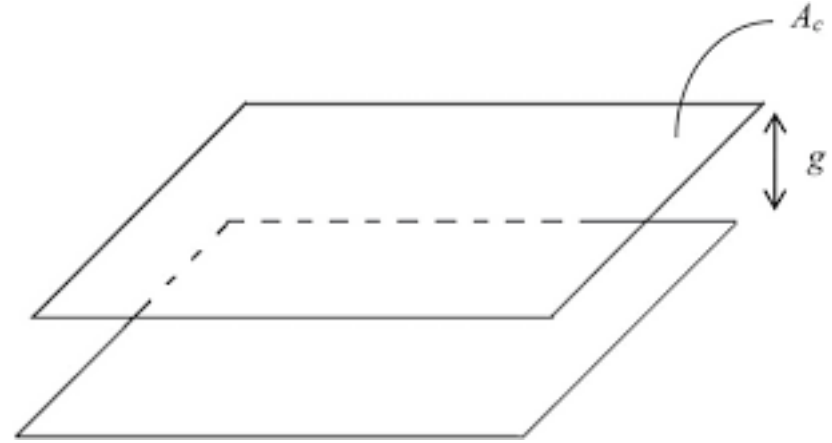


Guided waves between parallel planes

EM wave is propagating between pair of parallel perfectly conducting planes of infinite extent in the y- and z- directions

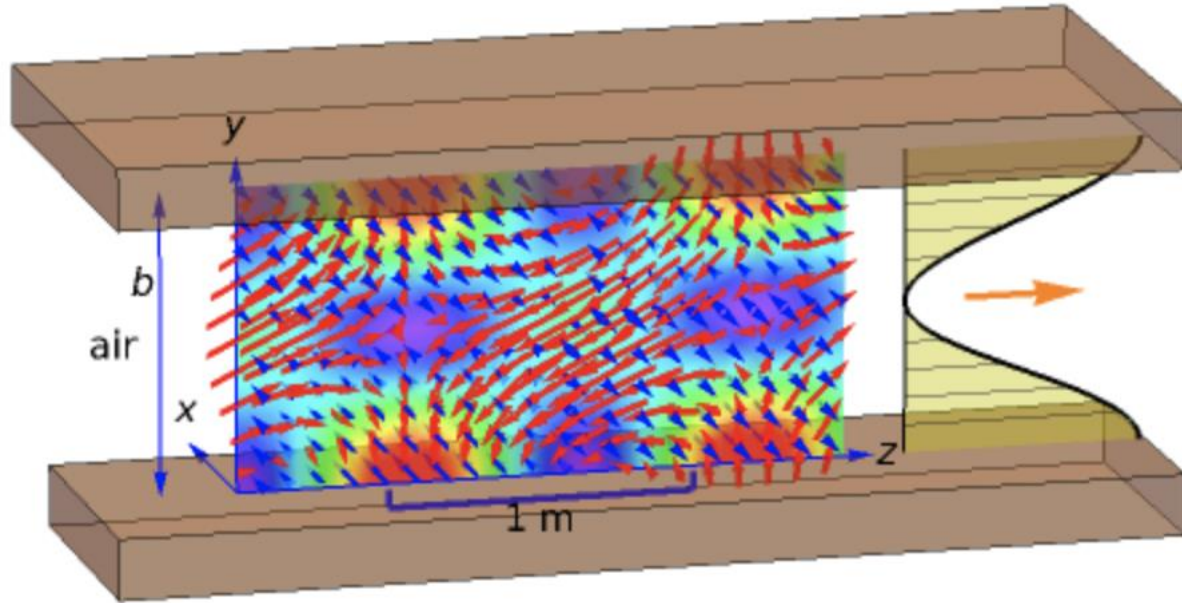
Its breadth along X-Axis



The wave is assumed to propagate along the Z-direction

Guiding Structure is filled with air as dielectric as shown in figure

Guided waves between parallel planes



Maxwell's Equations

Differential Form

(1) $\vec{\nabla} \cdot \vec{D} = \rho$ or $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ Gauss's law

(2) $\vec{\nabla} \cdot \vec{B} = 0$ Gauss's law for magnetism

(3) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ Faraday's law of induction

(4) $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ or $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ Ampère's law

- Together with the Lorentz force these equations form the basic of the classic electromagnetism $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ Lorentz Force

$$\vec{D} = \epsilon_0 \vec{E}$$

ϵ_0 =permittivity of free space

$$\vec{B} = \mu_0 \vec{H}$$

μ_0 =permeability of free space

ρ = electric charge density (As/m³)

J = electric current density (A/m²)

D = electric flux density/displacement field (Unit: As/m²)

E = electric field intensity (Unit: V/m)

H = magnetic field intensity (Unit: A/m)

B = magnetic flux density (Unit: Tesla=Vs/m²)

The wave equation for TE and TM Waves

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z \quad \text{For TE Wave} \quad (E_z = 0)$$

$$\nabla^2 E_z = -\omega^2 \mu \epsilon E_z \quad \text{For TM Wave} \quad (H_z = 0)$$

Expanding $\nabla^2 E_z$ in rectangular coordinate system

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \epsilon E_z \quad \text{--- (10)}$$

Since the wave is propagating in the 'z' direction we have the operator

$$E_z = E_{0z} e^{-\gamma z}$$

Where E_{0z} = Maximum value of Electric field along z – direction

$$\gamma = \alpha + j\beta \quad \gamma - \text{Propagation Constant} \left(\frac{1}{m} \right)$$

$$\alpha - \text{Attenuation Constant} \left(\frac{dB}{m} \text{ or } \frac{Np}{m} \right) \quad \beta - \text{Phase Shift Constant} \left(\frac{rad}{m} \right)$$

$$\frac{\partial E_z}{\partial z} = E_{0z} e^{-\gamma z} (-\gamma) = -\gamma E_z \text{ so that } \frac{\partial}{\partial z} = -\gamma$$

$$\frac{\partial^2 E_z}{\partial z^2} = E_{0z} e^{-\gamma z} (-\gamma)(-\gamma) = \gamma^2 E_z \text{ so that } \frac{\partial^2}{\partial z^2} = \gamma^2$$

$$\frac{\partial^2}{\partial z^2} = \gamma^2$$

Substituting above operator in Equation 10

$$\frac{\partial^2 \mathbf{E}_z}{\partial x^2} + \frac{\partial^2 \mathbf{E}_z}{\partial y^2} + \gamma^2 \mathbf{E}_z = -\omega^2 \mu \epsilon \mathbf{E}_z$$

$$\frac{\partial^2 \mathbf{E}_z}{\partial x^2} + \frac{\partial^2 \mathbf{E}_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \epsilon) \mathbf{E}_z = 0 \text{ --- (11)}$$

Let $\gamma^2 + \omega^2 \mu \epsilon = h^2$, be a constant , then

Equation 11 can be rewritten as

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0 \text{ --- (12) for TE Wave}$$

Similarly

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \text{ --- (13) for TM Wave}$$

By solving the above partial differential equations for E_z and H_z

Using Maxwell's equation, it is possible to find the various components along X and Y directions. $[E_x, H_x, E_y, H_y]$

From Maxwell's First Equation(Ampere's Law)

$$\nabla \times \mathbf{H} = \mathbf{j}\omega\epsilon\mathbf{E}$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \mathbf{j}\omega\epsilon[i\mathbf{E}_x + j\mathbf{E}_y + k\mathbf{E}_z]$$

Replacing $\frac{\partial}{\partial z} = -\gamma$, we get

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ H_x & H_y & H_z \end{vmatrix} = \mathbf{j}\omega\epsilon[i\mathbf{E}_x + j\mathbf{E}_y + k\mathbf{E}_z]$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ H_x & H_y & H_z \end{vmatrix} = j\omega\epsilon[iE_x + jE_y + kE_z]$$

$$i\left[\frac{\partial H_z}{\partial y} + \gamma H_y\right] - j\left[\frac{\partial H_z}{\partial x} + \gamma H_x\right] + k\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] = i[j\omega\epsilon E_x] + j[j\omega\epsilon E_y] + k[j\omega\epsilon E_z]$$

Equating coefficients of i, j and k (after expanding),we get

Equating coefficients of i, j and k (after expanding),we get

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\epsilon E_x \text{ --- (14)}$$

$$\frac{\partial H_z}{\partial x} + \gamma H_x = -j\omega\epsilon E_y \text{ --- (15)}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \text{ --- (16)}$$

Similarly from Maxwell's Second Equation(Faraday's Law)

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu[iH_x + jH_y + kH_z]$$

Replacing $\frac{\partial}{\partial z} = -\gamma$, we get

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu[iH_x + jH_y + kH_z]$$

$$i\left[\frac{\partial E_z}{\partial y} + \gamma E_y\right] - j\left[\frac{\partial E_z}{\partial x} + \gamma E_x\right] + k\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right] = -i[j\omega\mu H_x] - j[j\omega\mu H_y] - k[j\omega\mu H_z]$$

Equating coefficients of i, j and k (after expanding),we get

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \text{ --- (17)}$$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = +j\omega\mu H_y \text{ --- (18)}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \text{ --- (19)}$$

Combining Equation 14 and 18 to eliminate H_y , we get an expression for E_x .

From Equation 18

$$H_y = \frac{1}{j\omega\mu} \frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\mu} E_x$$

Substituting for H_y in Equation 14, we get

$$\frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu} \frac{\partial E_z}{\partial x} + \frac{\gamma^2}{j\omega\mu} E_x = j\omega\epsilon E_x$$

$$E_x \left[j\omega\epsilon - \frac{\gamma^2}{j\omega\mu} \right] = \frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu} \frac{\partial E_z}{\partial x}$$

Multiplying by $j\omega\mu$, we get

$$E_x [-\omega^2\mu\epsilon - \gamma^2] = j\omega\mu \frac{\partial H_z}{\partial y} + \gamma \frac{\partial E_z}{\partial x}$$

$$E_x [-(\omega^2\mu\epsilon + \gamma^2)] = j\omega\mu \frac{\partial H_z}{\partial y} + \gamma \frac{\partial E_z}{\partial x}$$

Where $\gamma^2 + \omega^2 \mu \epsilon = h^2$

Dividing by $-h^2$ throughout, we get

$$E_x = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y} - - - - (20)$$

Similarly remaining terms by solving Equations 15 and 17, we get

$$E_y = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x} - - - - (21)$$

Similarly remaining terms by solving Equations 15 and 17, we get

$$H_x = \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial x} + \frac{j\omega\epsilon}{h^2} \frac{\partial E_z}{\partial y} - - - - (22)$$

Similarly remaining terms by solving Equations 14 and 18, we get

$$H_y = \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial y} + \frac{j\omega\epsilon}{h^2} \frac{\partial E_z}{\partial x} - - - - (23)$$

These equations (20, 21, 22 and 23) give a general relationship for field components within a Waveguide.

Summary of field Components with in a Rectangular Waveguide

$$E_x = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y} \text{ --- --- (20)}$$

$$E_y = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x} \text{ --- --- (21)}$$

$$H_x = \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial x} + \frac{j\omega\epsilon}{h^2} \frac{\partial E_z}{\partial y} \text{ --- --- (22)}$$

$$H_y = \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial y} + \frac{j\omega\epsilon}{h^2} \frac{\partial E_z}{\partial x} \text{ --- --- (23)}$$

Parallel Conducting plates (Guided waves)

Since the space between the planes is infinite in extent in the Y - direction,

- There are no boundary conditions to be met in this direction
- It can be assumed that the field is uniform or constant in the Y- direction.
- Hence the derivatives with respect to **y** can be put to 0.
- In the X direction there are certain boundary conditions which must be met.

Summary of field Components within a Guided Structure (Parallel Plates)

$$E_x = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x}$$

$$E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial x}$$

$$H_y = \frac{-j\omega\epsilon}{h^2} \frac{\partial E_z}{\partial x}$$