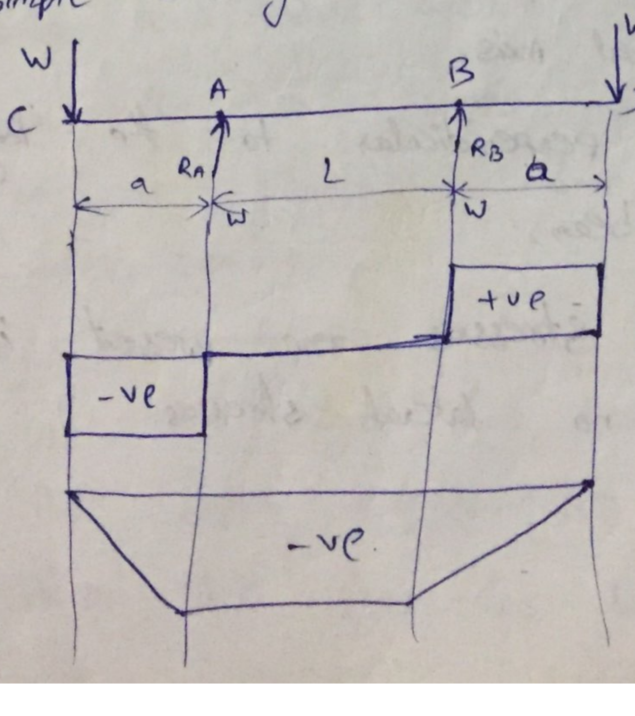
THEORY OF SIMPLE BENDING

Bending stresses:-

When some external load acts on a beam, the shear force and bending moments are set up at all sections of the beam. Due to the shear force and bending moment, the beam undergoes some deformation. The material of the beam will offer resistance or stresses against these deformations. These stresses with certain assumptions can be calculated. The stresses introduced by bending moment are known as bending stresses.

PURE BENDING OR SIMPLE BENDING:-

The length of a beam is subjected to a constant bending moment and no shear force (i.e zero shear force), then the stresses will be setup in that length of the beam due to B.M. only and that length of the beam is said to be in pure bending (or) simple bending.



$R_A + R_B = w + w$

T.M.A.A

$\circlearrowleft = \circlearrowright$

$w(L+a) = R_B L + w \cdot a$

$wL + \cancel{wa} = R_B \cancel{L} + \cancel{w \cdot a}$

$\boxed{R_B = W}$

$R_A = -R_B + 2W$

$\boxed{R_A = W}$

$BM_D = 0$

$BM_B = -w \times a$

$BM_A = -w(a+L) + wL = -wa$

$BM_C = -\cancel{2w} - w(2a+L) + w(L+a) + wa$

$= 0$

ASSUMPTIONS IN THEORY OF SIMPLE BENDING:-

The following assumptions are made in theory of simple bending.

1) The material of the beam is homogeneous (material is same kind throughout) and Isotropic (Elastic properties in all directions are equal).

2) The value of young's modulus of elasticity is same in tension and compression.

3) The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.

4) The radius of curvature is large compared with the dimensions of the cross section.

5) Each layer of the beam is free to expand or contract, independently of the layer above or below ~~from~~ to the neutral axis.

6) The load acts perpendicular to the longitudinal axis of the beam.

7) only longitudinal stresses are present. in the beam there is no lateral stresses.

Bending stress:-

The internal stress produced in the beam to resist the applied bending moments is known as bending stress. It is denoted by σ_b.

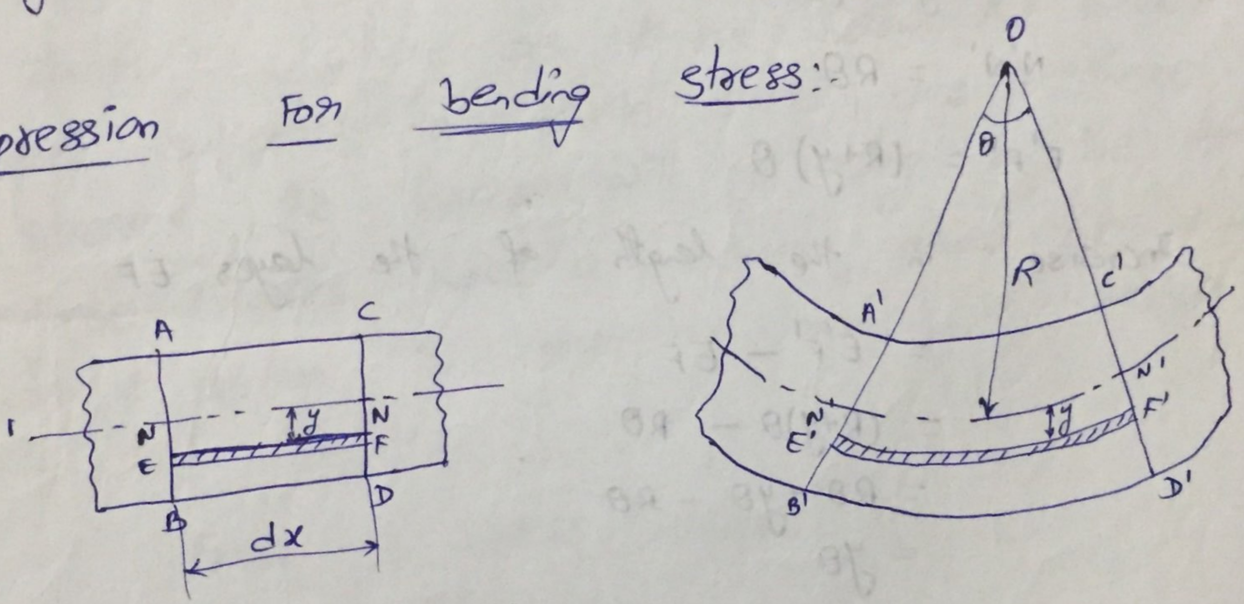
mathematically $\sigma_b = \frac{E}{R} \times y$.

E = Young's modulus of Elasticity.

R = Radius of curvature after bending.

y = Distance of outtermost layer from neutral Axis.

Expression For bending stress:-



Consider a small section of beam at a distance 'y' from the neutral axis subjected to a bending moment M. as shown in above fig. before bending and after bending.

The top layer such as AC has deformed to the shape A'C'. This layer has been shortened in its length. The bottom layer BD has deformed to the shape B'D'. This layer has been elongated. At a level b/w the top & bottom of the beam, there will be a layer which is neither shortened nor elongated. This layer is known as neutral layer. ~~(axis)~~

From figure.

original length of the Neutral Axis (NN) = δx

After bending the length of Neutral Axis N'N' will remain unchanged. But the length of E'F' will increase.

Hence $N'N' = NN = \delta x$

But from fig (B)

$$N'N' = R\theta$$

$$E'F' = (R+y)\theta$$

Increase in the length of the layer EF

$$= E'F' - EF$$

$$= (R+y)\theta - R\theta$$

$$= \cancel{R\theta} + y\theta - \cancel{R\theta}$$

$$= y\theta$$

strain in layer EF = $\dfrac{\text{Increase in length}}{\text{original length}}$

$$= \frac{y\theta}{R\theta} = \frac{y}{R}$$

we know

modulus of Elasticity (E) = $\dfrac{\text{stress}}{\text{strain}}$

$$E = \frac{\sigma_b}{\frac{y}{R}} = \frac{R\,\sigma_b}{y}$$

$$\frac{\sigma_b}{y} = \frac{E}{R}$$

$$\boxed{\sigma_b = \frac{E}{R} \times y}$$

Moment of Resistance :-

Due to the pure bending, the layers above the Neutral axis are subjected to Compressive stress & where as layers below the neutral axis are subjected to tensile stress. due to these stresses the forces will be acting on the layers. these forces have a moment about the N.A. for a section is known as moment of resistance of that section.

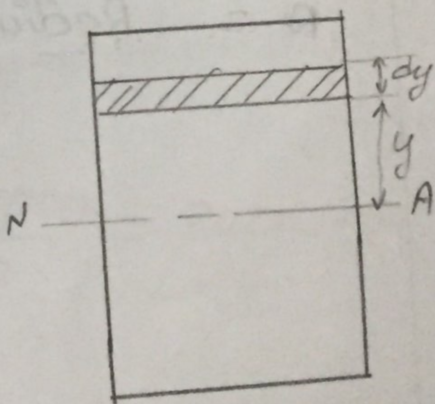
Force = stress × Area

$= \frac{E}{R} \times y \times dA$

moment = Force × $\perp^{lar}$ distance

$= \frac{E}{R} \times y \times dA \times y$

$= \frac{E}{R} y^2 dA$



Total moment $= \int M$

$= \int \frac{E}{R} y^2 dA$

$= \frac{E}{R} \int y^2 dA$

$\boxed{\therefore I = y^2 dA}$

$$M = \frac{E}{R} \times I$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}} \longrightarrow \text{Bending equation}$$

The above equation is known as Bending equation

where

M = Maximum bending moment (N-mm)

I = Moment of Inertia (mm^4)

σ = Bending stress (N/mm^2)

y = Distance from the neutral axis to the outermost layer (mm)

E = Young's modulus (N/mm^2)

R = Radius of Curvature (mm)

Q) A steel plate of width 120mm and of thickness 20mm is bent into a circular arc of radius 10m. Determine the maximum stress induced and the bending moment which will produce the maximum stress. Take $E = 2\times10^5$ N/mm².

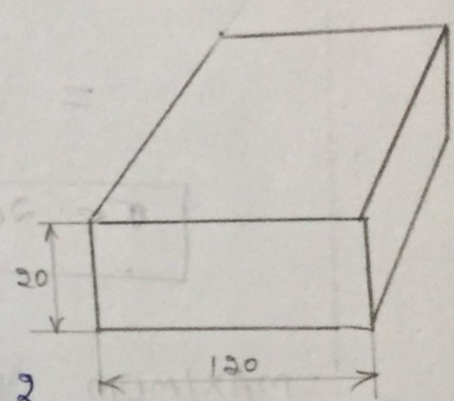
Sol Given data :-

width of steel plate (b) = 120mm

thickness of plate (t) = 20mm

Radius of curvature (R) = 10m

modulus of Elasticity (E) = 2×10^5 N/mm².



we know

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$M = \frac{E}{R} \times I$$

$$I = \frac{bt^3}{12} = \frac{120\times(20)^3}{12} = \frac{960000}{12} = 80,000\,mm^4$$

$$M = \frac{2\times10^5}{10\times1000} \times 80,000$$

$$= \frac{1.6\times10^{10}}{1\times10^4} = 1600000\,N\text{-}mm = 1600\,N\text{-}m$$

$$\boxed{M = 1.6\,KN\text{-}m}$$

we Also know

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$y = \frac{20}{2} = 10mm$$

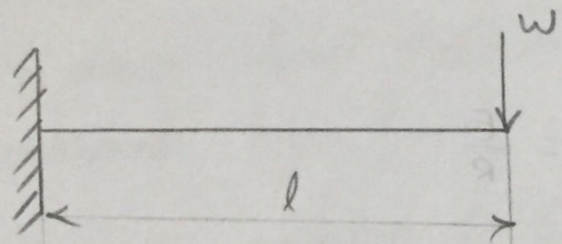
$$\sigma = \frac{2\times10^5}{10\times10^3} \times 10$$

$$= \frac{2\times10^6}{1\times10^4}$$

$$\boxed{\sigma = 200\ N/mm^2}$$

MAXIMUM BENDING MOMENT FOR DIFFERENT BEAMS:-

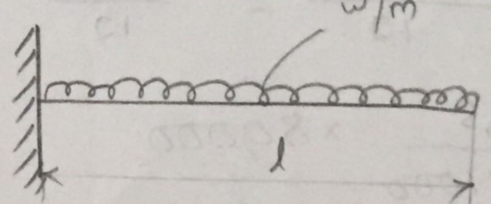
1)



max. bending moment $= w\times l$.

$$\boxed{M = w.l}$$

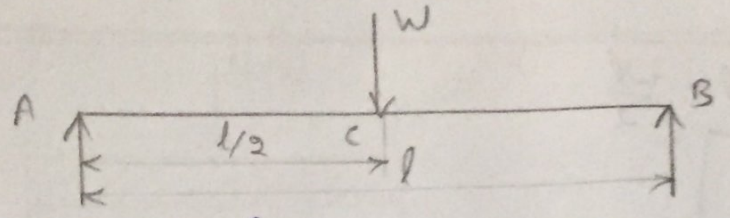
2)



maximum bending moment $(M) = \underbrace{(w\times l)}_{\text{load}} \times \underbrace{\frac{l}{2}}_{1^{\text{lar}}\ \text{distance}}$

$$\boxed{M = \frac{wl^2}{2}}$$

3)



Sum of upward forces = Sum of downward forces

$R_A + R_B = W$

Sum of Anticlock wise moments = Sum of clockwise moments

Taking moments about A

$R_B \times l = W \times l/2$

$\boxed{R_B = \frac{W}{2}}$

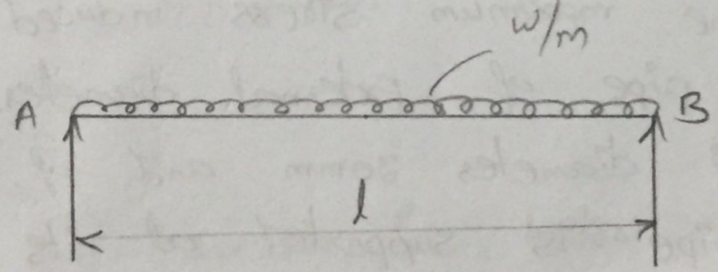
$R_A = W - \frac{W}{2}$

$\boxed{R_A = \frac{W}{2}}$

maximum Bending moment (m) = $\frac{W}{2} \times \frac{l}{2}$

$\boxed{m = \frac{Wl}{4}}$

4)



↑ forces = ↓ forces

$R_A + R_B = w \times l$

↺ moments = ↻ moments

$R_B \times l = w \times l \times \frac{l}{2}$

$R_B = \frac{w l^{\cancel{2}}}{2 \cancel{l}}$

$\boxed{R_B = wl/2}$

$$R_A = wl - \frac{wl}{2}$$

$$\boxed{R_A = \frac{wl}{2}}$$

maximum bending moment $(m) = R_B \times \frac{l}{2} - \left(\frac{w \times l}{2}\right) \times \frac{\frac{l}{2}}{2}$

$$= \frac{wl}{2} \times \frac{l}{2} - \frac{wl}{2} \times \frac{l}{4}$$

$$= \frac{wl^2}{4} - \frac{wl^2}{8}$$

$$= \frac{8wl^2 - 4wl^2}{32}$$

$$= \frac{\cancel{4}^{1} wl^2}{\cancel{32}_{8}}$$

$$\boxed{m = \frac{wl^2}{8}}$$

Q) Calculate the maximum stress induced in a cast iron pipe of External diameter 40mm, and internal diameter 20mm and of length 4m when the pipe is supported at its ends and carries a point load of 80N at its centre.

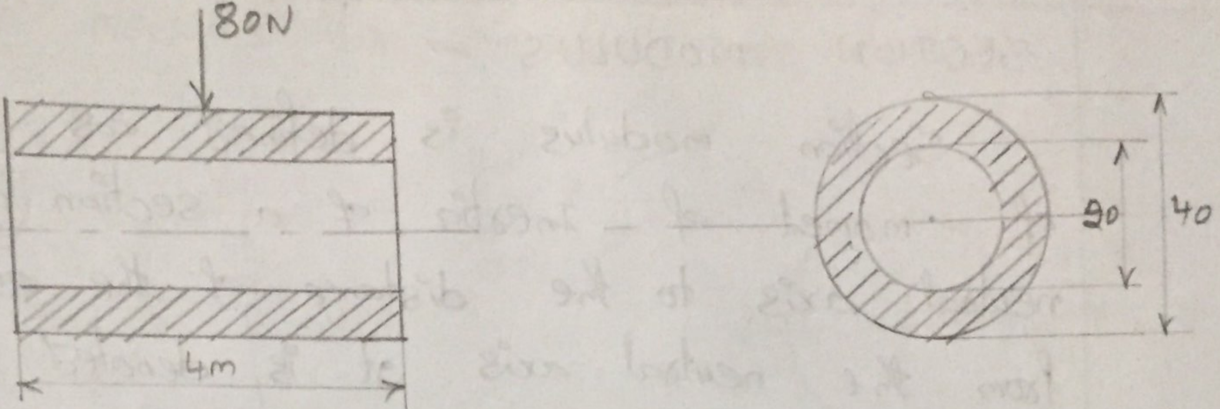
Sol Given data :-

External diameter (D) = 40mm

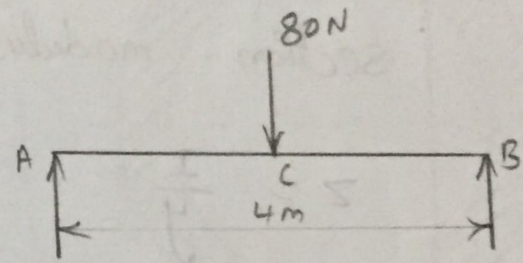
Internal diameter (d) = 20 mm

length of pipe (l) = 4m = 4000mm

point Load (w) = 80N



In the above problem the given beam it is in simply supported in nature and the point load is acting at the centre of the beam.



we know the maximum bending moment for S.S.B carrying point load at the centre is

$$m = \frac{wl}{4} = \frac{80 \times 4000}{4}$$

$$= 80000 \text{ N-mm}$$

$$\frac{m}{I} = \frac{\sigma}{y}$$

moment of Inertia for hollow circular section is given by $I = \frac{\pi(D^4 - d^4)}{64} = \frac{\pi(40^4 - 20^4)}{64}$

$$= \frac{7539822.36}{64} = 117809.72 \text{ mm}^4$$

$$y = \frac{40}{2} = 20\text{mm}$$

$$\sigma = \frac{m}{I} \times y \Rightarrow \frac{80000 \times 20}{117809.72}$$

$$\boxed{\sigma = 13.58 \text{ N/mm}^2}$$

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SECTION MODULUS :-

Section modulus is defined as the ratio of moment of Inertia of a section about the neutral axis to the distance of the outermost layers from the neutral axis. It is denoted by 'Z'

Hence mathematically.

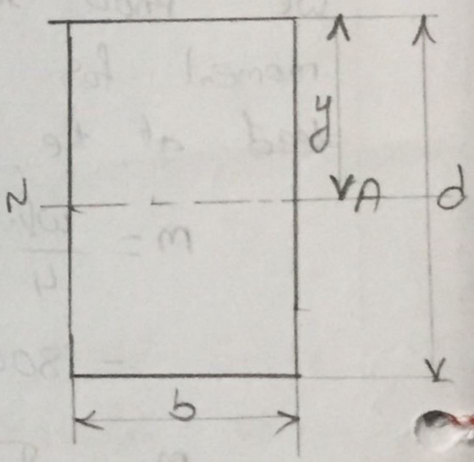
$$\text{Section modulus } (Z) = \frac{I}{y} =$$

Section modulus for Rectangular section :-

$$Z = \frac{I}{y}$$

$$= \frac{\frac{bd^3}{12}}{\frac{d}{2}} = \frac{bd^{\cancel{3}}}{\cancel{12}\,6} \times \frac{\cancel{2}\,1}{\cancel{d}}$$

$$\boxed{Z = \frac{bd^2}{6}}$$

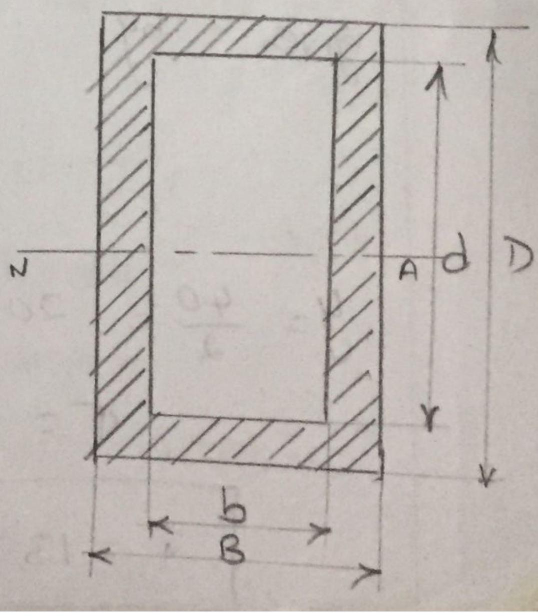


Section modulus for Hollow Rectangular section :-

$$Z = \frac{I}{y} = \frac{\frac{BD^3}{12} - \frac{bd^3}{12}}{\frac{D}{2}}$$

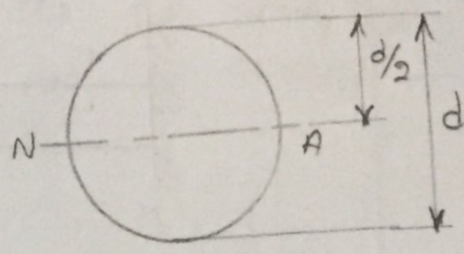
$$\frac{BD^3 - bd^3}{\cancel{12}\,6} \times \frac{\cancel{2}\,1}{D}$$

$$\boxed{Z = \frac{BD^3 - bd^3}{6D}}$$



Section modulus for circular cross - section :-

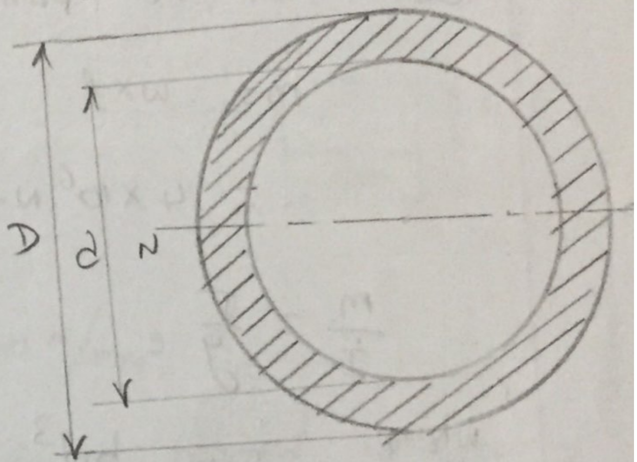
$$z = \frac{I}{y} = \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^{\cancel{4}\,3}}{\cancel{64}_{32}} \times \frac{2^1}{\cancel{d}}$$



$$\boxed{z = \frac{\pi d^3}{32}}$$

Section modulus for Hollow circular section :-

$$z = \frac{I}{y} = \frac{\frac{\pi D^4}{64} - \frac{\pi d^4}{64}}{\frac{D}{2}}$$



$$\frac{\pi (D^4 - d^4)}{\cancel{64}_{32}} \times \frac{2^1}{D}$$

$$\boxed{z = \frac{\pi (D^4 - d^4)}{32 D}}$$

Q) A Cantilever of length 2m fails when a load of 2KN is applied at the free end. of the section of the beam is 40mm x 60mm, find the stress at the failure.

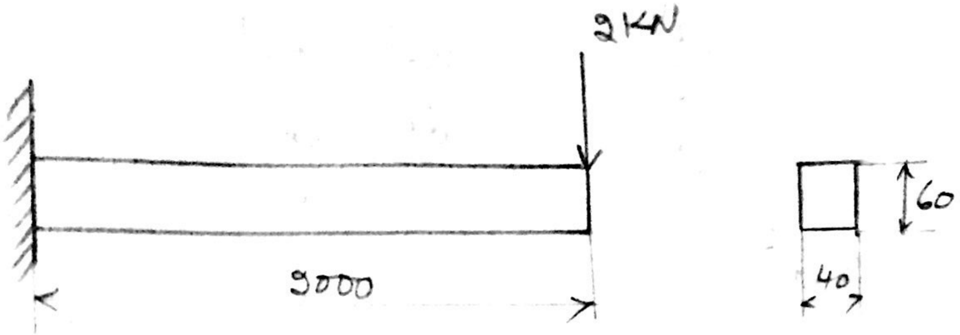
Sol Given data :-

length (l) = 2m = 2000mm

point Load (W) = 2KN = 2000N

breadth (b) = 40mm

Depth (d) = 60mm



we know maximum bending moment for cantilever beam when the point load is acting at free end

$$m = w \times l = (2 \times 10^3)(2000)$$

$$= 4 \times 10^6 \text{ N-mm}$$

$$\frac{m}{I} = \frac{\sigma}{y}$$

where $I = \frac{bd^3}{12} = \frac{40(60)^3}{12} = 720000 \text{ mm}^4$

$$y = \frac{60}{2} = 30\text{mm}$$

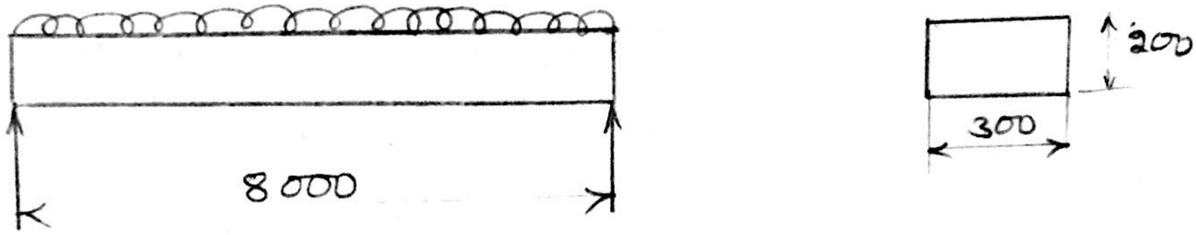
$$\sigma = \frac{m}{I} \times y \Rightarrow \frac{4 \times 10^6 \times 30}{720000}$$

$$= \frac{120000000}{720000}$$

$$\boxed{\sigma = 166.66 \text{ N/mm}^2}$$

8) A rectangular beam 200mm deep and 300mm wide is simply supported over a span of 8m. What uniformly distributed load per meter the beam may carry. If the bending stress is not to exceed $120 \, N/mm^2$.

Sol Given data:



Bending stress $(\sigma_b) = 120 \, N/mm^2$.

we know the maximum bending moment in case of simply supported beam carries point load it is given by

$$m = \frac{wl^2}{8} = \frac{w(8000)^2}{8} = \frac{64000000\,w}{8}$$

$$m = 8 \times 10^6 \, w.$$

we also know

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$I = \frac{bd^3}{12} = \frac{300(200)^3}{12} = 2 \times 10^8 \, mm^4$$

$$y = \frac{200}{2} = 100 \, mm$$

$$m = \frac{\sigma}{y} \times I$$

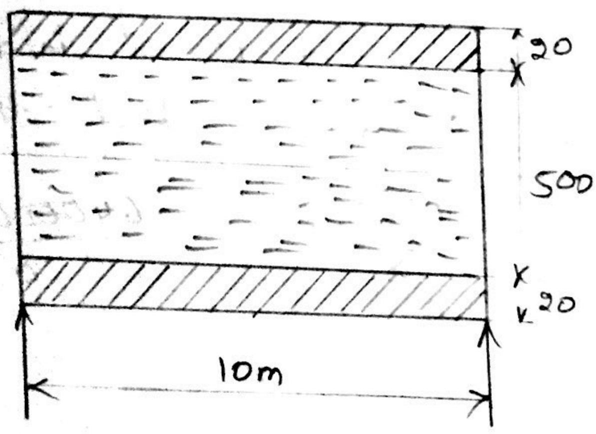
$$8\times10^6 w = \frac{120}{100}\times 2\times10^8$$

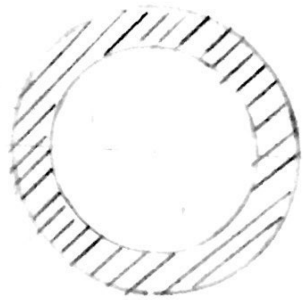
$$= \frac{2.4\times10^{10}}{8\times10^6\times100} = \frac{2.4\times\cancel{10^{10}}^{2}}{8\times\cancel{10^8}} = \frac{240}{8}$$

$$\boxed{w = 30\ N/mm}$$

Q) A water main of 500mm internal diameter and 20mm thick is running full. The water main is of cast iron and is supported at two points 10m apart. Find the maximum stress in the metal. The cast iron and water weigh 72000 N/m^3 & 10,000 N/m^3 resp.

Sol Given data.





Density of cast iron (ρ) = 72000 N/m^3

Density of water (ρ) = 10,000 N/m^3

The above mentioned beam it is simply supported carries with uniformly distribut load in nature. ~~beam~~ because the water is flowing throughout the length of the beam. For this condition we know

$$m = \frac{wl^2}{8}$$

But in the above eqn we don't know the value of "m" as well as value of "w" ~~the~~ but we have the parameters for calculating w.

we know

$$\text{weight density} = \frac{\text{weight}}{\text{volume}}$$

$$\text{weight} = \text{weight density} \times \text{Volume}$$

$$W_{water} = 10000 \times (\text{Area} \times \text{length})$$

$$= 10{,}000 \times \left[\frac{\pi}{4}(0.5)^2 \times 1\right]$$

∴ Here Consider per meter length only.

$$= 10{,}000 \times 0.1963$$

$$\boxed{W_{water} = 1963.49\ N}$$

$$\text{weight of Cast iron pipe} = \rho \times \text{Volume}$$

$$\cancel{= 72000 \times \left(\frac{\pi}{4}(0.54)^2 \times 1\right)}$$

$$\cancel{= 72000 \times 0.2290}$$

$$\cancel{\boxed{W_{C.I} = 16489.59\ N.}}$$

$$= 72000 \left[\frac{\pi}{4}(0.54^2 - 0.5^2) \times 1\right]$$

$$= 72000 \left[\frac{\pi}{4}(0.0416)\right]$$

$$= 72000\,(0.0326)$$

$$\boxed{W_{pipe} = 2352.42\ N}$$

Total weight of the pipe = weight of water + wt. of C.I pipe

$= 1963.49 + 2352.42$

$$\boxed{W_{Total} = 4315.91 \text{ N}}$$

we know.

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$I = \frac{\pi (D^4 - d^4)}{64} = \cancel{\pi(5404)}$$

$$= \frac{\pi (540^4 - 500^4)}{64} = \frac{7.0781 \times 10^{10}}{64}$$

$$I = 1105966278 \text{ mm}^4$$

$$y = \frac{540}{2} = 270 \text{mm}.$$

maximum bending moment for the above mentioned beam

$$M = \frac{wl^2}{8} = \frac{4315.91 (10)^2}{8}$$

$\boxed{\therefore w \text{ is in N/m run}}$

$$= \frac{431591}{8} = 53948.87 \text{ N-m}$$

$$= 53948.87 \times 10^3 \text{ N-mm}$$

$$\sigma = \frac{M}{I} \times y = \frac{53948.87 \times 10^3 \times 270}{1105966278} = \frac{1.45 \times 10^{10}}{1105966278}$$

$$\boxed{\sigma = 13.17 \text{ N/mm}^2}$$

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Q) A timber beam of rectangular section of length 8m is simply supported. The beam carries a U.D.L of 12 KN/m run over the entire length and a point load of 10KN at 3m from left support. If the depth is two times the width and the stress in the timber is not to exceed 8 N/mm². Find the suitable dimensions of the section.

Sol Given data :-

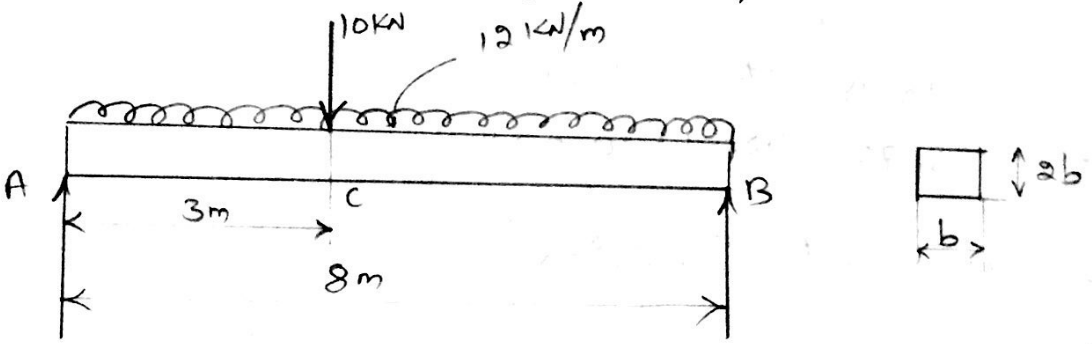
Length of the beam $(l) = 8m$

uniformly distributed Load$(\omega) = 12 KN/m$

point Load $(W) = 10KN$

Depth $(d) = 2b$

stress $(\sigma) = 8 N/mm^2$.



we know

$\uparrow$ forces $= \downarrow$ forces

$R_A + R_B = (12 \times 8) + 10$

$= 96 + 10$

$R_A + R_B = 106 KN$ —— ①

Sum of anticlockwise moments = sum of clockwise moments

Taking moments of A.

$$R_B \times 8 = (10 \times 3) + (12 \times 8)\left(\frac{8}{2}\right)$$

$$8R_B = 30 + 384$$

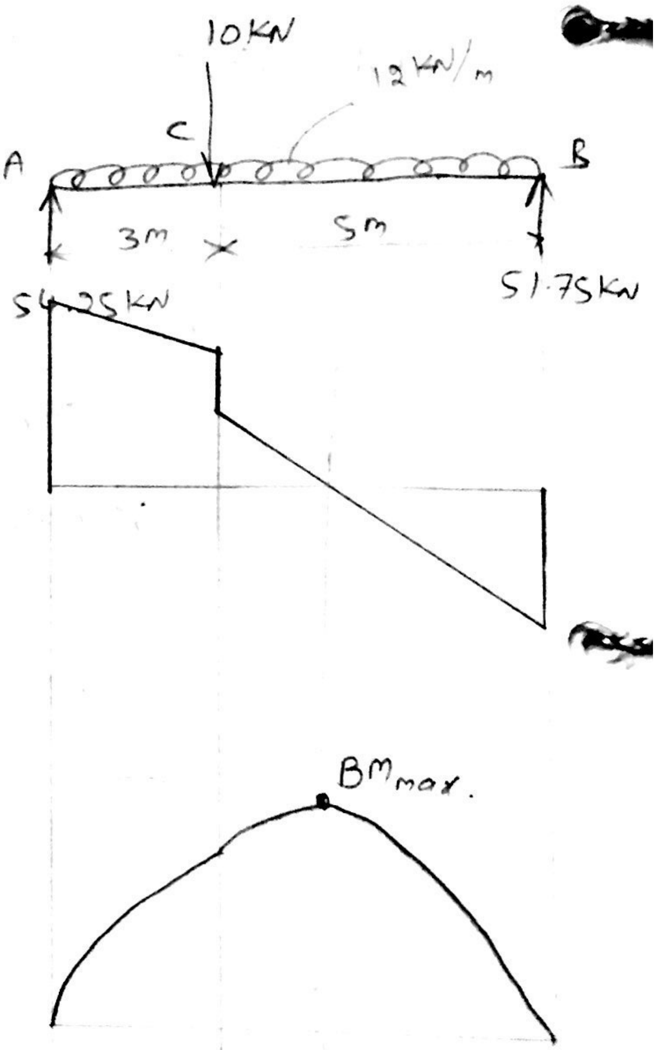
$$R_B = 414/8$$

$$\boxed{R_B = 51.75\text{ KN}} \quad \text{— ②}$$

substitute R_B value in eqn ① to get R_A

$$R_A = 106 - 51.75$$

$$\boxed{R_A = 54.25\text{ KN}}$$



Shear force Calculations:

$$SF_B = -51.75\text{ KN}$$

$$SF_C = -51.75 + (12 \times 5)$$

$= 8.25\text{ KN}$ without point Load.

$$= 8.25 + 10$$

$= 18.25$ with point Load.

$$SF_A = -51.75 + (12 \times 8) + 10 + 54.25$$

$$= 0$$

Bending moment Calculations:-

$$BM_B = 0$$

$$BM_C = (51.75 \times 5) - (12 \times 5) \times \frac{5}{2}$$

$$= 258.75 - 150$$

$$= 108.75$$

$$BM_A = 0$$

whenever the shear force is zero there should be max. bending moment.

$$BM_{max} = (51.75 \times x) - (\cancel{12}^{6} \times x)\left(\frac{x}{\cancel{2}}\right)$$

$SF_x = 0$

$0 = -51.75 + (12 \times x)$

$12x = +51.75$

$x = \frac{51.75}{12}$

$\boxed{x = 4.3125 m}$

$$= (51.75 \times 4.3125) - 6\,(4.3125)^2$$

$$= 223.1718 - 111.5859$$

$\boxed{BM_{max} = 111.5858 \text{ KN-m}}$

we know

$$\frac{M}{I} = \frac{\sigma}{y}$$

$\boxed{\therefore d = 2b}$

$$\frac{111.5858 \times 10^3 \times 10^3}{\frac{bd^3}{12}} = \frac{8}{\left(\frac{d}{2}\right)}$$

$$\frac{111.5858 \times 10^6 \times 12}{b\,(2b)^3} = \frac{\cancel{2} \times 8}{\cancel{2}b}$$

$$\frac{1339030350}{8b^4} = \frac{8}{b}$$

$$\frac{1339030350}{64} = \frac{b^{\cancel{4}\,3}}{\cancel{b}}$$

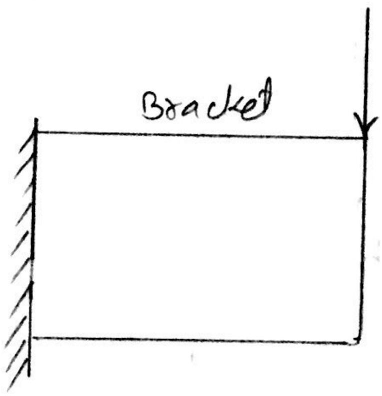
$$b = \sqrt[3]{20922349.22}$$

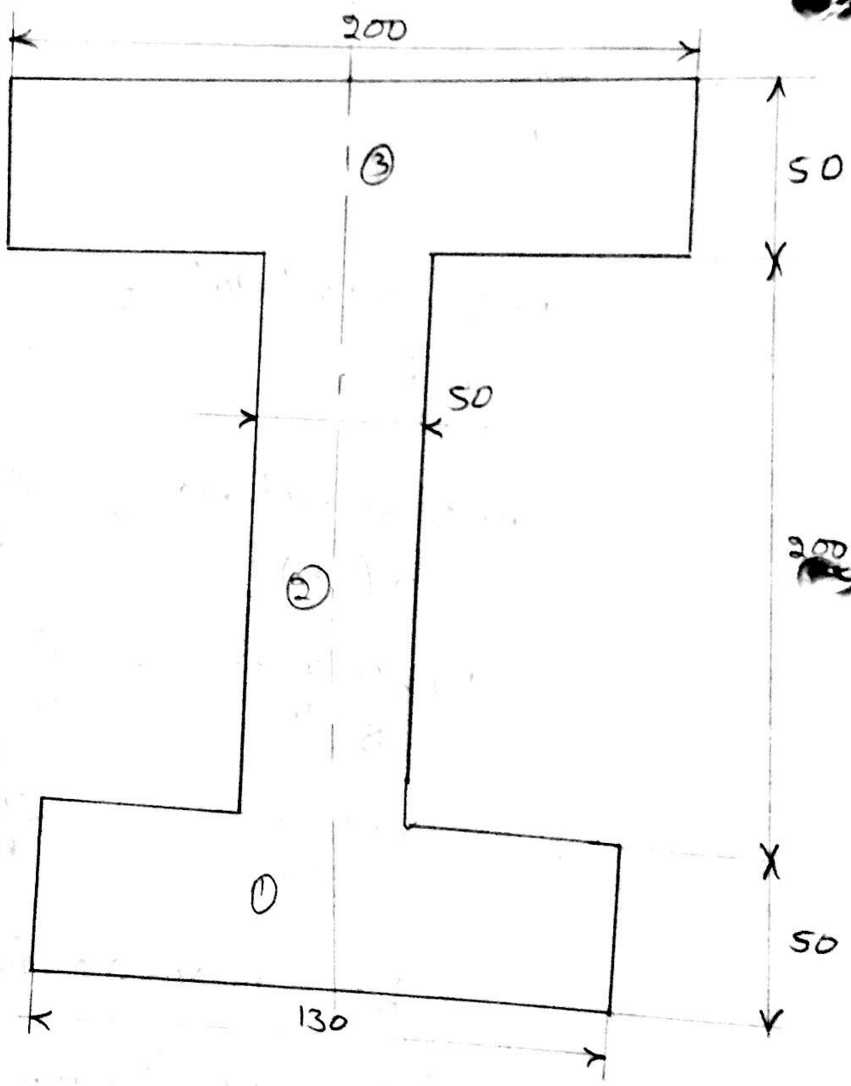
$\boxed{b = 275.55 \text{ mm}}$

$d = 2b = 2 \times 275.55$

$\boxed{d = 551.10 \text{ mm}}$

A Cast iron bracket subjected to bending & it has the cross-section of I-form with unequal flanges. The dimensions of the section are shown in fig. Find the position of the neutral axis and moment of inertia of the section about the neutral axis. If the maximum bending moment on the section is 40MN-mm, Determine the maximum bending stress. what is the nature of the stress.





we know

$$\frac{M}{I} = \frac{\sigma}{y}$$

Here except m value all values are unknown.

For calculating moment of Inertial value first we have to calculate $\bar{y}$.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$A_1 = 130 \times 50 = 6500\,mm^2$; $y_1 = \frac{50}{2} = 25\,mm$

$A_2 = 200 \times 50 = 10{,}000\,mm^2$; $y_2 = 50 + \frac{200}{2} = 150\,mm$

$A_3 = 200 \times 50 = 10{,}000\,mm^2$; $y_3 = 200 + 50 + \frac{50}{2} = 275\,mm$.

$$\bar{y} = \frac{(6500 \times 25) + (10{,}000 \times 150) + (10{,}000 \times 275)}{6500 + 10000 + 10000}$$

$$= \frac{162500 + 1500000 + 2750000}{26500} = \frac{4412500}{26500}$$

$\boxed{\bar{y} = 166.5\,mm} \rightarrow$ from bottom.

$\boxed{I = I_{GG} + Ah^2}$ $\boxed{\therefore h = y - \bar{y}}$

$$I_1 = I_{GG_1} + A_1 h_1^2$$

$$= \frac{b_1 d_1^3}{12} + A_1 (y_1 - \bar{y})^2$$

$$= \frac{(130)(50)^3}{12} + 6500(25 - 166.5)^2$$

$$= 1354166.667 + 130144625$$

$I_1 = 131498791.7\,mm^4$

$$I_2 = I_{GG_2} + A_2 h_2^2$$

$$= \frac{50(200)^3}{12} + (10{,}000)(150 - 166.5)^2.$$

$$= 33333333.33 + 2722500$$

$$I_2 = 36055833.33 \, mm^4$$

$$I_3 = I_{GG_3} + A_3 h_3^2$$

$$= \frac{200(50)^3}{12} + (10000)(275 - 166.5)^2$$

$$= 2083333.33 + 117722500$$

$$I_3 = 119805833.3 \, mm^4$$

Total moment of Inertia $(I) = I_1 + I_2 + I_3$

$$= 131498791.7 + 36055833.33 + 119805833.3$$

$$I = 287360458.4 \, mm^4.$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \times y \Rightarrow \frac{40 \times 10^6 \times 166.5}{287360458.4} = \frac{666 \times 10^7}{287360458.4}$$

$$\boxed{\sigma = 23.17 \, N/mm^2}$$

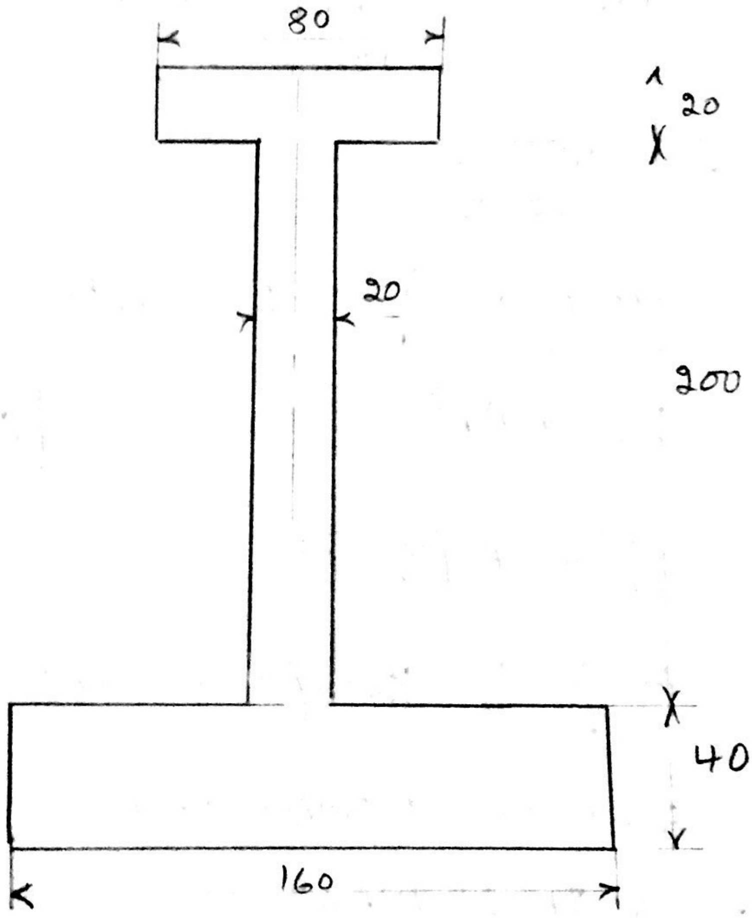
The above stress will be ~~Tensile~~ Compressive because in the Cantilever beam the upper layers are subjected to Tensile whereas the bottom layers are subjected to Compressive stress.

Q) A Cast iron beam is of I-section as shown in fig. The beam is simply supported on a span of 5m. If the tensile stress is not to exceed $20 N/mm^2$, find the safe uniformly load which the beam can carry. Find also the maximum Compressive stress.

sol

length of the beam (L) = 5m

Tensile stress $(\sigma_t) = 20 N/mm^2$



we know

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$A_1 = 160 \times 40 = 6400 mm^2 \; ; \; y_1 = \frac{40}{2} = 20mm$

$A_2 = 20 \times 200 = 4000 mm^2 \; ; \; y_2 = 40 + \frac{200}{2} = 140mm$

$A_3 = 80 \times 20 = 1600 mm^2 \; ; \; y_3 = 40 + 200 + \frac{20}{2} = 250mm$

$$\bar{y} = \frac{(6400 \times 20) + (4000 \times 140) + (1600 \times 250)}{6400 + 4000 + 1600}$$

$$= \frac{128000 + 560000 + 400000}{12000}$$

$$= \frac{1088000}{12000}$$

$\boxed{\bar{y} = 90.66 \; mm}$ — from bottom

$I = I_{GG} + Ah^2$ $\quad \boxed{\therefore h = y - \bar{y}}$

$$I_1 = \frac{b_1 d_1^3}{12} + A_1 (y_1 - \bar{y})^2$$

$$= \frac{160(40)^3}{12} + 6400(20 - 90.66)^2$$

$$= 853333.33 + 31960177.78$$

$$I_1 = 32813511.11 \; mm^4$$

$$I_2 = \frac{20(200)^3}{12} + 4000(140 - 90.66)^2$$

$$= 13333333.33 + 9737742.4.$$

$$I_2 = 23071075.73 mm^4.$$

$$I_3 = \frac{80(20)^3}{12} + 1600(250-90.66)^2$$

$$= 53333.33 + 40622776.96$$

$$= 40676110.29 \, mm^4$$

Total $M.O.I \ (I) = I_1 + I_2 + I_3$

$$= \cancel{853333.3} = 32813511.11 + 23071075.73 + 40676110.29$$

$$I = 96560697.13 \, mm^4$$

For simply supported beam the layers above the N.A are subjected to Compressive stress. whereas the layers below the N.A are subjected to Tensile stresses.

$$\frac{M}{I} = \frac{\sigma}{y}$$

For simply supported beam carries udl throughout its length. The max ~~beam~~ bending moment it is given by

$$M = \frac{wl^2}{8}$$

$$\frac{wl^2}{8} = \frac{20}{90.66} \times 96560697.13$$

$$= \frac{1931213943}{90.66} \times \frac{8}{(5000)^2}$$

$$= \frac{1.54 \times 10^{10}}{22665 \times 105}$$

$$\boxed{w = 6.79 \, N/mm}$$

$$M = \frac{wl^2}{8} = \frac{6.79\,(5000)^2}{8}$$

$$M = 21233178.91 \text{ N-mm}$$

$$\sigma_c = \frac{M}{I} \times y_c$$

For calculating compressive stress we have to consider y value above the neutral axis, because the layer above the N.A subjected to compressive stress

$$= \frac{21233178.91}{96560697.13} \times 169.34$$

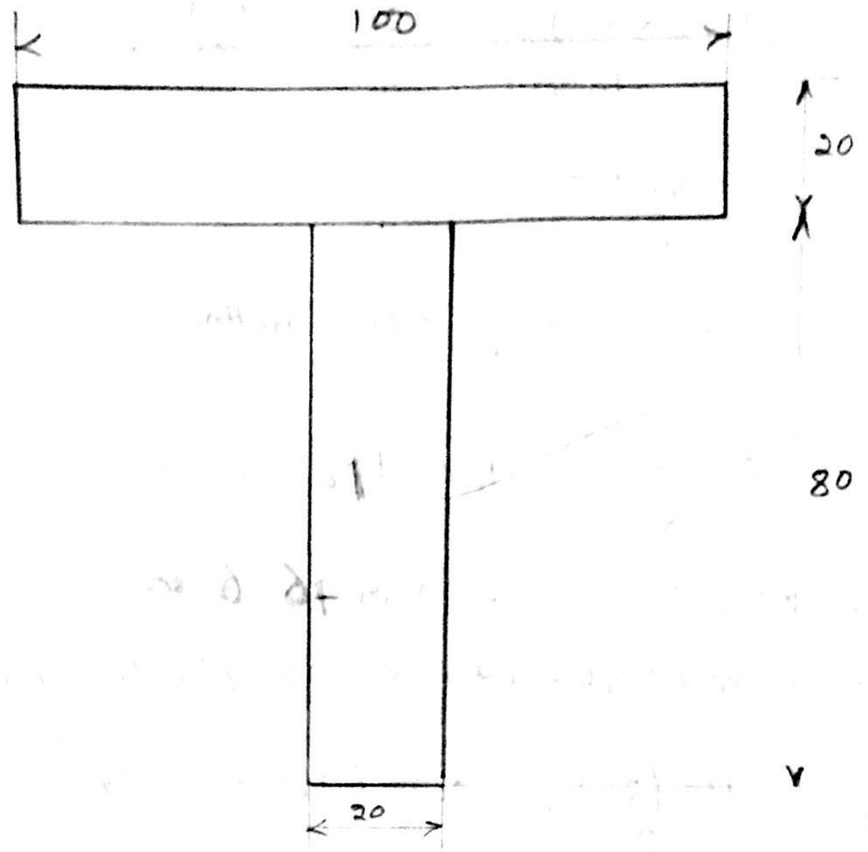
$$y_c = 260 - 90.66$$

$$y_c = 169.34 \text{ mm}$$

$$= \frac{3595626517}{96560697.13}$$

$$\boxed{\sigma_c = 37.23 \text{ N/mm}^2}$$

A Cast Iron beam is of T-section as shown in figure. The beam is simply supported on a span of 8m. The beam carries a uniformly distributed load of 1.5 kN/m length on the entire span. Determine the maximum tensile and maximum compressive stresses.



For simply supported beam carries udl, maximum Bending moment is given by

$$m = \frac{\omega l^2}{8} = \frac{1.5 \times 10^3 \times (8)^2}{8} = \frac{96000}{8}$$

$$= 12000 \text{ N-m}$$

$$= 12000 \times 10^3 \text{ N-mm}$$

we know $I = I_{GG} + Ah^2$ $\boxed{\therefore h = y - \bar{y}}$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$A_1 = 20 \times 80 = 1600 mm^2$; $y_1 = \frac{80}{2} = 40 mm$

$A_2 = 100 \times 20 = 2000 mm^2$; $y_2 = 80 + \frac{20}{2} = 90 mm.$

$$\bar{y} = \frac{(1600 \times 40) + (2000 \times 90)}{1600 + 2000} = \frac{64000 + 18 \times 10^4}{3600}$$

$$= \frac{244000}{3600}$$

$\boxed{\bar{y} = 67.77\ mm}$ from bottom

$$I_1 = \frac{20(80)^3}{12} + 1600(\cancel{8}40 - 67.77)^2$$

$$= 853333.33 + 1233876.64$$

$$I_1 = \cancel{1092345.67\ mm^4}\ 2087209.97\ mm^4$$

$$I_2 = \frac{100(20)^3}{12} + 2000(90 - 67.77)^2$$

$$= 66666.66 + 988345.8$$

$$I_2 = 1055012.46$$

$$I = I_1 + I_2$$

$$= \overset{2087209.97}{\cancel{1092345.67}} + 1055012.46$$

$\boxed{\cancel{I = 2147358.13\ mm^4}}$ $\boxed{I = 3142222.43\ mm^4}$

we know

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \times y$$

$$= \frac{12 \times 10^6}{\cancel{2147358.13}\ 3142222.43} \times 67.77$$

$$\sigma = 258.81\ N/mm^2.$$

The above calculated "σ" value is tensile in nature because we considered the y value from the bottom. The layers below the N.A subjected to tensile stress whereas the layers above the N.A subjected to compressive stress in case of simply supported beam.

similarly

$$\sigma_t = \frac{M}{I} \times y_t$$

$y_t = 100 - 67.77$
$= 32.23$

$$= \frac{12 \times 10^6}{3142222.43} \times 32.23$$

$$= \frac{386760000}{3142222.43}$$

$$\boxed{\sigma_t = 123.08 \ N/mm^2}$$

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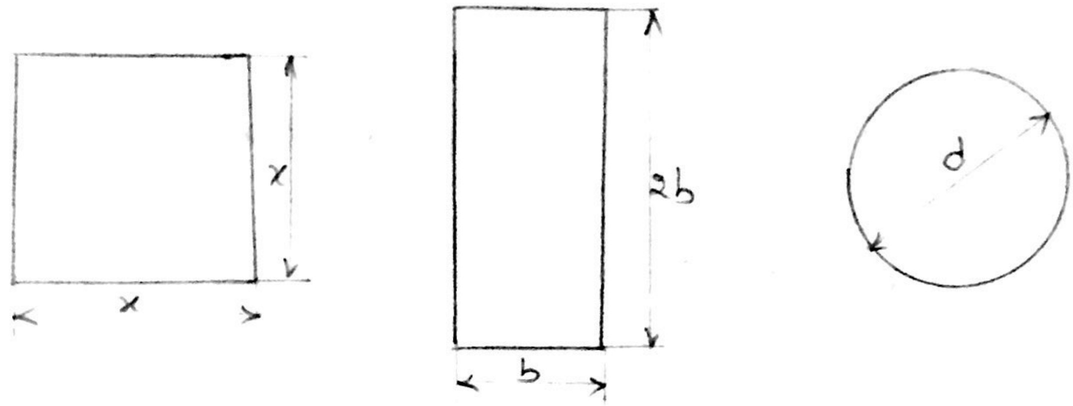
Three beams have the same length, same allowable bending stress and same bending moment. The cross section of the beams are a square, rectangle with depth twice the width and a circle. Find the ratios of weights of the circular and the rectangular beams with respect to square beams.



we know $\frac{M}{I} = \frac{\sigma}{y}$

$M_{square} = M_{circular} = M_{Rect}$

$\sigma_{square} = \sigma_{rect} = \sigma_{circular}$

$M_{square} = M_{Rectangle}$

$$\frac{I_R}{y_R}\cancel{\sigma_S} = \frac{I_R}{y_R}\cancel{\sigma_R}$$

$$\frac{\frac{x\,x^3}{12}}{\frac{x}{2}} = \frac{\frac{b(2b)^3}{12}}{\frac{2b}{2}} \Rightarrow \frac{x^{\cancel{4}\,3}}{\cancel{12}\,6} \times \frac{\cancel{2}}{\cancel{x}} = \frac{\cancel{8}\,2\,b^{\cancel{4}\,3}}{\cancel{12}\,3} \times \frac{1}{\cancel{b}}$$

$$\frac{x^3}{6} = \frac{2b^3}{3}$$

$$b^3 = \frac{\cancel{3} \times x^3}{2 \times \cancel{6}\,2} \Rightarrow b = \sqrt[3]{\frac{1}{4}x^3} = (0.25\,x^3)^{1/3}$$

$$\boxed{b = 0.629\,x}$$

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$$M_{square} = M_{circular}$$

$$\frac{I_B}{y_B} \times \cancel{\sigma_B} = \frac{I_c}{y_c} \times \cancel{\sigma_c}$$

$$\frac{\frac{x x^3}{12}}{\frac{x}{2}} = \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} \Rightarrow \frac{\cancel{x^4}^{3}}{\cancel{12}_{6}} \times \frac{\cancel{2}^{1}}{\cancel{x}} = \frac{\pi \cancel{d^4}^{3}}{\cancel{64}_{32}} \times \frac{\cancel{2}^{1}}{\cancel{d}}$$

$$\frac{x^3}{6} = \frac{\pi d^3}{32}$$

$$\frac{32 x^3}{6\pi} = d^3 \Rightarrow (1.6976 x^3)^{1/3}$$

$$\boxed{d = 1.1929 x}$$

we know

$$\omega_{square} = \omega_{Rectangle}$$

$$\boxed{\omega = \frac{W}{V}}$$

$$\frac{\text{weight of square beam}}{\text{Area of square beam} \times \cancel{l}} = \frac{\text{wt of Rect beam}}{\text{Area of Rect beam} \times \cancel{l}}$$

$$\frac{\text{wt. of square beam}}{\text{wt. of Rect beam}} = \frac{\text{Area of square beam}}{\text{Area of Rect beam.}}$$

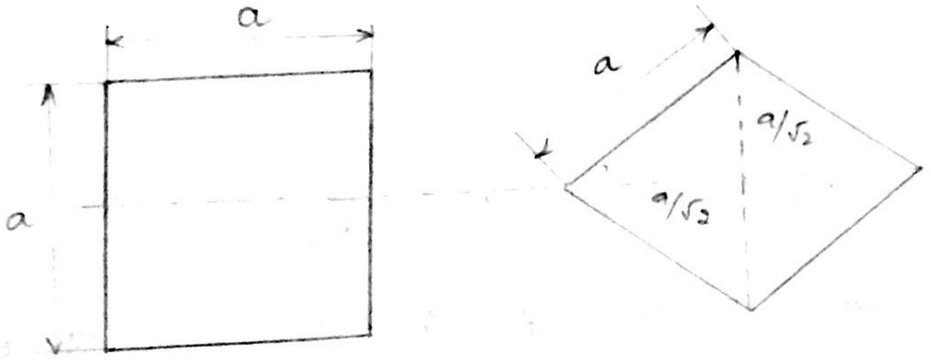
$$\frac{\text{wt. of Rectangular beam}}{\text{wt. of square beam}} = \frac{(0.629 \times x)(2 \times 0.629 x)}{x \times x}$$

$$\boxed{\frac{W_R}{W_S} = 0.7912.}$$

$$\frac{W_{circular}}{W_{square}} = \frac{\frac{\pi d^2}{4}}{x \times x} = \frac{\frac{\pi}{4}(1.1929 x)^2}{x^2} = \frac{1.117 \cancel{x^2}}{\cancel{x^2}}$$

$$\boxed{\frac{W_{circular}}{W_{square}} = 1.117}$$

Q) A beam is of square section of the side 'a'. If the permissible bending stress is 'σ' find the moment of resistance when the beam section is placed such that (i) two sides are horizontal, (ii) one diagonal is vertical. Find also the ratio of moments of the resistance of the section in two positions.



$\sin 45 = \frac{opp}{a}$

$\frac{1}{\sqrt{2}} \times a = opp$

moment of resistance :- when two sides are horizontal

$$M = \frac{I}{y} \times \sigma$$

$$= \frac{\frac{a \times a^3}{12}}{\frac{a}{2}} \times \sigma \Rightarrow \frac{\cancel{a^4}^{3}}{\cancel{12}_{6}} \times \frac{\cancel{2}^{1}}{\cancel{a}} \times \sigma$$

$$M_1 = \frac{a^3}{6} \times \sigma$$

$\frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}} = \frac{2a}{\sqrt{2}}$

when one diagonal is vertical.

$$= \frac{I}{y} \times \sigma \Rightarrow \frac{\cancel{2}^{1} \times \frac{bh^3}{\cancel{12}_{6}}}{\frac{a}{\sqrt{2}}} = \frac{1}{6}\left(\frac{2a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right)^3 \times \frac{\sqrt{2}}{a}$$

$$= \frac{1}{6} \times \frac{2a \times a^3}{(\sqrt{2})^3 \times a} = \frac{a^3}{3\sqrt{2}(\sqrt{2})^2} = \frac{a^3}{6\sqrt{2}} \times \sigma$$

$$M_2 = \frac{a^3}{6\sqrt{2}} \times \sigma$$

Ratio of moment of Resistance of the section in two positions.

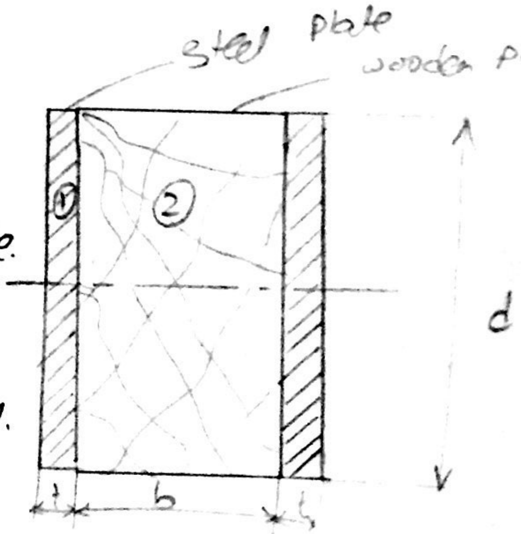
$$\frac{M_1}{M_2} = \frac{\sigma \times \frac{a^3}{6}}{\sigma \times \frac{a^3}{6\sqrt{2}}} = \cancel{\frac{a^3}{6} \times \sigma} \times \frac{6\sqrt{2}}{\cancel{a^3 \times \sigma}} = \sqrt{2}.$$

$$\boxed{\frac{M_1}{M_2} = 1.414}$$

COMPOSITE BEAMS (FLITCHED BEAMS) :-

A beam made up of two or more different materials assumed to be rigidly connected together and behaving like a single piece is known as Composite beam or a wooden flitched beam.

Consider the Composite beam as shown in fig.



Let E_1 = Young's modulus of steel plate.

I_1 = M.O.I of steel about N.A.

M_1 = moment of Resistance of steel.

E_2 = young's modulus of wood

I_2 = M.O.I of wood about N.A.

M_2 = moment of Resistance of wood.

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we know for the composite section strain should be same in both beams.

$$e_1 = e_2$$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\sigma_1 = \frac{E_1}{E_2} \times \sigma_2$$

$$\boxed{\therefore \frac{E_1}{E_2} = m.}$$

modular ratio.

$$\boxed{\sigma_1 = m\sigma_2}$$

$$\boxed{\frac{M}{I} = \frac{\sigma}{y}}$$

moment of Resistance of steel and wood are

$$M_1 = \frac{\sigma_1}{y_1} \times I_1 \qquad M_2 = \frac{\sigma_2}{y_2} \times I_2$$

$$M = M_1 + M_2$$

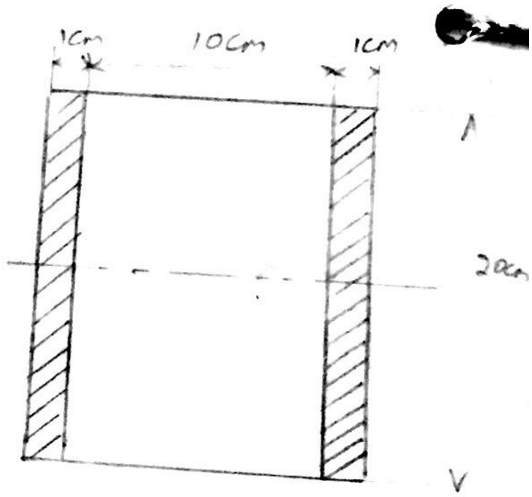
$$= \frac{m\sigma_2}{y} \times I_1 + \frac{\sigma_2}{y} \times I_2$$

$$= \frac{\sigma_2}{y}\left[mI_1 + I_2\right]$$

$$\boxed{\therefore I = mI_1 + I_2}$$

$$\boxed{M = \frac{\sigma_2}{y} \times I.}$$

A flitched beam consist of a wooden joist 10cm wide and 20cm deep strengthened by two steel plates 10mm thick and 20cm deep as shown in fig. If the maximum stress in the wooden joist is $7\ N/mm^2$, find the corresponding maximum stress attained in steel. Find also the moment of resistance of the composite section. Take young's modulus for steel $= 2\times10^5\ N/mm^2$ and for wood $= 1\times10^4\ N/mm^2$.



$b_2 = 10cm$

$d_2 = 20cm$

width of one steel plate $(b_1) = 10mm$.

depth of " " " $(d_1) = 20cm$.

No. of plates $= 2$.

max. stress in wood $= 7\ N/mm^2$

$\sigma_{steel} = ?$

$E_s = 2\times10^5\ N/mm^2$

$E_w = 1\times10^4\ N/mm^2$

we know $\frac{M}{I} = \frac{\sigma}{y}$

$$I_1 = \frac{bd^3}{12} = \frac{2\times10\times(200)^3}{12} = 1333.33\times10^4\ mm^4$$

$$I_2 = \frac{100\ (200)^3}{12} = 6666.66\times10^4\ mm^4$$

we know Total moment of inertia for flitched beam

$$I = mI_1 + I_2 \qquad m = \frac{E_1}{E_2} = \frac{2\times10^5}{1\times10^4} = 20$$

$$= 20(1333.33)\times10^4 + 6666.66\times10^4$$

$$= 26666.6\times10^4 + 6666.66\times10^4$$

$$= 33333.2\times10^4 \text{ mm}^4.$$

$$M = \frac{\sigma_2}{y}\times I$$

$$= \frac{7\times 33333.2\times10^4}{100}$$

$$= \frac{233332.82}{100}$$

$$= 2333.32\times10^4 \text{ N-mm}$$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\sigma_1 = \frac{E_1}{E_2}\times\sigma_2$$

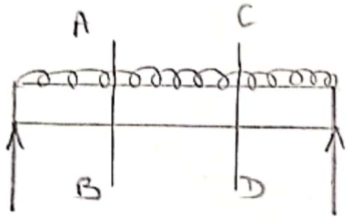
$$= 20\times7$$

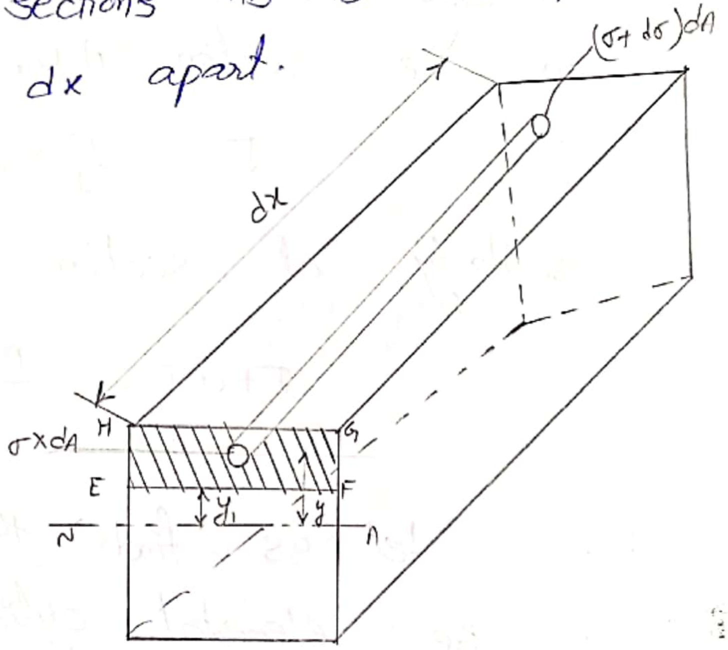
$$= 140 \text{ N/mm}^2$$

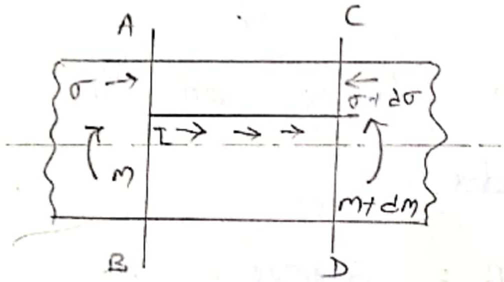
SHEAR STRESSES IN BEAMS

Shear stress at a Section :-

The below fig shows a simply supported beam carrying a uniformly distributed load. For a uniformly distributed load, the shear force and bending moment will vary along the length of the beam. Consider two sections AB and CD of this beam at a distance dx apart.







Let it is required to find the shear stress on the section AB at a distance y_1 from the neutral axis. Fig (c) shows the cross-section of the beam. On the cross-section of the beam let EF be a line at a distance y_1 from the neutral axis. Now consider the part of the beam above the level EF and between the sections AB and CD. This part of the beam may be taken to consists of an infinite numbers of elemental cylinders each of area dA and length dx. Consider one such elemental cylinder at a distance y from neutral axis.

The bending stress at a distance y from the neutral axis given by

$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma = \frac{M}{I} \times y.$$

Bending stress on the end of elemental cylinder on the section AB, (where bending moment is M) will be

$$\sigma = \frac{M}{I} \times y$$

similarly at section CD.

$$\sigma + d\sigma = \frac{M + dM}{I} \times y.$$

Now let us find the forces on the two ends of the elemental cylinder.

$$\boxed{\frac{F}{A} = \sigma}$$

Force on the section AB = Stress × Area of elemental cyl'd

$$F_{AB} = \sigma \times dA$$

$$= \frac{M}{I} \times y \times dA$$

similarly force on the section CD = $(\sigma + d\sigma) \times dA$

$$F_{CD} = \frac{M + dM}{I} \times y \times dA$$

At the two ends of the elemental cylinder; the forces are different. They are acting along the the same line but are in opposite direction. Hence there will be unbalanced force on the elemental cylinder.

Net unbalanced force on the elemental cylinder

$$= F_{CD} - F_{AB}$$

$$= \frac{(M+dM)}{I} \times y \times dA - \frac{M}{I} \times y \times dA$$

$$= \frac{\cancel{MydA}}{\cancel{I}} + \frac{dMydA}{I} - \frac{\cancel{MydA}}{\cancel{I}}$$

$$F = \frac{dM \times y \times dA}{I}$$

Total unbalanced force (F)

$$\int F = \int \frac{dM}{I} y \, dA$$

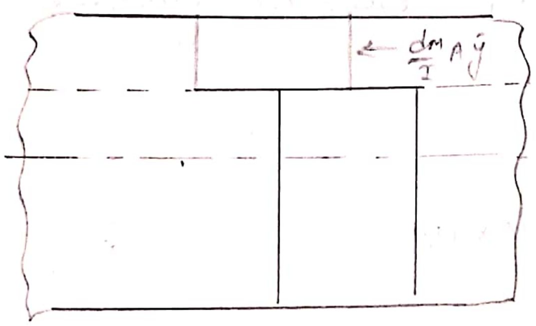
$$\boxed{\therefore \int y \, dA = A\bar{y}}$$

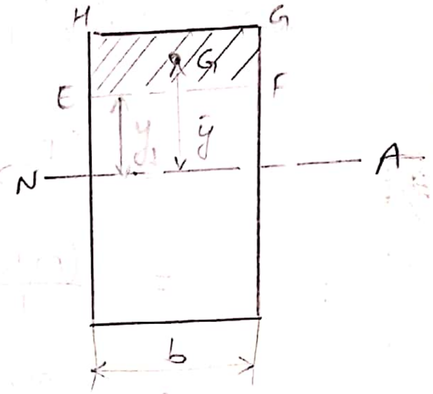
$$= \frac{dM}{I} \times A\bar{y}$$

where A = Area of section above the level EF

$\bar{y}$ = Dist of the C.G of the area A from N.A

Due to the total unbalanced force acting on the part of beam above the level EF and b/w the sections AB and CD ~~as shown in below fig~~ the beam may fail due to shear. as shown in fig. Hence in order the above part may not fail by shear, the horizontal section of the beam at the level EF must offer a shear resistance. This shear resistance at least must be equal to total unbalanced force to avoid failure due to shear.





Shear force due to shear stress (τ)

= shear stress × shear Area.

= $\tau \times b \times dx$

Total unbalanced force = shear force

$$\frac{dm}{I} \times A\bar{y} = \tau \times b \times dx$$

$$\boxed{\therefore \frac{dm}{dx} = F}$$

$$\frac{dm}{dx} \frac{A\bar{y}}{Ib} = \tau$$

$$\boxed{\tau = \frac{FA\bar{y}}{Ib}}$$

where

τ = shear stress

F = shear force.

A = Area of the section above y_1

$\bar{y}$ = Distance of C.G. of area A from N.A.

I = M.O.I of the total section.

b = Actual width of the section.

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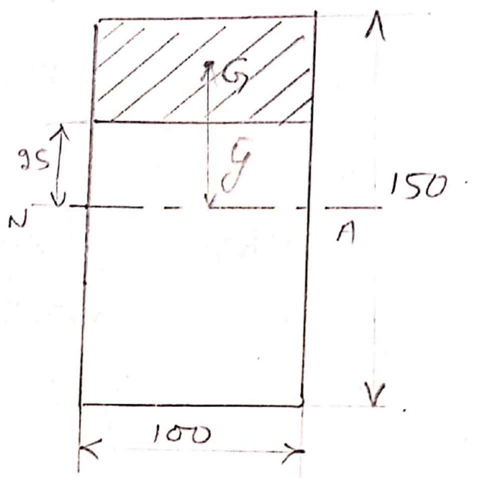
Q) A wooden beam 100mm wide and 150mm deep is simply supported over a span of 4m. If shear force at a section of the beam is 4500N, Find the shear stress at a distance of 25mm above the N.A.

Sol Given data.



width of the beam (b) = 100mm

Depth of the beam (d) = 150mm

length of the beam (l) = 4m.

Shear force (F) = 4500N

Shear stress at a dist 25mm..

we know.

$$\tau = \frac{FA\bar{y}}{Ib}$$

Area of beam above y_1

$$A = 100 \times 50 = 5000 \, mm^2.$$

$$\bar{y} = 25 + \frac{50}{2} = 50mm$$

$$I = \frac{bd^3}{12} = \frac{100(150)^3}{12}.$$

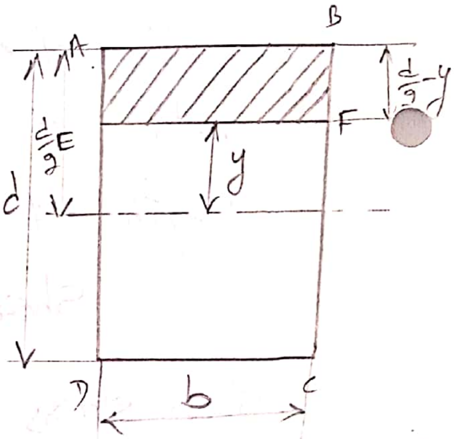
$$= 28.125 \times 10^6 \, mm^4$$

$$\tau = \frac{4500 \times 5000 \times (25 + \frac{50}{2})}{28.125 \times 10^6 \times 100} = \frac{11.25 \times 10^8}{28.125 \times 10^8}$$

$$\tau = 0.4 \, N/mm^2.$$

Shear stress distribution for Rectangular ~~different~~ sections:-

The below fig shows a rectangular section of a beam of width b and depth d. Let F is the shear force acting at the section. Consider a level EF at a distance y from the neutral axis.



We know

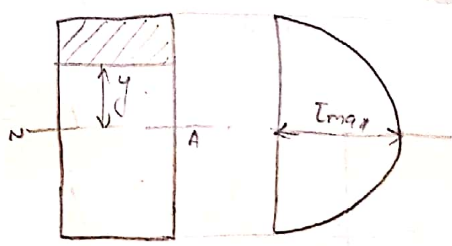
Shear stress at a distance y from neutral axis

$$\tau = \frac{FA\bar{y}}{Ib}$$

Here Area(A) = $b \times \left(\frac{d}{2}-y\right)$

$$\bar{y} = y + \left(\frac{\frac{d}{2}-y}{2}\right) \Rightarrow y + \left(\frac{d}{2}-y\right)\frac{1}{2} \Rightarrow y + \frac{d}{4} - \frac{y}{2}$$

$$= \frac{y}{2} + \frac{d}{4} \Rightarrow \frac{1}{2}\left(\frac{d}{2}+y\right)$$



$$\tau = \frac{F \times b\left(\frac{d}{2}-y\right)\left(\frac{d}{2}+y\right)\frac{1}{2}}{Ib}$$

$$= \frac{F\cancel{b}}{2}\left(\frac{d^2}{4}-y^2\right) \times \frac{1}{I\cancel{b}} = \frac{F}{2I}\left(\frac{d^2}{4}-y^2\right)$$

We see that as τ increases y decreases.

At the top edge $y = \frac{d}{2}$

$$\tau = \frac{F}{2I}\left[\frac{d^2}{4} - \left(\frac{d}{2}\right)^2\right] = 0.$$

At the neutral axis $y = 0$

$$= \frac{F}{2I}\left(\frac{d^2}{4} - 0\right)$$

$$= \frac{F}{2I}\left(\frac{d^2}{4}\right) \qquad \because I = \frac{bd^3}{12}$$

$$= \frac{F}{\cancel{2}\frac{bd^3}{\cancel{12}_6}}\left(\frac{d^2}{4}\right) \Rightarrow \frac{{}^3\cancel{6}Fd^{\cancel{2}}}{\cancel{4}_2\,bd^{\cancel{3}}} = \frac{3}{2}\frac{F}{bd}$$

$$\tau_{max} = \frac{3}{2}\frac{F}{bd} \Rightarrow 1.5\frac{F}{bd}$$

Now average shear stress

$$\tau_{avg} = \frac{\text{shear Force}}{\text{Area of Cross section}} = \frac{F}{b \times d}$$

Now

$$\tau_{max} = 1.5\,\tau_{avg}.$$

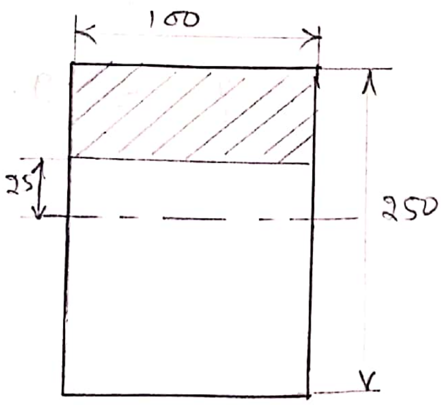
Q) A rectangular beam 100mm wide and 250mm deep is subjected to a maximum shear force of 50 KN. Determine.
i) Average shear stress ii) maximum shear stress &
iii) shear stress at a distance 25mm above the neutral axis

Sol Given data :-

width (w) = 100mm

Depth (d) = 250 mm

shear force (F) = 50 KN



we know

i) Average shear stress (τ_{avg}) $= \frac{F}{bd} = \frac{50\times10^3}{100\times250}$

$$= \frac{\cancel{50000}^{2}}{\cancel{25000}} = 2\ N/mm^2.$$

ii) maximum shear stress (τ_{max})

$$\tau_{max} = 1.5\ \tau_{avg}$$

$$= 1.5 \times 2$$

$$= 3\ N/mm^2.$$

iii) The shear stress at a distance 25mm from neutra axis

$$\tau = \frac{FA\bar{y}}{Ib} \quad (\text{or})$$

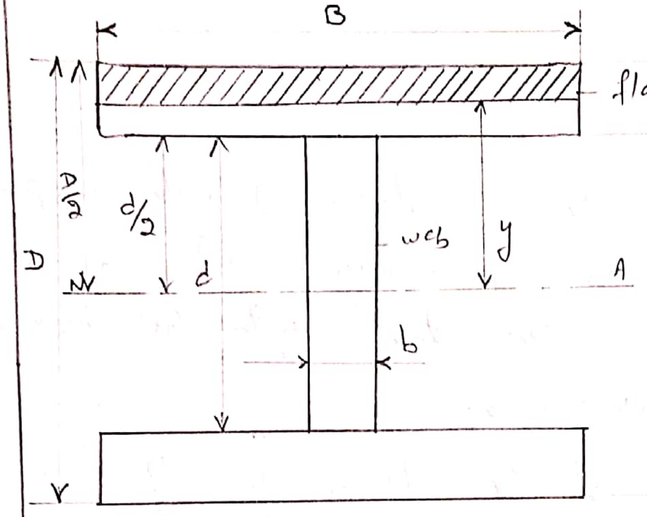
$$\tau = \frac{F}{2I}\left[\frac{d^2}{4} - y^2\right]$$

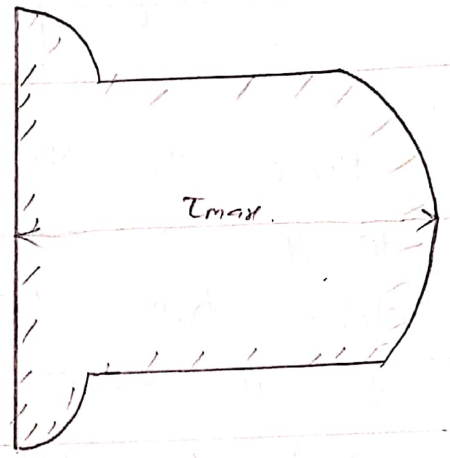
$$= \frac{50000}{2\times\frac{100(250)^3}{12}}\left[\frac{250^2}{4} - 25^2\right]$$

$$= 1.92\times10^{-4}\left[15625 - 625\right]$$

$$\tau = 2.88\ N/mm^2$$

Shear stress distribution for I- section :-





In this case the shear distribution in the web and shear distribution in the flange are to be calculated separately. 1) Let us first calculate the shear distribution in the flange.

we know shear stress at a distance y from N.A.

$$\tau = \frac{FA\bar{y}}{Ib}$$

In this case.

width of the section $(b) = B$.

Area of the section $(A) = B \times \left(\frac{D}{2} - y\right)$

Dist. of the C.G of the shaded area from N.A.

$$\bar{y} = y + \frac{\left(\frac{D}{2} - y\right)}{2} \Rightarrow y + \frac{1}{2}\left(\frac{D}{2} - y\right)$$

$$= y + \frac{D}{4} - \frac{y}{2} \Rightarrow \frac{D}{4} + \frac{y}{2}$$

$$\bar{y} = \frac{1}{2}\left(\frac{D}{2} + y\right)$$

The shear stress in the flange becomes

$$\tau = \frac{F \times B \times \left(\frac{D}{2} - y\right) \times \frac{1}{2}\left(\frac{D}{2} + y\right)}{I \times B}$$

$$= \frac{FB}{2}\left(\frac{D}{2}-y\right)\left(\frac{D}{2}+y\right) \times \frac{1}{IB}$$

$$= \frac{F}{2I}\left(\frac{D^2}{4}-y^2\right)$$

Hence the variation of shear stress (τ) with respect to y in the flange is parabolic. It is also clear from the above eqn that with the increase of y, shear stress decreases.

$\Rightarrow$ for the upper edge of the flange $y = \frac{D}{2}$.

$$= \frac{F}{2I}\left(\frac{D^2}{4} - \left(\frac{D}{2}\right)^2\right) \Rightarrow \frac{F}{2I}\left(\frac{D^2}{4} - \frac{D^2}{4}\right)$$

$$= 0.$$

$\Rightarrow$ for the lower edge of the flange :-

$$y = \frac{d}{2}$$

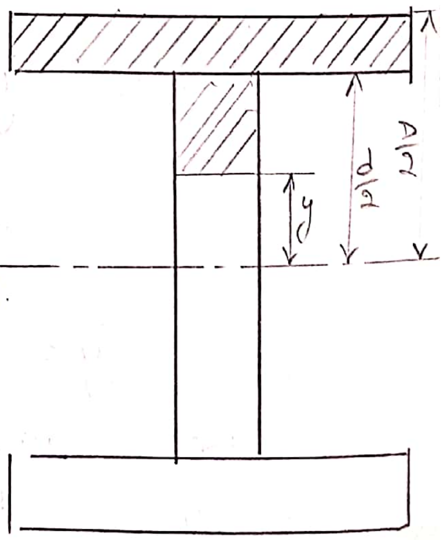
$$= \frac{F}{2I}\left(\frac{D^2}{4} - \left(\frac{d}{2}\right)^2\right)$$

$$= \frac{F}{2I}\left(\frac{D^2}{4} - \frac{d^2}{4}\right).$$

$$= \frac{F}{8I}\left(D^2 - d^2\right).$$

2) Now shear stress distribution in the web :-

Here $A\bar{y}$ is made up of two parts i.e



$A\bar{y}$ = moment of the flange area about N.A + moment of shaded area of web about N.A.

$$= B\left(\frac{D}{2}-\frac{d}{2}\right)\left[\frac{\left(\frac{D}{2}-\frac{d}{2}\right)}{2}+\frac{d}{2}\right] + b\left(\frac{d}{2}-y\right)\left(y + \frac{\left(\frac{d}{2}-y\right)}{2}\right)$$

$$= B\left(\frac{D}{2}-\frac{d}{2}\right)\left(\frac{D}{4}-\frac{d}{4}+\frac{d}{2}\right) + b\left(\frac{d}{2}-y\right) * \left(y+\left(\frac{d}{4}-\frac{y}{2}\right)\right)$$

$$= B\left(\frac{D}{2}-\frac{d}{2}\right)\frac{1}{2}\left(\frac{D}{2}-\frac{d}{2}+d\right) + b\left(\frac{d}{2}-y\right)\left(\frac{d}{4}+\frac{y}{2}\right)$$

$$= \frac{B}{2}\left(\frac{D}{2}-\frac{d}{2}\right)\left(\frac{D}{2}+\frac{d}{2}\right) + b\left(\frac{d}{2}-y\right)\frac{1}{2}\left(\frac{d}{2}+y\right)$$

$$= \frac{B}{2}\left(\frac{D^2}{4}-\frac{d^2}{4}\right) + \frac{b}{2}\left(\frac{d^2}{4}-y^2\right)$$

$$A\bar{y} = \frac{B}{8I}\left(D^2-d^2\right) + \frac{b}{2}\left(\frac{d^2}{4}-y^2\right)$$

$$\tau = \frac{FA\bar{y}}{Ib}$$

$$= \frac{F \times \frac{B}{8I}\left(D^2-d^2\right) + \frac{b}{2}\left(\frac{d^2}{4}-y^2\right)}{Ib}$$

$$\boxed{\tau = \frac{F}{Ib}\left[\frac{B\left(D^2-d^2\right)}{8} + \frac{b}{2}\left(\frac{d^2}{4}-y^2\right)\right]}$$

At $y = 0$; shear stress is max.

$$= \frac{F}{Ib}\left[\frac{B\left(D^2-d^2\right)}{8} + \frac{bd^2}{8}\right] \Rightarrow \frac{F}{8Ib}\left[BD^2 - Bd^2 + bd^2\right]$$

At $y = \frac{d}{2}$

$$= \frac{F}{Ib}\left[\frac{B\left(D^2-d^2\right)}{8} + \frac{b}{2}\left(\frac{d^2}{4}-\frac{d^2}{4}\right)\right]$$

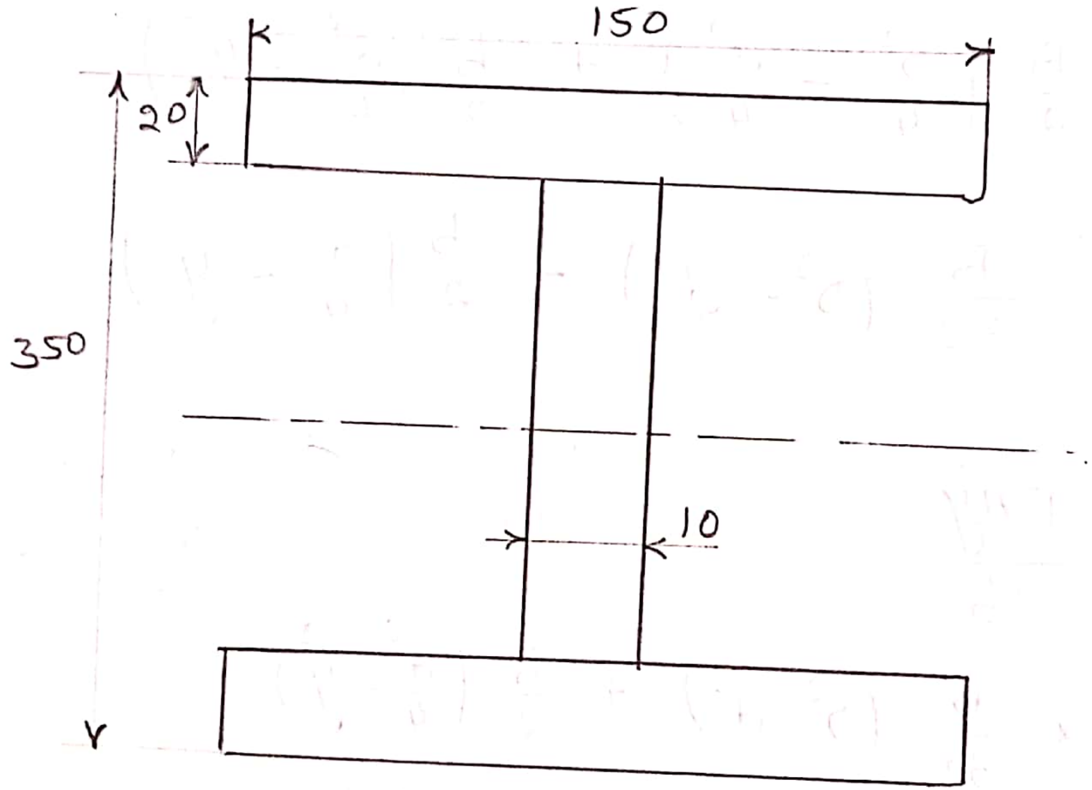
$$= \frac{F}{Ib}\left[\frac{B\left(D^2-d^2\right)}{8}\right] = \frac{B}{b} \times \frac{F}{8I}\left(D^2-d^2\right)$$

From the above eqn

It is clear that the stress at the junction changes abruptly from $\frac{F}{8I}\left(D^2-d^2\right)$ to $\frac{B}{b} \times \frac{F}{8I}\left(D^2-d^2\right)$

Q) An I- section beam 350mm x 150mm has a web thickness of 10mm and a flange thickness of 20mm. If the shear force acting on the section is 40KN, find the maximum shear stress developed in I-section. Sketch the shear stress distribution across the section.

Sol



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Sol Given data :-

Depth $(D) = 350 mm$

width $(B) = 150mm$.

web thickness$(b) = 10mm$

Flange thickness ($= 20mm$

shear force $(F) = 40000N$

moment of inertia of the section about neutral axis.

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{150(350)^3}{12} - \frac{140 \times 310^3}{12}$$

$$= 535937500 - 347561666.7$$

$$= 188.37 \times 10^6 mm^4$$

Shear stress distribution in the flange.

$$\tau = \frac{F}{2I}\left(\frac{D^2}{4} - y^2\right)$$

For upper edge of the flange $(y = \frac{D}{2}) = 0$.

For lower edge of the flange $(y = \frac{d}{2})$

then we have

$$= \frac{F}{8I}(D^2 - d^2) \Rightarrow \frac{40000}{8 \times 188.37 \times 10^6}(350^2 - 310^2)$$

$$= \frac{40000}{1507.006 \times 10^6}(26400)$$

$$= \frac{1056 \times 10^6}{1507.006 \times 10^6} = 0.7007 \; N/mm^2.$$

shear stress distribution in the web :-

$$\tau_{web} = \frac{F}{Ib}\left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

At the top edge of the web. $y = \frac{d}{2}$

$$= \frac{F}{Ib}\left[\frac{B}{8}(D^2 - d^2)\right]$$

$$= \frac{40000}{188.37 \times 10^6 \times 10}\left[\frac{150}{8}(350^2 - 310^2)\right]$$

$$= \frac{1.584 \times 10^{11}}{1.5069 \times 10^{10}}$$

$$\tau = 10.48 \ N/mm^2$$

τ_{max} $y = 0.$

$$\tau_{max} = \frac{F}{Ib}\left[\frac{B}{8}(D^2 - d^2) + \frac{bd^2}{8}\right]$$

$$= 10.48 + \frac{10(310)^2}{8} \times \frac{40000}{188.37 \times 106 \times 10}$$

$$= 10.48 + \frac{3.844 \times 10^{10}}{1.50696 \times 10^{10}}$$

$$= 10.48 + 2.55$$

$$\tau_{max} = 13.03 \ N/mm^2$$