Thermo Dynamics

Introduction :-

Therme (Heat) (Greek word)

→ Thermodynamics is branch of science which deals with the relations among heat, work and properties of system which are in equilibrium.

→ Thermodynamics basically describes four laws, Zeroth law, first law, second law and third law.

→ The Zeroth law deals with thermal equilibrium and establishes a concept of temperature.

→ The second law indicates the limit of converting heat into work and introduces the principle of increase of entropy.

→ Third law defines absolute zero of entropy.

These laws are based on experimental observations and have no mathematical proof. These laws are based on logical reasoning.

Thermodynamic Systems :-

System :-

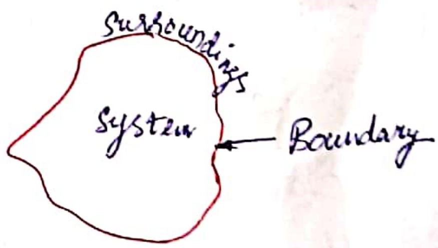


Fig → System

A system is a finite quantity of matter or prescribed region of space.

Boundary :-

→ The actual or hypothetical envelope enclosing the system is the boundary of the system.

→ The boundary may be fixed or it may ~~to~~ move, as and when a system containing a gas is compressed or expanded.

→ The boundary may be real or imaginary.

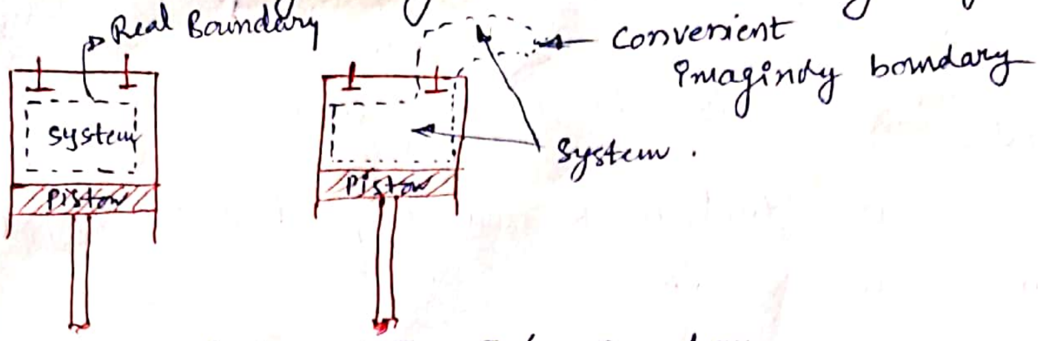


Fig → The Real and Imaginary Boundary.

Surroundings:-

→ Any thing outside of the system is called surrounding.

→ Boundary seperates system and surroundings.

System + Surroundings = Universe.

Closed System :-

Energy can cross.
mass can't cross.

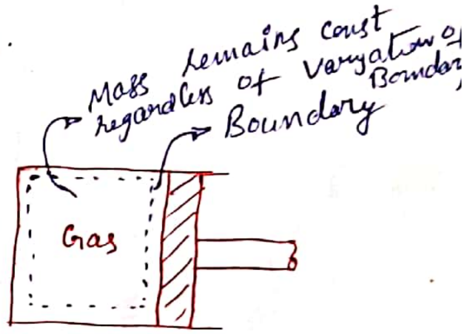


Fig → Closed System.

→ If the boundry of the system is impervious to the flow of matter, it is called closed system.

→ In this system energy can enter & leaves the system.

→ An example of this system is mass of gas & vapour contained in an engine cylinder, the boundery of which is drawn by cylinder walls, the cylinder head and piston crown.

→ Hence the boundary is continuos and no matter may enter or leave.

Open System :-

An open system is one in which both energy and matter flows into & out of the system. Most of the engineering systems or open.

ex:- Air Compressor / Turbines, boilers - - etc.

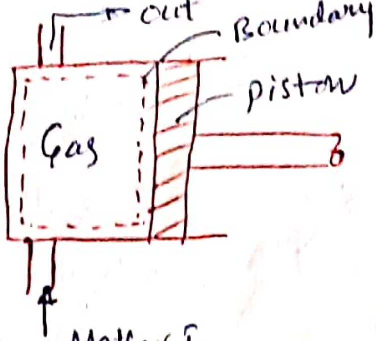


Fig → open System.

Isolated System :-

An isolated system is that system which exchanges neither energy nor matter with any other system & environment.

Ex:- Thermos flask.

Adiabatic system :-

mass can cross (work)
Energy can't cross. (Heat)

→ An adiabatic system is one which is thermally insulated. It can however exchange work with its surroundings.

→ If adiabatic system does not exchange work it will become isolated system. Ex:- piston working inside a insulated cyl.

Phase :-

A phase is a quantity of matter which is homogeneous throughout in chemical composition and physical structure.

Based on phase, systems are of 2 types.

1) Homogeneous system :-

A system which consist of single phase is known as homogeneous system.

Ex:- Mixture of air and water vapour, water and milk, octane & butane

2) Heterogeneous system :-

A system which consists of two or more phases is known as heterogeneous system.

Ex:- Water and steam, ice and water, water and oil.

Macroscopic and Microscopic point of View :-

Thermodynamics studies are undertaken by following 2 different approaches.

1. Macroscopic approach – Macro means big and total
2. Microscopic approach – Micro means small.

Macroscopic – Engg. approach	Microscopic scie. approach
→ In this certain quantity of matter is considered without taking into account ~~of~~ the events occuring at molecular level. Another words this approach to thermodynamics is concerned with gross or overall behaviour. This is known as classical T.D.	→ The approach considers that system is made up of a large no. of discrete molecules. These molecules have diff velocities and energies. The values of energies are constantly changing with time. This approach to thermodynamics which is concerned directly with the structure of matter is known as statistical T.D.
→ This ~~system~~ approach requires simple mathematical formulae.	→ This approach requires advanced statistical and mathematical methods.
→ The values of properties of the system are their average value. For ex consider a gas in a closed container. The pr of gas is avg value of pr. exerted by millions of individual molecules. Illy Temp. These properties like pr, Temp can be measured very easily and changes in properties felt by our senses.	→ The properties like vel, momentum impulse, force of impact -- etc which describe the molecules cannot be easily measured with instruments. Our senses cannot feel them
→ Few properties are required to describe a syst. So it is simple	→ Large no of variables are needed to describe a system. So it is complicated.

Pure Substance :-

A pure substance is one which has a homogeneous and invariable chemical composition even though there is a change of phase.

ex:- liquid, water, mixture of ice and water.

Thermodynamic Equilibrium :-

→ A body is said to be in thermal equilibrium if it obeys three types of equilibrium namely thermal, mechanical and chemical equilibrium.

1. Thermal Equilibrium :-

The temperature of the system doesnot change with the time and has ~~all~~ same value at all points of the system.

2. Mechanical Equilibrium :-

There are no unbalanced forces with in the system or between the surroundings. The pressure in the system is same at all the points and doesnot change with respect to time.

3. chemical equilibrium :-

No chemical reaction takes place in the system and chemical composition which is same throughout the system.

Properties of System :-

1. Intensive properties :-

These properties doesnot depend on mass of the system.

Ex:- Pressure, Temperature

2. Extensive Properties :-

These properties depend on the mass of the system.

Ex:- volume, weight, density ---etc.

Extensive properties are often devided by mass associated with them to obtain the intensive properties.

Ex:- $\frac{V}{m}$ = v = specific volume, which is independent of mass.

State :-

→ State is the condition of the system at an instant of time.

or

Each unique condition of the system is called a state.

→ Each property has a single value at each state.

→ A variable is a property, if and only if, it has a single value at each equilibrium state.

Process :-

→ A process occurs when the system undergoes a change in a state.

→ A process may be non-flow in which fixed mass with in the defined boundary is undergoing a change of state.

Ex:- A substance which is being heated in a closed cylinder undergoes a non-flow process. All closed systems undergoes non-flow process.

→ A process may be a flow process in which mass is entering and leaving through the boundary of a system.

Ex:- All open systems undergoes flow process.

Scanned with CamScanner

Quasi-static process :-

→ Quasi means almost. A quasi static process is also called reversible process.

→ This process is a succession of equilibrium states and infinite slowness is its characteristic feature.

Cycle :-

→ Any process or series of processes whose start & end states are identical is called as cycle.

→ The process through which the system has passed can shown on state diagram.

Fig shows a cycle a system commencing at condition 1, changes in Pressure and volume through a path 123 and returns to initial condition 1

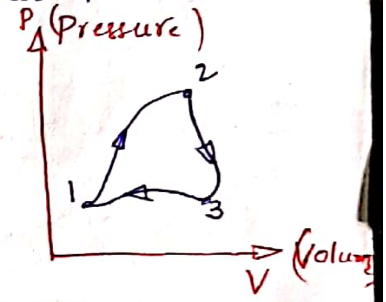


Fig → Cycle of operations.

Point function :-

There are certain properties which can be located on a graph by a point. Those properties are called as point function.

Ex:- Pressure, Temperature, Volume - - - etc.

$$\int_1^2 dV = V_2 - V_1 \rightarrow \text{An exact differential.}$$

path function :-

There are certain properties which cannot be located on a graph by a point, but are given by area. In that case the area on the graph pertaining to particular process is a function of path of the process. Those properties are called as path functions.

Ex:- Heat and work. These are in exact differentials.

$$\int_1^2 \delta Q \neq Q_2 - Q_1, \quad \int_1^2 \delta W \neq W_2 - W_1 \text{ in exact differential.}$$

Temperature :-

→ Temperature is a thermal state of a body which distinguishes a hot body from cold body.

→ Instruments for measuring ordinary temperatures are known as thermometers and those for measuring high temperatures are known as pyrometers.

→ It has been found that gas will not occupy any volume at certain temperature. This temperature is known as absolute zero temperature.

→ The point of absolute temperature is found to occur at 273.15°C below the freezing point of water.

∴ Absolute temperature = Thermometer reading in °C + 273.15

→ Absolute temperature can be represented by degree Kelvin (K)

Zeroth Law of T.D :- If 2 thermodynamic systems are each in thermal equilibrium with a third, then they are in thermal equilibrium with each other.

→ A temp. scale of certain thermometer is given by the relation $t = a \log p + b$ where a and b are const and p is the thermometric property of the fluid in thermometer. If at ice point and steam point thermometric property are found to be 1.5 and 7.5 resly. what will be the temp corresponding to the thermometric property of 3.5 on Celsius scale

Sol

$$t = a \log p + b$$

on Celsius scale

Ice point = $0^\circ c$, Steam point = $100^\circ c$

from given condition

$$0 = a \log 1.5 + b$$

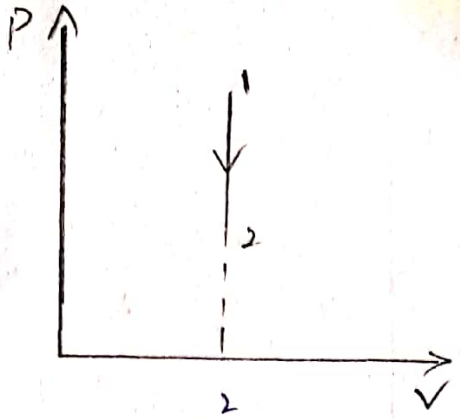
$$100 = a \log 7.5 + b$$

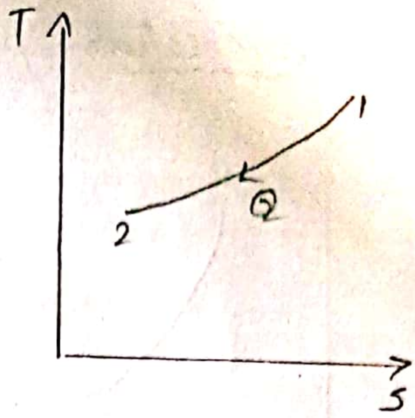
$$\Rightarrow a = 143.06 \quad b = -25.30$$

when $p = 3.5$

$$t = 52.66^\circ c$$

1. Constant volume process:-





$$\oint_1^2 W_{1-2} = pdv$$

$$\oint W = 0$$

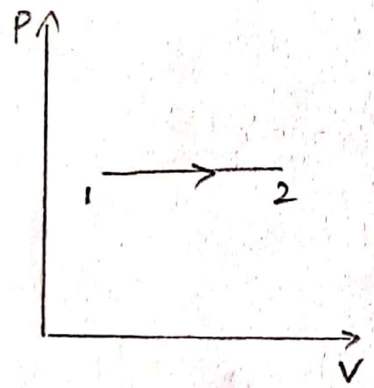
$$\partial Q - \partial W = dU$$

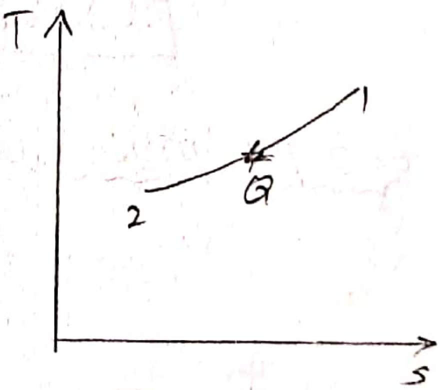
$$\partial Q = \partial W^{\nearrow 0} + dU$$

$$\delta Q = du$$

$$\boxed{\therefore \delta Q = mC_v dT}$$

2. constant pressure process:-





$$PV \propto T \qquad \frac{V}{T} = const$$

$$\partial Q - \partial W = dU$$

$$\partial W = \int_1^2 Pdv = P(V_2 - V_1)$$

$$\partial Q = P(V_2 - V_1) + (U_2 - U_1)$$

$$= (P_2V_2 + U_2) - (P_1V_1 + U_1)$$

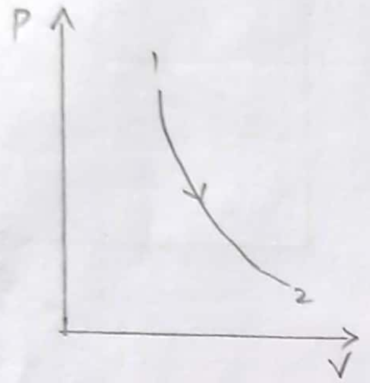
$$\partial Q = H_2 - H_1$$

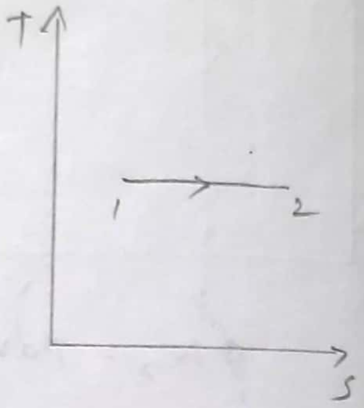
$H = mC_p dT$

$$C_p = \frac{dh}{dT}$$

$$C_v = \frac{du}{dT}$$

3) constant temperature process :-





$pV dT$

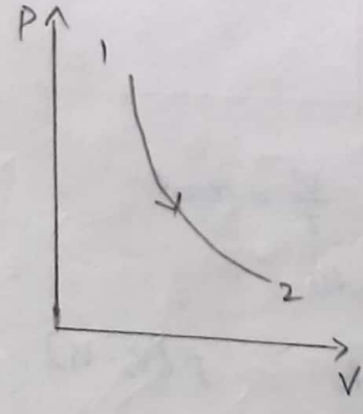
$pV = C \Rightarrow p = \frac{C}{V}$

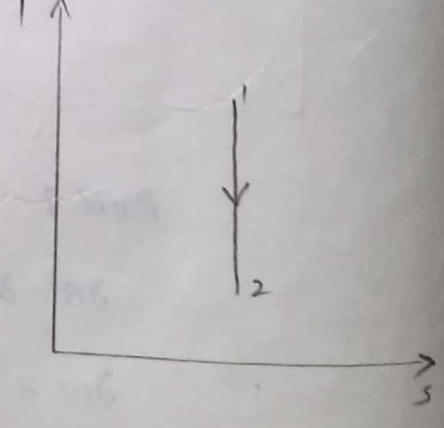
$$W = \int p \cdot dV = C\int_1^2 \frac{1}{V} dV = C \ln\left(\frac{V_2}{V_1}\right)$$

$$dU = mC_v(T_2 - T_1) = 0$$

$$\therefore \partial Q = \partial W$$

4) constant entropy process (Adiabatic) :-





$$PV^{\gamma} = const$$

$$P = \frac{c}{V^{\gamma}} = c \cdot V^{-\gamma}$$

$$W = c\int_1^2 V^{-\gamma} dV$$

$$= c\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_1^2$$

$$= PV^{\gamma}\left[\frac{V_2^{-\gamma+1} - V_1^{-\gamma+1}}{-\gamma+1}\right]$$

$$= \left[\frac{P_2 V^{\gamma} V_2^{-\gamma+1} - P_1 V^{\gamma} V_1^{-\gamma+1}}{-\gamma+1}\right]$$

$$\partial Q = 0$$

$$\Rightarrow \frac{P_2V_2 - P_1V_1}{-\gamma+1}$$

$$\therefore \partial W = dU$$

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

UNIT-1. Reciprocating Air Compressors

Introduction:-

→ An air compressor is a machine to compress the air and to raise its pressure.

→ It sucks air from atmosphere, compresses it and then delivers the same under a high pressure to storage vessel.

→ From storage vessel it may be conveyed by pipeline to a place where the supply of compressed air is required.

→ Since the compression of air requires some work to be done on it, therefore a compressor must be driven by some prime mover.

Applications of Compressed air:-

→ It is used for many purposes such as for operating pneumatic drills, riveters, road drills, paint spraying, in starting and supercharging of internal combustion engines, in gas turbine plants jet engines --etc.

→ It is also utilised in the operation of lifts, rams, pumps ---etc.

→ In industry compressed air is used for producing blast of air in blast furnaces.

Classification of Air Compressors :-

The air compressors may be classified in many ways, but the following are important from subject point of view.

1. According to working :-

a) Reciprocating air compressor

b) Rotary air compressor.

2. According to action :-

a) Single acting air compressor

b) Double acting air compressor

3. According to number of stages :-

a) Single stage air compressor

b) Multi stage air compressor.

Terminology in Air Compressors :-

1. Inlet pressure :- (Suction pressure)

It is the absolute pressure of air at the inlet of a compressor.

2. Discharge pressure :-

It is the absolute pressure of air at the outlet of a compressor.

3. Compression Ratio (Pressure Ratio):-

→ It is the ratio of discharge pressure to inlet pressure.

→ The ratio is always greater than unity because the delivery pressure is greater than suction pressure.

4. Compressor Capacity :-

It is the volume of air delivered by the compressor, and is expressed in m^3/min or m^3/sec.

5. Free Air Delivery :-

→ It is the actual volume delivered by a Compressor when reduced to the normal temperature and pressure condition.

→ The capacity of compressor generally given in terms of free air delivery.

6. Swept volume :- (Displacement)

It is the volume of air sucked by compressor during its suction stroke.

Mathematically

Swept volume, $V_s = \frac{\pi}{4} d^2 L$ (for single acting)

where d = Dia of cylinder bore

L = Length of piston stroke.

7. Mean effective pressure :-

In air compressor the pressure acting on the piston keeps on changing with the movement of piston in the cylinder.

→ The mean effective pressure of the cylinder compressor is found mathematically by dividing the workdone per cycle to the swept volume.

$$P_{mean} = \frac{\text{Work done}}{\text{Swept Volume}}$$

* Working of Single Stage Reciprocating Air Compressor :-

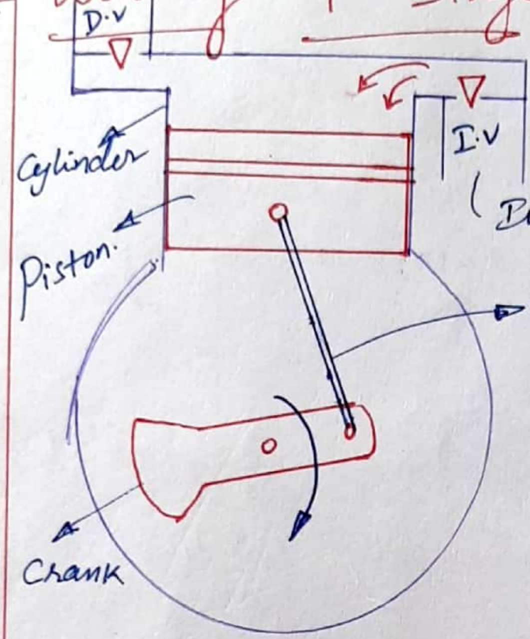


Fig:- Suction Stroke

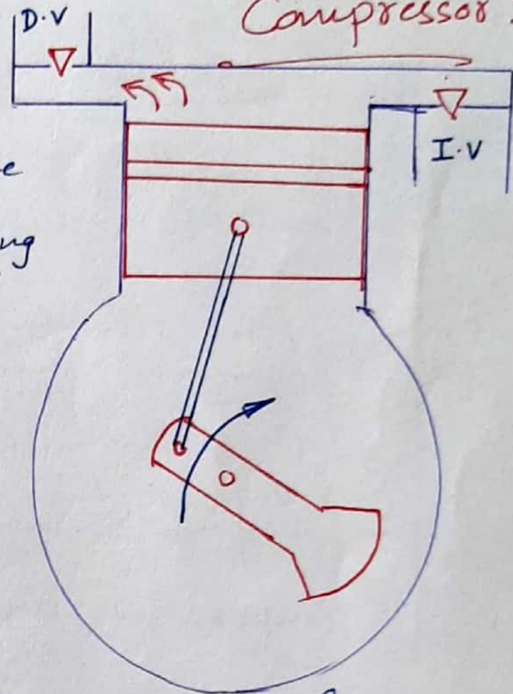


Fig:- Delivery Stroke.

→ A single stage reciprocating air compressor consists of a cylinder, piston, crank, connecting rod, I.V, D.V as shown in figure.

→ when piston moves downwards, the pressure inside the cylinder falls below the atmospheric pr. Due to this pressure difference, inlet valve (I.V) gets opened and air is sucked in to the cylinder at inlet pressure.

Scanned with CamScanner

→ Now when the piston moves upwards, the pressure inside the cylinder goes on increasing till it reaches discharge pressure.

→ At this stage, the discharge valve gets opened and air is delivered to the contained.

→ At the end of delivery stroke, a small quantity of air at high pressure is left in clearance space.

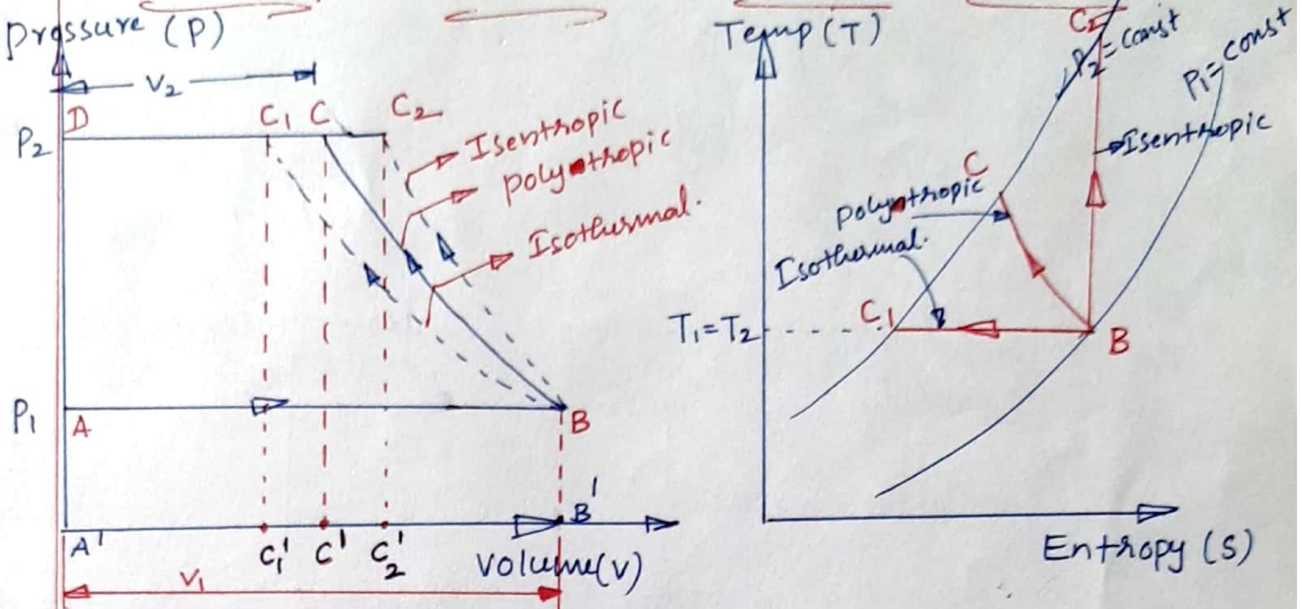
→ As the piston starts its suction, the air contained in clearance space expands till its pressure falls below atmosphere.

→ At this stage, the inlet valve (Iv) gets opened as a result of which fresh air is sucked into the cylinder and the cycle is repeated.

NOTE → ① In single acting reciprocating ~~pump~~ compressor the suction, compression and delivery of air takes place in 2-strokes of piston or one revolution of crank shaft.

② In double acting, suction, compression and delivery takes place on both sides of piston. So it supplies double the volume of air than single acting reciprocating compressor.

* Work done by a Single stage Reciprocating Air Compressor without Clearance volume :-



A → B = Suction

B → C = compression

C → D = Discharge

Consider a single acting reciprocating air compressor without clearance volume.

Let P_1 = Initial pressure of air (Before compression)

V_1 = Initial volume of air (" ")

T_1 = Initial temp of air (" ")

P_2, V_2, T_2 are corresponding values of air at delivery

r = pressure ratio i.e $\frac{P_2}{P_1}$

P–V & T–S diagram of air compressor are shown in the fig. During return stroke of the piston, the air is compressed at const temp.

The compression continues till the pressure (P_2) in the cylinder is sufficient to force open the delivery valve at C. After that no more compression takes place.

Pg No-(4)

→ Now during remaining stroke of compression, the compressed air is delivered till the piston head reaches the cylinder end.

→ After that the air is sucked from atmosphere during suction stroke AB at pressure P_1.

→ The compression of air may be isothermal or polytropic or isentropic.

1. **Workdone during isothermal compression :- ($PV = Const$)**

Isothermal compression & delivery of air is shown by BC_1 & C_1D.
C_1D represents the volume of air delivered.

Workdone by air compressor per cycle W = Area under the curve ABC_1D.

($\log_e^{10} = 2.3 \log_{10}$)

$$W = \text{Area } ABC_1D$$

$$= \text{Area } A'DC_1C_1' + \text{Area } C_1BB'C_1' - \text{Area } A'ABB'$$

$$= P_2 v_2 + 2.3 P_2 v_2 \log\left(\frac{v_1}{v_2}\right) - P_1 v_1$$

$$= 2.3\, P_1 v_1 \log\left(\frac{v_1}{v_2}\right) = 2.3 P_1 v_1 \log\left(\frac{P_2}{P}\right)$$

$(\because P_1 v_1 = P_2 v_2)$

$$= 2.3\, P_1 v_1 \log(r) = 2.3\, mRT_1 \log r.$$

2. **Workdone during polytropic compression :- ($Pv^n = Const$)**

polytropic compression is shown by line BC.

Workdone by air compressor per cycle W = Area under the curve ABCD.

$$W = \text{Area } ABCD$$

$$= \text{Area } A'DCC' + \text{Area } CBB'C' - \text{Area } A'ABB'$$

pg ④

$$W = P_2 v_2 + \frac{P_2 v_2 - P_1 v_1}{n-1} - P_1 v_1$$

$$= P_2 v_2 - P_1 v_1 \left[1 + \frac{1}{n-1}\right]$$

$$W = \frac{n}{n-1}\left(P_2 v_2 - P_1 v_1\right)$$

$$= \frac{n}{n-1} P_1 v_1 \left[\frac{P_2 v_2}{P_1 v_1} - 1\right]$$

for polyntropic process, $P_1 v_1^n = P_2 v_2^n$

$$\frac{v_2}{v_1} = \left(\frac{P_1}{P_2}\right)^{1/n}$$

$$\therefore W = \frac{n}{n-1} P_1 v_1 \left[\left(\frac{P_2}{P_1}\right)\left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} - 1\right]$$

$$= \frac{n}{n-1} P_1 v_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$\boxed{W = \frac{n}{n-1} mRT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] = \frac{n}{n-1} mR(T_2 - T_1)}$$

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{P_2}{P_1} \cdot \frac{T_1}{T_2} \rightarrow ①$$

$$P_1 v_1^n = P_2 v_2^n$$

$$\Rightarrow \frac{v_1}{v_2} = \left(\frac{P_2}{P_1}\right)^{1/n} \rightarrow ②$$

$① = ②$

$$\frac{P_2}{P_1} \cdot \frac{T_1}{T_2} = \left(\frac{P_2}{P_1}\right)^{1/n}$$

$$\frac{T_1}{T_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1-n}{n}}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

3. Workdone during isentropic compression ($PV^\gamma = const$):-

→ The isentropic compression is shown by curve BC_2.

→ workdone on air per cycle during isentropic compression may be calculated in the same way as polyntropic Comp^n.

workdone on air per cycle $W = \frac{\gamma}{\gamma-1} mRT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$

$$W = \frac{\gamma}{\gamma-1} mR(T_2 - T_1)$$

We know that $C_p - C_v = R$ & $\frac{C_p}{C_v} = \gamma$

$$R = C_p\left(1 - \frac{C_v}{C_p}\right)$$

$$R = C_p\left(1 - \frac{1}{\gamma}\right) = C_p\left(\frac{\gamma-1}{\gamma}\right)$$

$$\therefore w/d = \frac{\gamma}{\gamma-1} m C_p \left(\frac{\gamma-1}{\gamma}\right)(T_2 - T_1) = \boxed{mC_p(T_2 - T_1) = W}$$

Scanned with CamScanner

NOTE :- → Workdone on air is minimum when the comp^n is isothermal and it is maximum when the comp^n is isentropic.

→ Inorder to perform isothermal process, the compression should be very slow so that the temperature is maintained const. which is not possible in actual practice.

→ However isothermal compression is achieved if

① the cooling (air/water) is done during comp^n.

② By injecting the cold water during comp^n.

③ By adding intercoolers in multistage comp^n.

* **Power Required to Drive Single Stage Air Compressor:**

$$\text{power} = \frac{\text{Workdone} \times \text{Number of working Strokes/min}}{60} \text{ watts}$$

$$= \frac{W \times N_w}{60} \text{ watts}$$

$N_w = N$ for single acting

$= 2N$ for double acting

where N = Speed of Compressor in r.p.m.

∴ Isothermal power = $\frac{W_{isothermal} \times N_w}{60}$ watts

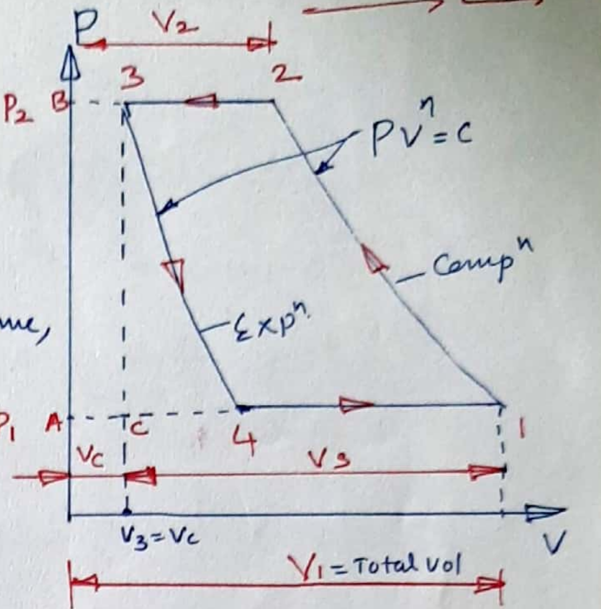
Isentropic power = $\frac{W_{isentropic} \times N_w}{60}$ watts

Indicated Power = $\frac{W_{polytropic} \times N_w}{60}$ watts

→ Indicated power is also known as power of compressor.

* Workdone by Reciprocating Air Compressor with Clearance Volume:-

Consider a reciprocating air compressor with clearance volume



Let

P_1, V_1, T_1 are pressure, volume, Temp of air before compression.

P_2, V_2, T_2 are corresponding values at delivery.

$V_1 - V_4$ = Effective swept vol.

$V_1 - V_3$ = Swept vol.

$V_3 = V_c$ = Clearance vol.

r = pressure ratio = $\frac{P_2}{P_1}$

V_c = Clearance volume ~~= $V_1 - V_3$~~

V_s = Stroke/swept volume = $V_1 - V_c$

n = polytropic index for comp^n & exp^n.

→ During return stroke the air is compressed by its major part i.e compression stroke 1-2. This stroke continues till the pressure reaches P_2.

→ This pressure P_2, force the delivery valve at 2.

→ During remaining stroke of compression stroke, compressed air is delivered till the piston reaches 3.

→ At this stage there will be some air (equal to clearance volume) left in the clearance space will expand during outward stroke of piston i.e exp^n stroke 3-4.

→ This exp^n continues till the pressure P_1 in the cylinder is sufficient to force open the I.V at 4. After this the air is sucked from atmosphere during suction stroke 4-1 at pressure P_1.

Let assume the compression and expansion may be polytropic.

Workdone by the compressor per cycle = W = Area 1-2-3-4.

$$= \text{Area } A12B - \text{Area } A43B.$$

$$= \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] - \frac{n}{n-1} P_4 V_4 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W = \frac{n}{n-1} P_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] (V_1 - V_4)$$

Note → V_4 is called expanded clearance volume

$(V_1 - V_4)$ is called effective swept volume.

Need for Multistage Compression :-

In single stage compression, if the requirement of pressure of compressed air is more than that case, we employ a large pressure ratio (8 to 10 bar) in a single cylinder.

If we employ a single stage compression for producing higher pressure ratio, it suffers from following drawbacks.

① The size of cylinder is too large.

② Due to compression, there is rise in temp of air. It is difficult to reject heat from the air in small time available for compression.

③ Due to higher air temp at the end of compⁿ, it may heat up cylinder head or burn the lubricating oil.

Advantages of Multi Stage Compression :-

1. The workdone per kg of air is reduced in multistage compression with intercooler as compared to single stage compression for the same air delivery.

2. It improves volumetric efficiency for same pr. ratio

3. The size of 2 cylinders (HP & L.P) may be adjusted to suit the volume and pressure of air.

4. It reduces leakage loss

5. It produces more uniform torque and hence smaller size fly wheel is required.

6. It produces effective lubrication because of lower temperature range

7. It reduces the cost of compressor.

2-Stage Reciprocating Air Compressor With Intercooler :-

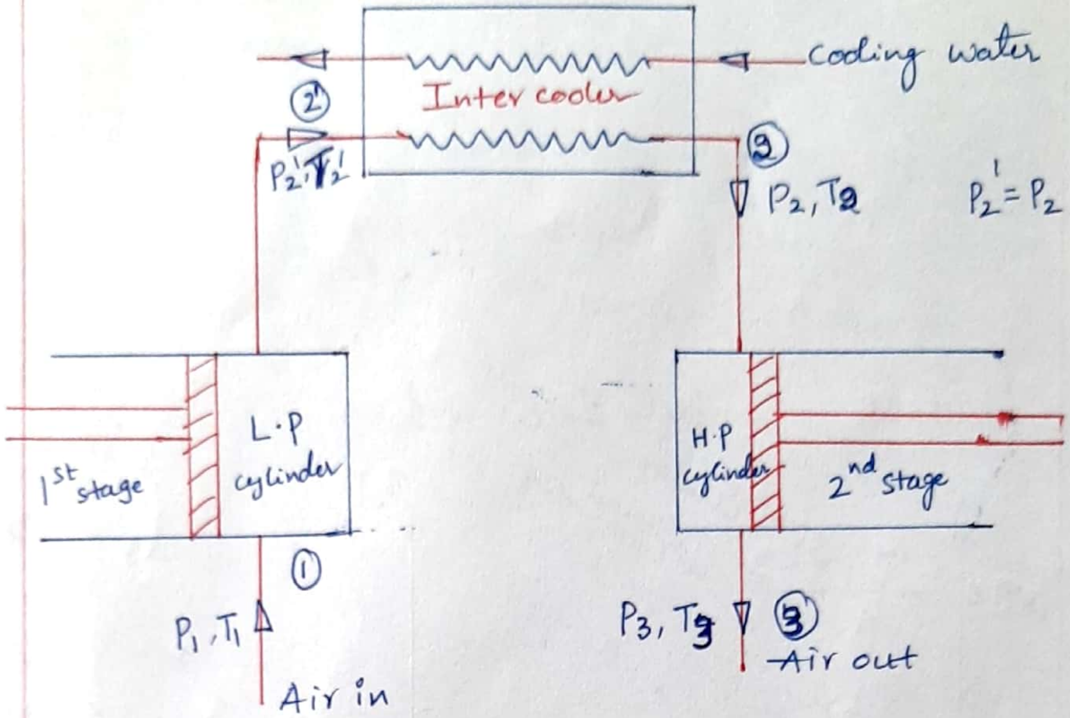


Fig → 2 stage Reciprocating Air Compressor with Intercooler.

→ Fresh air is sucked from atmosphere in low pressure cylinder during its suction stroke at pressure P_1, temperature T_1.

→ The air after compression in L.P. cylinder from $1 \to 2'$, is delivered to the intercooler at pressure P_2', temp T_2'.

→ Now the air is cooled in intercooler from $2'$ to 2 at const pressure P_2 and from temperature T_2' to T_2.

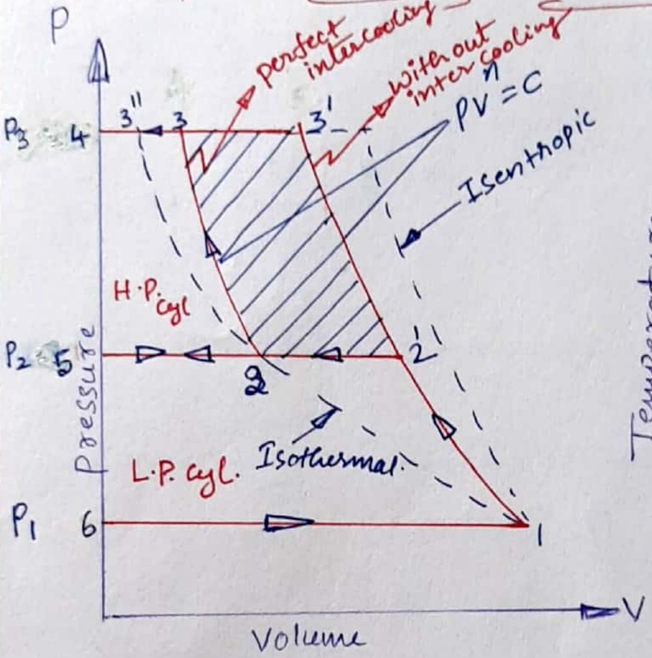
→ After that the air is sucked in the high pressure cylinder during its suction stroke.

→ Finally the air is further compressed in H.P cylinder from $2 \to 3$ and its pressure is P_3, temperature T_3.

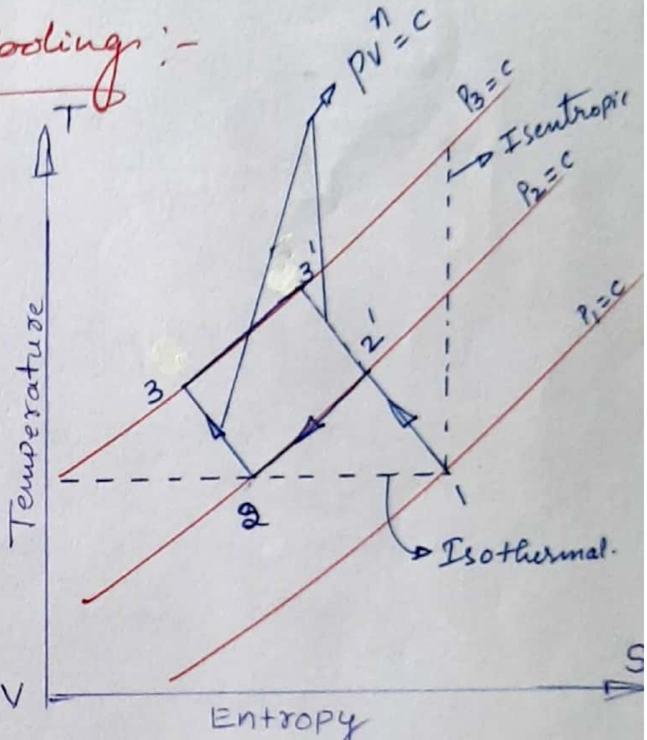
Assumptions:-

1. The effect of Clearance is neglected.
2. There is no pressure drop in intercooler
3. The Compression in both cylinders is polytropic
4. The suction and delivery of air takes place at constant Pressure.

1. Perfect / Complete Inter Cooling:-



Fig→ P-V diagram.



Fig→ T-S diagram

→ When the temperature of air leaving the intercooler T_3, is equal to original atmospheric air temp T_1, then the intercooling is known as perfect Intercooling

→ Cycle $613'4$ is that of single stage compressor.

→ cycles $612'5$ & $52'3'4$ are that of 2-stage compressor without inter cooling

→ cycles $612'5$ & 5234 are that of 2-stage compressor with perfect intercooling.

Case-I. Single Stage Compressor :-

As earlier stated cycle 613'4 is that of Single stage compressor.

For this cycle work done $W = \frac{n}{n-1} P_1V_1 \left[\left(\frac{P_3}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$

Delivery temp $T_3 = T_1 \left(\frac{P_3}{P_1}\right)^{\frac{n-1}{n}}$

Case-II ⓐ 2-stage Compressor without Inter cooler :-

61 2'5 - Low pressure cycle

52'3'4 - High pressure cycle

For this cycle

work done $W = \frac{n}{n-1} P_1V_1 \left[\left(\frac{P_2'}{P_1}\right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} P_2V_2 \left[\left(\frac{P_3'}{P_2'}\right)^{\frac{n-1}{n}} - 1 \right]$

$P_2' = P_2$, $P_3' = P_3$.

This workdone is also equal to workdone in case-I

Because of no intercooling

ⓑ 2-Stage compressor with perfect Intercooling :-

612'5 - Low pressure cycle

5234 - High pressure cycle

For this cycle

work done $W = \frac{n}{n-1} P_1V_1 \left[\left(\frac{P_2'}{P_1}\right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} P_2V_2 \left[\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 1 \right]$

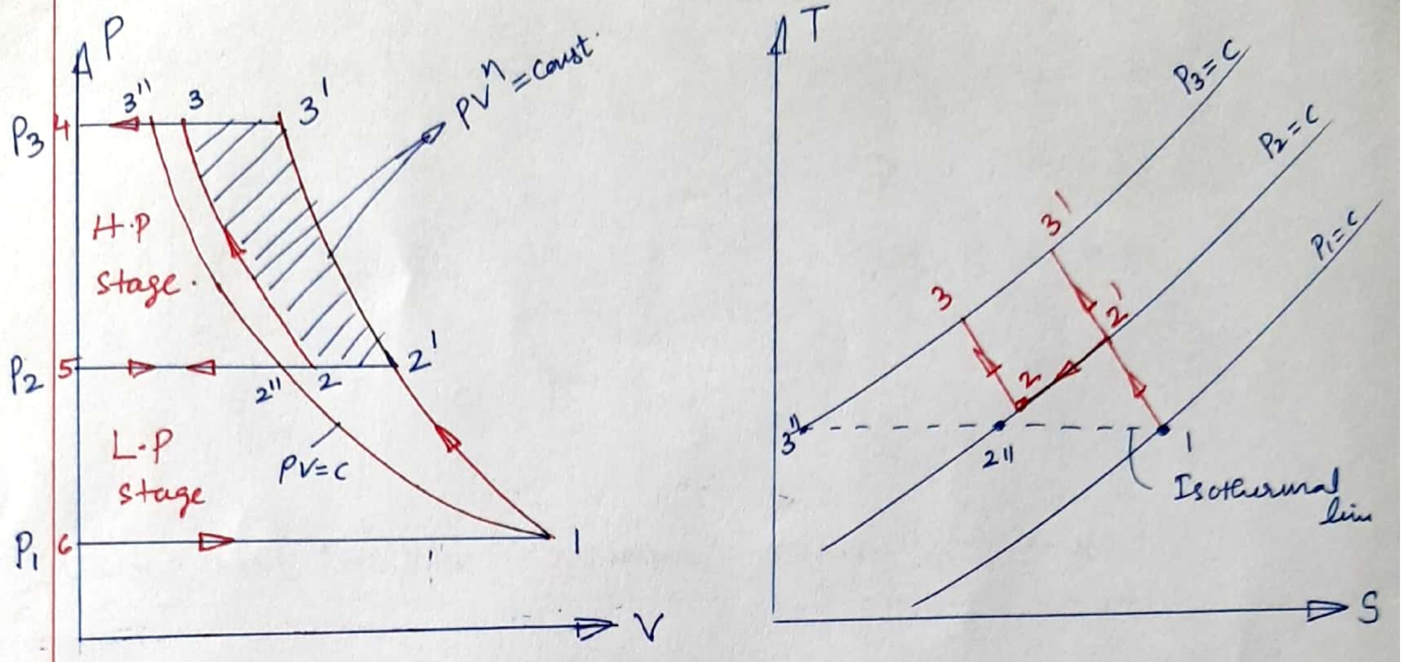
But Point 2 & 1' are on isothermal line

$P_1V_1 = P_2V_2$

$\therefore W = \frac{n}{n-1} P_1V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 2 \right]$

→ The area shaded 2'233' which is the work saving which occurs as a result of using intercooler.

2-Stage Air Compressor with Imperfect Cooling:-



$$\text{workdone} = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2'}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} P_2 V_2 \left[\left(\frac{P_3'}{P_2} \right)^{\frac{n-1}{n}} - 1 \right]$$

$\because P_2' = P_2$

$$W = \frac{n}{n-1} \left[P_1 V_1 \left(\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right) + P_2 V_2 \left(\left(\frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 1 \right) \right]$$

Minimum work required for 2-Stage Reciprocating Air Compressor:-

We know that for 2-stage reciprocating air compressor, with perfect intercooling

$$W = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 2 \right]$$

If the intake pressure P_1 and delivery pressure P_3 are fixed, then least value of intermediate pressure 'P_2' may be obtained by differentiating the above equation wrt P_2 and equate to zero. At this pressure P_2, work required is minimum.

$$\frac{dW}{dP_2} = 0 \Rightarrow \frac{d}{dP_2} \left\{ \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 2 \right] \right\} = 0$$

Let $\frac{n-1}{n} = a$

$$\frac{d}{dP_2} \left[\left(\frac{P_2}{P_1}\right)^{a} + \left(\frac{P_3}{P_2}\right)^{a} - 2 \right] = 0$$

$$P_1^{-a} \cdot a P_2^{a-1} + P_3^{a} \cdot -a P_2^{-a-1} - 0 = 0$$

$$a P_1^{-a} P_2^{a-1} = + a P_3^{a} P_2^{-a-1}$$

$$\frac{P_2^{a-1}}{P_2^{-a-1}} = \frac{P_3^{a}}{P_1^{-a}}$$

$$P_2^{a-1+a+1} = (P_3 P_1)^a$$

$$P_2^{2a} = (P_3 P_1)^a \Rightarrow P_2^2 = P_1 P_3 \Rightarrow P_2 = \sqrt{P_1 P_3}$$

$$\frac{P_2}{P_1} = \frac{P_3}{P_2} = \left(\frac{P_3}{P_1}\right)^{1/2}$$

Substitute $\frac{P_3}{P_2} = \frac{P_2}{P_1}$ in workdone to get W_{min}

$$W_{min} = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 2 \right]$$

$$= \frac{2n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

= 2 x Work required for each stage.

Substitute $\frac{P_2}{P_1} = \left(\frac{P_3}{P_1}\right)^{1/2}$ in the above eqn.

$$W = 2 \times \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_3}{P_1}\right)^{\frac{n-1}{2n}} - 1 \right]$$

Similarly for 3-S compressor

$$\frac{P_2}{P_1} = \frac{P_3}{P_2} = \frac{P_4}{P_3} = \left(\frac{P_4}{P_1}\right)^{1/3} \quad \&$$

Minimum work for 3-stage compressor

$$W = 3 \times \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_4}{P_1}\right)^{\frac{n-1}{3n}} - 1 \right]$$

NOTE:- In general

$$\frac{P_2}{P_1} = \frac{P_3}{P_2} = \frac{P_4}{P_3} = \cdots = \frac{P_{q+1}}{P_q} = \left(\frac{P_{q+1}}{P_1}\right)^{\frac{1}{q}}$$

$$W = q \times \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_{q+1}}{P_1}\right)^{\frac{n-1}{qn}} - 1 \right]$$

Heat Rejected in a Reciprocating Air Compressor:-

The total heat rejected in a reciprocating air compressor is the sum of heat rejected during polytropic compression per kg and heat rejected in the intercooler per kg of air.

W.K.T

Heat rejected during polytropic compression per kg of air $q_1 = \frac{\gamma - n}{\gamma - 1} \times \text{workdone}$

$$= \frac{\gamma - n}{\gamma - 1} \times \frac{R(T_2' - T_1)}{(n-1)}$$

$C_p - C_v = R \quad C_p/C_v = \gamma$

$$\frac{C_p - C_v}{C_v} = \frac{R}{C_v}$$

$\gamma - 1 = R/C_v \Rightarrow R = C_v(\gamma - 1)$

$$q_1 = \frac{\gamma - n}{\gamma - 1} \, C_v(\gamma - 1) \, \frac{(T_2' - T_1)}{n-1}$$

$$q_1 = C_v \cdot \frac{\gamma - n}{n-1} \left(T_2' - T_1\right)$$

Heat rejected in intercooler per kg of air,

$$q_2 = C_p\left(T_2' - T_2\right)$$

$\therefore$ Total heat rejected $q = q_1 + q_2$

$$q = C_v \cdot \frac{\gamma - n}{n-1}\left(T_2' - T_1\right) + C_p\left(T_2' - T_2\right)$$

for perfect intercooling $T_2 = T_1$

$$q = (T_2' - T_1)\left[\left(\frac{\gamma - n}{n-1}\right)C_v + C_p\right]$$

Free Air Delivery :-

→ It is the amount of atmoshpheric air that can be sucked by the compressor during suction at 1atm pressure, 20°C, 100% dry air when the compressor motor running at 100% of rated R.P.M.

→ FAD is an important purchasing parameter and it measures the capacity of compressor interms of air flow it handle.

Efficiencies of compressor :-

① Isothermal efficiency :-

In a compressor the work is minimum when the compression follows isothermal process. The ratio of isothermal work to actual work is called isothermal efficiency.

$$\eta_{iso} = \frac{\text{Isothermal work}}{\text{Actual work}}$$

② $$\eta_{vol} = \frac{\text{Effective Swept volume}}{\text{Swept volume}} = \frac{V_1 - V_4}{V_1 - V_3}$$

③ $$\eta_{mech} = \frac{\text{Indicated power of a Compressor}}{\text{Shaft power}}$$

Scanned with CamScanner

After cooling in Reciprocating Air Compressor:-

→ The Compressed air discharged from air compressor is hot.

→ Compressed air at this temperature contains large quantity of water in the vapour form.

→ Due to after cooler water vapour can be condensed into water.

Advantages:-

1) To reduce final temperature of air thus tank size can be reduced & greater amount of air can be stored for the same size
2) It increases volumetric efficiency
3) Reduces risk of fire.
4) protect the equipment from excess heat.

Pg No 12

Volumetric Efficiency :-

The volumetric efficiency of a compressor is defined as ratio of free air delivery to the displacement of the compressor. or

It is the ratio of effective swept volume to the swept volume.

$$\eta_{vol} = \frac{\text{Effective Swept vol}}{\text{Swept vol}} = \frac{V_1 - V_4}{V_1 - V_3}$$

→ Clearance ratio is defined as clearance vol to the swept volume.

$$\text{Clearance ratio} = K = \frac{V_c}{V_1 - V_3} = \frac{V_c}{V_s}$$

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3} = \frac{(V_1 - V_3) + (V_3 - V_4)}{V_1 - V_3} = 1 + \frac{V_3 - V_4}{V_1 - V_3}$$

$$= 1 + \frac{V_3}{V_1 - V_3} - \frac{V_4}{V_1 - V_3} = 1 + \frac{V_3}{V_1 - V_3} - \frac{V_4}{V_1 - V_3}\left(\frac{V_3}{V_3}\right)$$

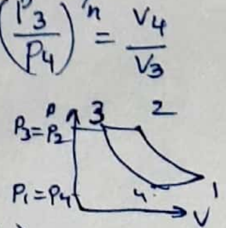
$$= 1 + \frac{V_3}{V_1 - V_3} - \frac{V_3}{V_1 - V_3}\left(\frac{V_4}{V_3}\right)$$

$$P_1 V_1^n = P_2 V_2^n \qquad \left(\frac{P_2}{P_1}\right)^{1/n} = \frac{V_1}{V_2}$$

$$\eta_{vol} = 1 + K - K\left(\frac{V_4}{V_3}\right)$$

$$P_3 V_3^n = P_4 V_4^n \qquad \left(\frac{P_3}{P_4}\right)^{1/n} = \frac{V_4}{V_3}$$

$$\eta_{vol} = 1 + K - K\left(\frac{P_3}{P_4}\right)^{1/n}$$



$$= 1 + K - K\left(\frac{P_2}{P_1}\right)^{1/n} = 1 + K - K\left(\frac{V_1}{V_2}\right)$$

$$\therefore \eta_{vol} = 1 + K - K\left(\frac{V_1}{V_2}\right)$$

→ In actual practice air that is sucked in during suction stroke gets heated up while passing through the hot valves and casing in contact with hot cylinder walls.

Thus ambient conditions are different from conditions obtained at state 1.

P_{amb} = Pr of ambient air

T_{amb} = Temp of ambient air

$$\frac{P_{amb} \cdot V_{amb}}{T_{amb}} = \frac{P_1 (V_1 - V_4)}{T_1}$$

$$\Rightarrow V_{amb} = \frac{P_1}{T_1} \cdot \frac{T_{amb}}{P_{amb}} \cdot (V_1 - V_4)$$

Volumetric efficiency refer to ambient conditions may be written as

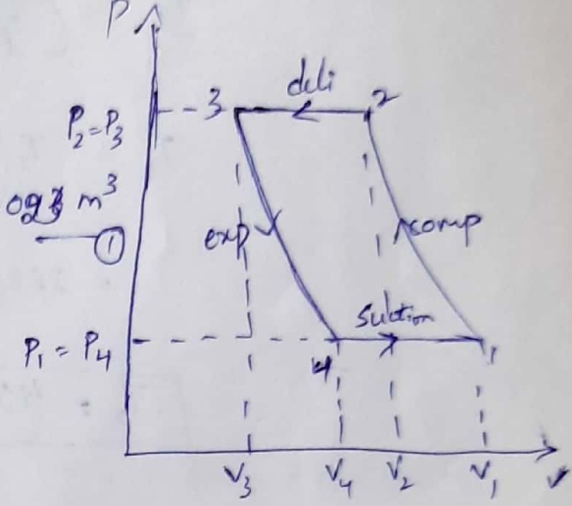
$$\eta_{vol(amb)} = \frac{V_{amb}}{V_1 - V_3} = \frac{P_1}{T_1} \cdot \frac{T_{amb}}{P_{amb}} \left(\frac{V_1 - V_4}{V_1 - V_3}\right)$$

$$\eta_{vol(amb)} = \frac{P_1}{T_1} \cdot \frac{T_{amb}}{P_{amb}} \left(1 + K - K\left(\frac{V_1}{V_2}\right)\right]$$

or

$$\eta_{vol(amb)} = \frac{P_1}{T_1} \frac{T_{amb}}{P_{amb}} \left(1 + K - K\left(\frac{P_2}{P_1}\right)^{1/n}\right)$$

(4) A single stage double acting air compressor is required to deliver 14 m^3 air per min measured at 1.013 bar and 15°c. The delivery pressure is 7 bar and speed is 300 rpm take clearance volume as 5% of swept volume and index n is 1.3. calculate (i) swept volume of cylinder (ii) delivery temperature (iii) Indicated power.



So:

$$V_1 - V_4 = 14\ m^3/min = \frac{14}{600}\ m^3 = 0.0233\ m^3 \longrightarrow (1)$$

$$P_1 = 1.013\ bar = 1.013 \times 10^5\ N/m^2$$

$$T_1 = 15°K = 288K$$

$$P_2 = 7\ bar = 7 \times 10^5\ N/m^2$$

$$N = 300\ rpm$$

$$V_c = \frac{5}{100} V_s = 0.05 V_s$$

$$n = 1.3$$

$$V_s = V_1 - V_3 = V_1 - V_c$$

$$V_s = V_1 - 0.05 V_s$$

$$V_1 = 1.05 V_s \longrightarrow (2)$$

$$P_3 V_3^n = P_4 V_4^n$$

$P_4 = P_1$. $P_3 = P_2$

$V_3 = V_c$

$$\left(\frac{V_4}{V_c}\right)^n = \left(\frac{P_2}{P_1}\right)$$

$$V_4 = V_c \left(\frac{P_2}{P_1}\right)^{1/n}$$

$$= 0.05 V_c \left(\frac{7}{1.013}\right)^{1/1.3}$$

$$V_4 = 0.22 V_s \longrightarrow (3)$$

Scanned with CamScanner

② & ③ in ①

$1.05V_s - 0.22V_s = 0.023$

$V_s(1.05 - 0.22) = 0.023$

$\therefore V_s = 0.027\ m^3$

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$

$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$

$= 288\left(\frac{7}{1.013}\right)^{\frac{1.3-1}{1.3}}$

$\therefore T_2 = 450K$

work done $= \frac{n}{n-1} P_1(V_1 - V_4)\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$

$= \frac{1.3}{1.3-1} \times 1.013\times10^5 (0.023)\left[\left(\frac{7}{1.013}\right)^{\frac{1.3-1}{1.3}} - 1\right]$

$\therefore W = 5675.79\ N\text{-}m$

$P = \frac{W \times N_w}{60}$ — 2N

$= \frac{5675.79 \times 2 \times 300}{60}$

$\therefore P = 56.75KW$

Scanned with CamScanner

(5) A single stage double acting air compressor has a Free Air delivery (F.A.D) of 14 m^3/min measured at 1.013 bar and 15°c. The pressure and temperature in cylinder during induction are 0.95 bar and 32°c. The delivery pressure is 7 bar and index of compression and expansion n = 1.3. The clearance is 5% of swept volume. calculate (i) Indicated power required (ii) volumetric efficiency.

Sol

$$V_1 - V_4 = 14\, m^3/min = \frac{14}{600}\, m^3 = 0.023\, m^3$$

$$P_1 = 0.95\, bar = 0.95 \times 10^5\, N/m^2$$

$$T_1 = 32°c = 305K \qquad P_2 = 7\, bar = 7 \times 10^5\, N/m^2$$

$$n = 1.3$$

$$V_c = 0.05\, V_s \Rightarrow \frac{V_c}{V_s} = 0.05 = K$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$T_2 = 305 \left(\frac{7}{0.95}\right)^{\frac{1.3-1}{1.3}}$$

$$\therefore T_2 = 483.57\, K$$

$$W = \frac{n}{n-1} P_1 (V_1 - V_4) \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.3}{1.3-1} \times 0.95 \times 10^5 \times (14) \left[\left(\frac{7}{0.95}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$W = 33.74 \times 10^5\, N\text{-}m/min$$

$$power = \frac{33.74 \times 10^5}{60} = 56.23\, KW.$$

$$\eta_{vol} = 1 + K - K\left(\frac{P_2}{P_1}\right)^{1/n}\left[\frac{P_a}{P_a} \cdot \frac{T_a}{T_1}\right)$$

$$= 1 + 0.05 - 0.05\left(\frac{7}{0.95}\right)^{1/1.3}\left[\frac{0.95\times10^5}{1.013\times10^5} \times \frac{288}{305}\right)$$

$$= 0.844$$

$$\therefore \eta_{vol} = 84.4\%$$

⑥ A single stage single acting air compressor delivers 0.6 Kg of air per min at 6 bar. The temperature and pressure at end of suction stroke are 30°c and 1 bar. The bore and stroke of compressor are 100mm and 150mm respectively. The clearance volume is 3% of swept volume and $n = 1.3$, find i) volumetric efficiency of compressor (ii) power required to drive the compressor if mechanical efficiency is 85% iii) speed of compressor in rpm.

Sol:

$m = 0.6$ Kg/min

$P_2 = 6$ bar

$T_1 = 30°c$

$P_1 = 1$ bar

$d = 100mm, \quad L = 150mm$

$V_c = 0.03\, V_s \Rightarrow K = \frac{V_c}{V_s} = 0.03$

$n = 1.3$

$$\eta_{vol} = 1 + K - K\left(\frac{P_2}{P_1}\right)^{1/n}$$

$$= 1 + 0.03 - 0.03 \left(\frac{6}{1}\right)^{1/1.3}$$

$$= 0.910$$

$$\therefore \eta_{vol} = 91\%$$

ii) $$\text{work done} = \frac{n}{n-1} P_1 (V_1 - V_4) \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{n}{n-1} mRT_1 \left(\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right)$$

$$= \frac{1.3}{1.3-1} \times 0.6 \times 287 \times 303 \left(\left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right)$$

$$W = 1.1 \times 10^5 \ N\text{-}m/min$$

$$I.P = \frac{1.1 \times 10^5}{60} = 1929.6 \ W = 1.92 \ KW$$

$$\eta_{mech} = \frac{I.P}{\text{Shaft power}}$$

$$S.P = \frac{1929.6}{0.85} = 2270 W$$

$$= 2.27 KW$$

iii) Speed of compressor

$$\eta_{vol} = \frac{FAD}{V_s}$$

$$P = \frac{W \times N}{60}$$

$$\frac{60P}{W}$$

$$\frac{60 \times 1926.6}{1.1 \times 10^5}$$

$$F.A.D = P_1 (V_1 - V_4) = mRT_1$$

$$FAD = \frac{mRT_1}{P_1}$$

$$= \frac{0.6 \times 287 \times 303}{1 \times 10^5} = 0.52 \ m^3/min$$

$$\eta_{vol} = \frac{0.52}{\frac{\pi}{4} d^2 L \times N}$$

$$0.91 = \frac{0.52}{\frac{\pi}{4} \times 100^2 \times 150 \times N}$$

N

(7) A single stage single acting compressor delivers 14 m^3/min From 1 bar to 7 bar. The speed of compressor is 310 rpm $n = 1.35$ and clearance volume is 5% of swept volume. Assume the temperature stroke length is 1.5 times the diameter. Assume the temperature and pressure of air at suction are same as atmospheric air. Determine the bore and stroke of compressor.

Sol

$FAD = 14\ m^3/min$

$P_1 = 1$ bar

$P_2 = 7$ bar

$N = 310$ rpm

$n = 1.35$

$V_c = 5\%\ V_s \Rightarrow K = \frac{V_c}{V_s} = 0.05$

$L = 1.5D$

$$\eta_{vol} = \frac{FAD}{V_s}$$

$$\eta_{vol} = 1 + K - K\left(\frac{P_2}{P_1}\right)^{1/n}$$

$$= 1 + 0.05 - 0.05\left(\frac{7}{1}\right)^{1/1.35}$$

$$= 0.838$$

$$\eta_{vol} = 83.8\%$$

Scanned with CamScanner

$$\eta_{vol} = \frac{FAD}{V_s}$$

$$V_s = \frac{FAD}{\eta_{vol}}$$

$$\left(\frac{\pi}{4} d^2 L\right) N = \frac{14}{0.838}$$

$$\frac{\pi}{4} \times d^2 \times 1.5d \times 310 = \frac{14}{0.838}$$

$$365.21\, d^3 = 16.70$$

$$\boxed{\therefore d = 0.35 m}$$

$$L = 1.5 D$$

$$= 1.5 \times 0.35$$

$$\boxed{\therefore L = 0.53 m}$$

(8) Air at 103 KPa and 27°c is drawn LP cylinder of 2-stage air compressor and is isentropically compressed to 700 Kpa. The air is then cooled at constant pressure to 37°c in an inter cooler and is then again compressed isentropically to 4 MPa in HP cylinder and then delivered at that pressure. Determine the power required to run the compressor if it has to deliver 30 m³ of free air per hour at inlet condition.

Sol

$P_1 = 103\ KPa$

$T_1 = 27°c$

$P_2 = 700\ KPa$

$T_2' = 37°c$

$P_3 = 4\ MPa$

FAD = $V_1 = 30\ m^3/hr$

isentropically, $\gamma = 1.4$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = 300 \times \left(\frac{700}{103}\right)^{\frac{1.4-1}{1.4}}$$

$$T_2 = 518.7\ ^\circ K$$

$$\frac{T_3}{T_2'} = \left(\frac{P_3}{P_2'}\right)^{\frac{\gamma-1}{\gamma}} = T_3 = 310 \times \left(\frac{4000}{700}\right)^{\frac{1.4-1}{1.4}}$$

$$T_3 = 510.07 K$$

$$\text{Work done} = \frac{\gamma}{\gamma-1}\left[P_2V_2 - P_1V_1\right] + \frac{\gamma}{\gamma-1}\left[P_3V_3 - P_2'V_2'\right]$$

$$= \frac{\gamma}{\gamma-1}\left\{mR(T_2-T_1)\right. + \frac{\gamma}{\gamma-1} mR(T_3 - T_2')$$

$$m = \frac{P_1V_1}{RT_1} = \frac{103\times10^3 \times 30}{287 \times 300} = 9.96 \times 10^{-3}$$

$$= \frac{\gamma}{\gamma-1} mR\left((T_2-T_1) + (T_3 - T_2')\right)$$

$$= \frac{1.4}{1.4-1} \times 9.96\times10^{-3} \times 287\left[(518.7-300) + (510.07-310)\right]$$

$$W = 4.18\ kJ/s$$

(9) A 2-stage single acting compressor takes in air at the rate of 0.2 m^3/s, the intake pressure and temperature are 0.1 mpa and 16°c. The air is compressed to a final pressure of 0.7 mpa. The intermediate pressure is ideal and intercooling is perfect. ~~The compressor~~ $n = 1.25$ and $N = 600$ rpm. neglecting the clearance. determine i) intermediate pressure

2) total volume of each cylinder 3) power required to drive the compressor. 4) heat rejection in inter cooler.

Sol

$V_1 = FAD = 0.2\ m^3/s$

$P_1 = 0.1\ MPa$

$P_3 = 0.7\ MPa$

$T_1 = 16°C$

$n = 1.25$

$N = 600\ rpm$

$C_p = 1.005\ KJ/kg\ K$

$R = 0.287\ KJ/kg\cdot K$

for perfect inter cooling

$T_2 = T_1$, ① $P_2 = \sqrt{P_1 P_3}$

$\therefore P_2 = \sqrt{0.1 \times 0.7} = 0.26\ MPa$

② V_1 = Vol of L.P cyl

V_2 = vol of HP cyl.

$$V_1 = \frac{0.2\ m^3/s}{\frac{600}{60}} = 0.02\ m^3$$

$T_1 = T_2$ (100% I.C)

$P_1 V_1 = P_2 V_2$

$$V_2 = \frac{P_1 V_1}{P_2} = \frac{0.1 \times 0.02}{0.26}$$

$\therefore V_2 = 0.0076\ m^3$

③ $$W = \frac{2n}{n-1} P_1 V_1 \left[\left(\frac{P_3}{P_1}\right)^{\frac{n-1}{2n}} - 1 \right]$$

$$= \frac{2 \times 1.25}{1.25 - 1} \times 100 \times 0.02 \left[\left(\frac{0.7}{0.1}\right)^{\frac{1.25-1}{2 \times 1.25}} - 1 \right]$$

$W = 4296.28\ N\text{-}m \Rightarrow 42.9 \times 10^3\ N\text{-}m/s$

(9) Heat rejected

$$\frac{T_2'}{T_1} = \left(\frac{P_2'}{P_1}\right)^{\frac{n-1}{n}}$$

$$T_2' = 289 \times \left(\frac{0.26}{0.1}\right)^{\frac{1.25-1}{1.25}}$$

$$T_2' = 349.85\ K$$

$$m = \frac{P_1 V_1}{R T_1} = \frac{0.1\times10^6 \times 0.02}{0.287\times289} = 24.11\ kg/s$$

$$\therefore Q = m C_p (T_1 - T_2')$$

$$= 24.11 \times 1.005 (289 - 349.85)$$

$$= -1146$$

(10) A 2-stage air compressor with complete inter cooling delivers air to the mains at a pressure of 30 bar. The suction conditions are 1 bar and 15°c (27°c). If the both the cylinders have same stroke, find the ratio of cylinder diameters, assume the index for compression is 1.3

Sol

$P_3 = 30$ bar

$P_1 = 1$ bar

$T_1 = 15°c$

$L_1 = L_2$

$n = 1.3$

for complete intercooling

$T_1 = T_2 = 288\ K$

$P_2 = \sqrt{P_1 P_3} = \sqrt{30\times1}$

$\therefore P_2 = 5.47$ bar

V_1 = Vol of L.P

V_2 = vol of H.P

$$\frac{V_1}{V_2} = \frac{\frac{\pi}{4} d_1^2 L_1 N_1}{\frac{\pi}{4} d_2^2 L_2 N_2} \Rightarrow \frac{d_1}{d_2} = \sqrt{\frac{V_1}{V_2}} \quad \text{—(1)}$$

$$P_1 V_1^n = P_2' V_2'^n$$

1-2'

$$\left(\frac{V_1}{V_2'}\right) = \left(\frac{P_2}{P_1}\right)^{1/n} \quad \text{—(2)}$$

2'-2

$$\frac{V_2'}{T_2'} = \frac{V_2}{T_2} \Rightarrow \frac{V_2'}{V_2} = \frac{T_2'}{T_2} \quad \text{—(3)}$$

$$\frac{T_2'}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$\frac{T_2'}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$\Rightarrow T_2' = 288 \times \left(\frac{5.47}{1}\right)^{\frac{1.3-1}{1.3}}$$

1.249

$$T_2' = 426.28 \, K$$

$T_2 =$

$$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{1/n} \cdot \frac{T_2'}{T_2}$$

$$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{1/n} \cdot \frac{T_2'}{T_2}$$

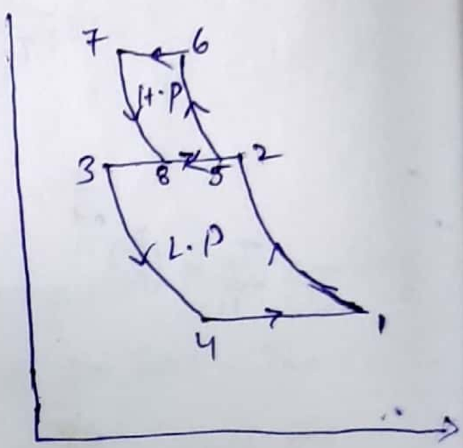
$$= \left(\frac{5.47}{1}\right)^{1/1.3} \times \frac{426.28}{288}$$

$$\frac{V_1}{V_2} = 5.46$$

$$\frac{d_1}{d_2} = \sqrt{5.46}$$

$$\therefore \frac{d_1}{d_2} = 2.33$$

(11) In a single acting 2-stage reciprocating air compressor 4.5 kg of air per min are compressed from 1.013 bar and 15°c through a pressure ratio of 9 to 1. Both the stages have same pressure ratio and law of compression ratio $PV^{1.3} = C$. If the intercooling is complete. calculate the indicated power, cylinder swept volume, assume that the clearance volume of both the stages are 5% of their swept volumes and speed of compressor is 300rpm.



Sol:

$m = 4.5\ kg/min$

$P_s = 1.013$ bar

$T_s = 15°c$

Pressure ratio $= \frac{9}{1} = \frac{P_d}{P_s}$

$n = 1.3$ $\Rightarrow P_d = 9P_s$

$V_c = 0.05 V_s$

$N = 300$ rpm

Pr ratio is same for both stages $\Rightarrow \frac{P_i}{P_s} = \frac{P_d}{P_i}$ 3

Inter cooling Complete $=$ $P_i = \sqrt{P_s P_d}$

$P_i^2 = P_s \cdot P_d$

$P_d = 9P_s$

$P_i^2 = 9P_s^2 \Rightarrow \boxed{P_i = 3P_s}$

$$\frac{T_i}{T_s} = \left(\frac{P_i}{P_s}\right)^{\frac{n-1}{n}}$$

$$T_i = 288 \times \left(\frac{3P_s}{P_s}\right)^{\frac{1.3-1}{1.3}}$$

$$\therefore T_i = 371.10\ K$$

$$\frac{T_d}{T_i} = \left(\frac{P_d}{P_i}\right)^{\frac{n-1}{n}} \Rightarrow T_d = 371.10 \times (3)^{\frac{1.3-1}{1.3}}$$

$$T_d = 478.18\ K$$

$$\text{Work done} = \frac{n}{n-1} P_s V_s \left[\left(\frac{P_i}{P_s}\right)^{\frac{n-1}{n}} - 1 + \left(\frac{P_d}{P_i}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W/d = \frac{2n}{n-1} P_s V_s \left[\left(\frac{P_d}{P_s}\right)^{\frac{n-1}{2n}} - 1\right]$$

$P_s V_s = m R T_s$

$1.013 \times 10^5 \times V_s = 4.5 \times 287 \times 288$

$$V_s = \frac{4.5 \times 287 \times 288}{1.013 \times 10^5}$$

$V_s = 3.67\ m^3/min$

$$= \frac{2 \times 1.3}{1.3-1} \times 1.013 \times 10^5 \times 3.67 \left[\left(\frac{9P_s}{P_s}\right)^{\frac{1.3-1}{2 \times 1.3}} - 1\right]$$

$= 929747.22\ N\text{-}m/min$

$\text{power} = 929747.22/60$

$\therefore \text{power} = 15.49\ KW$

$$K = \frac{V_c}{V_s} = 0.05$$

$$1 + 0.05 - 0.05\left(\frac{3P_s}{P_s}\right)^{\frac{1.3-1}{1.3}} = 0.985$$

(L.P) $\eta_{vol} = 1 + K - K\left(\frac{P_i}{P_s}\right)^{\frac{n-1}{n}} = 0.985 \Rightarrow 98.5\%$

(HP) $\eta_{vol} = 1 + K - K\left(\frac{P_d}{P_i}\right)^{\frac{n-1}{n}} = 0.985 = 98.5\%$

L.P

$\eta_{vol} = \frac{FAD}{V_s}$

$0.985 = \frac{3.68}{V_s}$

$V_s = \frac{3.68}{0.985}$

$V_s = 3.73\ m^3/min$

$\therefore V_s = 0.0124\ m^3$

FAD

$P_s \overbrace{(V_1 - V_4)}^{FAD} = mRT_s$

$FAD = \frac{mRT_s}{P_s}$

$= \frac{4.5 \times 287 \times 288}{1.013 \times 10^5}$

$FAD = 3.68\ m^3/min$

$V_{s(HP)} = ?$

$\eta_{vol} = 0.985$

$P_i V = mRT_i$

$V = \frac{mRT_i}{P_i} = \frac{4.5 \times 287 \times 288}{3 \times 1.013 \times 10^5} = 1.59\ m^3/min$

($P_i = 3P_s$)

$\eta_{vol} = \frac{FAD}{V_s} \Rightarrow V_s = \frac{1.59}{0.985} = 1.61\ m^3/min$

$\therefore V_s = 0.0053\ m^3$

(12) A single acting 2-stage compressor with complete inter cooling delivered 10.5 kg/min at 16 bar. The suction occurs at 1 bar and 27°C, n is 1.3, N is 440 rpm: calculate

a) power required to drive the compressor

b) Isothermal efficiency c) free air delivery

d) heat transfered in inter cooler e) calculate swept volume and clearance volume of each cylinder. If clearance ratio's for L.P and HP are 0.04 & 0.06 respectively.

5Q

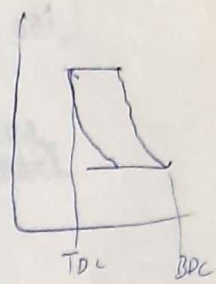
$m = 10.5\ kg/min$

$P_d = 16\ bar$

$P_s = 1\ bar$

$T_s = 27^\circ c$, $n = 1.3$, $N = 440\ rpm$,

$K_{L.P} = 0.04$, $K_{HP} = 0.06$.



$$P_i = \sqrt{P_s P_d}$$

$$P_i = 4\ bar$$

$$\frac{T_i}{T_s} = \left(\frac{P_i}{P_s}\right)^{\frac{n-1}{n}} \Rightarrow T_i = 300 \times \left(\frac{4}{1}\right)^{\frac{1.3-1}{1.3}}$$

$$\therefore T_i = 413.10\ K$$

$$\frac{T_d}{T_i} = \left(\frac{P_d}{P_i}\right)^{\frac{n-1}{n}} = T_d = 413.10 \times \left(\frac{16}{4}\right)^{\frac{1.3-1}{1.3}}$$

$$\therefore T_d = 568.84\ K$$

$$\text{work done} = \frac{2n}{n-1} P_s V_s \left[\left(\frac{P_d}{P_s}\right)^{\frac{n-1}{2n}} - 1\right]$$

$$= \frac{2 \times 1.3}{1.3-1} \times 1 \times 10^5 \times 9.04 \left[\left(\frac{16}{1}\right)^{\frac{1.3-1}{2 \times 1.3}} - 1\right]$$

$P_s V_s = m R T_s$

$1 \times 10^5 \times V_s = 10.5 \times 287 \times 300$

$V_s = 9.04\ m^3/min$

$$W/d = 2953743.38\ N\text{-}m/min$$

$$\text{power} = \frac{2953743.38}{60}$$

$$\therefore P = 49.22\ kW$$

(2) $\eta_{iso} = \frac{\text{Isothermal work}}{\text{Indicated work}} \times 100$

Isothermal work (~~Indicated~~) $= P_s V_s \ln\left(\frac{P_d}{P_s}\right)$

$= 1\times10^5 \times 9.04 \ln\left(\frac{16}{1}\right)$

$= 2506420.20$ N-m/min

$\eta_{iso} = \frac{2506420.20}{2953743.38} = 0.848$

$\therefore \eta_{iso} = 84.8\%$

(3) Free air delivery

$P_s(V_1 - V_4) = mRT_s$

$V_1 - V_4 = \frac{mRT_s}{P_s}$

$= \frac{10.5 \times 287 \times 300}{1\times10^5}$

$\therefore$ FAD $= 9.04\ m^3/min$

(4)

$\frac{T_2}{T_s} = \left(\frac{P_i}{P_s}\right)^{\frac{n-1}{n}}$

$T_2 = 300 \times \left(\frac{4}{1}\right)^{\frac{1.3-1}{1.3}} = 413.10\ K$

$Q = mC_p(T_s - T_2)$

$= 10.5 \times 1005 (300 - 413.10)$

$= -1193487.75$ J/min

$= -19891\ W \Rightarrow 19.89\ kW$ (-ve)

$$\eta_{vol} = 1 + K - K \left(\frac{P_i}{P_s}\right)^{\frac{n-1}{n}}$$

$$= 1 + 0.04 - 0.04 \left(\frac{4}{1}\right)^{\frac{1}{1.3}}$$

$$= 0.923 \Rightarrow 92.3\%$$

$$V_{s(LP)} = \frac{FAD}{\eta_{vol}} = \frac{9.04}{0.923} = 9.79\ m^3/min$$

$$\therefore V_{s(LP)} = 0.022\ m^3$$

$$\eta_{vol} = 1 + K - K \left(\frac{P_d}{P_i}\right)^{1/n}$$

$$= 1 + 0.06 - 0.06 \left(\frac{16}{4}\right)^{\frac{1}{1.3}}$$

$$\eta_{vol} = 88.5\%$$

$$P_i \overset{FAD}{V} = mRT_s$$

$V_{s(HP} = 0.000$

$$V = \frac{10.5 \times 287 \times 300}{4 \times 10^5}$$

$$\therefore V = 2.26\ m^3/min$$

$$V_s = \frac{2.26}{0.88} = 2.55\ m^3/min$$

$$= \frac{2.55}{400}\ m^3$$

$$\therefore V_s = 0.00057\ m^3$$