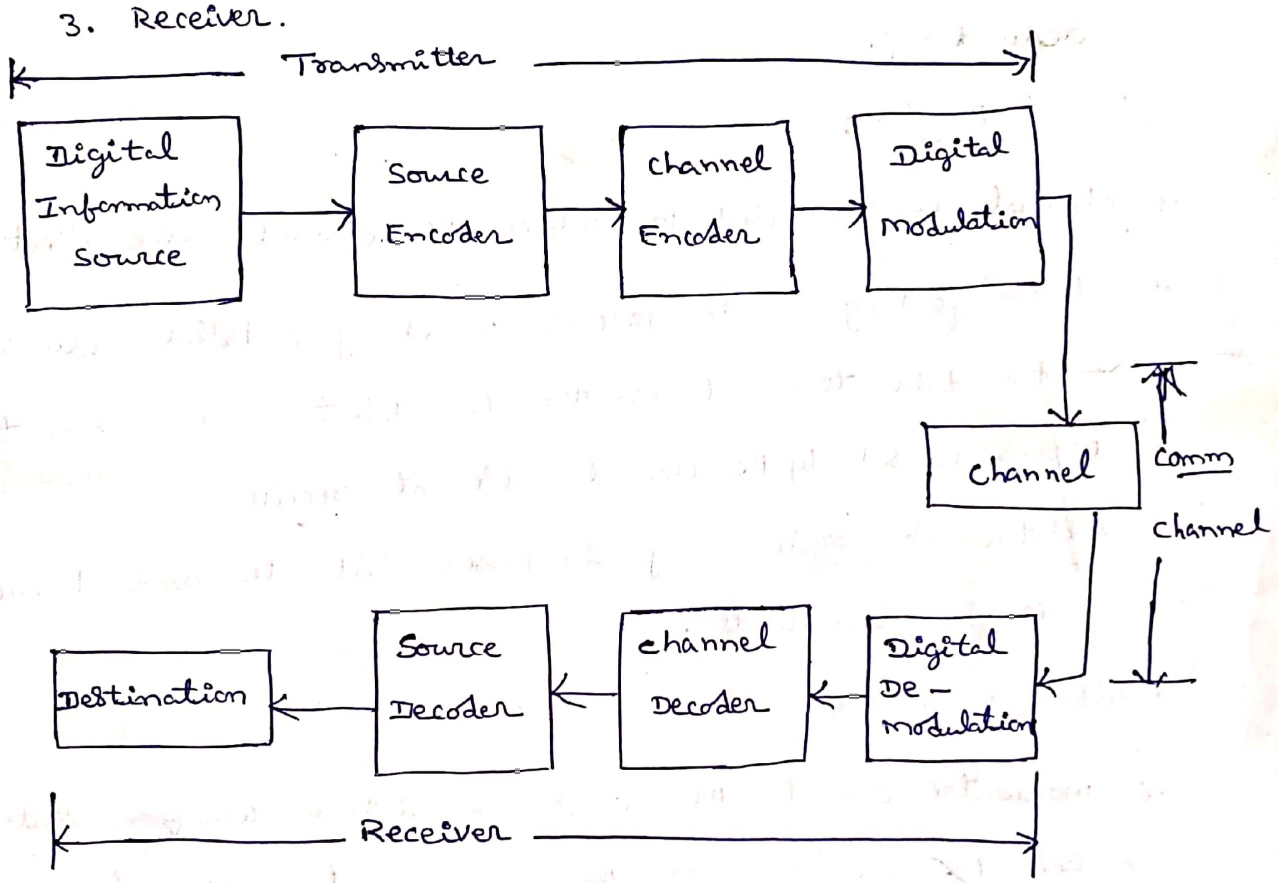
Digital Communication Systems

UNIT - I

Elements of Digital communication System:

* The block diagram shown below consists of three main blocks 1. Transmitter 2. Communication channel and 3. Receiver.



Digital Information Source:

* The source of information is assumed to be digital ie symbols, letters.
* If input is analog signal, then it is converted into digital form by using sampler and quantizer.
* The source of information are human voice, Television pictures teletype data, atmospheric temperature and pressure etc.

Source Encoder :

* Digital information coming out of source consists of lots of redundancy which when transmitted as it is results in "Improper utilization of bandwidth" hence results in poor efficiency.
* The objective of source encoder is to eliminate or reduce redundancy.

Channel Encoder :

* Channel encoder used to reduce the channel noise effect
* Channel coding is the process of adding controlled redundancy to the data to be transmitted to detect and or correct the errors caused by the channel noise at receiver.
* Addition of redundancy increases bit rate and hence increase bandwidth.

Digital modulator :

* modulator converts the bit stream into a waveform suitable for transmission over the communication channels.

Eg : Free - Space ; twisted wire cables, Coaxial cable, waveguide, optical fiber cable etc.

↓

Communication channel Eg:

Eg : ASK, FSK, PSK, QPSK etc.

Communication channel : It is the media through which signal can be transmitted.

Digital Demodulation :

* Demodulator converts the waveform into digital data (optimum detectors are used to minimize the probability of error).

Channel Decoder :

* The sequence of numbers then passed through the channel decoder.
* Decoder detects the error in the received data & correct the error.
* To reconstruct the original information sequence from the knowledge of the code used by the channel encoder and the redundancy contained in the received data.

Source Decoder :

* Decoder converts the codes back to symbols ie converts digital information to discrete symbols.
* At the end if an analog signal is desired then source decoder tries to decode the sequence from the knowledge of the encoding algorithm.
* and which results in the approximate replica of the input at the transmitter end.

Destination

* Finally we get the desired signal in desired format analog or digital.

Advantages and Disadvantages of Digital communication:

Advantages

* The digital communication systems are simpler and cheaper compared to analog communication systems because of the advances made in the IC technologies.
* In digital communication the speech, video and other data may be merged and transmitted over a common channel using multiplexing.
* Using data encryption, only permitted receivers may be allowed to detect the transmitted data. This property is of its most importance in military applications.
* Since the transmission is digital and the channel encoding is used, therefore noise does not accumulate from repeater to repeater in long distance communications.
* Since the transmitted signal is digital in nature, therefore a large amount of noise interference may be tolerated.
* Since in digital communication, channel coding is used therefore the errors may be detected and corrected in the receivers.
* Digital communication is adaptive to other advanced branches of data processing such as DSP, image processing and data compression etc.

Disadvantages of Digital communication :

* Due to analog to digital conversion, the data rate becomes high. Therfore more transmission bandwidth is required for digital communication.
* Digital communication needs synchronization in case of synchronous modulation.

Comparison of Analog and Digital communication Systems :

S.NO	Parameter	AC System	DC System
1	Bandwidth	Less	more
2	Error Detection & correction	Not Possible	Possible
3.	Immune to Noise	Less	more
4.	complexity	more	Less
5.	Cost	more	Less
6.	Quality of Reconstruction	Good	Very Good
7.	Synchronization	Not required	Required
8.	Data Security	Not possible	Possible
9.	Flexibility & Reliability	Less	more
10.	Power required	more	Less
11.	Multiplexing	FDM (Complex Process)	TDM (Best Suited)
12.	Implementation	Difficult to implement analog CKts	Easier to Implement digital CKts.
13.	Programmable	Not possible	Possible
14.	Amplitude & Time	Amplitude and time in message vary continuously w.r.t. to time	Both Amplitude & time take discrete values.

Analog to Digital conversion:

* In communication systems, some times it happens that the analog signal to digital signal for transmission.
* In such cases we have to convert an analog signal into digital signal ie we have to convert a continuous time signal in the form of digits.

Sampling : Sampling is a process where an analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time. ie process of converting continous time signal into discrete time signal.

* According to sampling theorem let us consider analog signal, we mark the time instants t_0, t_1, t_2 ---- at equal time interval along the time axis.

* At each of these time instants, the magnitude of the signal is measured and thus samples of the signal are taken.
* Fig (b) shows a representation of the signal of fig (c) in terms of its samples.

* In fig (b) is defined only at the sampling instants. This means that it no longer is a continous function of time but rather it is a discrete time signal

* By using quantization, the total amplitude range which the signal may occupy is divided into a number of standard levels.

* In fig (c), amplitudes of the signal $x(t)$ lies in the range $(-mp, mp)$ which is partitioned into 'L' intervals, each of magnitude $\Delta v = \frac{2mp}{L}$

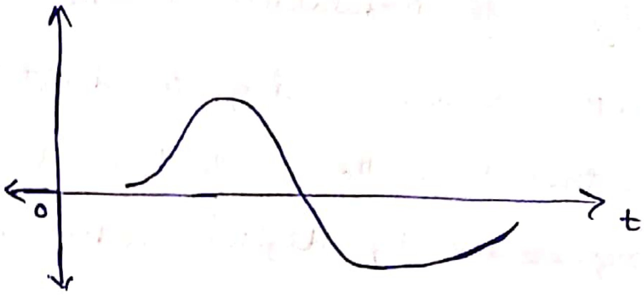


Fig (a) An Analog signal

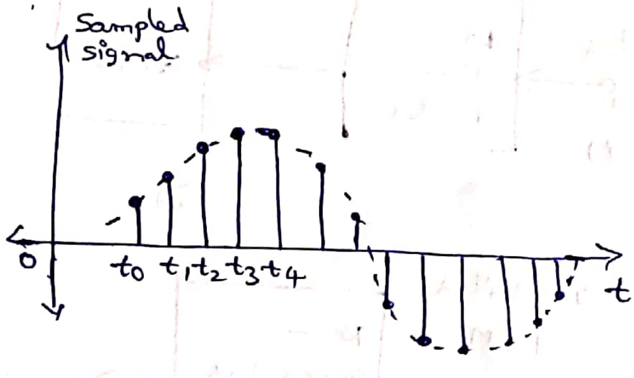


Fig (b) Samples of analog signal

* Now each sample is approximated (or) rounded off to the nearest Quantized level as shown in figure.

* Since each Sample is now approximated to one of the 'L' numbers, therefore the information is digitized.

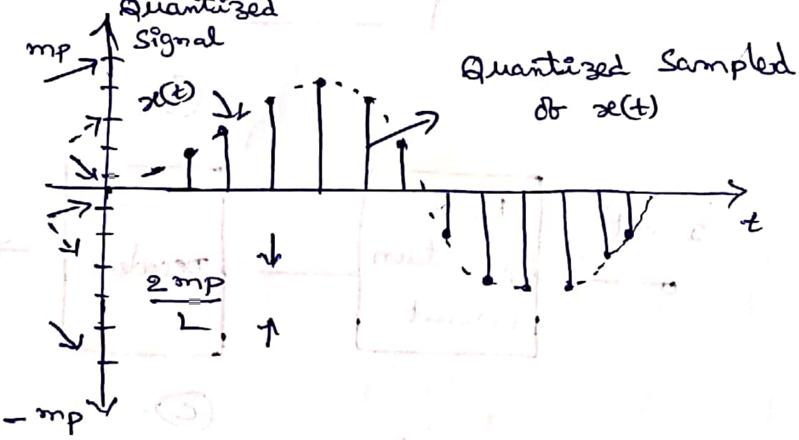


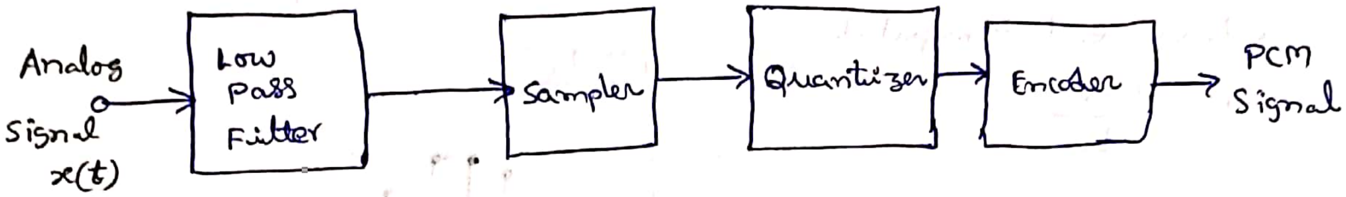
Fig (c) Quantization.

* The quantized signal is an approximation of the original signal.

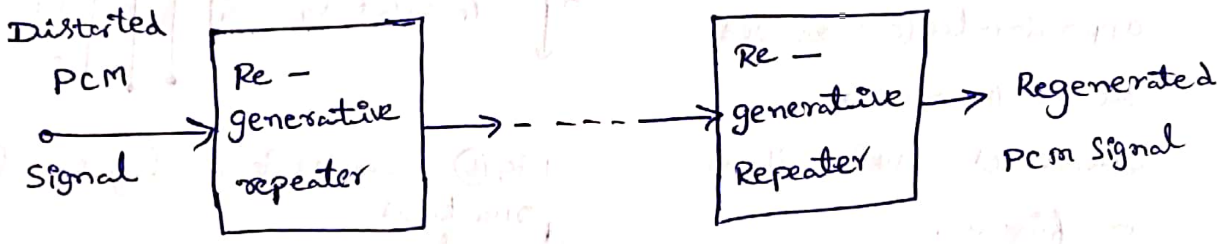
* In order to improve the accuracy of the quantized signal to any desired degree simply by increasing the number of levels. 'L'.

Pulse code Modulation (PCM):

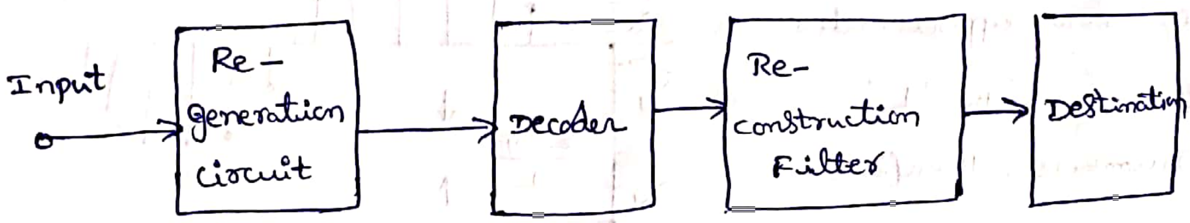
* PCM is an analog to digital converter where the information contained in the instantaneous samples of an analog signal are represented by digital codes in a serial bit stream manner.



(a)



(b)



(c)

The basic elements of a PCM System (a) Transmitter (b) Transmission Path (c) Receiver

* Few Important points

1. PCM is a type of pulse modulation like PAM, PWM(or) PPM but there is an important difference between them PAM, PWM (or) PPM are 'analog pulse modulation' Systems where as PCM is a digital pulse modulation System.

2. The PCM output is in the coded digital form. It is in the form of digital pulses of constant amplitude, width and position.

Scanned with CamScanner

3. The information is transmitted in the form of code words. I - (5)

A PCM system consists of a PCM encoder and a PCM decoder.

4. The essential operations in the PCM transmitter are sampling quantizing and encoding.

5. All the operations are usually performed in the same circuit called as analog to digital converter.

6. It should be understood that the PCM is not modulation in the conventional sense.

7. Because in modulation, one of the characteristics of the carrier is varied in proportion with the amplitude of the modulating signal.

PCM Transmitter :

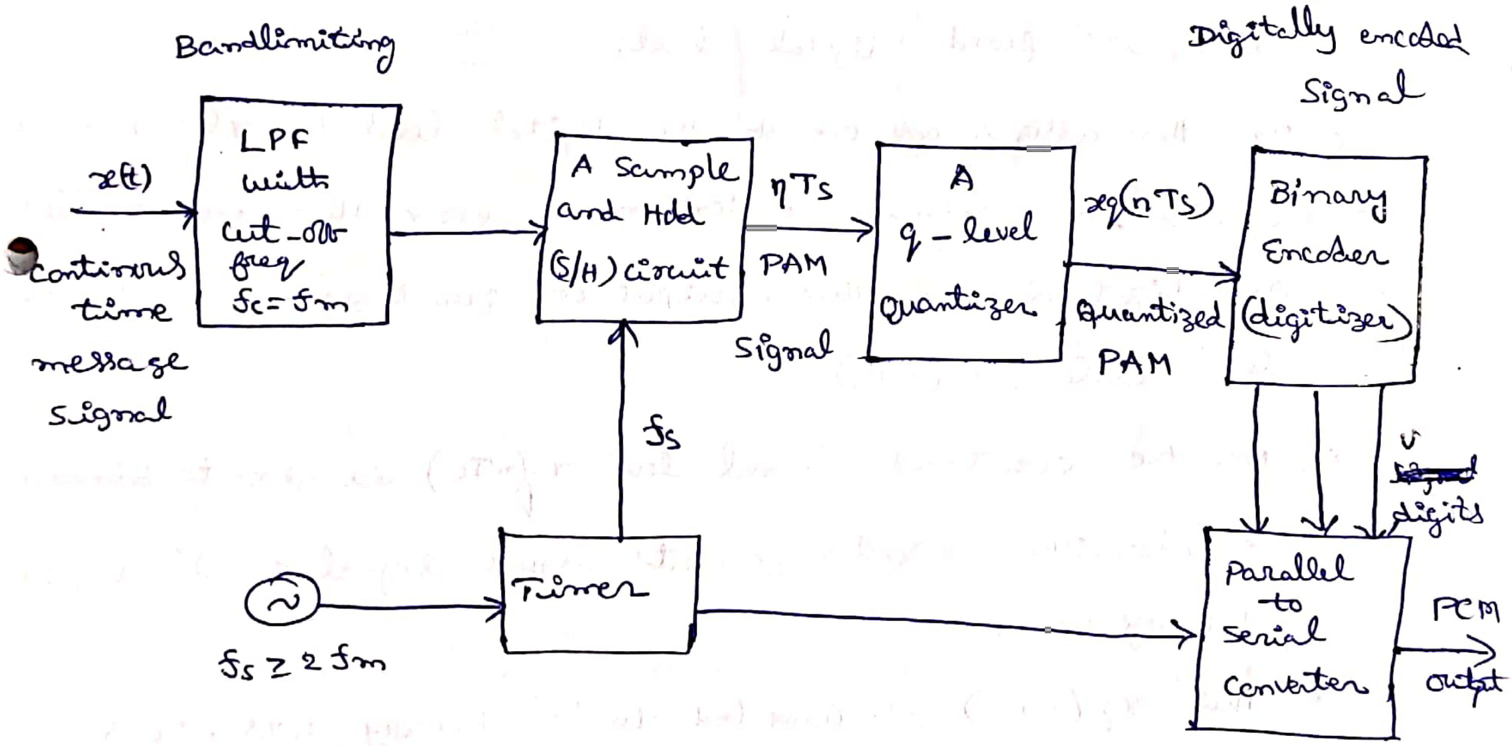


Fig : PCM generator. $r = u f_s$

* The signal $x(t)$ is first passed through the low pass filter of cutoff frequency f_m Hz.

* This low pass filter blocks all the frequency components which are lying above 'f_m Hz'.

* The signal $x(t)$ is bandlimited to f_m Hz.

* The sample and hold circuit then samples this signal at the rate of 'f_s'.

* Sampling frequency 'f_s' is selected sufficiently above nyquist rate to avoid aliasing ie

$$f_s \geq 2 f_m .$$

* The output of sample and hold circuit is denoted by $x(nT_s)$. This signal $x(nT_s)$ is discrete in time and continuous in amplitude.

* A q-level quantizer compares input $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level to $x(nT_s)$ with its fixed digital levels.

* It then assigns any one of the digital level to $x(nT_s)$ which results in minimum distortion or error. This error is called Quantization error. Thus, output of quantizer is a digital level called $x_q(nT_s)$.

* Now the quantized signal level $x_q(nT_s)$ is given to binary encoder. This encoder converts input signal to 'v' digits binary word.

* Thus $x_q(nT_s)$ is converted to 'v' binary bits. This encoder is also known as digitizer.

PCM Transmission Path :

* The path between the PCM transmitter and PCM receiver over which the PCM signal travel, is called as "PCM transmission path".
* The most important feature of PCM system lies in its ability to control the effects of distortion and noise when the PCM wave travels on the channel.
* The repeaters are spaced close enough to each other on the transmission path.
* The regenerative performs three basic operations namely equalization, timing and decision making.

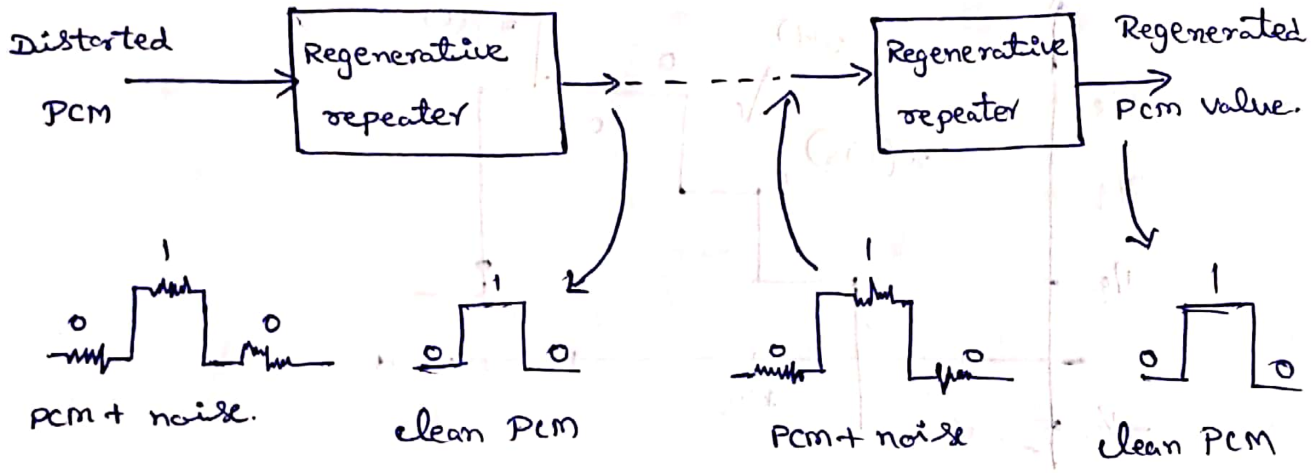


Fig : PCM Transmission Path.

* Hence each repeater actually reproduces the clean noise free PCM signal from the PCM signal distorted by the channel noise.
* This improves the performance of PCM in presence of noise.

PCM Receiver :

* The regenerator at the start of PCM receiver reshapes the pulse and removes the noise.
* This signal is then converted to parallel digital words for each sample.

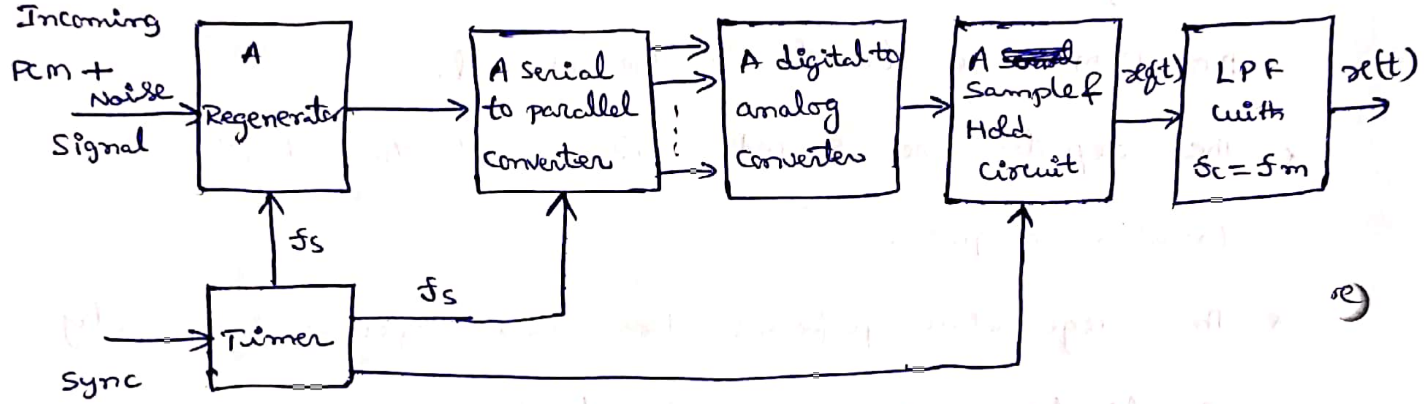


Fig : PCM Receiver

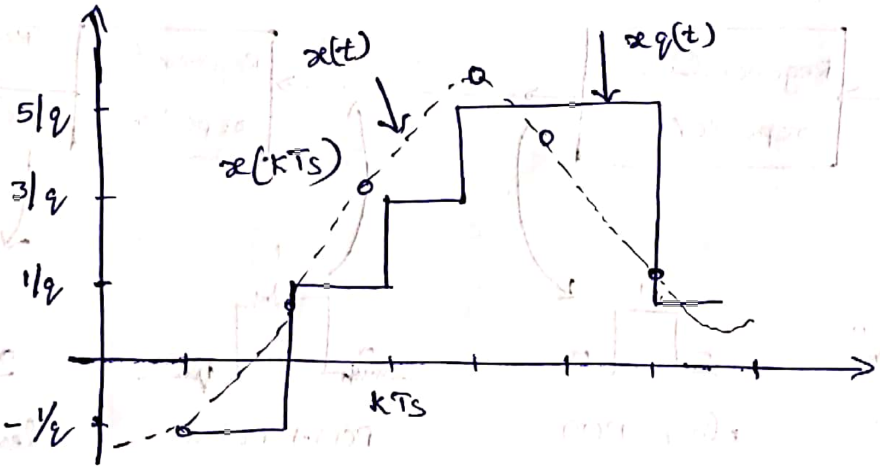


Fig : Reconstructed waveform.

* The digital word is converted to its analog value denoted as $x_q(t)$ with the help of a sample and hold circuit.
* This signal at the output of sample and hold circuit, it allowed to pass through a low pass reconstruction filter to get the appropriate original message signal denoted as $x(t)$.

Application of PCM:

* Digital information is better transmitted in its digital form because converting the signal to analog and sending it through an analog network can be costly.
* Digital data is easily compressed; therefore it can be transmitted using a small bandwidth.
* Because of the nature of devices used to boost the signal strength during transmission, error performance is much improved when compared with analog.
* It is also better to transmit information in digital form because computer components used in the transmission process are very reliable.
* The microphone and line-in circuits on a sound card generate PCM samples, and all sound cards require PCM for output.
* Compressed audio formats such as MP3 and AAC are converted to PCM first and the second card converts the PCM to analog for the speakers.

Limitations of PCM:

* Choosing a discrete value near the analog signal for each sample (Quantization error)
* Between samples no measurement of the signal is made; due to the sampling theorem this results in any frequency above fs (sampling frequency) being distorted (or) lost completely.

Companding in PCM Systems :

* Companding is nonuniform quantization. It is required to be implemented to improve the signal to quantization noise ratio of weak signals.
* we know that the quantization noise is given by

$$N_q = \frac{\Delta^2}{12}$$

* This shows that in the uniform quantization, once the step size is fixed, the quantization noise power remains constant.
* However the signal power is not constant, it is proportional to the square of signal amplitude.
* Hence signal power will be small for weak signals, but quantization noise power is constant.
* Therfore the signal to quantization noise for the weak signals is very poor. This will affect the quality of signal.
* Companding is a term derived from two words ie compression and expansion as under.

Companding = compressing + Expanding

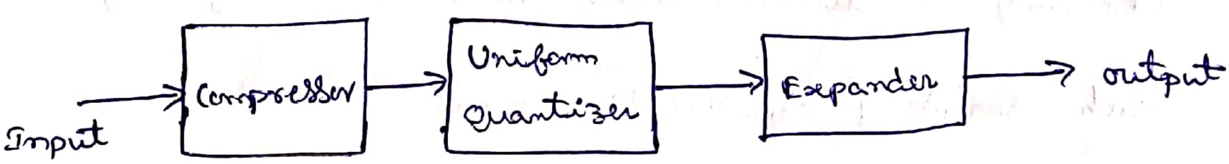


Fig : A companding model.

* The weak signals are amplified and strong signals are attenuated before applying them to a uniform quantizer. This process is called as 'compression' and the block called 'compressor'.

* At the receiver exactly opposite is followed which is called expansion. The circuit used for expansion at the receiver is combined to be called '<u>Companding</u>'.

* For providing expansion is called as an '<u>expander</u>'.

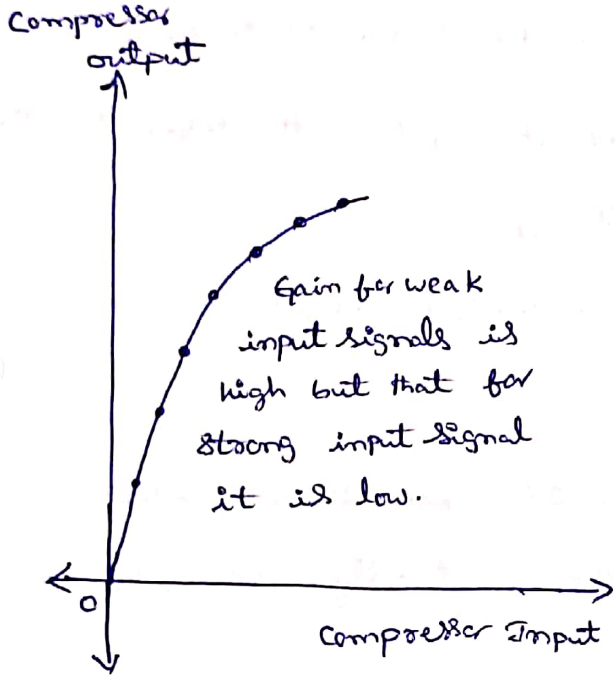


Fig: Compressor characterstics

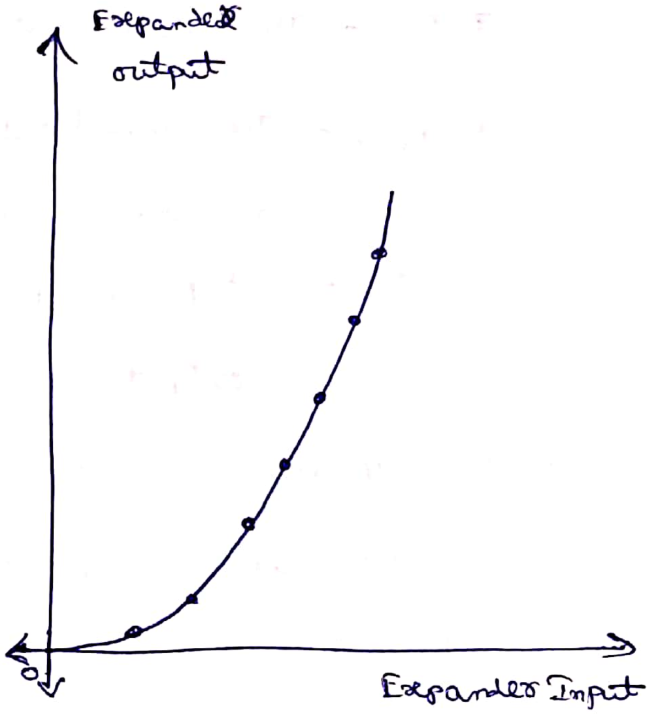


Fig: Expander characterstics.

<u>Compressor characterstics</u>:

* Ideally, we need a linear compressor charactorstics for small amplitudes of the input signal. There are two methods.

  ① μ - law Companding ② A - Law Companding

<u>μ - Law Companding</u>:

* In the μ - law Companding, the compressor characterstics is continous.

* It is approximately linear for smaller values of input levels and logarthemic for high input levels.

* The μ-law compressor characteristic is mathematically expressed as:

$$\frac{|y|}{x_{max}} = z(x) = (\operatorname{sgn} x)\,\frac{\ln\left(1+\mu|x|/x_{max}\right)}{\ln(1+\mu)}$$

where $0 \le |x|/x_{max} \le 1$

$z(x)$ – The output and 'x' is the input to the compressor

$|x|/x_{max}$ – The normalized value of input with respect to the maximum value 'x_{max}'.

$(\operatorname{sgn} x)$ – ± 1 ie Positive & negative values of input and output.

* The μ-law compressor characteristics for different values of 'μ' is shown in below figure, The practically used value of 'μ' is 255.

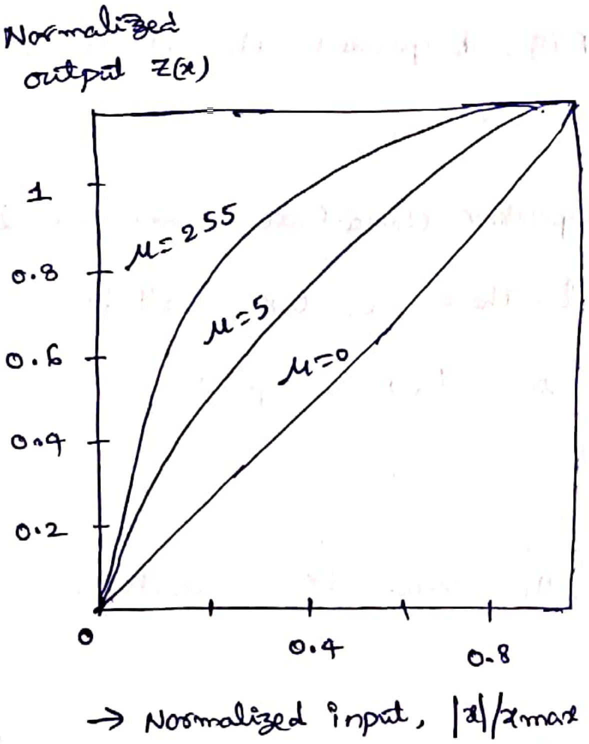


Fig (a): Compressor characteristic of a μ-law compressor

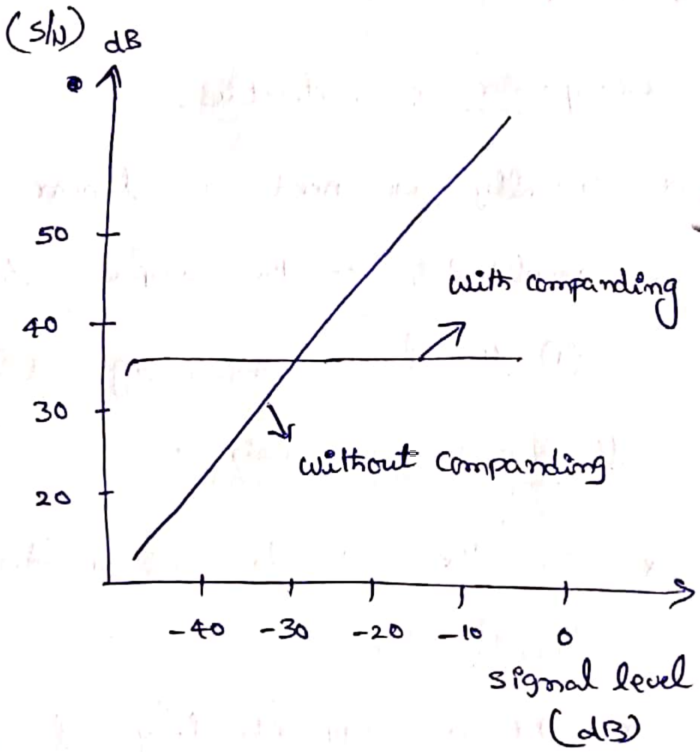


Fig (b): PCM performance with μ-law companding

* The characteristics of $\mu = 0$ correspons to the uniform quantization.
* The μ-law Companding is used for speech and music signals.
* Fig 6 shows the variation of signal to quantization noise ratio with respect to signal level, with and without companding.
* The SNR is almost constant at all the signal levels when companding is used.

A-Law Companding :

* In the A-law Companding, the compressor characteristic is piecewise, made up of a linear segment for low level inputs and a logarthemic segment for high level inputs.
* Below figure shows A-law compressor characteristics for different values of 'A'.

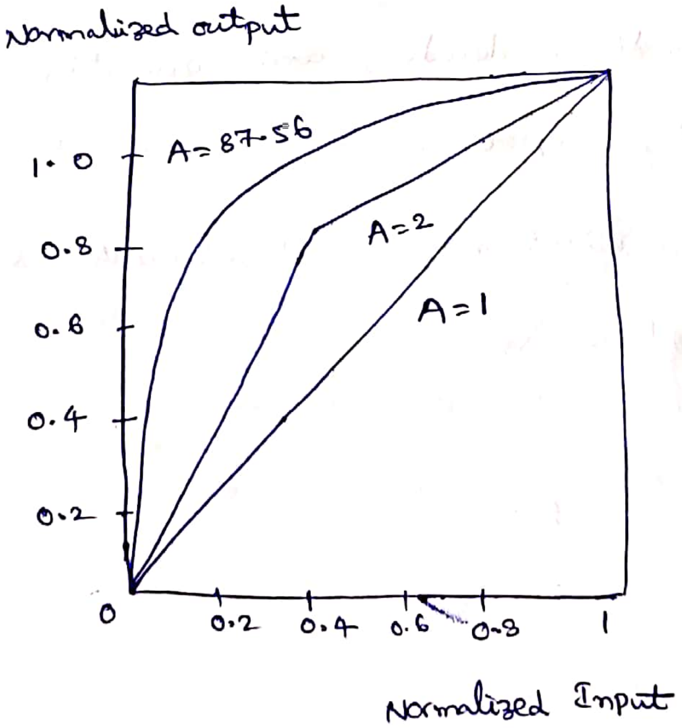


Fig : Compressor characteristic of A-law compressor.

* If $A = 1$, the characterstics is linear which correspons to a uniform quantization.

* The practically used value of 'A' is 87.56.

* The A-law companding is used for PCM telephone systems in Europe.

* The linear segment of the characterstics is for low level inputs whereas the logarthemic segments is for high level input.

* It is mathematically expressed as

$$\frac{Z(x)}{x_{max}} = \begin{cases} \dfrac{A|x|/x_{max}}{1+\log_e A} & \text{for } 0 \le \dfrac{|x|}{x_{max}} \le 1 \\[2ex] \dfrac{1+\log_e\left[A|x|/x_{max}\right]}{1+\log_e A} & \text{for } \dfrac{1}{A} \le \dfrac{|x|}{x_{max}} \le 1 \end{cases}$$

Drawbacks of PCM:

1. The encoding, decoding and quantizing circuitry of PCM is complex.

2. PCM requires a large bandwidth as compared to the other systems.

*. let us define

Delta Modulation:

* In PCM that it transmits all the bits which are used to code a sample.
* Hence, signalling rate and transmission channel bandwidth are quite large in PCM. to overcome this problem delta modulation is used.

Working Principle:

* Delta modulation transmits only one bit per sample.
* Here, the present sample value is compared with the previous sample value and this result whether the amplitude is increased or decreased is transmitted.
* Input signal $x(t)$ is approximated to step signal by the delta modulator. This step size is kept fixed.
* The difference between the input signal $x(t)$ and stair case approximated signal is confined to two levels ie '$+\Delta$' and '$-\Delta$'.
* Now, if the difference is positive, then approximated signal is increased by one step ie 'Δ'. If the difference is negative, then approximated signal is reduced by 'Δ'.
* when the step is reduced, '0' is transmitted and if the step is increased '1' is transmitted.
* Hence for each sample, only one binary bit is transmitted.

Mathematical Expressions :

* The principle of delta modulation can be explained with the help of few equations.

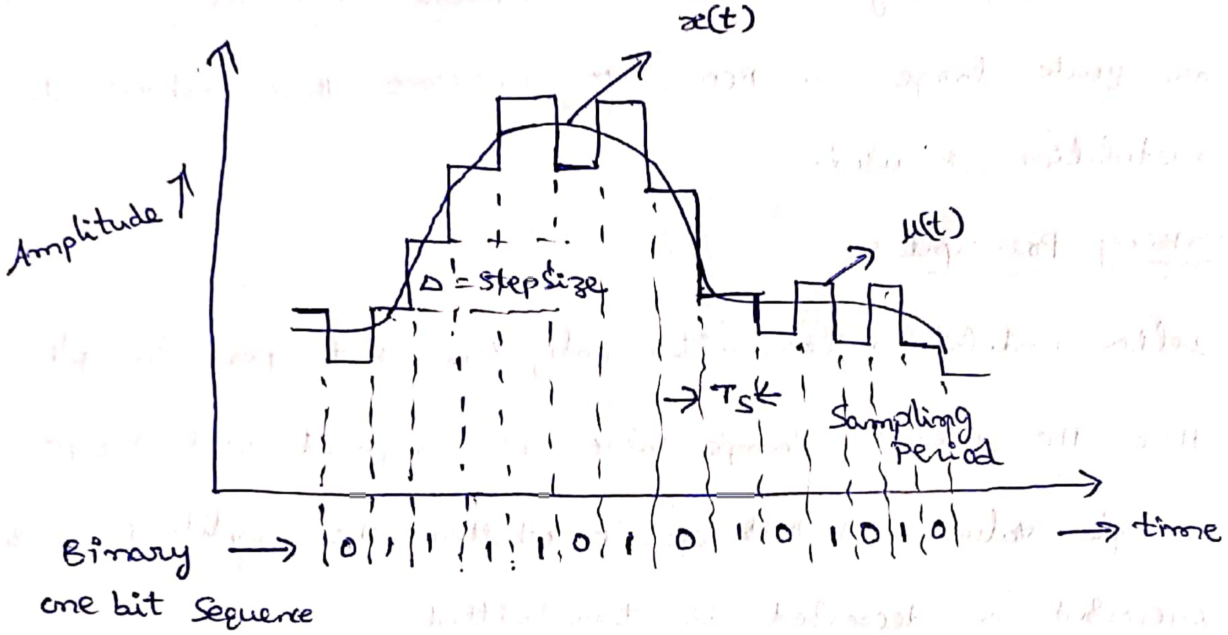


Fig : Delta modulation waveform

* The error between the sampled value of x(t) and last approximated sample is given as

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \rightarrow (1)$$

where $e(nT_s)$ = error at present sample

$x(nT_s)$ = Sampled signal of $x(t)$

$\hat{x}(nT_s)$ = Last sample approximation of the staircase waveform.

* Assume that $u(nT_s)$ as the present sample approximation of staircase output, then

$$u[(n-1)T_s] = \hat{x}(nT_s) \rightarrow (2)$$

= last sample approximation of staircase waveform

* Let us define quantity of $b(nTs)$ is such away that

$$b(nTs) = \Delta \, Sgn[e(nTs)] \rightarrow (3)$$

* This means that depending on the sign of error $e(nTs)$ the sign of step size 'Δ' is decided. we can write

$$b(nTs) = \begin{cases} +\Delta & \text{if } x(nTs) \geq \hat{x}(nTs) \\ -\Delta & \text{if } x(nTs) < \hat{x}(nTs) \end{cases}$$

* Also if $b(nTs) = +\Delta$ then a binary '1' is transmitted

$b(nTs) = -\Delta$ then a binary '0' is transmitted

Ts = Sampling Interval.

Transmitter part:

* Below fig shows the transmitter ie generation of delta modulated signal.

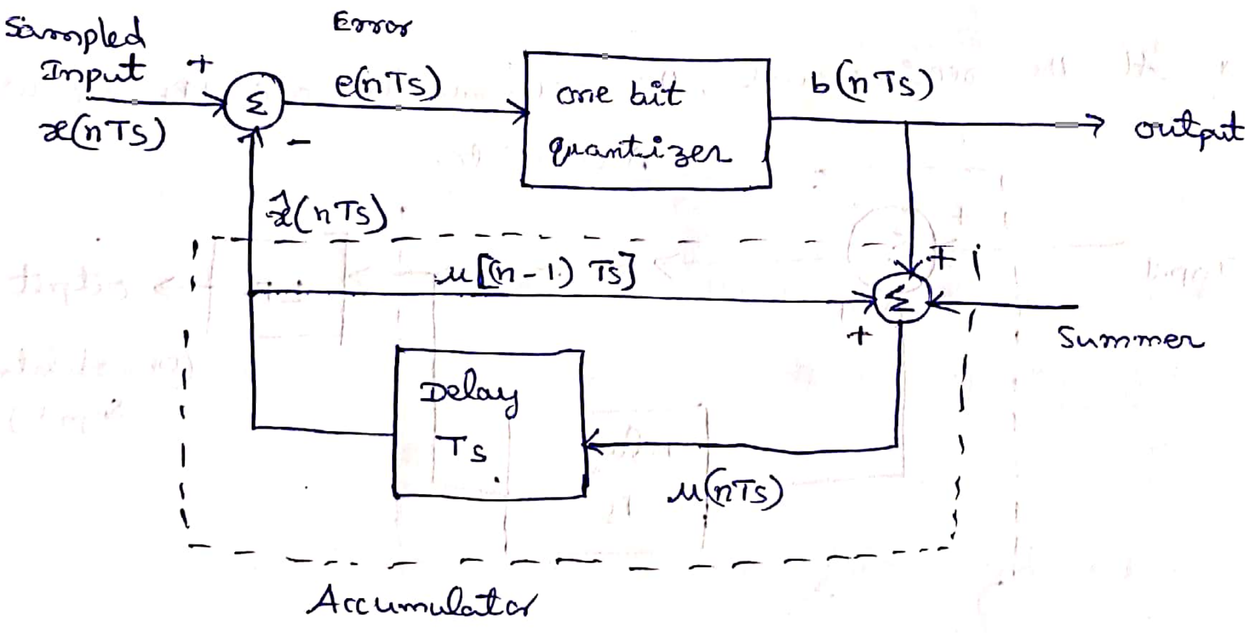


Fig : Delta modulation transmitter

* The summer in the accumulator adds quantizer output ($\pm\Delta$) with previous sample approximation.

* This gives present sample approximation ie.

$$u(nT_s) = u(nT_s - T_s) + [\pm \Delta]$$

or $$u(nT_s) = u[(n-1)T_s] + b(nT_s)$$

* The previous sample approximation $u[(n-1)T_s]$ is restored by delaying one sample period 'T_s'.
* The sampled input signal $x(nT_s)$ and staircase approximated signal $\hat{x}(nT_s)$ are subtracted to get error signal $e(nT_s)$.
* Thus depending on the sign of $e(nT_s)$, one bit quantizer generates an output of $+\Delta$ or $-\Delta$.
* If the step size is $+\Delta$, then binary '1' is transmitted and if it is $-\Delta$, then binary '0' is transmitted.

Receiver Section:

* At the receiver end, the accumulator and LPF are used.

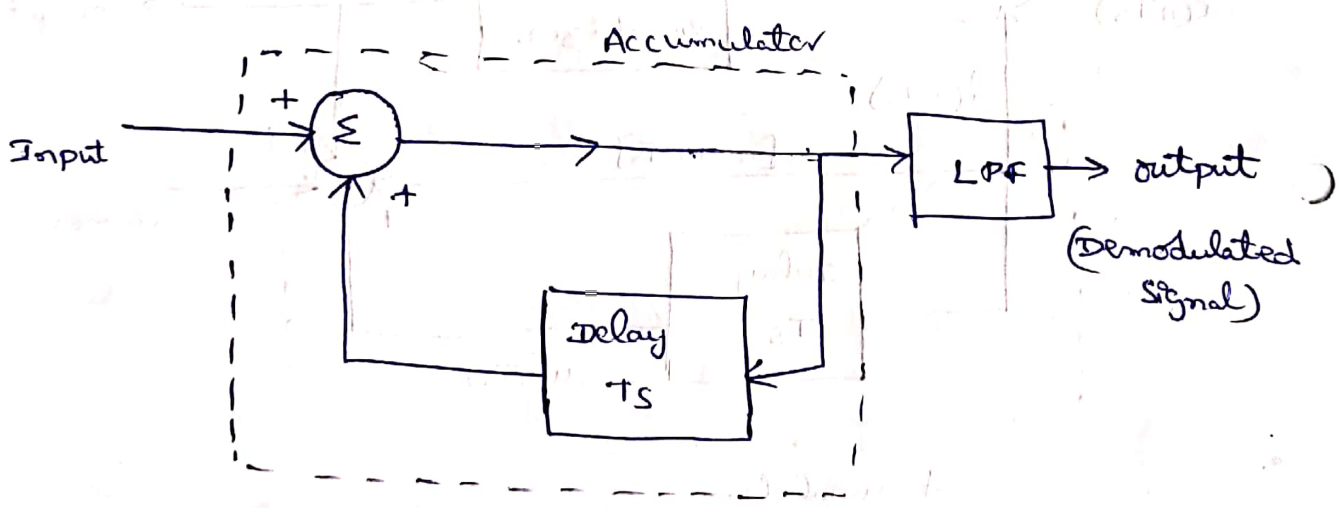


Fig: Delta modulation Receiver

* The accumulator generates the stair case approximated signal output and is delayed by one sampling period 'T_s'.
* It is then added to the input signal.

* If input is binary '1' then it adds to step to the previous output (which is delayed).
* If input is binary '0' then one step 'δ' is subtracted from the delayed signal.
* Also, the LPF has the cutoff frequency equal to highest frequency in $x(t)$.
* This LPF smoothens the staircase signal to reconstruct original message signal $x(t)$.

Advantages of Delta modulation:

* The delta modulation has certain advantages over PCM as

ⓐ The delta modulation transmits only one bit for one sample, therefore the signaling rate and transmission channel bandwidth is quite small for delta modulation compared to PCM

ⓑ The transmitter and receiver implementation is very much simple for delta modulation. There is no analog to digital converter required in delta modulation.

Drawbacks of Delta modulation:

* The delta modulation has two major drawbacks as under

① Slope overload distortion

② Granular (or) Idle noise.

Slope overload Distortion:

* This distortion arises because of large dynamic range of the input signal.
* The rate of rise of input signal $x(t)$ is so high that the staircase signal can't approximate it.
* The step size 'Δ' becomes too small for staircase signal $u(t)$ to follow the step segment of $x(t)$.

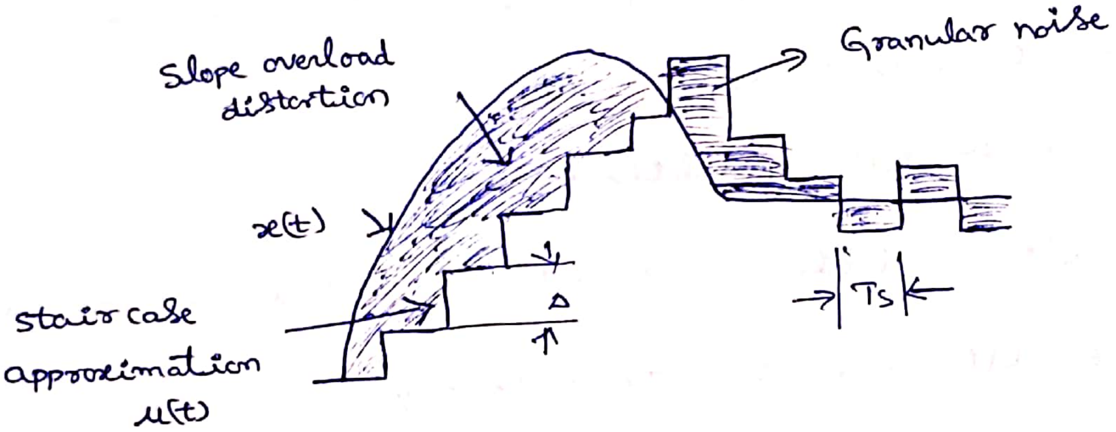


Fig : Quantization errors in delta modulation

* Hence, there is a large error between the staircase approximated signal and the original input signal $x(t)$.
* This error or noise is known as "Slope overload distortion"
* To reduce this error, the step size must be increased when slope of signal $x(t)$ is high.
* Since the step size of delta modulator remains fixed, its maximum or minimum slopes occur along straight lines.
* Therefore this modulator is known as "Linear Delta modulator (LDM)".

Granular or Idle Noise :

* Granular or Idle noise occurs when the step size is too large compared to small variations in the input signal.
* This means that for very small variations in the input signal, the staircase signal is changed by large amount (Δ) because of large step size.
* In fig when the input signal is almost flat, the staircase signal $u(t)$ keeps on oscillating by $\pm\Delta$ around the signal.
* The error between the input and approximated signal is called 'granular noise'. The solution to this problem is to make step size small.

Bit Rate (Signaling Rate) of Delta modulation :

Delta modulation bit rate (r) = Number of bits transmitted / Second

= Number of Samples / sec × No. of bits / sample

$= f_s \times 1 = f_s$

* Therefore the delta modulation bit rate is ($1/N$) times the bit rate of a PCM System, where 'N' is the number of bits per transmitted PCM codeword.
* Hence the channel bandwidth for the delta modulation system is reduced to a great extent as compared to that for the PCM System.

Adaptive Delta modulation :

* To overcome the quantization errors due to slope overload & granular noise, the step size (Δ) is made adaptive to variations in the input signal $x(t)$.

* Particularly in the steep segment of the signal $x(t)$, the step size is increased, Also if the input is varying slowly, the step size is reduced. This method is known as 'Adaptive Delta modulation (ADM)'.

* The Adaptive delta modulators can take continous changes in step size (or) discrete changes in step size.

Transmitter Section :

* In transmitter section the logic for step size control is added in the diagram.

* The step size increases or decreses according to a specified rule depending on the bit quantizer output.

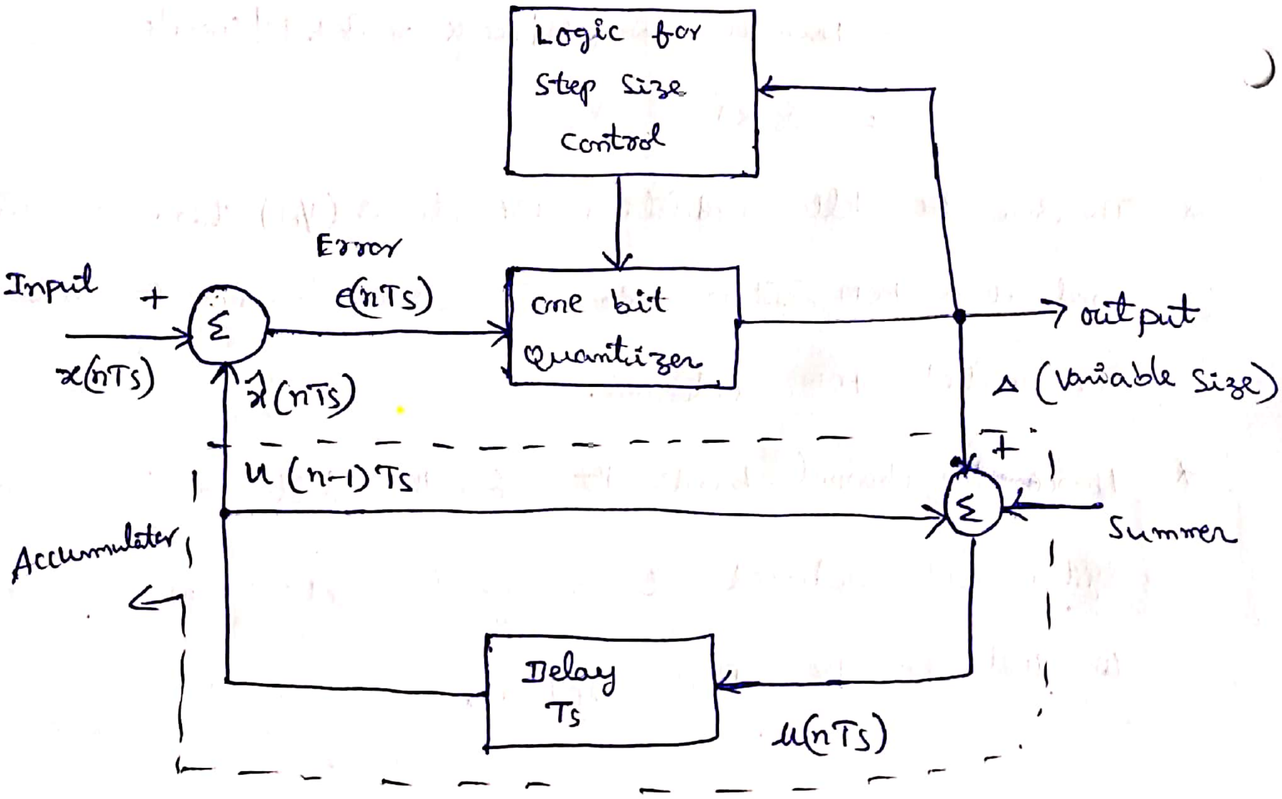


Fig : Adaptive Delta Modulator Transmitter

* If one bit quantizer output is high (ie 1), then step size may be doubled for next sample.
* If one bit quantizer output is low, then step size may be reduced by one step.

Receiver Section:

* In receiver section there are two portions. The first portion produces the step size from each incoming bit.
* Exactly the same process is followed as that in transmitter. The previous input and present input decides the step size.

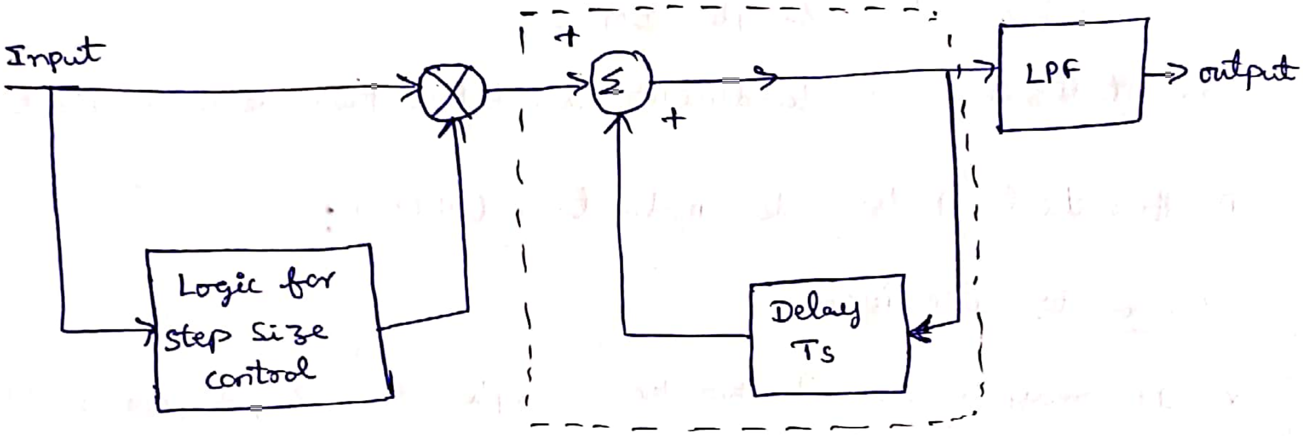


Fig: Adaptive Delta modulator Receiver

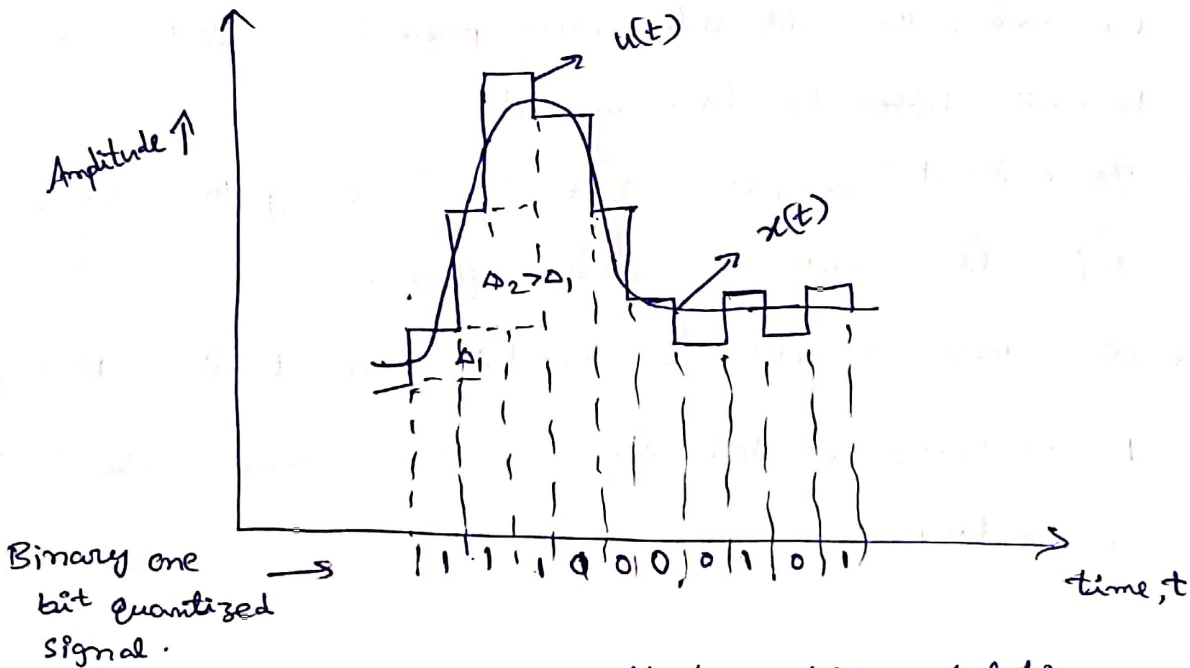


Fig: waveforms for Adaptive delta modulation.

* It is then applied to an accumulator which builds up staircase waveform.

* The LPF then smoothens out the staircase waveform to reconstruct the original signal.

Advantages of Adaptive Delta modulation:

1. The signal to noise ratio becomes better than ordinary delta modulation because of the reduction in slope overload distortion and idle noise
2. Because of the variable step size, the dynamic range of ADM is wider than simple DM.
3. Utilization of bandwidth is better than delta modulation.

Differential Pulse code modulation (DPCM):

Reason to use DPCM:

* It may be observed that the samples of a signal are highly correlated with each other.
* This is due to the fact that any signal does not change fast
* This means that its value from present sample to next sample does not differ by large amount.
* The adjacent samples of the signal carry the same information with a little difference.
* When these samples are encoded by a standard PCM system. the resulting encoded signal contains some redundant information.

Redundant Information in PCM :

* In below figure shows a continuous time signal $x(t)$ by dotted line. This signal is sampled by flat top sampling at intervals $T_s, 2T_s, 3T_s \ldots nT_s$.

* The sampling frequency is selected to be higher than nyquist rate. The samples are encoded by using 3 bit (8 levels) PCM.

* The encoded binary value of each sample is written on the top of the samples.

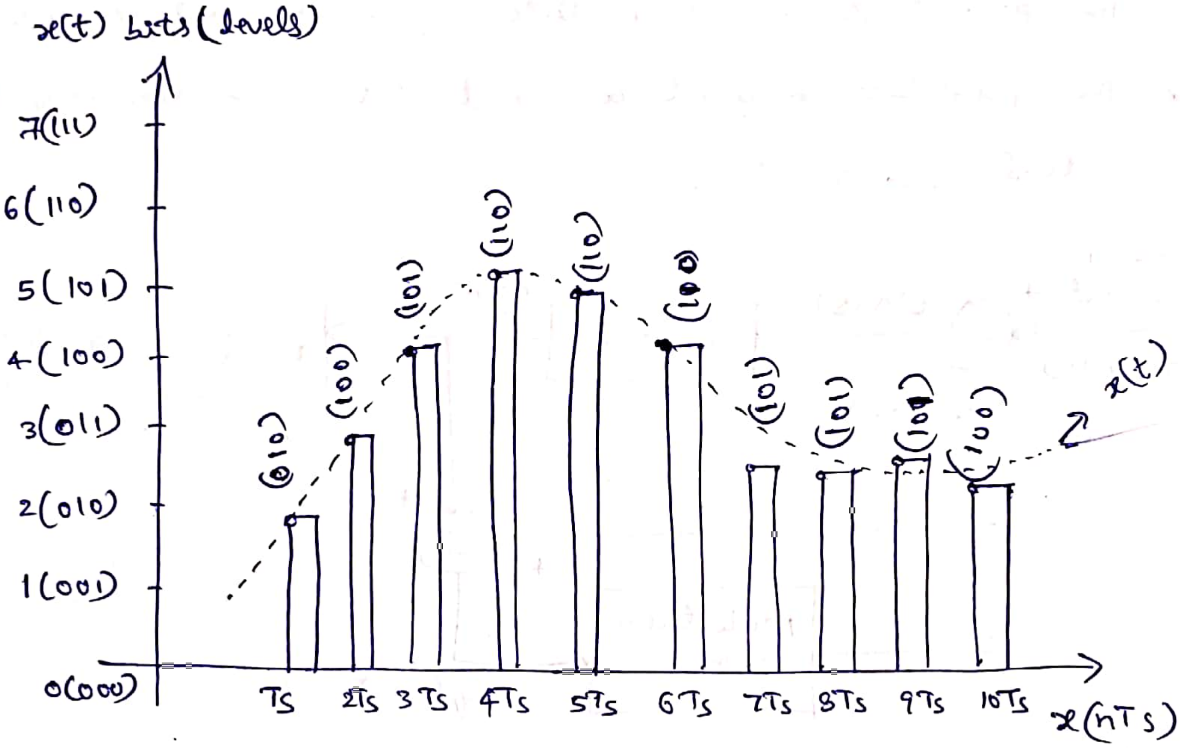


Fig : Redundant information in PCM

* The samples taken at 4TS, 5Ts and 6Ts are encoded to same value of (110). This information can be carried by one sample.

* But three samples are carrying the same information means that it is redundant.

* Consider another example of samples only due to last bit and first two bits are redundant, since they don't change in 9Ts & 10Ts.

* If this redundancy is reduced, then overall bit rate will decrease and number of bits required to transmit one sample will also be reduced.

* This type of digital pulse modulation scheme is known as Differential pulse code modulation (DPCM).

<u>Working Principle</u>:

* The DPCM works on the principle of prediction. The value of the present sample is predicted from the past samples.

* The prediction may not be exact but it is very close to the actual sample value.

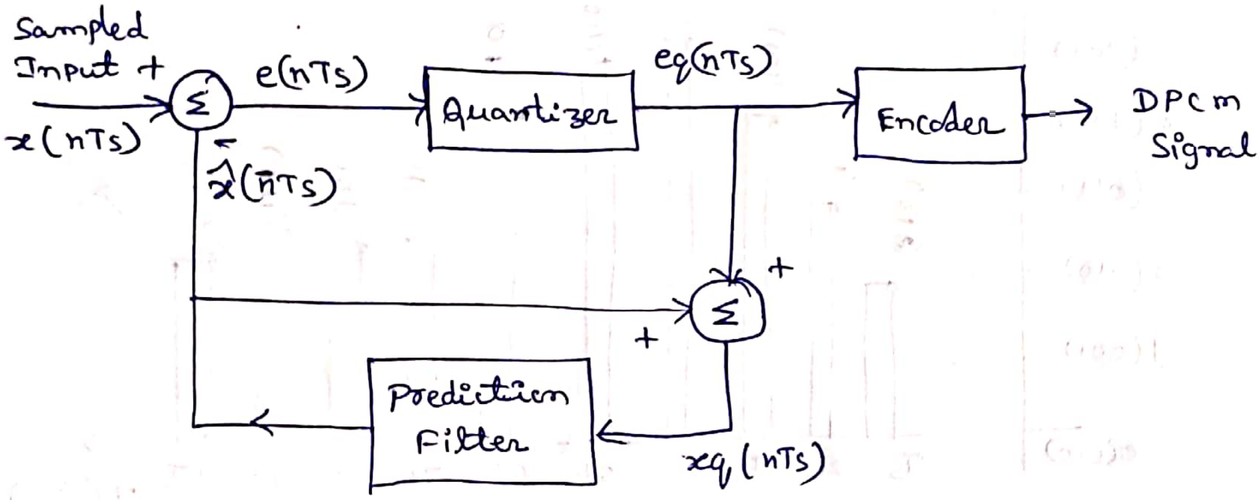


Fig: A DPCM Transmitter.

* The sampled signal is denoted by $x(nT_s)$ and the predicted signal is denoted by $\hat{x}(nT_s)$

* The comparator finds out the difference between and actual sample value $x(nT_s)$ and predicted sample value $\hat{x}(nT_s)$.

* This is known as prediction error and it is denoted by $e(nT_s)$. It can be defined as

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

* Thus, error is the difference between unquantized input sample $x(nT_s)$ and prediction of it $\hat{x}(nT_s)$. The predicted value is produced by using a prediction filter.

* The quantizer output signal gap $e_q(nT_s)$ and previous prediction is added and given as input to the prediction filter. This signal is called $x_q(nT_s)$.

* This makes the prediction more and more close to the actual sampled signal. The quantized error signal $e_q(nT_s)$ is very small.

* This can be encoded by using small number of bits. Thus number of bits per sample are reduced in DPCM.

* The quantizer output can be written as

$$e_q(nT_s) = e(nT_s) + q(nT_s) \longrightarrow ①$$

here $q(nT_s)$ is the quantization error.

* The prediction filter input $x_q(nT_s)$ is obtained by sum $\hat{x}(nT_s)$ and quantizer output ie

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s) \longrightarrow ②$$

Substitute eq ① in eq ② we get

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s) \longrightarrow ③$$

Recall

$$e(nT_s) = x(nT_s) - (\hat{x}T_s)$$

$$\therefore \quad e(nT_s) + \hat{x}(nT_s) = x(nT_s)$$

$\therefore$ The value of $e(nT_s) + \hat{x}(nT_s)$ from eq ③ we get

$$x_q(nT_s) = x(nT_s) + q(nT_s).$$

Reception of DPCM Signal :

* In below figure, the decoder first reconstructs the quantized error signal from incoming binary signal.

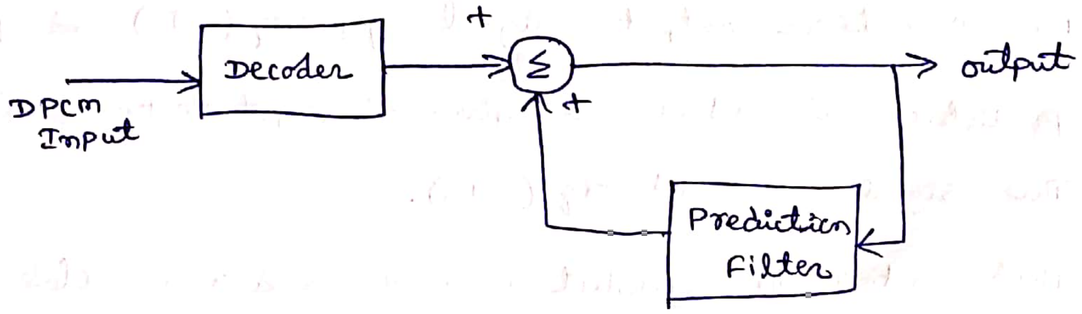


Fig : DPCM Receiver

* The prediction filter output and quantized error signals are summed up to give the quantized version of the original signal.
* Thus the signal at the receiver differs from actual signal by quantization error $q(nT_s)$, which is introduced permanently in the reconstructed signal.

SNR of DPCM :

* The output signal to quantization noise ratio for a DPCM system may be defined in a similar manner as that for a PCM system.

$$SNR = \frac{\text{Mean square value of signal}}{\text{Mean square value of quantization noise}} = \frac{\sigma_X^2}{\sigma_Q^2}$$

where σ_X^2 = Variance of original input signal $x(nT_s)$

σ_Q^2 = Variance of the quantization error $q(nT_s)$

$$SNR = \frac{\sigma_X^2}{\sigma_E^2} \times \frac{\sigma_E^2}{\sigma_Q^2}$$

σ_E^2 = Variance of the prediction error $e(nT_s)$

$\therefore$ $SNR = G_P (SNR)_P$

$G_P = \frac{\sigma_X^2}{\sigma_E^2}$ = Prediction gain $(SNR)_P = \frac{\sigma_E^2}{\sigma_Q^2}$ = Prediction error to quantization noise ratio.

Comparison between PCM, DM, ADM & DPCM:

S.NO.	Parameter of comparison	PCM	DM	ADM	DPCM
1.	Number of bits	It can use 4, 8 (or) 16 bits per sample	It uses only one bit for one sample	only one bit is used to encode one sample	Bits can be more than one but are less than PCM
2.	Levels and step size	The number of levels depend on no. of bits. Level size is kept fixed	Step size is kept fixed and can't be varied	According to the signal variation, step size varies (Adapted).	Here fixed no. of levels are used.
3.	Quantization error & distortion	Quantization error depends on no. of levels used.	Slope overload distortion & Idle noise are present	Quantization noise is present but other errors are absent	Slope overload distortion and quantization noise is present.
4.	Transmission Bandwidth	Highest bandwidth is required since no. of bits are high	Lowest bandwidth is required	Lowest bandwidth is required	Bandwidth required is lower than PCM.
5.	Feedback	There is no feedback in transmitter (or) Receiver	Feedback exists in transmitter	Feedback exists.	Here feedback exists
6.	Complexity of Implementation	System complex	Simple	Simple	Simple.

Advantages of DPCM :

1. As the difference between $x(nTs)$ and $\hat{x}(nTs)$ is being encoded and transmitted by the DPCM technique, a small difference voltage is to be quantized and encoded.

2. This will require less no. of quantization levels and hence less no. of bits to represent them.

3. Thus signaling rate and bandwidth of a DPCM System will be less than that of PCM.