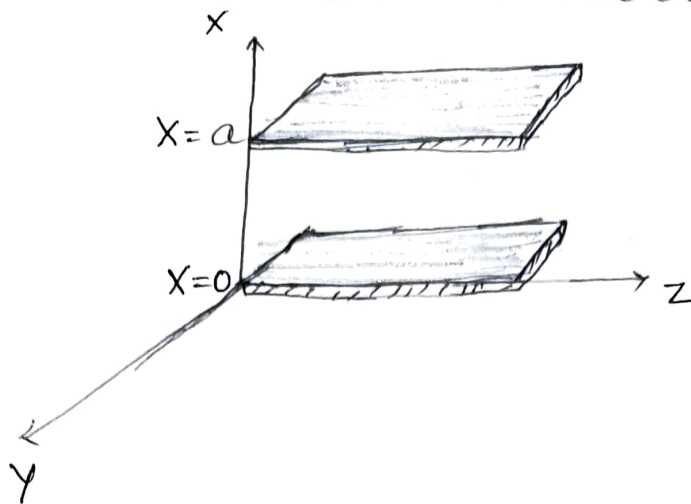




Let us consider a propagation of Em waves, between two parallel conducting planes of infinite extent in y and z direction and separated by 'a' mts in x-direction. Since the planes has been assumed as perfect conductor.

To find the field configuration in the region between the planes we have to solve the maxwells eqn and applying the boundary conditions.

$$E_{tan} = 0 \text{ and } H_{Normal} = 0$$

From maxwells eqn

$$\nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{--- (1)}$$

$$\text{and } \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{--- (2)}$$

For non conducting medium ($\sigma=0$)

$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} \Rightarrow \nabla \times \vec{H} = j\omega \epsilon \vec{E} \quad \text{--- (3)}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \Rightarrow \nabla \times \vec{E} = -j\omega \mu \vec{H} \quad \text{--- (4)}$$

we know the wave eqn for non conducting medium

$$\nabla^2 \vec{E} = -\omega^2 \mu \epsilon \vec{E} \quad \text{--- (5)}$$

$$\nabla^2 \vec{H} = -\omega^2 \mu \epsilon \vec{H} \quad \text{--- (6)}$$

Em wave eqn
 $\nabla^2 \vec{E} = j\omega \mu [\sigma + j\omega \epsilon] \vec{E}$
 (or)
 $\nabla^2 \vec{E} = \mu \sigma \frac{\partial \vec{E}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$

From (4)

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu \left[H_x \vec{a}_x + H_y \vec{a}_y + H_z \vec{a}_z \right]$$

$$\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right] \vec{a}_x + \left[-\frac{\partial E_x}{\partial z} + \frac{\partial E_z}{\partial x} \right] \vec{a}_y + \left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right] \vec{a}_z = -j\omega\mu \left[H_x \vec{a}_x + H_y \vec{a}_y + H_z \vec{a}_z \right]$$

Comparing on both sides

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \quad \text{--- (a)}$$

$$\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} = j\omega\mu H_y \quad \text{--- (b)}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad \text{--- (c)}$$

--- (7)

Similarly

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x \quad \text{--- (a)}$$

$$\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} = -j\omega\epsilon E_y \quad \text{--- (b)}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \quad \text{--- (c)}$$

--- (8)

Similarly from wave eqns. (from (5) & (6))

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2\mu\epsilon E$$

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = -\omega^2\mu\epsilon H$$

Assume that wave travels in +z direction. The variation of field component in this direction may be expressed in the form of $e^{\gamma z}$ i.e. $\gamma = \alpha + j\beta$.

The parallel plane is infinite extend in y-direction hence there are no boundary condition to be met in y-direction. and also field is uniform along y.

$\therefore$ no variation in y -direction i.e. $\frac{\partial}{\partial y} = 0$

We can define E and H in phasor form as

$$E = E_m e^{jz} \quad \text{and} \quad H = H_m e^{jz}$$

Hence eqn (7) to (8) becomes

from 7(a)

$$\left(\frac{\partial E_z}{\partial y} \right)_{=0} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$-\frac{\partial [E_m e^{jz}]}{\partial z} = -j\omega\mu H_x \Rightarrow \underline{-E_m e^{jz} (+j)} = -j\omega\mu H_x$$

$$j E_y = j\omega\mu H_x$$

$$\boxed{E_y = \frac{-j\omega\mu}{j} H_x} \quad \text{--- (9a)}$$

From 7(b)

$$\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} = j\omega\mu H_y$$

$$\frac{\partial E_z}{\partial x} - \frac{\partial [E_m e^{jz}]}{\partial z} = j\omega\mu H_y$$

$$\frac{\partial E_z}{\partial x} - [E_m e^{jz} (-j)] = j\omega\mu H_y \Rightarrow \partial E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y \quad \text{--- (9b)}$$

From 7(c)

$$\frac{\partial E_y}{\partial x} - \left(\frac{\partial E_x}{\partial y} \right)_{=0} = -j\omega\mu H_z \Rightarrow \frac{\partial E_y}{\partial x} = -j\omega\mu H_z \quad \text{--- (9c)}$$

From 8(a)

$$\left(\frac{\partial H_z}{\partial y} \right)_{=0} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x$$

$$-\frac{\partial [H_m e^{jz}]}{\partial z} = j\omega\epsilon E_x$$

$$-\underline{H_m e^{jz} (-j)} = j\omega\epsilon E_x \Rightarrow \partial H_y = j\omega\epsilon E_x$$

$$H_y = \frac{j\omega\epsilon}{j} E_x \quad \text{--- (10a)}$$

From 8(b)

$$\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} = -j\omega\epsilon E_y$$

$$\frac{\partial H_z}{\partial x} - \frac{\partial [H_m e^{jz}]}{\partial z} = -j\omega\epsilon E_y$$

$$\frac{\partial H_z}{\partial x} - \frac{\partial [H_m e^{jz} (-j)]}{\partial z} = -j\omega\epsilon E_y$$

$$\partial H_x + \frac{\partial H_z}{\partial x} = -j\omega\epsilon E_y \quad \text{--- (10b)}$$

From 8(c)

$$\frac{\partial H_y}{\partial x} - \left(\frac{\partial H_x}{\partial y} \right)_{=0} = j\omega\epsilon E_z \Rightarrow \frac{\partial H_y}{\partial x} = j\omega\epsilon E_z \quad \text{--- (10c)}$$

Solve the ⑨ ⑩ eqns we get E_x, E_y, H_x, H_y in terms of E_z and H_z since the resultant wave propagates in $+z$ direction.

From 9(a) $E_y = -\frac{j\omega\mu}{\gamma} H_x$

Sub this in ⑩b

$$\gamma H_x + \frac{\partial H_z}{\partial x} = -j\omega\epsilon \left[-\frac{j\omega\mu}{\gamma} H_x \right]$$

$$H_x \left[\gamma + \frac{\omega^2\mu\epsilon}{\gamma} \right] = -\frac{\partial H_z}{\partial x} \Rightarrow H_x = -\frac{\gamma}{\gamma^2 + \omega^2\mu\epsilon} \frac{\partial H_z}{\partial x}$$

$$H_x = -\frac{\gamma}{h^2} \frac{\partial H_z}{\partial x}$$

where $\gamma^2 + \omega^2\mu\epsilon = h^2$

Sub H_x value in 9(a)

$$E_y = -\frac{j\omega\mu}{\gamma} \left[-\frac{\gamma}{h^2} \frac{\partial H_z}{\partial x} \right]$$

$$E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

From ⑩(a)

$$H_y = \frac{j\omega\epsilon}{\gamma} E_x$$

Sub this in 9(b)

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu \left[\frac{j\omega\epsilon}{\gamma} E_x \right]$$

$$E_x \left[\gamma + \frac{\omega^2\mu\epsilon}{\gamma} \right] = -\frac{\partial E_z}{\partial x} \Rightarrow E_x = -\frac{\gamma}{\gamma^2 + \omega^2\mu\epsilon} \frac{\partial E_z}{\partial x}$$

$$E_x = -\frac{\gamma}{h^2} \frac{\partial E_z}{\partial x}$$

Sub this E_x value in ⑩(a)

$$\therefore H_y = \frac{j\omega\epsilon}{\gamma} \left[-\frac{\gamma}{h^2} \frac{\partial E_z}{\partial x} \right]$$

$$H_y = -\frac{j\omega\epsilon}{h^2} \frac{\partial E_z}{\partial x}$$

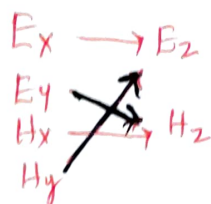
$$E_x = -\frac{\partial}{\partial x} \frac{\partial E_z}{\partial x}$$

$$E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

$$H_x = -\frac{\partial}{\partial x} \frac{\partial H_z}{\partial x}$$

$$H_y = -\frac{j\omega\epsilon}{h^2} \frac{\partial E_z}{\partial x}$$

— (I)



In above eqns, in general there must be 'z' component of either 'E' or 'H' otherwise all the components of 'E' and 'H' would be zero and there would be no fields at all in the region under consideration.

Thus it can be divided into different types.

(1) Transverse Electric Waves (TE) or 'H' waves

It means that the electric field strength (E) is entirely transverse (or) perpendicular to the direction of propagation, where the component of 'H' lies along the direction of resultant wave propagation i.e. $E_z = 0$ and $H_z \neq 0$

(2) Transverse Magnetic Waves (TM) or 'E' waves

magnetic field strength (H) is entirely ^{transverse} (or) perpendicular to the direction of propagation, where as the component of 'E' lies along the direction of wave propagation

i.e. $E_z \neq 0$ and $H_z = 0$.

(3) Transverse Electromagnetic Waves (TEM)

$E_z = 0$ and $H_z = 0$

Transverse Electric Waves (TE) (or) H waves

For TE wave $E_z = 0$ and $H_z \neq 0$

Sub in above eqn.

$$\boxed{E_x = 0}$$

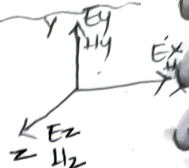
$$\boxed{H_y = 0}$$

$$\boxed{E_z = 0}$$

$$H_z = ?$$

$$E_y = ?$$

$$H_x = ?$$



The non zero values of the other component are obtained as follows.

$$\frac{\partial^2 E}{\partial x^2} + \gamma^2 E = -\omega^2 \mu \epsilon E$$

$$\frac{\partial^2 E}{\partial x^2} + (\gamma^2 + \omega^2 \mu \epsilon) E = 0$$

$$\frac{\partial^2 E}{\partial x^2} + h^2 E = 0$$

We know $E = E_m e^{-\gamma z}$

$$\therefore \frac{\partial^2 (E_m e^{-\gamma z})}{\partial x^2} + h^2 E_m e^{-\gamma z} = 0$$

$$\frac{\partial^2 E_m}{\partial x^2} + h^2 E_m = 0$$

Solution $E_m = A \cosh(hx) + B \sinh(hx)$

$$\text{let } E_y = E_m e^{-\gamma z}$$

$$E_y = [A \cosh(hx) + B \sinh(hx)] e^{-\gamma z} \quad \text{--- (1)}$$

apply boundary conditions to get constant A & B

$$E_y = 0 \text{ at } x = 0$$

$$E_y = 0 \text{ at } x = a$$

Sub in (1)

$$0 = A + 0 \Rightarrow A = 0$$

$$\therefore E_y = B \sinh(hx) e^{-\gamma z}$$

apply 2nd boundary condition.

$$E_y = 0 \text{ at } x = a$$

$$0 = B \sinh(ha) e^{-\gamma z} \Rightarrow B \sinh(ha) = 0$$

$$\sinh(ha) = 0$$

$$(ha) = m\pi \Rightarrow h = \frac{m\pi}{a}$$

$$\therefore \boxed{E_y = B \sin\left(\frac{m\pi}{a} x\right) e^{-\gamma z}}$$

We know

$$\nabla^2 E = \gamma^2 E$$

$$\nabla^2 E = -\omega^2 \mu \epsilon E$$

$$\frac{\partial^2 E}{\partial x^2} + \left(\frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} \right) = -\omega^2 \mu \epsilon E$$

$$\boxed{\frac{\partial^2 E}{\partial x^2} + \gamma^2 E = -\omega^2 \mu \epsilon E}$$

(from (1))

$$E_z = E_m e^{-\gamma z}$$

$$\frac{\partial E}{\partial z} = -E_m e^{-\gamma z}$$

$$\frac{\partial^2 E}{\partial z^2} = +E_m e^{-\gamma z}$$

$$\left(\frac{\partial^2 E}{\partial z^2} \right) = \gamma^2 E$$

From (I)

$$E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

$$\frac{\partial H_z}{\partial x} = \frac{h^2}{j\omega\mu} E_y$$

$$H_z = \int \frac{h^2}{j\omega\mu} E_y dx$$

$$H_z = \frac{h^2}{j\omega\mu} \int B \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z} dx$$

$$= \frac{-h^2}{j\omega\mu} B e^{-\gamma z} \frac{\cos\left(\frac{m\pi x}{a}\right)}{\frac{m\pi}{a}}$$

$$= -\left(\frac{m\pi}{a}\right) \frac{B e^{-\gamma z} \cos\left(\frac{m\pi x}{a}\right)}{j\omega\mu}$$

$$H_z = -\frac{m\pi}{j\omega\mu a} B \cos\left(\frac{m\pi x}{a}\right) e^{-\gamma z}$$

Sub this H_z value in (I)

$$H_x = -\frac{\gamma}{h^2} \frac{\partial H_z}{\partial x}$$

$$= -\frac{\gamma}{h^2} \frac{\partial}{\partial x} \left[-\frac{m\pi}{j\omega\mu a} B \cos\left(\frac{m\pi x}{a}\right) e^{-\gamma z} \right]$$

$$= +\frac{\gamma}{h^2} \frac{m\pi}{j\omega\mu a} B (-\sin\left(\frac{m\pi x}{a}\right)) \left(\frac{m\pi}{a}\right) e^{-\gamma z}$$

$$H_x = -\frac{\gamma}{j\omega\mu} B \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z}$$

Summarizing the components of 'E' and 'H'

$$E_x = 0, H_y = 0, E_z = 0.$$

$$E_y = B \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z}$$

$$H_x = -\frac{\gamma}{j\omega\mu} B \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z}$$

$$H_z = -\frac{m\pi}{j\omega\mu a} B \cos\left(\frac{m\pi x}{a}\right) e^{-\gamma z}.$$

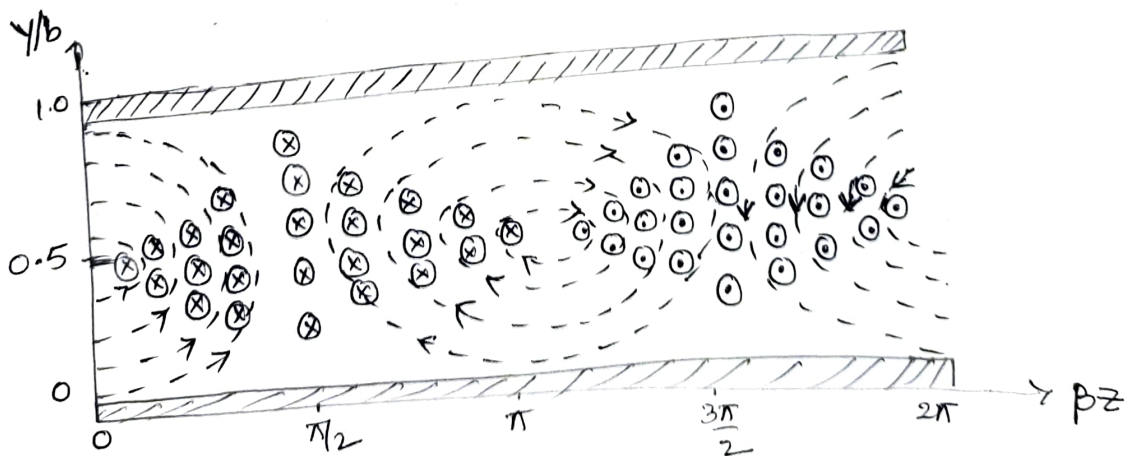
(Total of 6 Components)

The different values of 'm' in the above eqn define a particular field configuration or mode. The lowest value of 'm' is '1'. If $m=0$ then $E_y = H_x = H_z = 0$. We know that $\gamma = \alpha + j\beta$. For the parallel planes γ is purely real or purely imaginary depending upon the frequency of wave is transmitted.

- For some range of frequencies $\gamma = \alpha + j\beta$, means there is no phase shift only attenuation occurs ($\beta=0$) hence there is no wave propagation.
- For other range of frequencies $\gamma = j\beta$ i.e. there is no attenuation only phase shift ($\alpha=0$), it means there is wave propagation without attenuation. For these range of frequencies above eqn becomes.

$$\begin{aligned} E_y &= B \sin\left(\frac{m\pi}{a}x\right) e^{-j\beta z} \\ H_x &= -\frac{j\omega\mu}{\gamma} \sin\left(\frac{m\pi}{a}x\right) e^{-j\beta z} \\ H_z &= -\frac{B m\pi}{j\omega\mu a} \cos\left(\frac{m\pi}{a}x\right) e^{-j\beta z} \end{aligned}$$

The field distribution of TE₁₀ } $\begin{cases} \text{TE}_{mn} & m=1, n=0 \\ \text{The 2nd subscript 'n' is zero} \\ \text{for parallel waveguide} \end{cases}$



----- magnetic lines
 $\otimes \odot$ Electric field lines
 (X-axis into the page)

Transverse Magnetic Waves (TM Wave) or E Waves

For TM wave $H_z = 0$ and $E_z \neq 0$

Sub this in (I)

$$\begin{array}{l} E_y = 0 \\ H_x = 0 \end{array}$$

$$H_z = 0$$

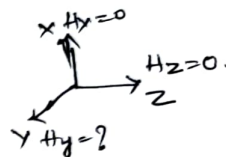
find $E_x = ?$

$$H_y = ?$$

$$E_z = ?$$

The non zero values of other components E_x, H_y, E_z are computed as follows.

The wave eqn



$$\frac{\partial^2 H}{\partial x^2} + \partial^2 H = -\omega^2 \mu \epsilon H$$

$$\frac{\partial^2 H}{\partial x^2} + (\gamma^2 + \omega^2 \mu \epsilon) H = 0$$

$$\frac{\partial^2 H}{\partial x^2} + h^2 H = 0 \quad \text{--- (1)}$$

we know that $H_y = H_m e^{-\gamma z}$

$$\frac{\partial^2}{\partial x^2} (H_m e^{-\gamma z}) + h^2 [H_m e^{-\gamma z}] = 0.$$

$$\frac{\partial^2 H_m}{\partial x^2} + h^2 H_m = 0.$$

general solution of the above eqn.

$$H_m = C \cosh(hx) + D \sinh(hx)$$

C & D are constant

$$H_y = H_m e^{-\gamma z}$$

$$H_y = [C \cosh(hx) + D \sinh(hx)] e^{-\gamma z} \quad \text{--- (2)}$$

In this boundary condition is, the normal component of H' must be zero i.e. $H_N = 0$ and $H_t \neq 0$ at surface.

Therefore C & D cannot be evaluated directly applying this condition. So, H_y will be converted in terms of ' E_z ' then apply the boundary condition.

i.e. $E_z = 0$ at $x=0$

and $E_z = 0$ at $x=a$

From eqn (100)

$$\begin{aligned}\frac{\partial H_y}{\partial x} &= j\omega \epsilon E_z \Rightarrow E_z = \frac{1}{j\omega \epsilon} \frac{\partial H_y}{\partial x} \\ &= \frac{1}{j\omega \epsilon} \frac{\partial}{\partial x} [C \cos(hx) + D \sin(hx)] e^{-jz} \\ \therefore E_z &= \frac{h}{j\omega \epsilon} [-C \sin(hx) + D \cos(hx)] e^{-jz}\end{aligned}$$

Now apply the boundary condition

$E_z = 0$ at $x=0$ (1st boundary condition)

$$0 = \frac{h}{j\omega \epsilon} [-C(0) + D(1)] e^{-jz} \Rightarrow \boxed{D=0}$$

$E_z = 0$ at $x=a$ (2nd boundary condition)

$$0 = \frac{h}{j\omega \epsilon} [-C \sin(ha) + \underbrace{D \cos(ha)}_{=0}] e^{-jz}$$

$$0 = -\frac{h}{j\omega \epsilon} C \sin(ha) e^{-jz}$$

$$C \sin(ha) = 0 \quad (\text{or})$$

$$C \neq 0, \sin(ha) = 0$$

$$ha = m\pi$$

$$h = \frac{m\pi}{a}$$

$$\therefore \boxed{E_z = -\frac{Ch}{j\omega \epsilon} \sin\left(\frac{m\pi}{a}x\right) e^{-jz}}$$

We know from (100)

$$\frac{\partial H_y}{\partial x} = j\omega \epsilon E_z$$

$$H_y = j\omega \epsilon \int E_z dx$$

$$= j\omega \epsilon \int -\frac{Ch}{j\omega \epsilon} \sin\left(\frac{m\pi}{a}x\right) e^{-jz} dx$$

$$= \cancel{j\omega \epsilon} e^{-jz} \cdot \frac{Ch}{\cancel{j\omega \epsilon}} \left[-\cos\left(\frac{m\pi}{a}x\right) \cdot \frac{1}{\left(\frac{m\pi}{a}\right)} \right]$$

$$\boxed{H_y = C \cos\left(\frac{m\pi}{a}x\right) e^{-jz}}$$

[We can ~~also~~ find H_y using eqn (1) also]

$$\begin{aligned}
 E_x &= -\frac{\partial}{\partial x} \frac{\partial E_z}{\partial x} \\
 &= -\frac{\partial}{\partial x} \frac{\partial}{\partial x} \left[-\frac{ch}{j\omega\epsilon} \sin\left(\frac{m\pi}{a}x\right) e^{-\gamma z} \right] \\
 &= +\frac{\partial}{\partial x} \frac{ch}{j\omega\epsilon} e^{-\gamma z} \cos\left(\frac{m\pi}{a}x\right) \cdot \left(\frac{m\pi}{a}\right)
 \end{aligned}$$

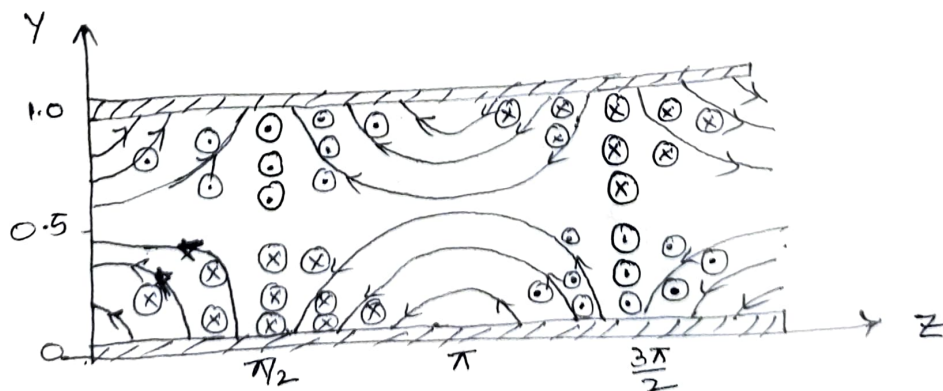
$$E_x = \frac{C\gamma}{j\omega\epsilon} \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z}$$

Summarize the components of 'E' and 'H'

$$\left. \begin{aligned}
 E_y &= 0, \quad H_x = 0, \quad H_z = 0 \\
 E_x &= \frac{C\gamma}{j\omega\epsilon} \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z} \\
 H_y &= C \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z} \\
 E_z &= -\frac{ch}{j\omega\epsilon} \sin\left(\frac{m\pi}{a}x\right) e^{-\gamma z}
 \end{aligned} \right\}$$

unlike TE waves, in case of TM waves there is a possibility of $m=0$. If $m=0$ then E_x and H_y present. Hence the lowest order that can be exist of TM wave is TM_0 Mode.

Fig: TM_{10} wave between parallel planes.



—— Electric field lines
 ⊙ ⊗ magnetic field lines
 (x-axis into paper)

Characteristics of TE and TM waves :

(1) Propagation Constant (γ)

We know that $h^2 = \gamma^2 + \omega^2 \mu \epsilon$

$$\gamma^2 = h^2 - \omega^2 \mu \epsilon$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \epsilon \quad \left(\because h = \frac{m\pi}{a}\right)$$

$$\text{or } \gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \epsilon} = \alpha + j\beta$$

i) At very high frequencies $\omega^2 \mu \epsilon \gg \left(\frac{m\pi}{a}\right)^2$, γ is imaginary

$$\therefore \gamma = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$

$$\alpha + j\beta = j\omega\sqrt{\mu\epsilon}$$

$$\therefore \beta = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$

The above eqn represents the wave propagate in Z-direction but there is no attenuation.

ii) ~~At~~ At low frequencies i.e. $\omega^2 \mu \epsilon \ll \left(\frac{m\pi}{a}\right)^2$, γ is real

$$\alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \epsilon}$$

If frequency is decreased, at a stage of critical frequency is reached.

The frequency at which $\gamma = 0$ is known as cut-off (critical) frequency i.e. at $f = f_c$

$$0 = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega_c^2 \mu \epsilon}$$

$$\omega_c^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2$$

$$4\pi^2 f_c^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}} \left(\frac{m\pi}{a}\right)$$

$$= \frac{v}{2a} \left(\frac{m\pi}{a}\right)$$

$$\left(\because \text{where } v = \frac{1}{\sqrt{\mu\epsilon}}\right) \\ \approx 3 \times 10^8 \text{ m/s}$$

$$f_c = \frac{vm}{2a}$$

$$\text{cut off wavelength } = \lambda_c = \frac{v}{f_c} \\ \therefore \lambda_c = \frac{2a}{m}$$

(2) Guide Wavelength (λ_g)

It can be defined as the distance travelled by the wave in order to undergo a phase shift of 2π radians

$$\lambda_g = \frac{2\pi}{\beta}$$

$$\lambda_g = \frac{2\pi}{\sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}}$$

At cutoff frequency i.e. at $f = f_c \Rightarrow \beta = 0$.

$$\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega_c^2 \mu \epsilon} = 0$$

$$\omega_c^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2$$

$$\begin{aligned} \therefore \lambda_g &= \frac{2\pi}{\sqrt{\omega^2 \mu \epsilon - \omega_c^2 \mu \epsilon}} \\ &= \frac{2\pi}{\omega \sqrt{\mu \epsilon} \left[\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} \right]} \end{aligned}$$

$$= \frac{2\pi v_p}{2\pi f \left[\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} \right]} \quad \left[\because v_p = \frac{1}{\sqrt{\mu \epsilon}} \right]$$

$$= \frac{\lambda}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}} = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} \quad \left[\because f_c \propto \frac{1}{\lambda_c} \right]$$

$$\therefore \lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

Squaring on both sides

$$\left(\frac{\lambda}{\lambda_g}\right)^2 = \left(\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}\right)^2$$

$$\left(\frac{\lambda}{\lambda_g}\right)^2 = 1 - \left(\frac{\lambda}{\lambda_c}\right)^2$$

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2} \Rightarrow$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$$

Where λ_g = guide wavelength
 λ_c = cutoff wavelength
 λ = wavelength

3) Phase Velocity (v_p)

The velocity of propagation is defined as the rate at which the wave changes its phase in terms of $d\phi$. (OR) The velocity with which constant phase points travels is called phase velocity (v_p).

$$v_p = \frac{\omega}{\beta}$$

$$= \frac{\omega}{\sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}}$$

at cutoff frequency $\omega_c^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2$

$$\therefore v_p = \frac{\omega}{\sqrt{\omega^2 \mu \epsilon - \omega_c^2 \mu \epsilon}}$$

$$= \frac{\omega}{\omega \sqrt{\mu \epsilon} \left[\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} \right]} = \frac{v_p}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{v}{\sqrt{1 - \left(\frac{\lambda_c}{\lambda}\right)^2}}$$



$$\therefore v_p = \frac{v}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{v}{\sqrt{1 - \left(\frac{\lambda_c}{\lambda}\right)^2}}$$

$v \rightarrow$ vel. of light

4) Group velocity (v_g) The velocity with which change in constant phase points travels is called group velocity.

$$v_g = \frac{d\omega}{d\beta}$$

we know $\beta = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$

$$\frac{d\beta}{d\omega} = \frac{1}{2} \left[\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2 \right]^{-1/2} \cdot 2\omega \mu \epsilon$$

$$= \frac{\omega \mu \epsilon}{\sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}} = \frac{\omega \mu \epsilon}{\sqrt{\omega^2 \mu \epsilon - \omega_c^2 \mu \epsilon}}$$

(at $f = f_c$)

$$\therefore \frac{d\omega}{d\beta} = \frac{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}{\sqrt{\mu \epsilon}} = v \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = v \sqrt{1 - \left(\frac{\lambda_c}{\lambda}\right)^2}$$

$$\therefore v_g = v \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = v \sqrt{1 - \left(\frac{\lambda_c}{\lambda}\right)^2}$$

$$v_g \approx 10^8 - 10^2$$

Transverse Electromagnetic Wave (TEM) :

12

It is a special case of mode in which neither E_z nor H_z components present i.e. $E_z=0$ and $H_z=0$.

Substitute $E_z=0$ and $H_z=0$ in eqn (I) all field components becomes zero. So, that electromagnetic wave is entirely transverse. TEM wave also referred as "principle waves".

In TM waves if $m=0$

$$\left. \begin{aligned} E_x &= \frac{\beta c}{\omega \epsilon} e^{-j\beta z} \\ H_y &= c e^{-j\beta z} \\ E_z &= 0 \end{aligned} \right\}$$

It also proves that $E_z=H_z=0$. It means that for this special case of TM waves, The 'E' in the ^{direction of} resultant wave propagation is also zero. So that EM wave is entirely transverse. Because of $m=0$, we get

$$\therefore \gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \epsilon} = j\sqrt{\omega^2 \mu \epsilon} = j\omega\sqrt{\mu \epsilon} = j\beta$$

$$\boxed{\beta = \omega\sqrt{\mu \epsilon}}$$

$$\therefore \text{velocity of propagation} = v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu \epsilon}}$$

$$\boxed{v_p = \frac{1}{\sqrt{\mu \epsilon}}}$$

$$\text{Guide Wave length} = \lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu \epsilon}} = \frac{2\pi}{2\pi f\sqrt{\mu \epsilon}}$$

$$\boxed{\lambda_g = \frac{v}{f}}$$

Unlike TE and TM waves, the velocity of TEM wave is independent of frequency i.e. if $m=0$ then $f_c=0$. This proves that the TEM waves, for all the frequency below zero can propagate along the guide.

Intrinsic Impedance of TEM wave

$$\eta_{\text{TEM}} = \left| \frac{E_x}{H_y} \right| = \frac{\frac{B_0}{\omega \epsilon} e^{j\beta z}}{e^{j\beta z}}$$

$$= \frac{B_0}{\omega \epsilon} = \frac{B_0 \omega \sqrt{\mu \epsilon}}{\omega \epsilon} = \sqrt{\frac{\mu}{\epsilon}}$$

$$\boxed{\eta_{\text{TEM}} = \sqrt{\frac{\mu}{\epsilon}}}$$

For free space $\eta =$

$$\eta = \sqrt{\frac{4\pi \times 10^{-7} \text{ H/m}}{8.854 \times 10^{-12} \text{ F/m}}}$$

$$= 377 \Omega.$$

It is same as that of the intrinsic impedance of uniform plane wave in an unbounded region.

Attenuation ^(OR) in Parallel Plane Guides

13 (1)

- TE, TM, TEM Under the assumption of that planes are perfect conductors.
- The conductivity of wave guide are very large but not infinite, thus there are some losses occurs.
- $e^{-\alpha z}$ is multiplied in all cases to calculate losses due to wave guide walls.

$$V = V_0 e^{-j\beta z} \quad \& \quad I = I_0 e^{-j\beta z}$$

Now multiply with $e^{-\alpha z}$

$$V = V_0 e^{-j\beta z} e^{-\alpha z} \quad \& \quad I = I_0 e^{-j\beta z} e^{-\alpha z}$$

Average power transmitted is

$$\begin{aligned} P_{av} = W_{av} &= \frac{1}{2} \operatorname{Re}\{V I\} \\ &= \frac{1}{2} \operatorname{Re}\{V_0 I_0 e^{-\alpha z} \cdot e^{-\alpha z}\} \\ &= \frac{1}{2} \operatorname{Re}\{V_0 I_0\} e^{-2\alpha z} \quad \text{--- (1)} \end{aligned}$$

The Rate of decrease of transmitted power along the line is obtained

$$\begin{aligned} \frac{dW_{av}}{dz} &= \frac{1}{2} \operatorname{Re}\{V_0 I_0\} e^{-2\alpha z} (-2\alpha) \\ &= -2\alpha W_{av} \quad \text{--- (2)} \end{aligned}$$

The decrease of transmitted power per unit length

$$-2\alpha W_{av} = \Delta W_{av} = 2\alpha W_{av} \quad \text{--- (3)}$$

$$\frac{\text{Power lost per unit length}}{\text{Power transmitted}} = \frac{2\alpha W_{av}}{W_{av}} = 2\alpha$$

$$\therefore \alpha = \frac{1}{2} \left[\frac{\text{Power lost per unit length}}{\text{power transmitted}} \right]$$

Attenuation of wave in parallel planes

At frequencies less than cutoff, the waves are not transmitted through wave guide but they are attenuated.

The attenuation in wave guide comes from two sources i) conductor losses ii) Dielectric losses.

- Assuming that planes are of infinite conductivity so that losses are zero and $\gamma = j\beta$ (ie. $\alpha = 0$).

In actual wave guides conductivity is hardly infinite. Therefore there are always some losses.

If α_d is attenuation constant for lossy dielectric and α_c " " for conducting plates.

$$\therefore \alpha = \alpha_c + \alpha_d$$

$$\gamma = \alpha + j\beta = (\alpha_c + \alpha_d) + j\beta$$

$$\therefore e^{-\gamma z} = e^{-(\alpha_c + \alpha_d + j\beta)z}$$

The voltage and current relations in phasor form along line

$$V = V_0 e^{-\gamma z} = V_0 e^{-\alpha z} e^{-j\beta z}$$

$$I = I_0 e^{-\gamma z} = I_0 e^{-\alpha z} e^{-j\beta z}$$

The average power transmitted

$$P_{av} = \operatorname{Re} \left\{ \frac{1}{2} V I \right\} = \operatorname{Re} \left\{ \frac{1}{2} V_0 I_0 e^{-\alpha z} e^{-\alpha z} \right\}$$

$$P_{av} = \frac{1}{2} \operatorname{Re} \{ V_0 I_0 \} e^{-2\alpha z} \quad \text{--- (1)}$$

The rate of decrease of power along the line

$$\frac{\partial P_{av}}{\partial z} = \frac{1}{2} \operatorname{Re} \{ V_0 I_0 \} e^{-2\alpha z} \cdot (-2\alpha)$$

$$= -2\alpha P_{av} \quad \text{--- (2)}$$

The decrease of transmitted Power per Unit length of line

$$\frac{\partial P_{av}}{\partial z} = -2\alpha P_{av}$$

From conservation of energy

The decrease of power transmitted per unit length should be equal to power lost or dissipated per unit length

$$\frac{\text{Power lost per unit length}}{\text{Power transmitted}} = \frac{+2\alpha P_{av}}{P_{av}} = 2\alpha$$

$$\therefore \alpha = \frac{1}{2} \left[\frac{\text{Power lost per unit length}}{\text{Power transmitted}} \right]$$

Attenuation Factor for the TEM Wave :

14 (2)

For TEM wave

$$\left. \begin{aligned} H_y &= C_4 e^{-j\beta z} \\ E_x &= \frac{\beta C_4}{\omega \epsilon} e^{-j\beta z} \end{aligned} \right\} \text{ (from TM waves)}$$

The linear current density in each of the conducting planes will be given by

$$J_s = \vec{n} \times \vec{H}_y = n H_y \sin \theta = H_y \quad \left[\begin{aligned} \theta &= 90^\circ \\ n &= 1 \end{aligned} \right]$$

$$|J_s| = |H_y| = C_4 \quad \text{--- (4)}$$

Power loss per square meter in each conducting plane is

$$\frac{1}{2} J_s^2 R_s = \frac{1}{2} C_4^2 R_s \quad \text{--- (5)}$$

$$\text{where } R_s = \sqrt{\frac{\omega \mu m}{2 \sigma m}}$$

$$Z_s = \sqrt{\frac{j \omega \mu m}{2 \sigma m}}$$

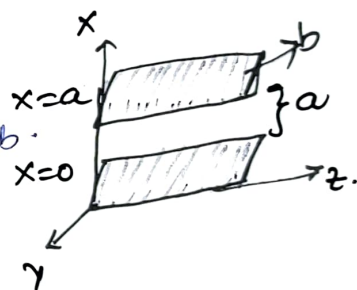
$R_s \rightarrow$ Resistive Component

$Z_s \rightarrow$ Surface Impedance

$\mu m, \sigma m \rightarrow$ Constants of metallic conductors.

Total power loss in lower & upper conducting surfaces per meter

$$\begin{aligned} \text{length for a width of 'b' mts.} &= 2 \left(\text{loss due to each plate} \right) \times b \\ &= 2 \left(\frac{1}{2} C_4^2 R_s \times 2b \right) \quad \text{(from (5))} \\ &= C_4^2 R_s b \quad \text{--- (6)} \end{aligned}$$



Total power transmitted = $P_{av} = \frac{1}{2} \operatorname{Re} \{ \vec{E}_x \times \vec{H}_y \}_z$ in z-direction

$$= \frac{1}{2} |E_x| |H_y| \sin \theta \quad [\theta = 90^\circ]$$

$$= \frac{1}{2} n H_y H_y \quad \left[\frac{E}{H} = n \right]$$

$$= \frac{1}{2} n H_y^2 = \frac{1}{2} n C_4^2 \quad \text{--- (7)}$$

For spacing 'a' mts. and cross section area of a width 'b' mts. of the guide is "ba" sq. mts.

$$\text{Total power transmitted} = \frac{1}{2} n C_4^2 ba \quad \text{--- (8)}$$

$$\alpha = \frac{1}{2} \left[\frac{\text{Power lost per unit length}}{\text{Power transmitted}} \right] = \frac{1}{2} \left[\frac{C_4^2 R_s b}{\frac{1}{2} n C_4^2 ba} \right] = \frac{R_s}{n a} = \frac{1}{n a} \sqrt{\frac{\omega \mu m}{2 \sigma m}}$$

Attenuation factor for TE wave

15 (3)

$$\left. \begin{aligned} E_y &= C_1 \sin\left(\frac{m\pi}{a}x\right) e^{-j\beta z} \\ H_x &= -\frac{\beta}{\omega\mu} C_1 \sin\left(\frac{m\pi}{a}x\right) e^{-j\beta z} \\ H_z &= -\frac{m\pi}{j\omega\mu a} C_1 \cos\left(\frac{m\pi}{a}x\right) e^{-j\beta z} \end{aligned} \right\} \text{(from TE waves)}$$

The amplitude of current density will exist only nearer to the wave guide walls i.e. $x=0$ and $x=a$

$$J = H_z = \frac{m\pi C_1}{\omega\mu a} \quad (\text{from above eqn})$$

$$\begin{aligned} \text{The power lost in each plane} &= \frac{1}{2} J_s^2 R_s \\ &= \frac{1}{2} \left(\frac{m\pi C_1}{\omega\mu a} \right)^2 \sqrt{\frac{\omega\mu}{2\sigma}} \end{aligned}$$

$$\begin{aligned} \text{Total Power lost} &= 2 (\text{Power lost in each plane}) \\ &= 2 \cdot \frac{1}{2} \left(\frac{m\pi C_1}{\omega\mu a} \right)^2 \sqrt{\frac{\omega\mu}{2\sigma}} \quad \text{--- (1)} \end{aligned}$$

The Power transmitted in z-direction per unit area

$$\begin{aligned} P_{av} &= \frac{1}{2} (\vec{E} \times \vec{H}) = \frac{1}{2} (E_y H_x) \\ &= \frac{1}{2} \left[C_1 \sin\left(\frac{m\pi}{a}x\right) \frac{\beta}{\omega\mu} C_1 \sin\left(\frac{m\pi}{a}x\right) \right] \\ &= \frac{1}{2} \left[\frac{\beta C_1^2}{\omega\mu} \sin^2\left(\frac{m\pi}{a}x\right) \right] \quad \text{--- (2)} \end{aligned}$$

power transmitted per unit width

$$\begin{aligned} &= \int_{x=0}^a P_{av} dx = \int_0^a \frac{\beta C_1^2}{2\omega\mu} \sin^2\left(\frac{m\pi}{a}x\right) dx \\ &= \frac{\beta C_1^2}{2\omega\mu} \left[\int_0^a \frac{1 - \cos\left(2\frac{m\pi}{a}x\right)}{2} dx \right] \\ &= \frac{\beta C_1^2}{4\omega\mu} \left[x - \frac{\sin\left(2\frac{m\pi}{a}x\right)}{2} \left(\frac{a}{m\pi} \right) \right]_0^a \\ &= \frac{\beta C_1^2}{4\omega\mu} [a - 0 - 0] = \frac{\beta C_1^2 a}{4\omega\mu} \\ &= \frac{\beta C_1^2 a}{4\omega\mu} \sqrt{\frac{\omega\mu}{2\sigma}} \quad \text{--- (3)} \end{aligned}$$

$$\begin{aligned} \alpha &= \frac{1}{2} \left[\frac{\text{Power lost}}{\text{Power transmitted}} \right] \\ &= \frac{1}{2} \left[\frac{\left(\frac{m\pi C_1}{\omega\mu a} \right)^2 \sqrt{\frac{\omega\mu}{2\sigma}}}{\frac{\beta C_1^2 a}{4\omega\mu} \sqrt{\frac{\omega\mu}{2\sigma}}} \right] \Rightarrow \alpha = \frac{2m^2\pi^2}{\omega^2\mu a^3} \sqrt{\frac{\omega\mu}{2\sigma}} \left[\frac{1}{\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2}} \right] \end{aligned}$$

Attenuation factor for TM waves

16 (4)

$$\left. \begin{aligned} H_y &= C_4 \cos\left(\frac{m\pi x}{a}\right) e^{-j\beta z} \\ E_x &= +\frac{\beta}{\omega\epsilon} C_4 \cos\left(\frac{m\pi x}{a}\right) e^{-j\beta z} \\ E_z &= -\frac{m\pi}{j\omega\epsilon a} C_4 \sin\left(\frac{m\pi x}{a}\right) e^{-j\beta z} \end{aligned} \right\} \text{ (TM wave field components)}$$

The Surface current is controlled by H_y at $x=0$ & $x=a$
thus the linear current density per unit length is

$$J_s = H_y = C_4$$

Power lost per unit length of conductor

$$= \frac{1}{2} J_s^2 R_s = \frac{1}{2} C_4^2 R_s = \frac{1}{2} C_4^2 \sqrt{\frac{\omega\mu m}{2\sigma m}}$$

$$(or) \frac{1}{2} C_4^2 R_s (2b)$$

Total power lost = 2 [power lost in individual conducting surface]

$$= 2 \left[\frac{1}{2} C_4^2 \sqrt{\frac{\omega\mu m}{2\sigma m}} \right] b = C_4^2 b \sqrt{\frac{\omega\mu m}{2\sigma m}} \quad \text{--- (1)}$$

Power transmitted per unit area

$$\begin{aligned} P_{av} &= \frac{1}{2} [\vec{E} \times \vec{H}] \\ &= \frac{1}{2} [E_x H_y] = \frac{1}{2} \left[\frac{\beta}{\omega\epsilon} C_4 \cos\left(\frac{m\pi x}{a}\right) \cdot C_4 \cos\left(\frac{m\pi x}{a}\right) \right] \\ &= \frac{1}{2} \frac{\beta}{\omega\epsilon} C_4^2 \cos^2\left(\frac{m\pi x}{a}\right) \end{aligned}$$

Power transmitted per unit width

$$\begin{aligned} P &= \int_{x=0}^a \int_{y=0}^b \frac{1}{2} \frac{\beta}{\omega\epsilon} C_4^2 \cos^2\left(\frac{m\pi x}{a}\right) dx dy \\ &= \frac{1}{4} \frac{\beta}{\omega\epsilon} C_4^2 ab \quad \text{--- (2)} \end{aligned}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

Attenuation = $\alpha = \frac{1}{2} \left(\frac{\text{Power lost}}{\text{Power transmitted}} \right)$

$$= \frac{1}{2} \left[\frac{C_4^2 \sqrt{\frac{\omega\mu m}{2\sigma m}} b}{\frac{1}{4} \frac{\beta}{\omega\epsilon} C_4^2 ab} \right] = \frac{2\omega\epsilon}{\beta a} \sqrt{\frac{\omega\mu m}{2\sigma m}}$$

$$\alpha = \frac{\sqrt{m}}{a^{3/2}} \sqrt{\frac{2\pi\mu m}{\sigma_m \mu \epsilon}} \sqrt{\frac{f/f_c}{1 - (f/f_c)^2}}$$

$$\alpha = \frac{2\omega\epsilon}{a} \left[\frac{1}{\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2}} \right] \sqrt{\frac{\omega\mu m}{2\sigma m}} \quad (OR)$$

Wave Impedance

$$Z = \frac{V}{I}$$

17

In Tx line power is transmitted along only one axis, but the EM waves transmitted in waveguides, it is a Three dimensional, the power may be transmitted along any or all of the three coordinates.

Wave Impedance at a point can be defined

$$\begin{aligned} Z_{xy}^+ &= \frac{E_x}{H_y} & Z_{yz}^+ &= \frac{E_y}{H_z} & Z_{zx}^+ &= \frac{E_z}{H_x} \\ Z_{yx}^+ &= -\frac{E_y}{H_x} & Z_{yz}^+ &= -\frac{E_y}{H_z} & Z_{xz}^+ &= -\frac{E_x}{H_z} \end{aligned}$$

These are (Wave impedances along its +ve direction)

The Impedance in the negative coordinates

$$\begin{aligned} Z_{xy}^- &= -\frac{E_x}{H_y} & Z_{yz}^- &= -\frac{E_y}{H_z} & Z_{zx}^- &= -\frac{E_z}{H_x} \\ Z_{yx}^- &= \frac{E_y}{H_x} & Z_{yz}^- &= \frac{E_z}{H_y} & Z_{xz}^- &= +\frac{E_x}{H_z} \end{aligned}$$

$Z^+ \rightarrow$ represents +ve direction

$Z^- \rightarrow$ Impedance in -ve direction

For TE waves:

$E_x = E_z = 0$ & $H_y = 0$ only E_y, H_x & H_z present

$\therefore Z_{xy} = Z_{zx} = \text{Expects } Z_{xz} = 0$. only $\left. \begin{matrix} Z_{yz} \\ Z_{yx} \end{matrix} \right\}$ exists

$$\therefore Z_{yx} = -\frac{E_y}{H_x} = \frac{j\omega\mu - \frac{\gamma \sin(\frac{m\pi}{a}x)}{\gamma - \frac{j}{\omega\mu} \frac{\gamma \sin(\frac{m\pi}{a}x)}} = \frac{j\omega\mu}{\gamma}$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\epsilon}$$

For Perfecting Conducting walls $\alpha = 0$.

$$\therefore \bar{\gamma} = j\bar{\beta} = j\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2}$$

i) $f < f_c \rightarrow \gamma$ is real so Impedance is pure real indicating that no power is transmitted

ii) $f > f_c \rightarrow \gamma$ is Imaginary

$$\gamma = j\beta = j \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$

$$\therefore Z_{yx} = \frac{j\omega\mu}{\gamma} = \frac{\omega\mu}{\sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}} = \frac{\omega\mu}{\omega\sqrt{\mu\epsilon} \left[\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} \right]} = \frac{\sqrt{\mu\epsilon}}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$\therefore$ Wave impedance is real and decreases.

$$Z_{yx} = \frac{\eta}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

TM waves

$$H_x = H_z = E_y = 0$$

$$Z_{yz} = Z_{zy} = Z_{zx} = Z_{xz} = 0$$

Z_{xy} } exist
 Z_{xz} }

$$Z_{xy} = \frac{E_x}{H_y} = \frac{\frac{\beta C_4 \cos\left(\frac{m\pi}{a}x\right)}{\omega\epsilon}}{\frac{C_4 \cos\left(\frac{m\pi}{a}x\right)}{j\epsilon}} = \frac{\beta}{\omega\epsilon} \quad (v_p = \sqrt{\frac{\mu\epsilon}{\epsilon}})$$

$$= \frac{\sqrt{\mu\epsilon}}{\epsilon} \frac{1}{v_p} = \frac{\sqrt{\mu\epsilon}}{\epsilon} = \sqrt{\frac{\mu}{\epsilon}} = \eta$$

(OR)

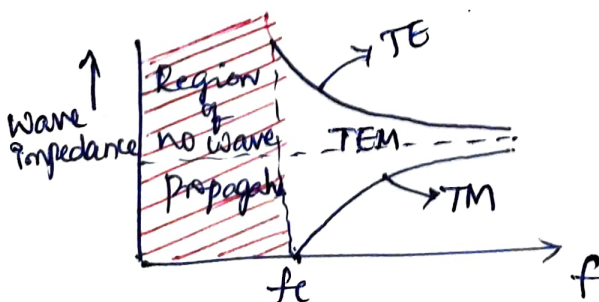
$$Z_{xy} = \frac{\beta}{\omega\epsilon}$$

at high frequency ($f > f_c$)

$$\therefore Z_{xy} = \frac{\sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}}{\omega\epsilon}$$

$$= \frac{\omega\sqrt{\mu\epsilon} \left[\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} \right]}{\omega\epsilon}$$

$$Z_{xy} = \eta \left[\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} \right]$$



① If a wave of 6 GHz is propagating between two parallel conducting plates separated by 30 mm. Find the cutoff wavelength, guide wavelength for TE₁ mode.

Sol $f = 6 \text{ GHz}$, $m = 1$.

$$a = 30 \text{ mm} = \frac{30}{1000} = 0.03 \text{ m}.$$

$$\lambda_c = \frac{2a}{m} = 2a = 2 \times 0.03 = 0.06 \text{ m}.$$

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8}{6 \times 10^9} = 0.05 \text{ m}$$

$$\text{Guide wavelength} = \lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} = \frac{0.05}{\sqrt{1 - \left(\frac{0.05}{0.06}\right)^2}} = 0.0905 \text{ m}$$

② When a wave of 6 GHz propagates in parallel conducting plates separated by 3 cm. Find the phase velocity, group velocity of the wave for the dominant wave?

Sol $f = 6 \text{ GHz}$

$$a = 3 \text{ cm} = \frac{3}{100} = 0.03 \text{ m}$$

$$\text{Free space wavelength} = \lambda = \frac{v}{f} = \frac{3 \times 10^8}{6 \times 10^9} = 0.05 \text{ m}$$

$$\lambda_c = 2a = 2(0.03) = 0.06 \text{ m}.$$

$$\text{Phase velocity} = v_p = \frac{v}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - \left(\frac{0.05}{0.06}\right)^2}} = 5.4 \times 10^8 \text{ m/s}$$

$$\text{Group velocity} = v_g = v \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2} = 3 \times 10^8 \sqrt{1 - \left(\frac{0.05}{0.06}\right)^2} = 1.66 \times 10^8 \text{ m/s}$$

③ When a wave of 6 GHz is to be propagated between two parallel conducting plates separated by 60 mm. Find the modes that will propagate through the guide.

Sol. $a = 60 \text{ mm} = \frac{60}{1000} = 0.06 \text{ m}$

$$f = 6 \text{ GHz}$$

* The modes which have cut off frequencies $<$ frequency ($f_c < f$)

of wave will propagate.

$$\text{For TE}_m \quad f_c = \frac{vm}{2a}$$

$$\text{For TE}_1 \rightarrow f_c = \frac{3 \times 10^8 \times 1}{2 \times 0.06} = 2.5 \text{ GHz}$$

$\therefore f > f_c \rightarrow \text{TE}_1$ propagate.

$$\text{For TE}_2 \rightarrow f_c = \frac{3 \times 10^8 \times 2}{2 \times 0.06} = 5 \text{ GHz}$$

$\therefore f > f_c \rightarrow \text{TE}_2$ propagate.

$$\text{For TE}_3 \rightarrow f_c = \frac{3 \times 10^8 \times 3}{2 \times 0.06} = 7.5 \text{ GHz}$$

$\therefore f < f_c \rightarrow \text{TE}_3$ does not propagate.

$$\text{For TE}_4 \rightarrow f_c = \frac{3 \times 10^8 \times 4}{2 \times 0.06} = 10 \text{ GHz}$$

$\therefore f < f_c \rightarrow \text{TE}_4$ does not propagate.