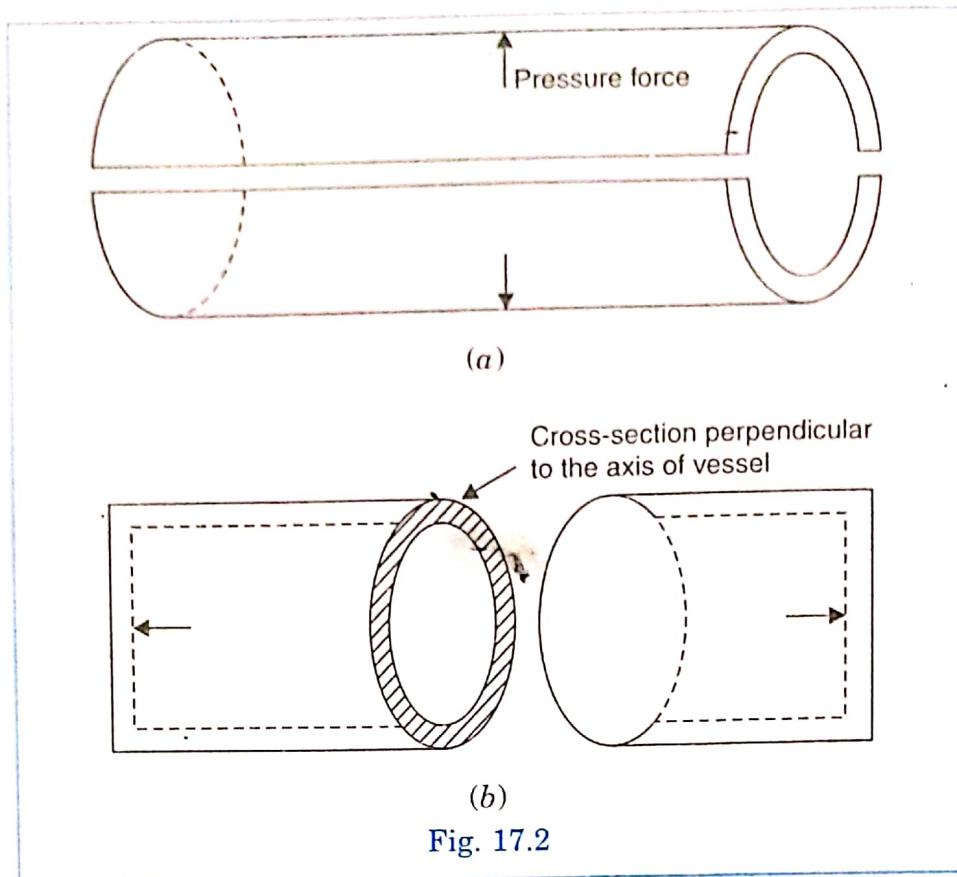


Fig. 17.2

17.3. STRESSES IN A THIN CYLINDRICAL VESSEL SUBJECTED TO INTERNAL PRESSURE

When a thin cylindrical vessel is subjected to internal fluid pressure, the stresses in the wall of the cylinder on the cross-section along the axis and on the cross-section perpendicular to the axis are set up. These stresses are tensile and are known as :

1. Circumferential stress (or hoop stress) and
2. Longitudinal stress.

The name of the stress is given according to the direction in which the stress is acting. The stress acting along the circumference of the cylinder is called circumferential stress whereas the stress acting along the length of the cylinder (*i.e.*, in the longitudinal direction) is known as longitudinal stress. The circumferential stress is also known as *hoop stress*. The stress set up in Fig. 17.2 (a) is circumferential stress whereas the stress set up in Fig. 17.2 (b) is longitudinal stress.

17.4. EXPRESSION FOR CIRCUMFERENTIAL STRESS (OR HOOP STRESS)

Consider a thin cylindrical vessel subjected to an internal fluid pressure. The circumferential stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place as shown in Fig. 17.3 (a).

The expression for hoop stress or circumferential stress (σ_1) is obtained as given below.

Let p = Internal pressure of fluid

d = Internal diameter of the cylinder

t = Thickness of the wall of the cylinder

σ_1 = Circumferential or hoop stress in the material.

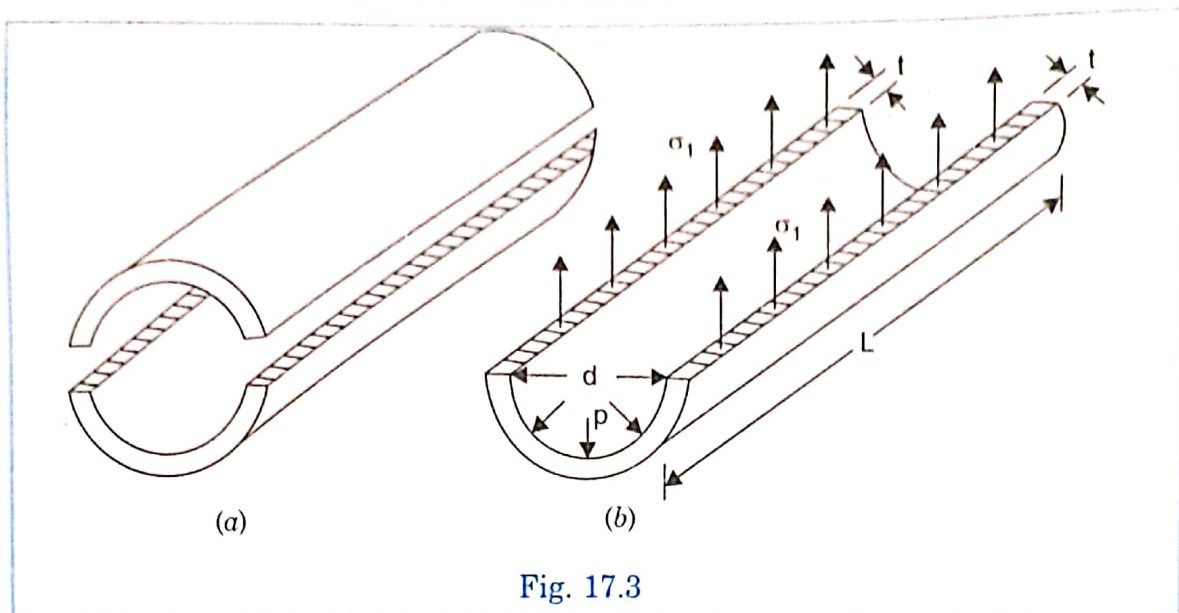


Fig. 17.3

The bursting will take place if the force due to fluid pressure is more than the resisting force due to circumferential stress set up in the material. In the limiting case, the two forces should be equal.

$$\begin{aligned}
 \text{Force due to fluid pressure} &= p \times \text{Area on which } p \text{ is acting} \\
 &= p \times (d \times L) \quad \dots(i) \\
 &(\because p \text{ is acting on projected area } d \times L)
 \end{aligned}$$

$$\begin{aligned}
 \text{Force due to circumferential stress} &= \sigma_1 \times \text{Area on which } \sigma_1 \text{ is acting} \\
 &= \sigma_1 \times (L \times t + L \times t) \\
 &= \sigma_1 \times 2Lt = 2\sigma_1 \times L \times t \quad \dots(ii)
 \end{aligned}$$

Equating (i) and (ii), we get

$$p \times d \times L = 2\sigma_1 \times L \times t$$

$$\therefore \sigma_1 = \frac{pd}{2t} \text{ (cancelling } L) \quad \dots(17.1)$$

This stress is tensile as shown in Fig. 17.3 (b).

17.5. EXPRESSION FOR LONGITUDINAL STRESS

Consider a thin cylindrical vessel subjected to internal fluid pressure. The longitudinal stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place along the section AB of Fig. 17.4 (a).

The longitudinal stress (σ_2) developed in the material is obtained as :

Let p = Internal pressure of fluid stored in thin cylinder

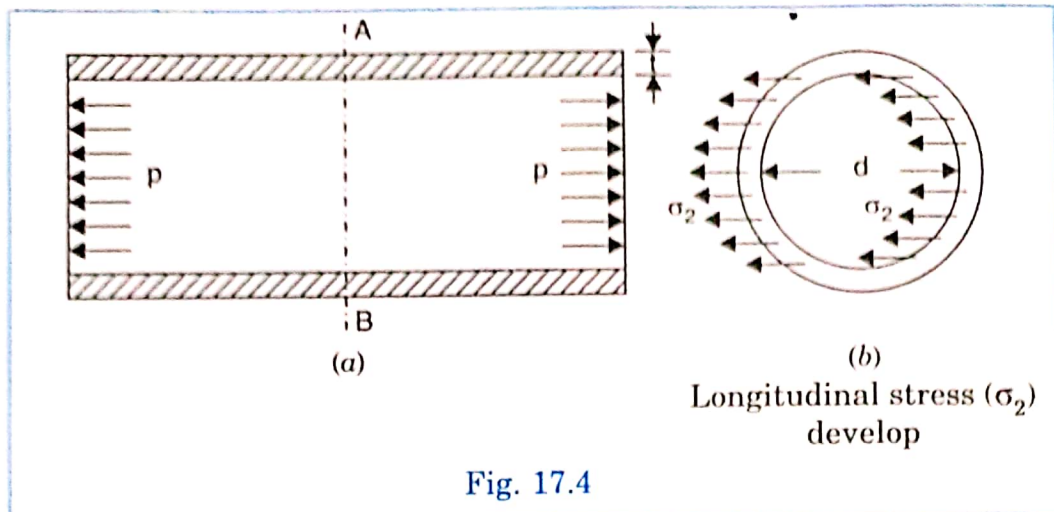
d = Internal diameter of cylinder

t = Thickness of the cylinder

σ_2 = Longitudinal stress in the material.

The bursting will take place if the force due to fluid pressure acting on the ends of the cylinder is more than the resisting force due to longitudinal stress (σ_2) developed in the material as shown in Fig. 17.4 (b). In the limiting case, both the forces should be equal.

$$\begin{aligned}
 \text{Force due to fluid pressure} &= p \times \text{Area on which } p \text{ is acting} \\
 &= p \times \frac{\pi}{4} d^2
 \end{aligned}$$



Resisting force = $\sigma_2 \times \text{Area on which } \sigma_2 \text{ is acting}$
 $= \sigma_2 \times \pi d \times t$

$\therefore$ Hence in the limiting case

Force due to fluid pressure = Resisting force

$$p \times \frac{\pi}{4} d^2 = \sigma_2 \times \pi d \times t$$

$$\therefore \sigma_2 = \frac{p \times \frac{\pi}{4} d^2}{\pi d \times t} = \frac{pd}{4t} \quad \dots(17.2)$$

The stress σ_2 is also tensile.

Equation (17.2) can be written as

$$\sigma_2 = \frac{pd}{2 \times 2t} = \frac{1}{2} \times \sigma_1 \quad \left(\because \sigma_1 = \frac{pd}{2t} \right)$$

or Longitudinal stress = Half of circumferential stress.

This also means that circumferential stress (σ_1) is two times the longitudinal stress (σ_2). Hence in the material of the cylinder the permissible stress should be less than the circumferential stress. Or in other words, the circumferential stress should not be greater than the permissible stress.

Maximum shear stress. At any point in the material of the cylindrical shell, there are two principal stresses, namely a circumferential stress of magnitude $\sigma_1 = \frac{pd}{2t}$ acting circumferentially and a longitudinal stress of magnitude $\sigma_2 = \frac{pd}{4t}$ acting parallel to the axis of the shell. These two stresses are tensile and perpendicular to each other.

$$\therefore \text{Maximum shear stress } \tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2} = \frac{pd}{8t} \quad \dots[17.2(A)]$$

Problem 17.1. A cylindrical pipe of diameter 1.5 m and thickness 1.5 cm is subjected to an internal fluid pressure of 1.2 N/mm². Determine :

- (i) Longitudinal stress developed in the pipe, and
- (ii) Circumferential stress developed in the pipe.

Sol. Given :

Dia. of pipe, $d = 1.5$ m
Thickness, $t = 1.5$ cm = 1.5×10^{-2} m
Internal fluid pressure, $p = 1.2$ N/mm²

As the ratio $\frac{t}{d} = \frac{1.5 \times 10^{-2}}{1.5} = \frac{1}{100}$, which is less than $\frac{1}{20}$, hence this is a case of thin cylinder.

Here unit of pressure (p) is in N/mm². Hence the unit of σ_1 and σ_2 will also be in N/mm².

- (i) The longitudinal stress (σ_2) is given by equation (17.2) as,

$$\begin{aligned}\sigma_2 &= \frac{p \times d}{4t} \\ &= \frac{1.2 \times 1.5}{4 \times 1.5 \times 10^{-2}} = 30 \text{ N/mm}^2. \quad \text{Ans.}\end{aligned}$$

- (ii) The circumferential stress (σ_1) is given by equation (17.1) as

$$\begin{aligned}\sigma_1 &= \frac{pd}{2t} \\ &= \frac{1.2 \times 1.5}{2 \times 1.5 \times 10^{-2}} = 60 \text{ N/mm}^2. \quad \text{Ans.}\end{aligned}$$

Problem 17.2. A cylinder of internal diameter 2.5 m and of thickness 5 cm contains a gas. If the tensile stress in the material is not to exceed 80 N/mm², determine the internal pressure of the gas.

Sol. Given :

Internal dia. of cylinder, $d = 2.5$ m
Thickness of cylinder, $t = 5$ cm = 5×10^{-2} m
Maximum permissible stress = 80 N/mm²

As maximum permissible stress is given. Hence this should be equal to circumferential stress (σ_1).

We know that the circumferential stress should not be greater than the maximum permissible stress. Hence take circumferential stress equal to maximum permissible stress.

$$\therefore \sigma_1 = 80 \text{ N/mm}^2$$

Let p = Internal pressure of the gas

Using equation (17.1),

$$\sigma_1 = \frac{pd}{2t}$$

or
$$p = \frac{2t \times \sigma_1}{d} = \frac{2 \times 5 \times 10^{-2} \times 80}{2.5} \quad \text{(Here unit of } \sigma_1 \text{ is in N/mm}^2\text{,}$$

hence unit of p will also be in N/mm²)
$$= 3.2 \text{ N/mm}^2. \quad \text{Ans.}$$

Problem 17.3. A cylinder of internal diameter 0.50 m contains air at a pressure of 7 N/mm² (gauge). If the maximum permissible stress induced in the material is 80 N/mm², find the thickness of the cylinder.

Sol. Given :

Internal dia. of cylinder, $d = 0.50$ m

Internal pressure of air, $p = 7$ N/mm²

Maximum permissible stress in the material means the circumferential stress (σ_1).

As stated earlier that the circumferential stress should not be greater than the maximum permissible stress. Hence take circumferential stress equal to maximum permissible stress.

$\therefore$ Circumferential stress, $\sigma_1 = 80$ N/mm²

Let t = Thickness of the cylinder

Using equation (17.1),

$$\sigma_1 = \frac{pd}{2t}$$

$$\therefore t = \frac{pd}{2 \times \sigma_1} = \frac{7 \times 0.50}{2 \times 80} = 0.021875 \text{ m}$$

(Here p and σ_1 are in N/mm², d is in m hence t will be in m)

= 2.188 cm. Ans.

If the value of t is taken 2.1875 cm, the stress induced will be 80 N/mm². If the value of t is less than 2.1875 cm, the stress induced will be more than 80 N/mm². But the stress induced should not be more than 80 N/mm². If the value of t is taken more than 2.1875 cm (say $t = 2.188$ cm), the stress induced will be less than 80 N/mm².

Hence take $t = 2.188$ cm or say **2.2 cm. Ans.**

Problem 17.4. A thin cylinder of internal diameter 1.25 m contains a fluid at an internal pressure of 2 N/mm². Determine the maximum thickness of the cylinder if :

(i) The longitudinal stress is not to exceed 30 N/mm².

(ii) The circumferential stress is not to exceed 45 N/mm².

Sol. Given :

Internal dia. of cylinder, $d = 1.25$ m

Internal pressure of fluid, $p = 2$ N/mm²

Longitudinal stress, $\sigma_2 = 30$ N/mm²

Circumferential stress, $\sigma_1 = 45$ N/mm²

Using equation (17.1),

$$\sigma_1 = \frac{pd}{2t}$$

$$\therefore t = \frac{p \times d}{2 \times \sigma_1} = \frac{2 \times 1.25}{2 \times 45} = 0.0277 \text{ m}$$

$$= 2.77 \text{ cm.} \quad \dots(i)$$

Using equation (17.2),

$$\sigma_2 = \frac{pd}{4t}$$

$$\therefore t = \frac{pd}{4 \times \sigma_2} = \frac{2 \times 1.25}{4 \times 30} = 0.0208 \text{ m}$$

$$= 2.08 \text{ cm.} \quad \dots(ii)$$

The longitudinal or circumferential stresses induced in the material are inversely proportional to the thickness (t) of the cylinder. Hence the stress induced will be less if the value of ' t ' is more. Hence take the maximum value of ' t ' calculated in equations (i) and (ii)

From equations (i) and (ii) it is clear that t should not be less than 2.77 cm.

Take $t = 2.80$ cm. **Ans.**

Problem 17.5. A water main 80 cm diameter contains water at a pressure head of 100 m. If the weight density of water is 9810 N/m^3 , find the thickness of the metal required for the water main. Given the permissible stress as 20 N/mm^2 .

Sol. Given :

Dia. of main, $d = 80$ cm

Pressure head of water, $h = 100$ m

Weight density of water, $w = \rho \times g = 1,000 \times 9.81 = 9810 \text{ N/m}^3$

Permissible stress $= 20 \text{ N/mm}^2$

Permissible stress is equal to circumferential stress (σ_1)

or $\sigma_1 = 20 \text{ N/mm}^2$

Pressure of water inside the water main,

$$p = \rho \times g \times h = wh = 9810 \times 100 \text{ N/m}^2$$

Here σ_1 is in N/mm^2 , hence pressure (p) should also be N/mm^2 . The value of p in N/mm^2 is given as

$$p = \frac{9810 \times 100}{1000^2 \text{ mm}^2} \text{ N/mm}^2 \quad (\because 1 \text{ m} = 1000 \text{ mm})$$

$$= 0.981 \text{ N/mm}^2$$

Let t = Thickness of the metal required.

Using equation (17.1),

$$\sigma_1 = \frac{p \times d}{2 \times t} \quad (\text{Here 'd' is in cm hence 't' will also be in cm})$$

$$\therefore t = \frac{p \times d}{2 \times \sigma_1} = \frac{0.981 \times 80}{2 \times 20} = 2 \text{ cm. } \mathbf{Ans.}$$

17.6. EFFICIENCY OF A JOINT

The cylindrical shells such as boilers are having two types of joints namely longitudinal joint and circumferential joint. In case of a joint, holes are made in the material of the shell for the rivets. Due to the holes, the area offering resistance decreases. Due to the decrease in area, the stress (which is equal to the force divided by the area) developed in the material of the shell will be more.

Hence in case of rivetted shell the circumferential and longitudinal stresses are greater than what are given by equations (17.1) and (17.2). If the efficiency of a longitudinal joint and circumferential joint are given then the circumferential and longitudinal stresses are obtained as :

Let η_l = Efficiency of a longitudinal joint, and

η_c = Efficiency of the circumferential joint.

Then the circumferential stress (σ_1) is given as

$$\sigma_1 = \frac{p \times d}{2t \times \eta_l} \quad \dots(17.3)$$

and the longitudinal stress (σ_2) is given as

$$\sigma_2 = \frac{p \times d}{4t \times \eta_c} \quad \dots(17.4)$$

Note. (i) In longitudinal joint, the circumferential stress is developed whereas in circumferential joint, the longitudinal stress is developed.

(ii) Efficiency of a joint means the efficiency of a longitudinal joint.

(iii) If efficiencies of a joint are given, thickness of the thin shell is determined from equation (17.3).

Problem 17.6. A boiler is subjected to an internal steam pressure of 2 N/mm². The thickness of boiler plate is 2.0 cm and permissible tensile stress is 120 N/mm². Find out the maximum diameter, when efficiency of longitudinal joint is 90% and that of circumferential joint is 40%.

Sol. Given :

Internal steam pressure, $p = 2 \text{ N/mm}^2$

Thickness of boiler plate, $t = 2.0 \text{ cm}$

Permissible tensile stress $= 120 \text{ N/mm}^2$

In case of a joint, the permissible stress may be circumferential stress or longitudinal stress.

Efficiency of longitudinal joint, $\eta_l = 90\% = 0.90$

Efficiency of circumferential joint, $\eta_c = 40\% = 0.40$.

Max. diameter for circumferential stress is given by equation (17.3).

$\therefore$ Using equation (17.3),

$$\sigma_1 = \frac{p \times d}{2 \times \eta_l \times t}$$

where $\sigma_1 =$ Given permissible stress $= 120 \text{ N/mm}^2$

$$\therefore 120 = \frac{2 \times d}{2 \times 0.90 \times 2.0} \quad (\text{Here } p \text{ and } \sigma_1 \text{ are in same units.})$$

Thickness is in cm hence 'd' will be in cm)

$$\therefore d = \frac{120 \times 2 \times 0.9 \times 2.0}{2} = 216.0 \text{ cm.} \quad \dots(i)$$

Max. diameter for longitudinal stress is given by equation (17.4).

$\therefore$ Using equation (17.4),

$$\sigma_2 = \frac{p \times d}{4 \times \eta_c \times t}$$

where $\sigma_2 =$ Given permissible stress
 $= 120 \text{ N/mm}^2$.

$$\therefore 120 = \frac{2 \times d}{4 \times 0.4 \times 2.0}$$

$$\therefore d = \frac{120 \times 4 \times 0.4 \times 2.0}{2} = 192 \text{ cm.} \quad \dots(ii)$$

The longitudinal or circumferential stresses induced in the material are directly proportional to diameter (d). Hence the stress induced will be less if the value of 'd' is less. Hence take the minimum value of 'd' calculated from equations (i) and (ii).

Maximum diameter of the boiler is equal to the minimum* value of diameter given by equations (i) and (ii).

Hence maximum diameter, $d = 192 \text{ cm}$. **Ans.**

(*Please note that if d is taken as equal to 216.0 cm, the longitudinal stress (σ_2) will be more than the given permissible value as shown below :

$$\sigma_2 = \frac{p \times d}{4 \times \eta_c \times t} = \frac{2 \times 216}{4 \times 0.4 \times 2.0} = 135 \text{ N/mm}^2.)$$

Problem 17.7. A boiler shell is to be made of 15 mm thick plate having a limiting tensile stress of 120 N/mm^2 . If the efficiencies of the longitudinal and circumferential joints are 70% and 30% respectively determine :

(i) The maximum permissible diameter of the shell for an internal pressure of 2 N/mm^2 , and

(ii) Permissible intensity of internal pressure when the shell diameter is 1.5 m.

Sol. Given :

Thickness of boiler shell, $t = 15 \text{ mm}$

Limiting tensile stress $= 120 \text{ N/mm}^2$

Limiting tensile stress may be circumferential stress or longitudinal stress.

Efficiency of longitudinal joint, $\eta_l = 70\% = 0.70$

Efficiency of circumferential joint, $\eta_c = 30\% = 0.30$.

(i) Maximum Permissible diameter for an internal pressure,
 $p = 2 \text{ N/mm}^2$.

The boiler shell should be designed for the limiting tensile stress of 120 N/mm^2 . First consider the limiting tensile stress as circumferential stress and then as longitudinal stress. The minimum diameter of the two case will satisfy the condition.

(a) Taking limiting tensile stress = Circumferential stress

$$\sigma_1 = 120 \text{ N/mm}^2.$$

$$\therefore \sigma_1 = 120 \text{ N/mm}^2$$

But σ_1 is also given by equation (17.3) as

$$\sigma_1 = \frac{p \times d}{2 \times \eta_l \times t}$$

$$\therefore 120 = \frac{2 \times d}{2 \times 0.7 \times 15} \quad (\text{Here } t \text{ is in mm hence 'd' will be in mm})$$

$$\therefore d = \frac{120 \times 2 \times 0.7 \times 15}{2} = 1260 \text{ mm} \quad \dots(i)$$

(b) Taking limiting tensile stress = Longitudinal stress

$$(\sigma_2) = 120 \text{ N/mm}^2.$$

$$\therefore \sigma_2 = 120 \text{ N/mm}^2$$

Using equation (17.4),

$$\sigma_2 = \frac{p \times d}{4 \times \eta_c \times t}$$

$$120 = \frac{2 \times d}{4 \times 0.30 \times 15}$$

or

$$\therefore d = \frac{120 \times 4 \times 0.3 \times 15}{2} = 1080 \text{ mm} \quad \dots(ii)$$

Thus the maximum diameter of the shell, in order both the conditions may be satisfied = 1080 mm. **Ans.**

(ii) Permissible intensity of internal pressure when the shell diameter is 1.5 m.

or $d = 1.5 \text{ m} = 1500 \text{ mm}.$

(a) Taking limiting tensile stress = Circumferential stress (σ_1)
 $= 120 \text{ N/mm}^2$

Using equation (17.3),

$$\sigma_1 = \frac{p \times d}{2 \times \eta_l \times t}$$

or $120 = \frac{p \times 1500}{2 \times 0.7 \times 15}$ (Here σ_1 is in N/mm^2 hence 'p' will be in N/mm^2)

$\therefore p = \frac{120 \times 2 \times 0.7 \times 15}{1500} = 1.68 \text{ N/mm}^2 \quad \dots(i)$

(b) Taking limiting tensile stress = Longitudinal stress (σ_2)
 $= 120 \text{ N/mm}^2$

Using equation (17.4),

$$\sigma_2 = \frac{p \times d}{4 \times \eta_c \times t}$$

$$120 = \frac{p \times 1500}{4 \times 0.30 \times 15}$$

$\therefore p = \frac{120 \times 4 \times 0.30 \times 15}{1500} = 1.44 \text{ N/mm}^2 \quad \dots(ii)$

Hence in order both the conditions may be satisfied the maximum permissible internal pressure is equal to the minimum* value of pressure given by (i) and (ii).

$\therefore$ Maximum permissible internal pressure = 1.44 N/mm^2

(*If p is taken equal to 1.68 N/mm^2 , then longitudinal stress (σ_2) will be,

$$\sigma_2 = \frac{p \times d}{4 \times \eta_c \times t} = \frac{1.68 \times 1500}{4 \times 0.30 \times 15} = 140 \text{ N/mm}^2.)$$

This value is more than the given limiting tensile stress.

Problem 17.8. A cylinder of thickness 1.5 cm, has to withstand maximum internal pressure of 1.5 N/mm^2 . If the ultimate tensile stress in the material of the cylinder is 300 N/mm^2 , factor of safety 3.0 and joint efficiency 80%, determine the diameter of the cylinder.

Sol. Given :

Thickness of cylinder, $t = 1.5 \text{ cm}$
 Internal pressure, $p = 1.5 \text{ N/mm}^2$
 Ultimate tensile stress $= 300 \text{ N/mm}^2$
 Factor of safety $= 3.0$

$\therefore$ Working stress, $\sigma_1 = \frac{\text{Ultimate tensile stress}}{\text{Factor of safety}} = \frac{300}{3} = 100 \text{ N/mm}^2$

Joint efficiency, $\eta = 80\% = 0.80$

Joint efficiency means the efficiency of longitudinal joint (or η_l),

$$\therefore \eta_l = 0.80.$$

The stress corresponding to longitudinal joint is given by equation (17.3).

Using equation (17.3),

$$\sigma_1 = \frac{p \times d}{2 \times \eta_l \times t}$$

$$100 = \frac{1.5 \times d}{2 \times 0.80 \times 1.5}$$

$$\therefore d = \frac{100 \times 2 \times 0.80 \times 1.5}{1.5} = 160 \text{ cm} = 1.6 \text{ m. Ans.}$$

Problem 17.9. Calculate : (i) the change in diameter, (ii) change in length and (iii) change in volume of a thin cylindrical shell 100 cm diameter, 1 cm thick and 5 m long when subjected to internal pressure of 3 N/mm^2 . Take the value of $E = 2 \times 10^5 \text{ N/mm}^2$ and Poisson's ratio, $\mu = 0.3$.

Sol. Given :

Diameter of shell, $d = 100 \text{ cm}$

Thickness of shell, $t = 1 \text{ cm}$

Length of shell, $L = 5 \text{ m} = 5 \times 100 = 500 \text{ cm}$

Internal pressure, $p = 3 \text{ N/mm}^2$

Young's modulus, $E = 2 \times 10^5 \text{ N/mm}^2$

Poisson's ratio, $\mu = 0.30$

(i) Change in diameter (δd) is given by equation (17.11) as

$$\begin{aligned}\delta d &= \frac{pd^2}{2tE} \left[1 - \frac{\mu}{2} \right] \\ &= \frac{3 \times 100^2}{2 \times 1 \times 2 \times 10^5} \left[1 - \frac{1}{2} \times 0.30 \right] \\ &= \frac{3}{40} [1 - 0.15] = \mathbf{0.06375 \text{ cm.}} \quad \text{Ans.}\end{aligned}$$

(ii) Change in length (δL) is given by equation (17.14) as

$$\begin{aligned}\delta L &= \frac{pdL}{2tE} \left[\frac{1}{2} - \mu \right] \\ &= \frac{3 \times 100 \times 500}{2 \times 1 \times 2 \times 10^5} \left[\frac{1}{2} - 0.30 \right] \\ &= \frac{15}{40} \times 0.20 = \mathbf{0.075 \text{ cm.}} \quad \text{Ans.}\end{aligned}$$

(iii) Change in volume (δV) is given by equation (17.18) as

$$\begin{aligned}\delta V &= V [2e_1 + e_2] \\ &= V \left[2 \frac{\delta d}{d} + \frac{\delta L}{L} \right] \quad \left(\because e_1 = \frac{\delta d}{d}, e_2 = \frac{\delta L}{L} \right)\end{aligned}$$

Substituting the values of δd , δL , d and L , we get

$$\begin{aligned}\delta V &= V \left[2 \times \frac{0.06375}{100} + \frac{0.075}{500} \right] \\ &= V [0.001275 + 0.00015] = 0.001425 V.\end{aligned}$$

But

$$\begin{aligned}V &= \text{Original volume} = \frac{\pi}{4} d^2 L \\ &= \frac{\pi}{4} \times 100^2 \times 500 \text{ cm}^3 = 3926990.817 \text{ cm}^3\end{aligned}$$

$$\therefore \delta V = 0.001425 \times 3926990.817 = \mathbf{5595.96 \text{ cm}^3.} \quad \text{Ans.}$$

Problem 17.10. A cylindrical thin drum 80 cm in diameter and 3 m long has a shell thickness of 1 cm. If the drum is subjected to an internal pressure of 2.5 N/mm^2 , determine (i) change in diameter, (ii) change in length and (iii) change in volume.

Take $E = 2 \times 10^5 \text{ N/mm}^2$: Poisson's ratio = 0.25.

Sol. Given :

Diameter of drum, $d = 80 \text{ cm}$

Length of drum, $L = 3 \text{ m} = 3 \times 100 = 300 \text{ cm}$

Thickness of drum, $t = 1 \text{ cm}$

Internal pressure, $p = 2.5 \text{ N/mm}^2$

Young's modulus, $E = 2 \times 10^5 \text{ N/mm}^2$

Poisson's ratio, $\mu = 0.25$

(i) Change in diameter (δd) is given by equation (17.11) as

$$\begin{aligned}\delta d &= \frac{pd^2}{2tE} \left(1 - \frac{1}{2} \times \mu \right) \\ &= \frac{2.5 \times 80^2}{2 \times 1 \times 2 \times 10^5} \left[1 - \frac{1}{2} \times 0.25 \right] \\ &= 0.04 [1 - 0.125] = \mathbf{0.035 \text{ cm.}} \quad \text{Ans.}\end{aligned}$$

(ii) Change in length (δL) is given by equation (17.14) as

$$\begin{aligned}\delta L &= \frac{pdL}{2tE} \left[\frac{1}{2} - \mu \right] \\ &= \frac{2.5 \times 80 \times 300}{2 \times 1 \times 2 \times 10^5} \left[\frac{1}{2} - 0.25 \right] = \mathbf{0.0375 \text{ cm.}} \quad \text{Ans.}\end{aligned}$$

(iii) Using equation (17.15) for volumetric strain $\left(\frac{\delta V}{V} \right)$, we have

$$\begin{aligned}\frac{\delta V}{V} &= 2 \frac{\delta d}{d} + \frac{\delta L}{L} \\ &= 2 \times \frac{0.035}{80} + \frac{0.0375}{300} \quad \left(\because \begin{array}{ll} \delta d = 0.035, & \delta L = 0.0375 \\ d = 80, & L = 300 \end{array} \right) \\ &= 0.000875 + 0.000125 = 0.001\end{aligned}$$

$$\therefore \delta V = 0.001 \times V$$

$$\text{where volume } V = \frac{\pi}{4} d^2 \times L = \frac{\pi}{4} \times 80^2 \times 300 = 1507964.473 \text{ cm}^3$$

$$\therefore \text{Change in volume, } \delta V = 0.001 \times 1507964.473 = \mathbf{1507.96 \text{ cm}^3.} \quad \text{Ans.}$$

Problem 17.11. A cylindrical shell 90 cm long 20 cm internal diameter having thickness of metal as 8 mm is filled with fluid at atmospheric pressure. If an additional 20 cm³ of fluid is pumped into the cylinder, find (i) the pressure exerted by the fluid on the cylinder and (ii) the hoop stress induced. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and $\mu = 0.3$.

Sol. Given :

Length of cylinder,	$L = 90 \text{ cm}$
Diameter of cylinder,	$d = 20 \text{ cm}$
Thickness of cylinder,	$t = 8 \text{ mm} = 0.8 \text{ cm}$
Volume of additional fluid	$= 20 \text{ cm}^3$

$$\begin{aligned}\text{Volume of cylinder,} \quad V &= \frac{\pi}{4} d^2 \times L = \frac{\pi}{4} \times 20^2 \times 90 \\ &= 28274.33 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{Increase in volume,} \quad \delta V &= \text{Volume of additional fluid} \\ &= 20 \text{ cm}^3\end{aligned}$$

(i) Let p = Pressure exerted by fluid on the cylinder
 $E = 2 \times 10^5 \text{ N/mm}^2$
 $\mu = 0.3$.

Now using equation (17.16), volumetric strain is given as

$$\frac{\delta V}{V} = 2 e_1 + e_2$$

$$\text{or } \frac{20}{28274.33} = 2 e_1 + e_2 \quad \dots(i)$$

But e_1 and e_2 are circumferential and longitudinal strains and are given by equation (17.6) and (17.8) respectively as

$$e_1 = \frac{pd}{2Et} \left[1 - \frac{1}{2} \times \mu \right]$$

$$\text{and } e_2 = \frac{pd}{2tE} \left(\frac{1}{2} - \mu \right).$$

Substituting these values in equation (i), we get

$$\begin{aligned} \frac{20}{28274.33} &= \frac{2pd}{2Et} \left[1 - \frac{1}{2} \times \mu \right] + \frac{pd}{2tE} \left[\frac{1}{2} - \mu \right] \\ &= \frac{2p \times 20}{2 \times 2 \times 10^5 \times 0.8} \left[1 - \frac{1}{2} \times 0.3 \right] + \frac{p \times 20}{0.8 \times 2 \times 10^5} \left[\frac{1}{2} - 0.3 \right] \end{aligned}$$

$$\text{or } 0.000707 = \frac{p}{8000} \times 0.85 + \frac{p}{8000} \times 0.20 = \frac{1.05p}{8000}$$

$$\therefore p = \frac{0.000707 \times 8000}{1.05} = 5.386 \text{ N/mm}^2. \quad \text{Ans.}$$

(ii) Hoop stress (σ_1) is given by equation (17.1) as

$$\sigma_1 = \frac{pd}{2t} = \frac{5.386 \times 20}{2 \times 0.8} = 67.33 \text{ N/mm}^2. \quad \text{Ans.}$$

Problem 17.12. A cylindrical vessel whose ends are closed by means of rigid flange plates, is made of steel plate 3 mm thick. The length and the internal diameter of the vessel are 50 cm and 25 cm respectively. Determine the longitudinal and hoop stresses in the cylindrical shell due to an internal fluid pressure of 3 N/mm². Also calculate the increase in length, diameter and volume of the vessel. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and $\mu = 0.3$.

Sol. Given :

Thickness, $t = 3 \text{ mm} = 0.3 \text{ cm}$

Length of the cylindrical vessel, $L = 50 \text{ cm}$

Internal diameter, $d = 25 \text{ cm}$

Internal fluid pressure, $p = 3 \text{ N/mm}^2$

Young's modulus, $E = 2 \times 10^5 \text{ N/mm}^2$

Poisson's ratio, $\mu = 0.3$

Let σ_1 = Hoop stress and

σ_2 = Longitudinal stress.

Using equation (17.1) for hoop stress,

$$\sigma_1 = \frac{p \times d}{2t} = \frac{3 \times 25}{2 \times 0.3} = 125 \text{ N/mm}^2. \quad \text{Ans.}$$

Using equation (17.2) for longitudinal stress,

$$\sigma_2 = \frac{p \times d}{4t} = \frac{3 \times 25}{4 \times 0.3} = 62.5 \text{ N/mm}^2. \quad \text{Ans.}$$

Using equation (17.5) for circumferential strain,

$$\begin{aligned}
 e_1 &= \frac{\sigma_1}{E} - \frac{\mu \times \sigma_2}{E} \\
 &= \frac{1}{E} [\sigma_1 - \mu \times \sigma_2] \\
 &= \frac{1}{2 \times 10^5} [125 - 62.5 \times 0.3] \quad (\because \mu = 0.3, \sigma_1 = 125 \text{ and } \sigma_2 = 62.5) \\
 &= \frac{1}{2 \times 10^5} (125 - 18.75) = \frac{106.25}{2 \times 10^5} \\
 &= 53.125 \times 10^{-5}.
 \end{aligned}$$

But circumferential strain is also given by equation (17.9) as

$$e_1 = \frac{\delta d}{d}$$

Equating the two values of circumferential strain e_1 , we get

$$\frac{\delta d}{d} = 53.125 \times 10^{-5}$$

$$\therefore \delta d = 53.125 \times 10^{-5} \times d = 53.125 \times 10^{-5} \times 25 = 0.0133 \text{ cm}$$

$\therefore$ Increase in diameter, $\delta d = 0.0133 \text{ cm}$. **Ans.**

Longitudinal strain is given by equation (17.7) as

$$\begin{aligned}
 e_2 &= \frac{\delta L}{L} = \frac{\sigma_2}{E} - \frac{\mu \times \sigma_1}{E} \\
 &= \frac{1}{E} [\sigma_2 - \mu \times \sigma_1] \\
 &= \frac{1}{2 \times 10^5} [62.5 - 125 \times 0.3] = \frac{1}{2 \times 10^5} [62.5 - 37.5] \\
 &= \frac{2.5}{2 \times 10^5} = 12.5 \times 10^{-5}
 \end{aligned}$$

$\therefore$ Increase in length, $\delta L = 12.5 \times 10^{-5} \times L$
 $= 12.5 \times 10^{-5} \times 50 = 0.00625 \text{ cm}$. **Ans.**

Volumetric strain is given by equation (17.16), as

$$\begin{aligned}
 \frac{\delta V}{V} &= 2 \frac{\delta d}{d} + \frac{\delta l}{l} \\
 &= 2e_1 + e_2 = 2 \times 53.125 \times 10^{-5} + 12.5 \times 10^{-5} \\
 &= 106.25 \times 10^{-5} + 12.5 \times 10^{-5} = 118.75 \times 10^{-5}
 \end{aligned}$$

$\therefore$ Increase in volume,

$$\begin{aligned}
 \delta V &= 118.75 \times 10^{-5} \times V \\
 &= 118.75 \times 10^{-5} \times \frac{\pi}{4} \times 25^2 \times 50 \quad \left(\because \text{volume} = \frac{\pi}{4} d^2 \times L \right) \\
 &= 29.13 \text{ cm}^3. \quad \text{Ans.}
 \end{aligned}$$

Problem 17.13. A cylindrical vessel is 1.5 m diameter and 4 m long is closed at ends by rigid plates. It is subjected to an internal pressure of 3 N/mm². If the maximum principal stress is not to exceed 150 N/mm², find the thickness of the shell. Assume $E = 2 \times 10^5 \text{ N/mm}^2$ and Poisson's ratio = 0.25. Find the changes in diameter, length and volume of the shell.

Sol. Given :

Dia., $d = 1.5 \text{ m} = 1500 \text{ mm}$

Length, $L = 4 \text{ m} = 4000 \text{ mm}$

Internal pressure, $p = 3 \text{ N/mm}^2$

Max. principal stress $= 150 \text{ N/mm}^2$

Max. principal stress means the circumferential stress

$\therefore$ Circumferential stress, $\sigma_1 = 150 \text{ N/mm}^2$

Value of $E = 2 \times 10^5 \text{ N/mm}^2$.

Poisson's ratio, $\mu = 0.25$

Let t = thickness of the shell,

δd = change in diameter,

δL = change in length, and

δV = change in volume.

(i) Using equation (17.1),

$$\sigma_1 = \frac{p \times d}{2t}$$

$$\therefore t = \frac{p \times d}{2 \times \sigma_1} = \frac{3 \times 1500}{2 \times 150}$$

(Here p and σ_1 are in same units, 'd'

is in mm hence 't' will be in mm)

$$= 15 \text{ mm. Ans.}$$

(ii) Using equation (17.11),

$$\delta d = \frac{pd^2}{2t \times E} \left(1 - \frac{1}{2} \times \mu \right)$$

$$= \frac{3 \times 1500^2}{2 \times 15 \times 2 \times 10^5} \left(1 - \frac{1}{2} \times 0.25 \right) = 0.984 \text{ mm. Ans.}$$

(iii) Using equation (17.14),

$$\delta L = \frac{p \times d \times L}{2t \times E} \left(\frac{1}{2} - \mu \right)$$

$$= \frac{3 \times 1500 \times 4000}{2 \times 15 \times 2 \times 10^5} \left(\frac{1}{2} - 0.25 \right)$$

$$= 0.75 \text{ mm. Ans.}$$

(iv) Using equation (17.17),

$$\frac{\delta V}{V} = \frac{p \times d}{2E \times t} \left(\frac{5}{2} - 2 \times \mu \right)$$

$$= \frac{3 \times 1500}{2 \times 2 \times 10^5 \times 15} \left(\frac{5}{2} - 2 \times 0.25 \right) = \frac{3 \times 1500 \times 2}{4 \times 10^5 \times 15}$$

$$\therefore \delta V = \frac{3}{2000} \times V = \frac{3}{2000} \times \left(\frac{\pi}{4} \times d^2 \times L \right)$$

$$= \frac{3}{2000} \times \left(\frac{\pi}{4} \times 1500^2 \times 4000 \right) = 10602875 \text{ mm}^3. \text{ Ans.}$$

$$\sigma_c = \frac{Pd}{2t}$$

$$\sigma_l = \frac{Pd}{4t}$$

if η given

$$\sigma_c = \frac{Pd}{2t} \eta_l$$

$$\sigma_l = \frac{Pd}{2t} \eta_c$$

$$e_c = \frac{Pd}{2tE} \left[1 - \frac{\mu}{2} \right]$$

$$e_l = \frac{Pd}{2tE} \left[\frac{1}{2} - \mu \right]$$

$$e_v = \frac{Pd}{2tE} \left[\frac{5}{2} - 2\mu \right]$$

$$e_v = 2e_c + e_l$$

$$V = A \times l \Rightarrow \frac{\pi}{4} d^2 \times l$$

$$e_c = \frac{\delta_c}{l}$$

$$e_l = \frac{\delta_l}{l}$$

$$\delta_v = \frac{\delta_v}{V}$$

17.10. THIN SPHERICAL SHELLS

Fig. 17.8 shows a thin spherical shell of internal diameter ' d ' and thickness ' t ' and subjected to an internal fluid pressure ' p '. The fluid inside the shell has a tendency to split the shell into two hemispheres along x - x axis.

The force (P) which has a tendency to split the shell

$$= p \times \frac{\pi}{4} d^2$$

The area resisting this force = $\pi.d.t$

$\therefore$ Hoop or circumferential stress (σ_1) induced in the material of the shell is given by,

$$\begin{aligned}\sigma_1 &= \frac{\text{Force } P}{\text{Area resisting the force } P} \\ &= \frac{p \times \frac{\pi}{4} d^2}{\pi.d.t} = \frac{p.d}{4t}\end{aligned}$$

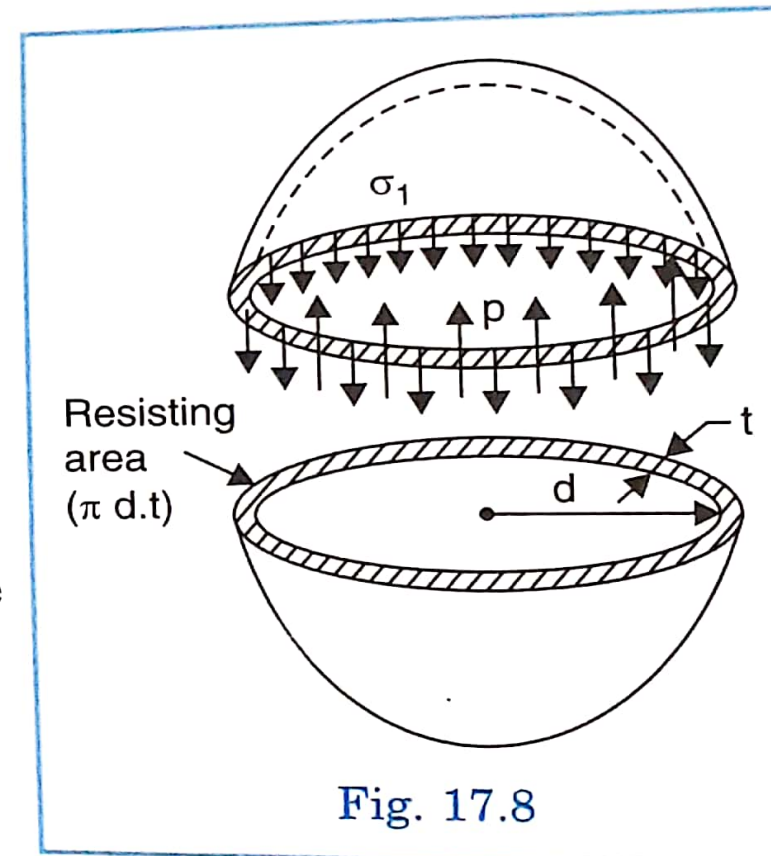


Fig. 17.8

The stress σ_1 is tensile in nature.

The fluid inside the shell is also having tendency to split the shell into two hemispheres along y-y axis. Then it can be shown that the tensile hoop stress will also be equal to $\frac{p.d}{4t}$. Let this stress is σ_2 .

$$\therefore \sigma_2 = \frac{p \times d}{4t}.$$

The stress σ_2 will be at right angles to σ_1 .

Problem 17.21. A vessel in the shape of a spherical shell of 1.20 m internal diameter and 12 mm shell thickness is subjected to pressure of 1.6 N/mm². Determine the stress induced in the material of the vessel.

Sol. Given :

Internal diameter, $d = 1.2 \text{ m} = 1.2 \times 10^3 \text{ mm}$

Shell thickness, $t = 12 \text{ mm}$

Fluid pressure, $p = 1.6 \text{ N/mm}^2$

The stress induced in the material of spherical shell is given by,

$$\sigma_1 = \frac{p.d}{4t} = \frac{1.6 \times 1.2 \times 10^3}{4 \times 12} = 40 \text{ N/mm}^2. \quad \text{Ans.}$$

Problem 17.22. A spherical vessel 1.5 m diameter is subjected to an internal pressure of 2 N/mm². Find the thickness of the plate required if maximum stress is not to exceed 150 N/mm² and joint efficiency is 75%.

Sol. Given :

Diameter of shell, $d = 1.5 \text{ m} = 1.5 \times 10^3 \text{ mm}$

Fluid pressure, $p = 2 \text{ N/mm}^2$

Stress in material, $\sigma_1 = 150 \text{ N/mm}^2$

Joint efficiency, $\eta = 75\% \text{ or } 0.75$

Let $t = \text{thickness of the plate.}$

The stress induced is given by,

$$\sigma_1 = \frac{p.d}{4t.\eta}$$

or
$$t = \frac{p.d}{4.\eta.\sigma_1} = \frac{2 \times 1.5 \times 10^3}{4 \times 0.75 \times 150} = 6.67 \text{ mm.} \quad \text{Ans.}$$

17.11. CHANGE IN DIMENSIONS OF A THIN SPHERICAL SHELL DUE TO AN INTERNAL PRESSURE

In previous article, we have seen that the stresses σ_1 and σ_2 at any point are equal (each equal to $p.d/4t$) and like. There is no shear stress at any point in the shell.

(Maximum shear stress = $\frac{\sigma_1 - \sigma_2}{2} = \frac{p \times d}{4t} - \frac{p \times d}{4t} = 0$). The stresses σ_1 and σ_2 are acting at right angles to each other.

$\therefore$ Strain in any one direction is given by,

$$e = \frac{\sigma_1}{E} - \frac{\mu \times \sigma_1}{E}$$

$$= \frac{\sigma_1}{E} - \frac{\mu \times \sigma_1}{E} \quad \left(\because \sigma_1 = \sigma_2 = \frac{p \times d}{4t} \right)$$

$$= \frac{\sigma_1}{E} (1 - \mu) = \frac{p \times d}{4tE} (1 - \mu)$$

We know that strain in any direction is also

$$= \frac{\delta d}{d}$$

$$\therefore \frac{\delta d}{d} = \frac{p \times d}{4tE} (1 - \mu) \quad \dots(i)$$

Volumetric strain $\left(\frac{dV}{V} \right)$

The ratio of change of volume to the original volume is known as volumetric strain. If V = original volume and dV = change in volume. Then $\frac{dV}{V}$ = volumetric strain.

Let V = Original volume

$$= \frac{\pi}{6} d^3 \quad \left(\because \text{For a sphere, } V = \frac{4}{3} \pi r^3 = \frac{\pi}{6} d^3 \right)$$

Taking the differential of the above equation, we get

$$dV = \frac{\pi}{6} \times 3d^2 \times d(d)$$

$$\text{Hence } \frac{dV}{V} = \frac{\frac{\pi}{6} \times 3d^2 \times d(d)}{\frac{\pi}{6} \times d^3}$$

$$= 3 \frac{d(d)}{d} \quad \dots(ii)$$

But from equation (i), we have

$$\frac{\delta d}{d} \text{ or } \frac{d(d)}{d} = \frac{p \times d}{4tE} (1 - \mu)$$

Substituting this value in equation (ii), we get

$$\frac{dV}{V} = \frac{3 \times p \times d}{4tE} (1 - \mu).$$

Problem 17.23. A spherical shell of internal diameter 0.9 m and of thickness 10 mm is subjected to an internal pressure of 1.4 N/mm². Determine the increase in diameter and increase in volume. Take $E = 2 \times 10^5$ N/mm² and $\mu = \frac{1}{3}$.

Sol. Given :

Internal diameter, $d = 0.9 \text{ m} = 0.9 \times 10^3 \text{ mm}$

Thickness of shell, $t = 10 \text{ mm}$

Fluid pressure, $p = 1.4 \text{ N/mm}^2$

Value of $E = 2 \times 10^5 \text{ N/mm}^2$

Value of $\mu = \frac{1}{3}$

Using the relation,

$$\frac{\delta d}{d} = \frac{p \times d}{4tE} (1 - \mu)$$
$$= \frac{1.4 \times (0.9 \times 10^3)}{4 \times 10 \times 2 \times 10^5} \left(1 - \frac{1}{3}\right) = 105 \times 10^{-6}$$

∴ Increase in diameter,

$$\delta d = 105 \times 10^{-6} \times 0.9 \times 10^3$$
$$= 94.5 \times 10^{-3} \text{ mm} = \mathbf{0.0945 \text{ mm.}} \quad \text{Ans.}$$

Now, volumetric strain $= 3 \times \frac{\delta d}{d}$

$$= 3 \times 105 \times 10^{-6} = 315 \times 10^{-6}$$

$$\frac{\delta V}{V} = 315 \times 10^{-6}$$

∴ Increase in volume,

$$\delta V = 315 \times 10^{-6} \times V$$
$$= 315 \times 10^{-6} \times \left(\frac{\pi}{6} d^3\right) \quad \left(\because V = \frac{\pi}{6} d^3 \text{ for a sphere}\right)$$
$$= 315 \times 10^{-6} \times \frac{\pi}{6} \times (0.9 \times 10^3)^3$$
$$= \mathbf{12028.5 \text{ mm}^3.} \quad \text{Ans.}$$