Unit - I

→ Theory of Machines is used to understand the relationship between the geometry and motion of parts of a machine (or) mechanism and forces which produce motion.

→ **Mechanism:** It is the arrangement of resistant bodies connected by movable joints and form a closed kinematic chain with one fixed member.

→ The purpose of mechanism is to transform motion by using relative internal motions.

Machine: It is an arrangement of parts of movable joints to transform motion and to produce work.

Ex:
1) Machines for power generation i.e. engines, turbines
2) Machines for production i.e. M/C tools, mills, coal mining m/c, Excavators
3) Machines for transportation
4) Machines for control computing & operation CNC
5) Cybernetic Machines.

Theory of Machine
- KOM
- DOM.
  - Statics.
  - Dynamics

Kinematics of Machines: It is the branch of theory of machines which deals with the study of relative (comparative) motion of parts of which the machines are constituted, neglecting consideration of forces producing it. This study is also known as pure mechanism of geometry of machines.

→ kinematics deals with the study of relative motion of various elements without considering the forces causing such motion from geometric point of view.

Dynamics of Machines: It is the study of motion of a machine under the forces acting on different parts of machines.

It is further subdivided into

Statics: It is branch of dynamics which deals with forces acting on various machine elements at rest. Here the mass of machine element is assumed to be zero.

Kinetics: It is the branch of dynamics which deals with inertia forces of various machine elements.

→ Inertia forces are produced due to mass and change in motion of parts of machines.

Structure: It is the combination of rigid bodies connected by joints. The purpose of structure is to carry loads or forces and transfer the same to the foundation or support.

Ex: Trusses, frames, columns, beams etc

Elective
ing the
point of view.

ment:- It is a part of a machine which has been manufactured without the operation of assembling

Ex:- bolt & nut are two machine elements. These are used to fasten machine parts & other machine elements together

Link is not same as element.

Link:- Link is a resistant body or assembly of resistant bodies which constitute part or part of a machines, connecting other parts which have motion relative to it.

→ Link may consist of number of parts connected in such a way that they form one unit and have no motion relative to each other.

Ex:- piston, piston rod, crosshead of steam engine will be taken as one link

Crankpin, crankweb, crankshaft, flywheel is one link.

→ Link is meant for transmitting motion (or) for guiding other links. Sometimes it serve as support.

Classification of link 1) Rigid 2) Flexible 3) Fluid.

Rigid link:- A link which does not undergo any deformation while transmitting motion is called rigid link

Flexible link:- A link which deform partly while transmitting motion is flexible link

Fluid link:- A link which deforms completely while transmitting motion is fluid link.

Forces:

→ Forces in mechanism arise from various sources such as force of gravity, assembly, applied loads, forces from energy transmission, frictional forces, spring forces, impact forces, force due to change in temperature.

→ All these forces are considered in final design of machine element for successful operation.

Types of forces:

1) **Applied forces:** The forces applied on system or body from outside is called applied forces.

2) **Inertia forces:** Inertia forces are arise due to mass of links of mechanisms and their acceleration.

3) **Frictional forces:** Frictional forces are out come of friction in the joints.

4) **Constraint forces:** The pair of action and reaction between any two bodies are called constraint forces.

Static force analysis:-

→ In the analysis of static forces, inertia forces are not considered.

→ If inertia of forces are considered the analysis is called dynamic force analysis.

→ Forces of gravity on machine parts are small compared with other static forces which are present & they are neglected in static force analysis.

Static Equilibrium:-

→ A body is in static equilibrium if it remains in its state of rest & motion.

i.e if it is in rest it tends to remain rest.

if it is moving (in translation, rotation) its linear of angular velocity tends to remain constant.

Condition for static Equilibrium

1) The vector sum of all forces acting on body is zero

2) The vector sum of all moments about any arbitrary points is zero.

Mathematically $\Sigma F = 0$ $\Sigma M = 0$.

In planar mechanism forces can be described by two dimension vectors i.e $\Sigma F_x = 0$ $\Sigma F_y = 0$

$\Sigma M_x = 0$

Equilibrium of forces:-

The following are four types of force members

1) **Two force member:-**

Figure shows a link under the action of two forces F_1 & F_2. vector sum of two forces must be zero. It should be in equilibrium when

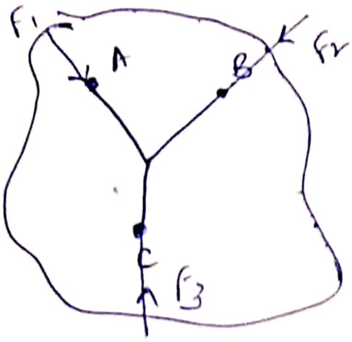
1) If two forces are of same magnitude
2) Two forces are collinear i.e act on same line.
3) Two forces are acting opposite in direction.

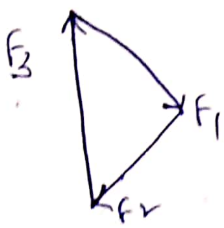
2) **Three force member:-**

In this system, the forces can be either parallel or non parallel. Now consider the non parallel forces

→ A member under the action of three forces F_1 F_2 F_3 shall be in equilibrium if

1) Resultant of three forces is zero
2) The three forces are concurrent i.e line of action of three forces at the same point.





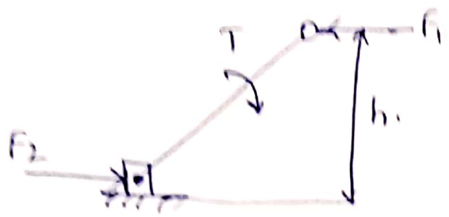
Resultant of three forces is zero if triangle of forces is closed.

Two force and torque

→ A member under the action of two forces and an applied torque shall be in equilibrium if

1) Two forces are equal in magnitude, parallel and opposite in direction.

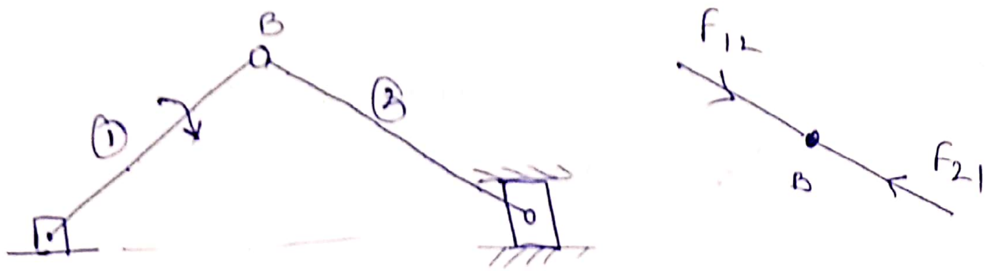
2) The two forces form a couple which is equal and opposite to the applied torque.



$$T = F_1 \times h = F_2 \times h$$

Force convention

The force applied by (1) on member (2) is represented by F_{12}. Similarly force applied by (2) on member (1) is represented by F_{21}



1) A force unknown in magnitude but known in direction is represented by solid straight line

Ex: ———

2) A force unknown in magnitude & direction is by wave line

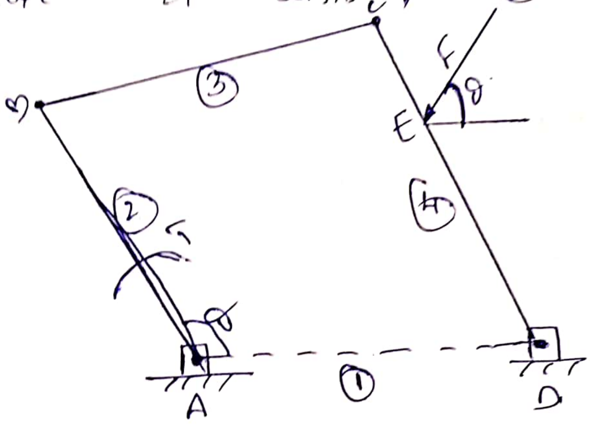
Ex: 〰〰〰

Static force analysis of four bar mechanism

→ A four link mechanism with four link joints is called four bar mechanism.

→ Individual parts & components of a mechanism are called bar.

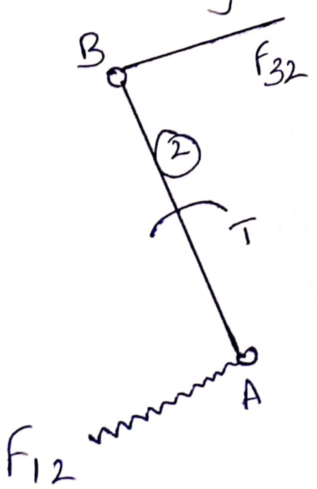
→ Four bar mechanism is simplest type of closed loop linkage. It consists of (3) movable link & (1) fixed link.



→ Consider a four bar mechanism subjected to a force (F) applied on link (4)

→ Our aim is to find the forces on various links including torque T on link (2)

Draw free body diagram of link (2), (3), (4)



Free Body diagram of link (2)

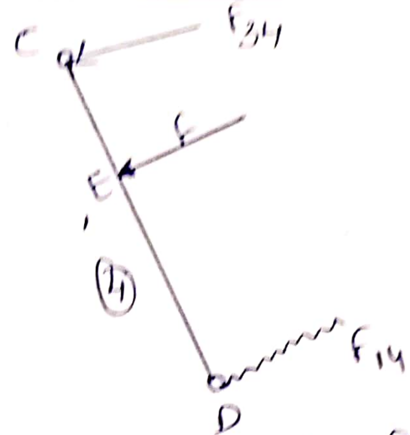
Member (2) is subjected to two forces F_{12}, F_{32}, & torque T as shown.

Free body diagram of link (3)

Member (3) is subjected to two forces F_{23} & F_{43}

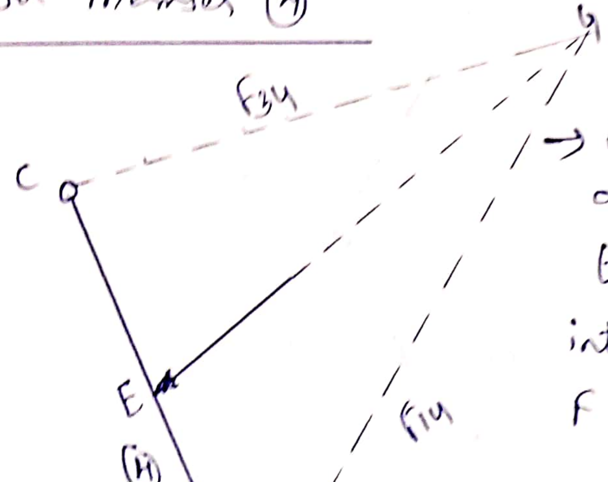
It is a two force member. For equilibrium these two forces must be equal i.e. ($F_{23} = F_{43}$) but magnitudes are unknown at this stage.

Free body diagram of link (4)



Member (4) is subjected to (3) forces F_{34}, F, F_{14}

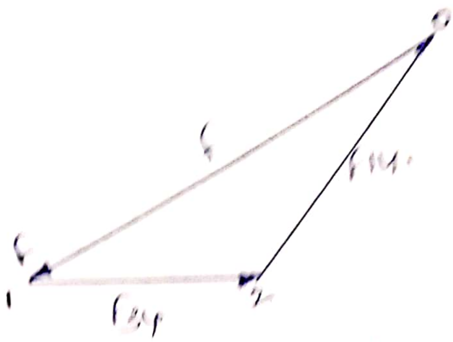
Force Polygon for member (4)



→ At point 'C' draw a line parallel to BC to represent F_{34} & intersect line of action of F at G.

→ Now LOA of F_{14} also pass through G.

Draw triangle of forces:



$F_{34} = -F_{43}$

$F_{23} = -F_{32}$

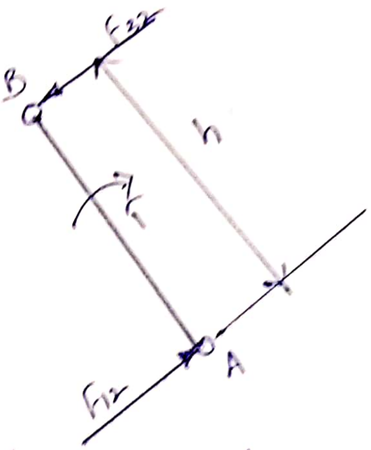
1) Draw force 'F' with some suitable scale making an angle θ.

2) from point ① draw a force vector F_{34} parallel to LOA of F_{34} in force polygon for member ④

3) from point ② draw a force vector F_{14} parallel to LOA of F_{14} in force polygon for member ④

4) Find F_{34}, F_{14} completely in magnitude & direction by measuring

Equilibrium of member ②



Member '2' is in equilibrium if F_{12} is equal, parallel & opposite to F_{32}.

$$T = F_{12} \times h = -F_{32} \times h.$$

...atic force analysis of slider crank mechanism

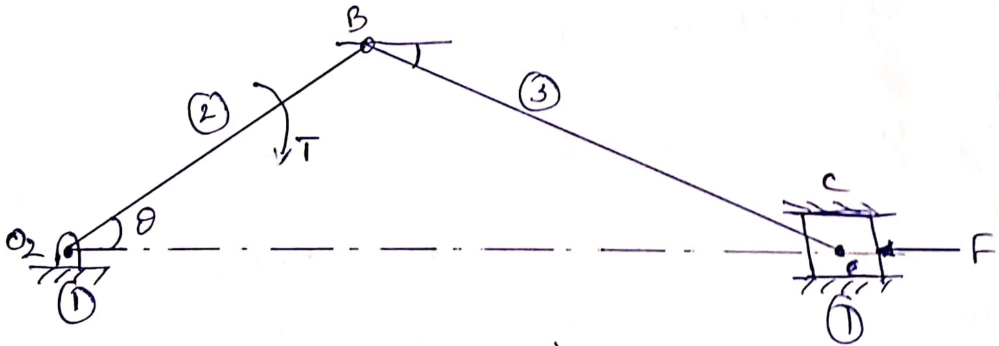
The static force analysis of slider crank mechanism is important as it appears in all of the IC Engine.

→ The basis element consist of three links connected with pin (or) revolute joints and one link that slides relative to one of neighbouring links. This is called slider crank mechanism.

Application:-

1) In IC Engines slider crank mechanism is used

2) It is used in electrical switch gears.

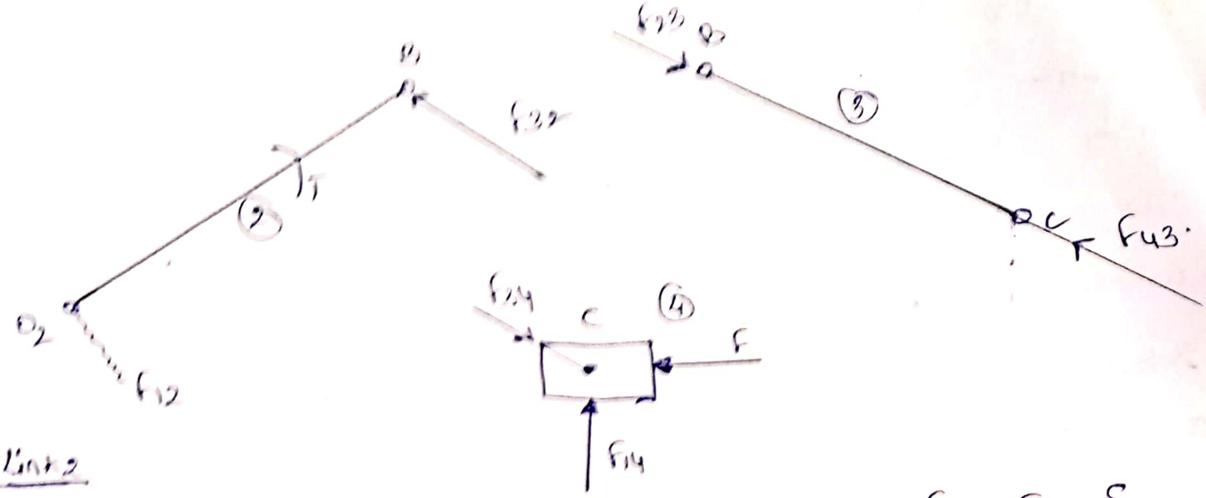


configuration diagram

→ A force F is applied to the piston and may be assumed to be the resultant of gas pressure

→ To keep system in equilibrium a couple of torque T is applied on link 2 through shaft O_2.

let us draw free body diagram of link (2), (3)



Link 2

→ Link '2' is subjected to two forces F_{12}, F_{32} & Torque T $F_{12} = F_{32}$ to balance force on link (2)

→ Thus link (2) has three unknown force F_{32} known in direction only F_{12} unknown magnitude & direction and unknown moment T

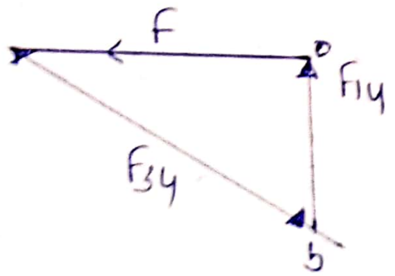
Link 3 → Link '3' is subjected to two forces F_{23} & F_{43} for equilibrium $F_{23} = -F_{43}$

Link 4 → Link (4) is subjected to 3 forces F, F_{34}, F_{14}

force 'F' is known in magnitude & direction

The two unknown forces for link (4) are F_{34} & F_{14}.

Draw force polygon for link (4) to find F_{34} & F_{14}



F_{34} & F_{14} mark & measure

(3)

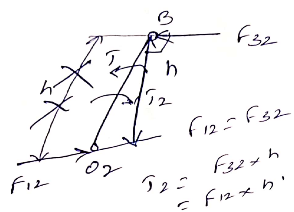
Torque (2) shall be in Equilibrium if

$F_{32} = -F_{12}$ i.e F_{12} is Equal, parallel & opposite to F_{32}

$\therefore$ $T = -F_{32} \times h$

$T_2 = -T$

Input torque



$F_{12} = F_{32}$

$T_2 = F_{32} \times h$

$= F_{12} \times h'$

Dynamic force analysis.

If the forces arrise due to mass of links of mechanisms and their accleration are called inertia forces & dynamic forces.

→ In general, the link is subjected to both static and inertia forces.

→ In high speed engines the accleration and inertia forces are very large when compare to static forces.

Ex: Reciprocating Engine.

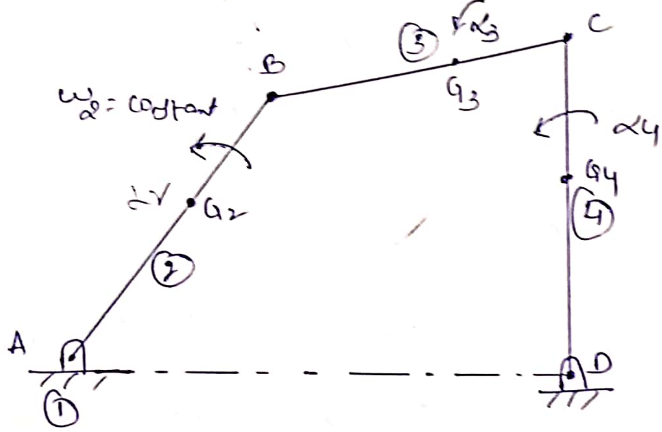
→ In slow speed engines the inertia forces are neglected. If inertia forces are considered. The analysis is called Dynamic force analysis.

D Alemberts Principle:

Dynamic force analysis for four bar mechanism.

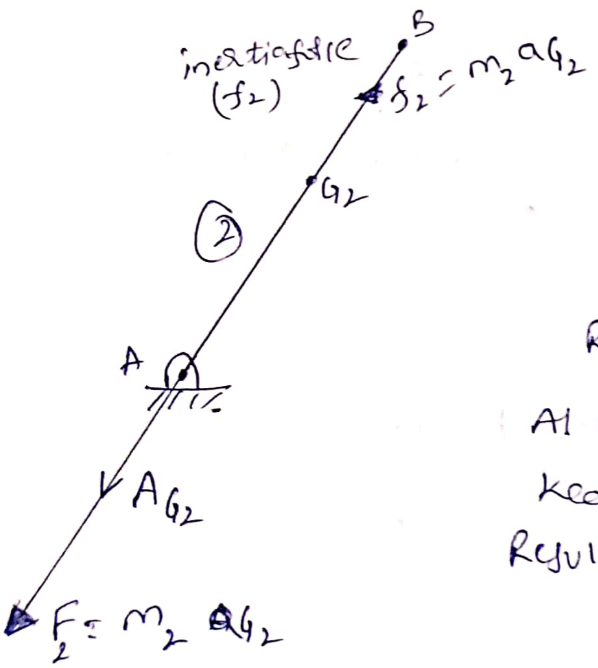
→ In order to design link and joints we must determine the loading conditions of each link & joint.

→ Consider a fourbar mechanism as shown.
Magnitude of ω_2 is known and constant.
centre of gravity of link 2, 3, 4 are G_2, G_3, G_4.



Our aim is to determine torque on link AB.

Draw free body diagram of link 2, 3, 4



a_{G_2} is accleration of centre of gravity

m_2 is mass of link 2

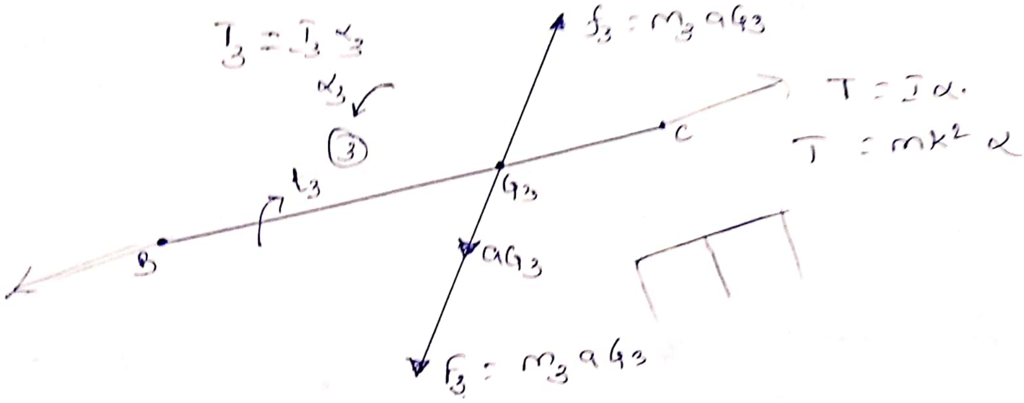
Resultant force $F_2 = m_2 a_{G_2}$.

As per D'alembert statement to keep link 2 in equilibrium

Resultant force F_2 = Inertia force f_2

$F_2 = -f_2$.

Free body diagram for link (3)



aG_3 is acceleration of centre of gravity of G_3.

Resultant force $F_3 = m_3 a_{G_3}$ Inertia force $f_3 = m_3 a_{G_3}$

m_3 is mass of link (3)

To maintain equilibrium $F_3 = -f_3$.

To produce α_3 there must be Resultant torque T_3

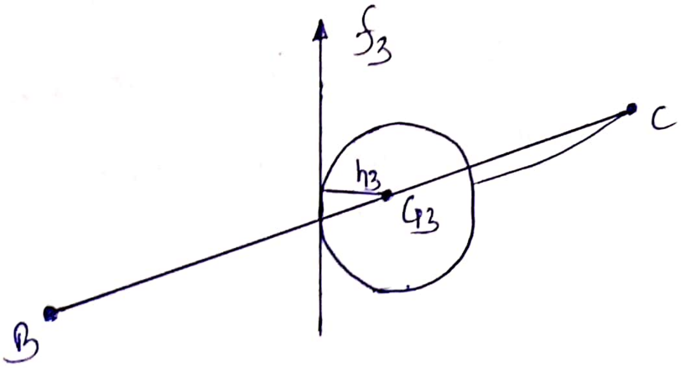
$\therefore T_3 = I_3 \alpha_3$. I_3 = Moment of Inertia of link (3)

To maintain equilibrium Resultant torque T_3 = Inertia torque (t_3)

i.e $T_3 = -t_3$.

Now replace inertia torque (t_3) & inertia force (f_3) by a single force f_3

The magnitude & direction of f_3 is same as above fig.

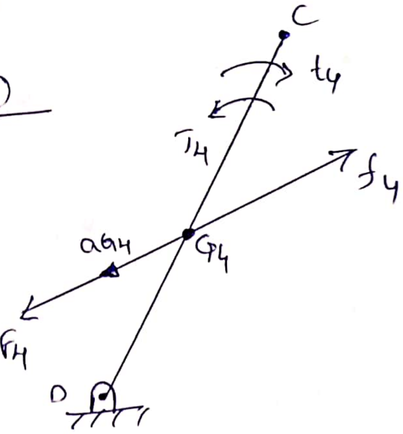


$t_3 = f_3 h_3$

$h_3 = \frac{t_3}{f_3} = \frac{I_3 \alpha_3}{m_3 a_{G_3}}$

To locate f_3 with G_3 as centre draw a circle of radius h_3 and f_3 is drawn tangent to the left side, since it must produce a torque about G_3 in the same direction of t_3.

Free body diagram of link (4)

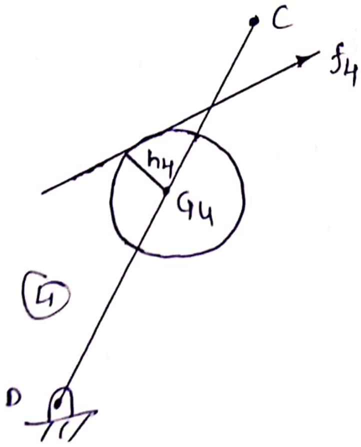


F_4 is resultant force $= m_4 a_{G4}$

f_4 is inertia force (equal & opposite to F_4)

T_4 is resultant torque $= I_{G4}\alpha_4$

t_4 is inertia torque (equal & opposite to T_4)



$$t_4 = f_4 h_4$$

$$h_4 = \frac{t_4}{f_4}$$

$$t_4 = \frac{I_4 \alpha_4}{m_4 a_{G4}}$$

To locate f_4, G_4 as centre draw a circle with radius h_4. f_4 is drawn tangent to the left side, since it must produce a torque about G_4 in the same direction of t_4.

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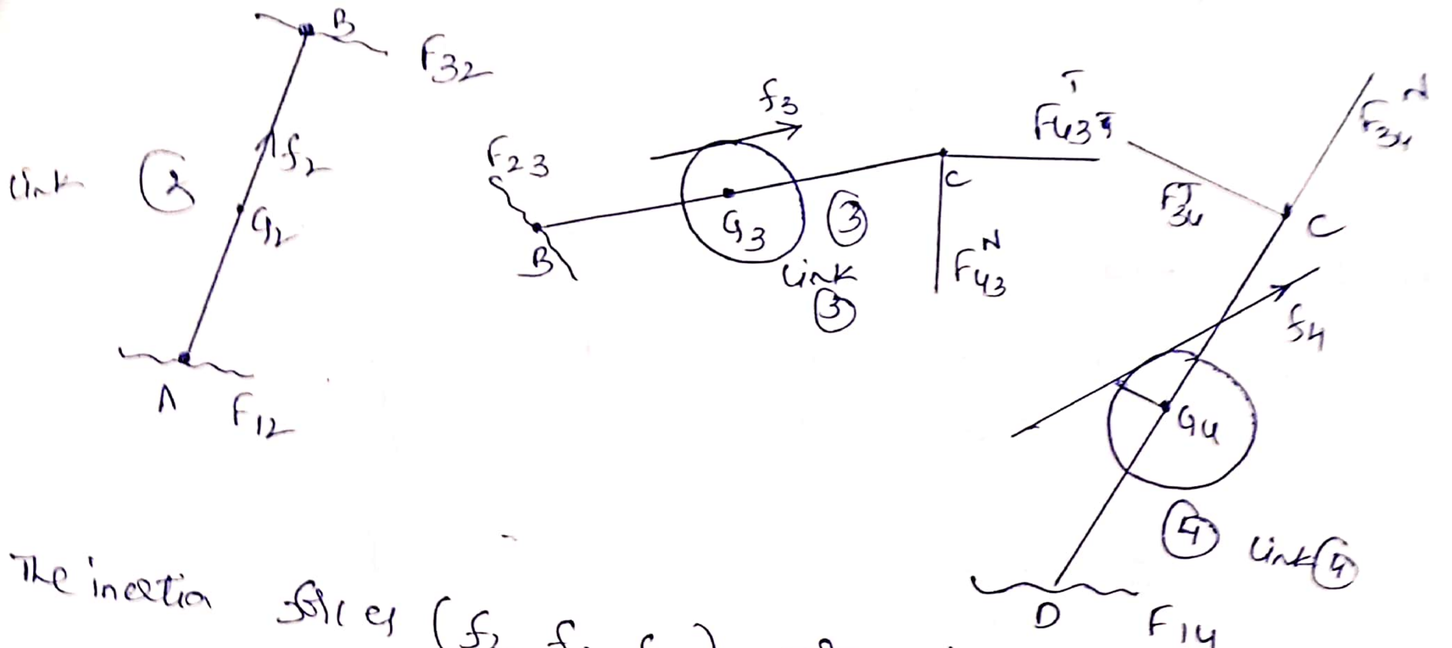
Find forces at each point and torque on link (2). Draw free body diagram of link 2, 3, 4.

(10)



→ The inertia forces (f_2, f_3, f_4) are treated as External force and each link is in equilibrium under the action of inertia force and unknown reaction.

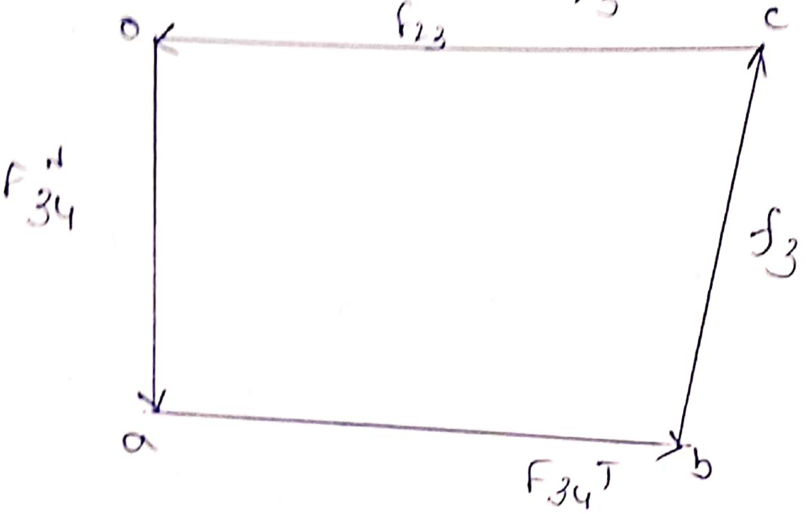
→ Similar to static force analysis the reaction can be found.

Consider link (4) and take moments ...

$$F_{43}^{n} \times CB = F_{34} \times h_{34}$$

$$F_{43}^{n} = \frac{F_{34} \times h_{34}}{CB}$$

Now draw force polygon of link (3). from force polygon we can find value F_{23}.

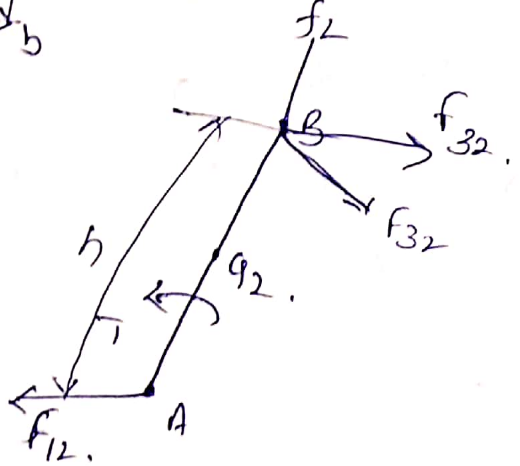


Now consider link 2

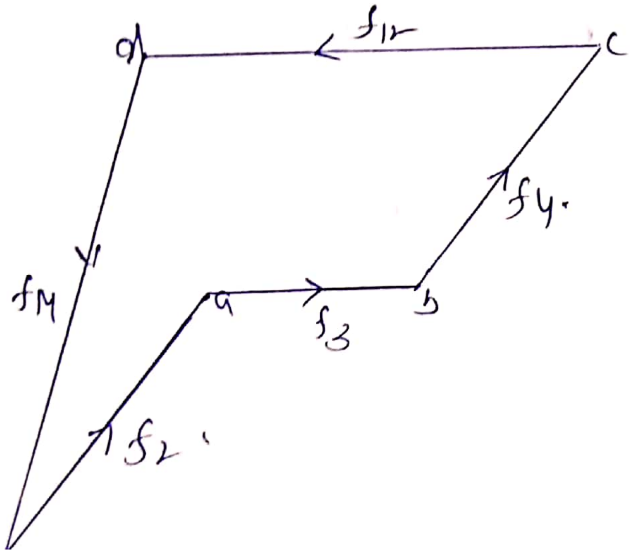
$F_{32} = -F_{23}$.

$F_{12} = -F_{32}$

from link 2 $T_2 = F_{12} \times h$.



F_{14} is obtained from force polygon by considering 2, 3, 4 as whole system.



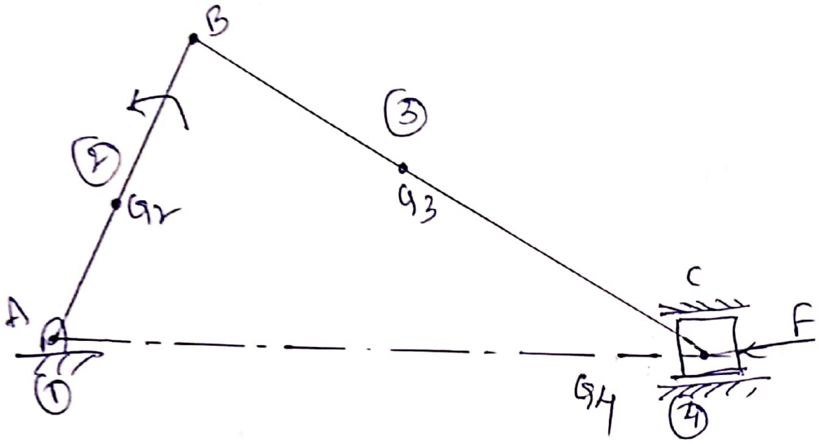
Dynamic force analysis of Slider Crank Mechanism

Consider a slider crank as shown.

Let F be the known force due to gas pressure on the piston which is constant whose magnitude & direction is known.

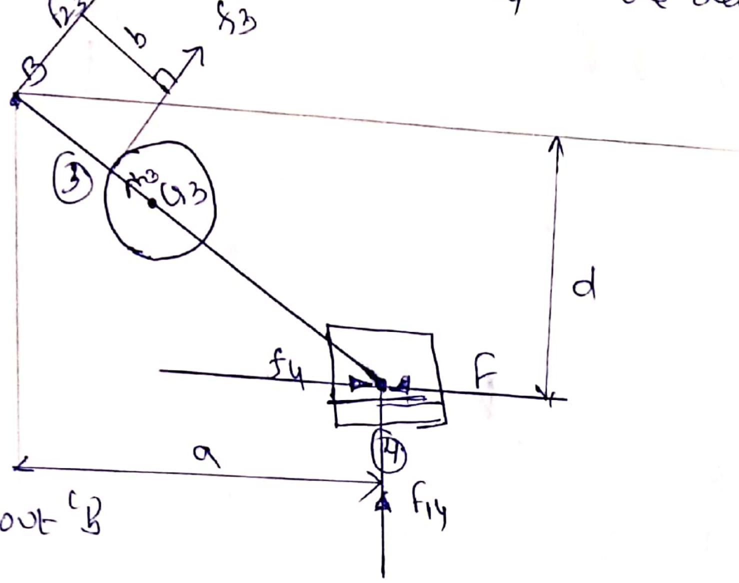
Let G_2, G_3, G_4 are centre of gravity of links (2), (3), (4)

Our aim is to find torque on link (2)



Draw combined free body diagram of link 3 & 4

Inertia force f_3, its moment arm relative to G_3
" " f_4 " " " G_4 are determined

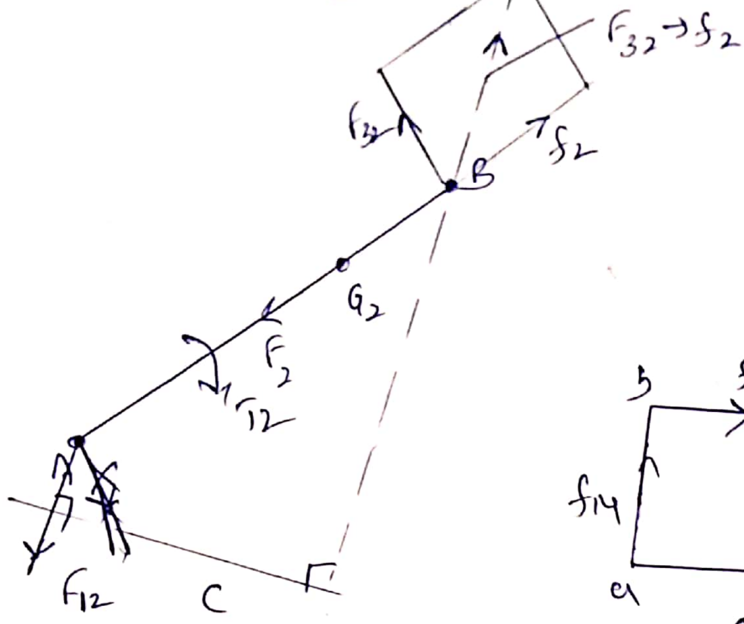


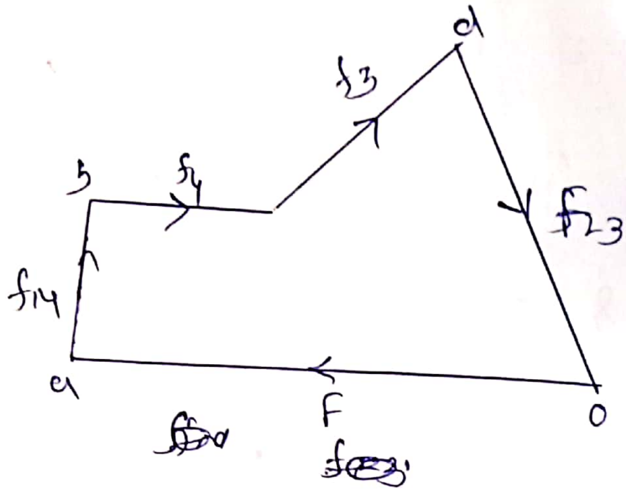
Taking moments about 'B'

$$F_{14} \times a + f_3 b + f_4 d - F d$$

$$F_{14} = \frac{Fd - f_3 b - f_4 . d}{a}$$

To find value of F_{23} draw a force Polygon.

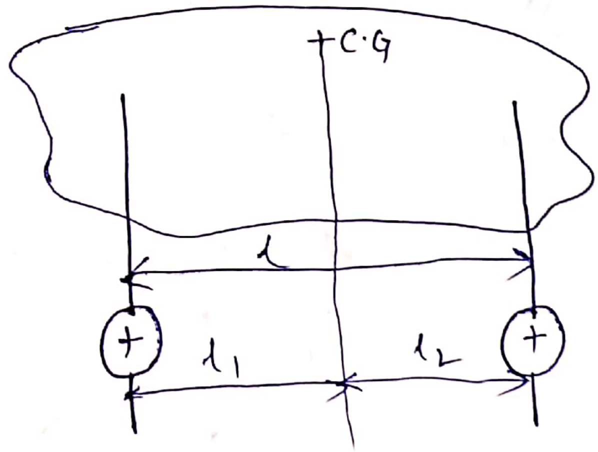




Find F_{23}.

$T_2 = F_{12} \times C$

Equivalent System:



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Gyroscope

Under the action of external forces only body spins about an axis and free to move in other direction this statement is called gyroscope

Applications

1) for directional control
   a) gyro compass: for airplane & ships
   b) inertial guidance control system for missiles & space travel
2) Gyroscopic effect occurs in the bearings of
   a) automobile when it makes a turn
   b) Jet engine shaft of the airplane changes directio

Definitions of Gyroscope

Gyroscopic Effect: A Gyroscope consist of a rotor mounted in the inner gimbal (ring). Inner gimbal is mounted in outer gimbal. outer gimbal is mounted on a fixed frame.

→ when rotor spins about x-axis with angular velocity ω rad/sec. The inner gimbal precesses (rotates) about y-axis

→ spatial mechanism is forced to turn about z-axis other than its own axis of rotation, and gyroscopic effect is thus set up

→ The resistance to this motion is called gyroscopic effect

Linear momentum:-

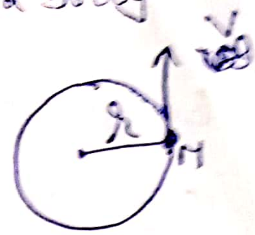
it is a vector quantity defined as product of a object mass and its velocity

$\therefore$ Linear momentum (p) = mass × velocity

$$\boxed{p = mv}$$

The Quantity of motion of a moving body measured by product of its mass & velocity

Angular momentum:- Rotational moment of a body is angular momentum.



The quantity of rotation of a body, which is the product of moment of inertia & its angular velocity (ω)

Moment of momentum is angular momentum

$\bar{p} = m \cdot v \cdot r$

$= m \cdot r \, r\omega$

$\bar{p} = mr^2\omega$ $\boxed{I = mr^2}$ Moment of inertia.

$\bar{p} = I\omega$

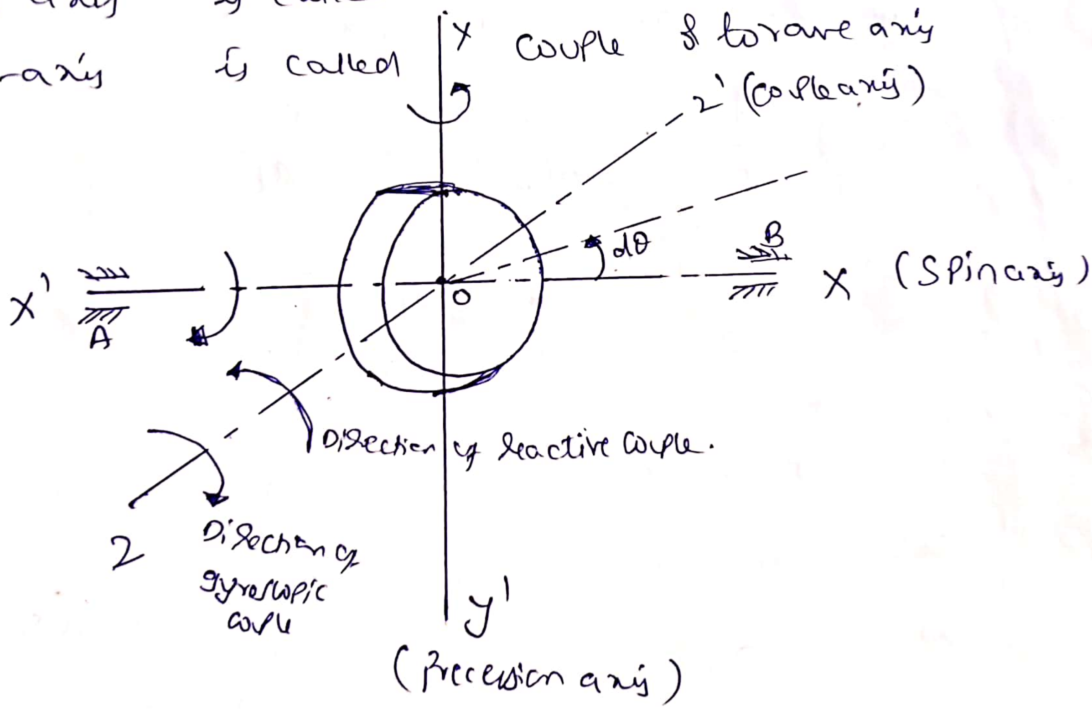
3. Gyroscopic couple:

→ Consider a rotary body of mass 'm' having radius of gyration 'k' mounted on the shaft supported at two bearings.

→ Let the rotor rotates (spins) about x-axis with constant angular velocity ω rad/sec. The X-axis is therefore called as spin axis

→ y-axis is called precession axis

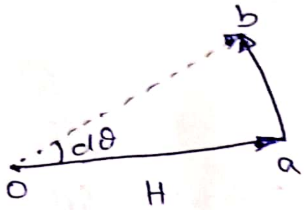
→ z-axis is called couple & torque axis



The angular momentum of rotating disc is $H = I\omega$.

Now the shaft axis (X-axis) moves through an angle $d\theta$ about y-axis in XOZ plane.

→ Then the angular momentum varies from H to ... where δH is change in angular momentum δH represented by vector ab.



$\therefore$ ab = $o.a \times \delta\theta$

$\delta H = H \times \delta\theta$

$= I\omega \cdot \delta\theta$

Now Rate of change of angular momentum is

$$C = \frac{dH}{dt} \Rightarrow \lim_{\delta t \to 0} \frac{I\omega\,\delta\theta}{\delta t}$$

$$= I\omega\left(\frac{d\theta}{dt}\right)$$

$$C = I\,\omega_s\,\omega_p$$

where C = gyroscopic couple (N-m)

ω_s = angular velocity of rotary body (rad/s)

ω_p = angular velocity of precession (rad/s)

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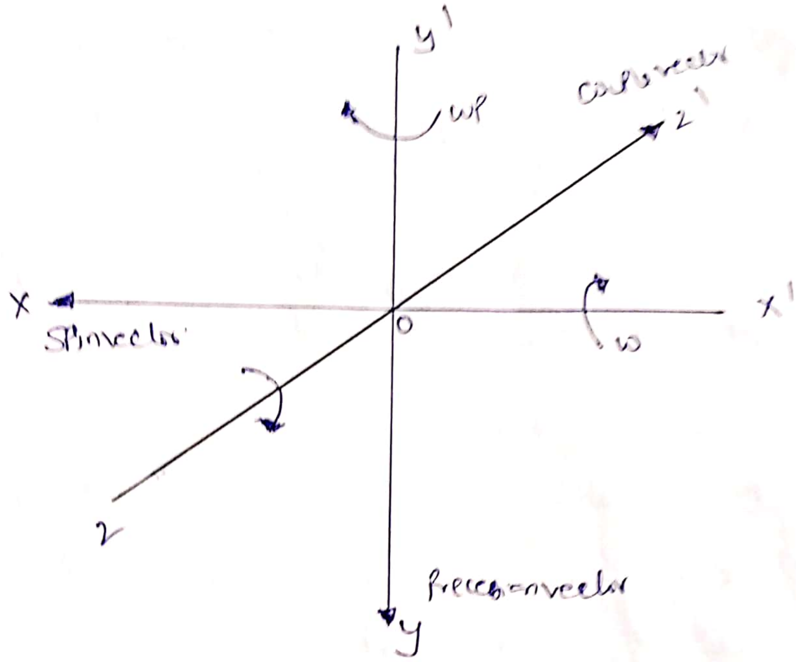
Direction of spin vector, precession vector, couple vector



To determine the direction of spin, precession, couple vector right hand screw rule is used. The fingers represent the rotation of disc and thumb shows the direction of spin, precession & torque vector.

Case 1)
Consider a rotor rotating in anticlockwise direction when seen from right. To determine active / reactive gyroscopic couple the following procedure is used.

a) Turn the spin vector through 90° in the direction of precession on XOZ plane

b) The turned spin vector is then correspond to the direction of active gyroscopic couple.

c) The reactive gyroscopic couple is opposite to active gyroscopic couple.

Gyroscopic Effect on Aeroplane

Aeroplanes are subjected to gyroscopic effect it taking off, landing & neglecting left or right turn air.

Let ω = Angular velocity of engine rotating parts in rad/s

m = Mass of engine & propeller in kg

k = Radius of gyration in m

I = Mass moment of inertia of engine & propeller in $kg\,m^2$

v = Linear velocity of aeroplane in m/s

R = Radius of curvature in m.

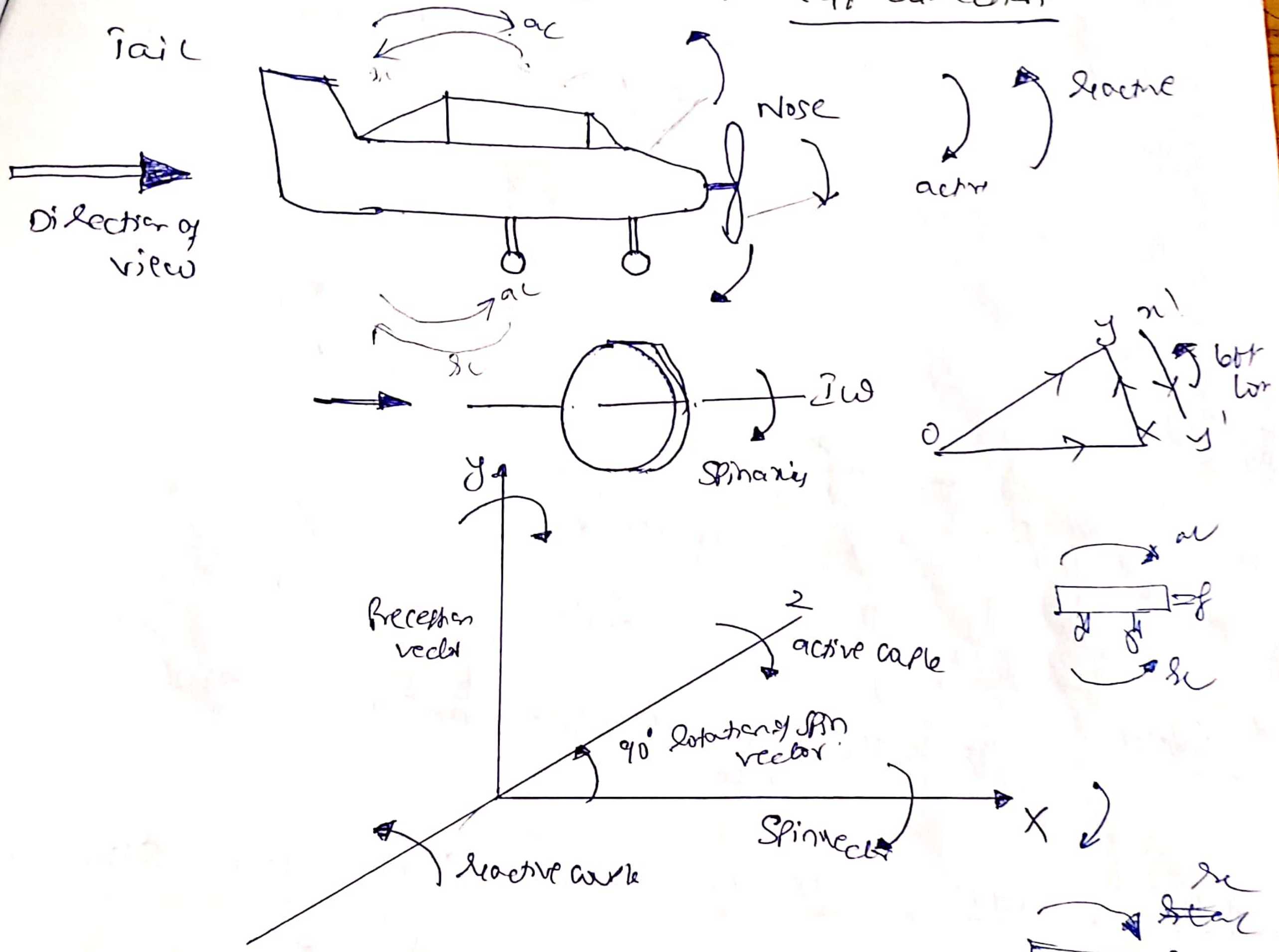
ω_p = Angular velocity of precession $= \frac{v}{R}$ rad/s

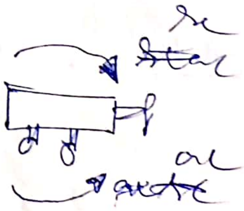
∴ Gyroscopic couple of aeroplane

$$\boxed{C = I\,\omega\,\omega_p}$$

Let us analyze effect of gyroscopic couple acting on the body for various conditions

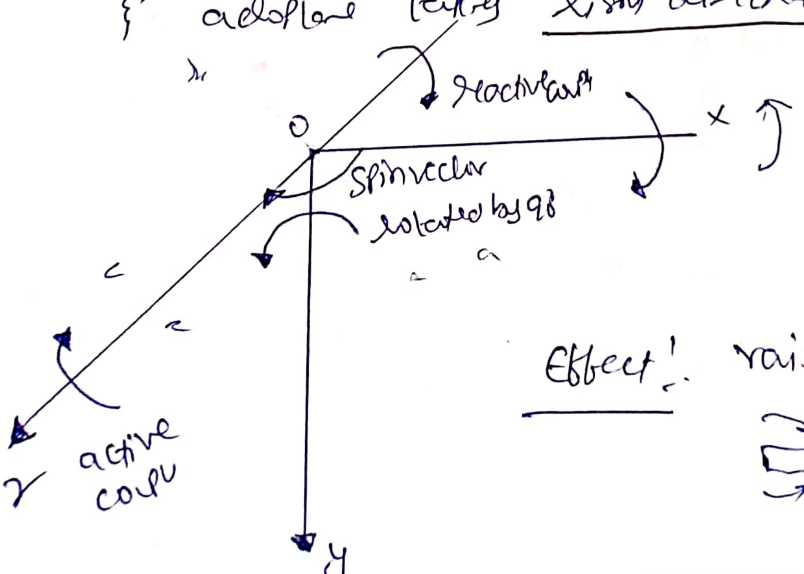
Case 1) Propeller rotates in clockwise when see from rear end & aeroplane turns left direction

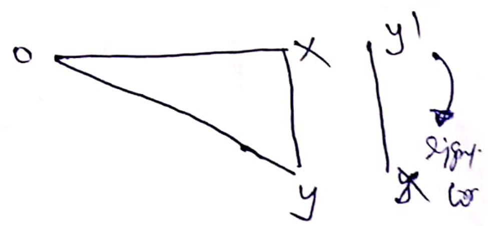




Effect:- Dip the tail & raise the nose.

Case 2 Propeller rotates in clockwise when see from rear end & aeroplane takes right direction





Effect:- raise the tail & dip the nose

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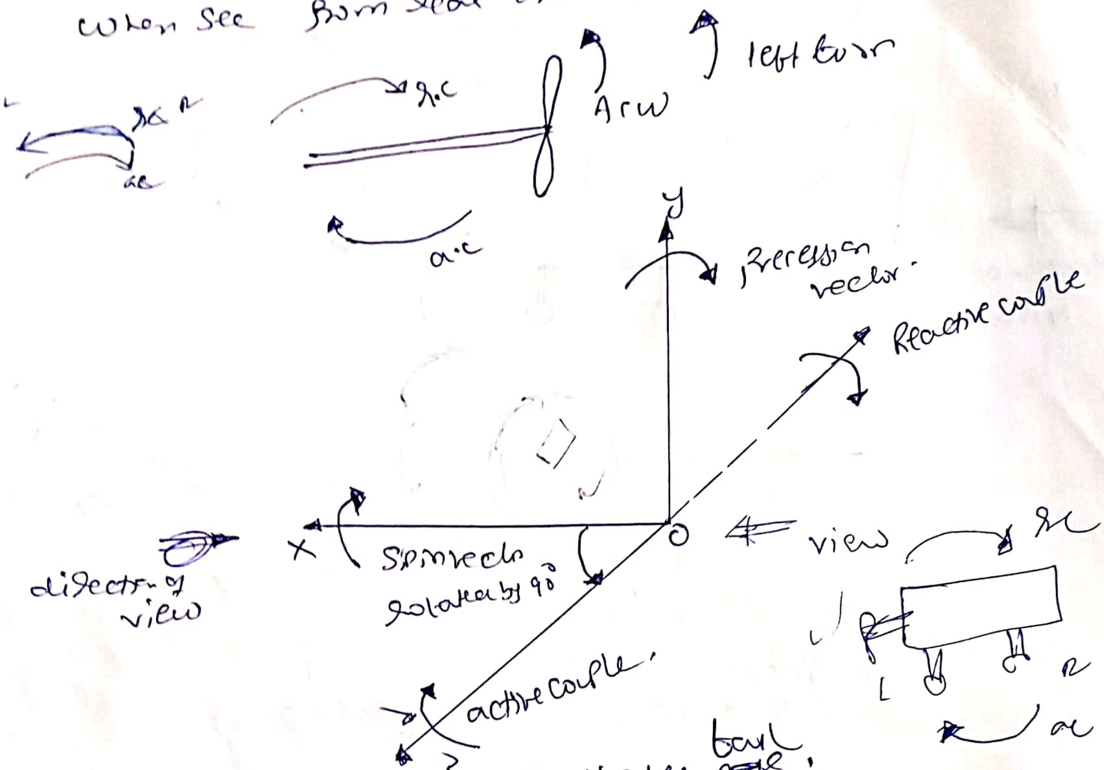
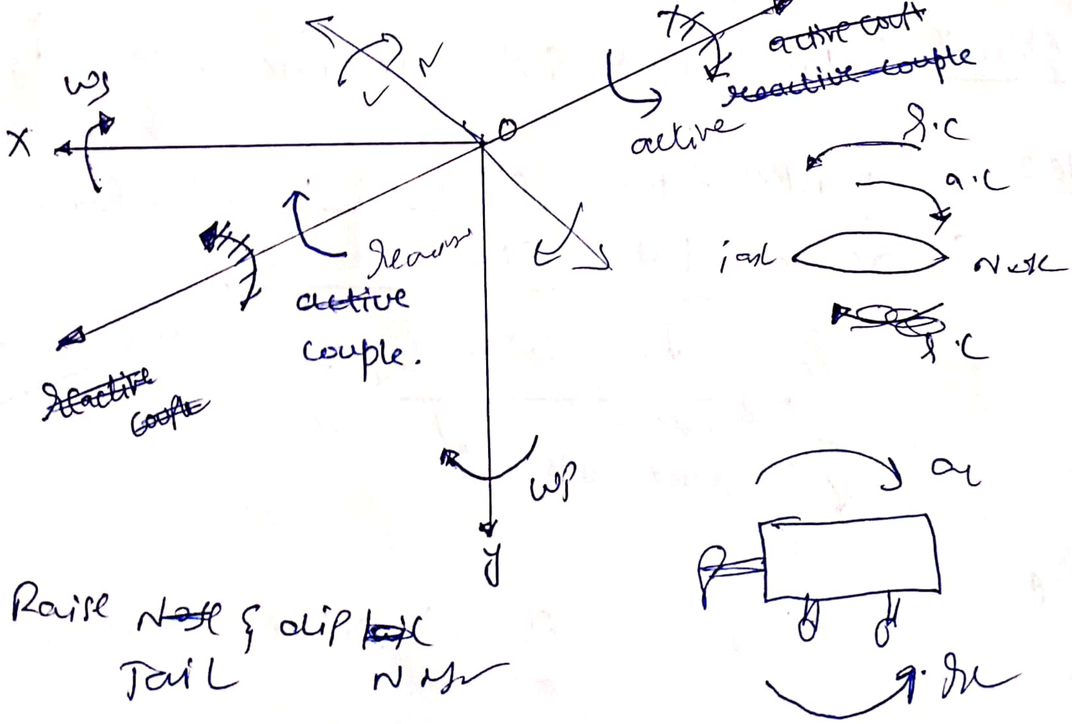
Case 3 Propeller rotates in Anticlockwise direction and aeroplane turns when see from rear end and left turn



Effect: raise the ~~tail~~ nose & dip the ~~nose~~ tail.

Case 4 Propeller Rotates Anticlockwise & Plane takes right turn.



Effect: Raise ~~Nose~~ Tail & dip ~~tail~~ Nose

Gyroscopic Effect on Ship

→ Gyroscope is used for stabilisation and directional control of a ship sailing in a rough sea.

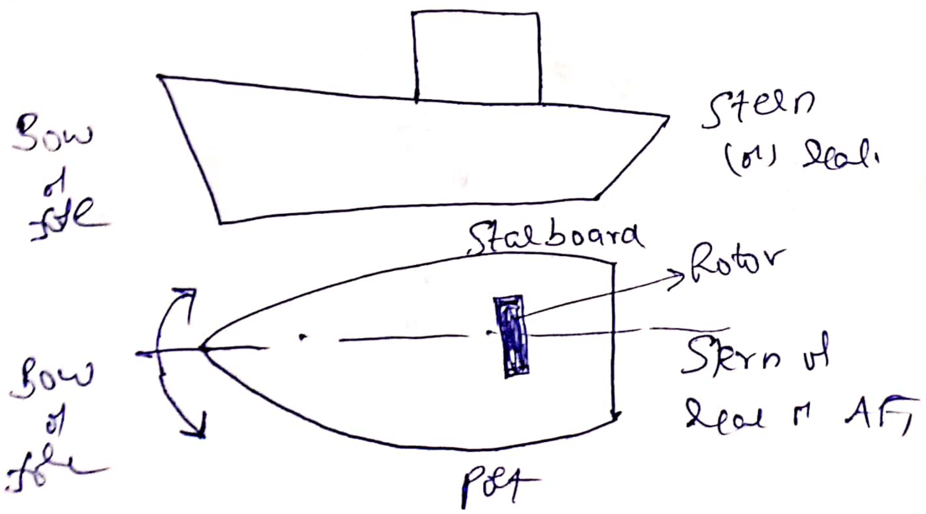
→ A ship while navigating in rough sea experience the three different types of motion

1) <u>Steering</u> - The turning of ship in a curve while moving forward
2) <u>Pitching</u> - The movement of ship up & down from horizontal position in a vertical plane about transverse axis
3) <u>Rolling</u>: Side way motion of ship about longitudinal axis
4) ~~Yawing~~:

for stabilisation of ship against any of the above motion gyroscope is used.

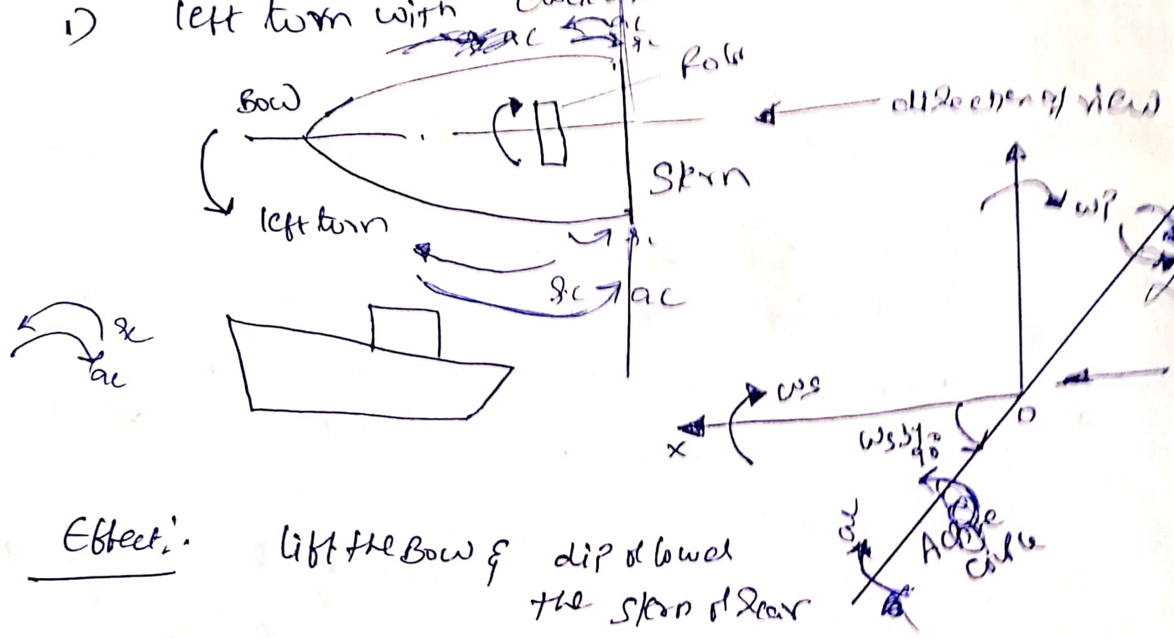
Ship terminology:

1) Bow - fore end of the ship
2) Stern - rear end of ship.
3) Starboard - It is right hand side of ship looking in direction of motion
4) Port - It is left hand side of ship looking in direction of motion.



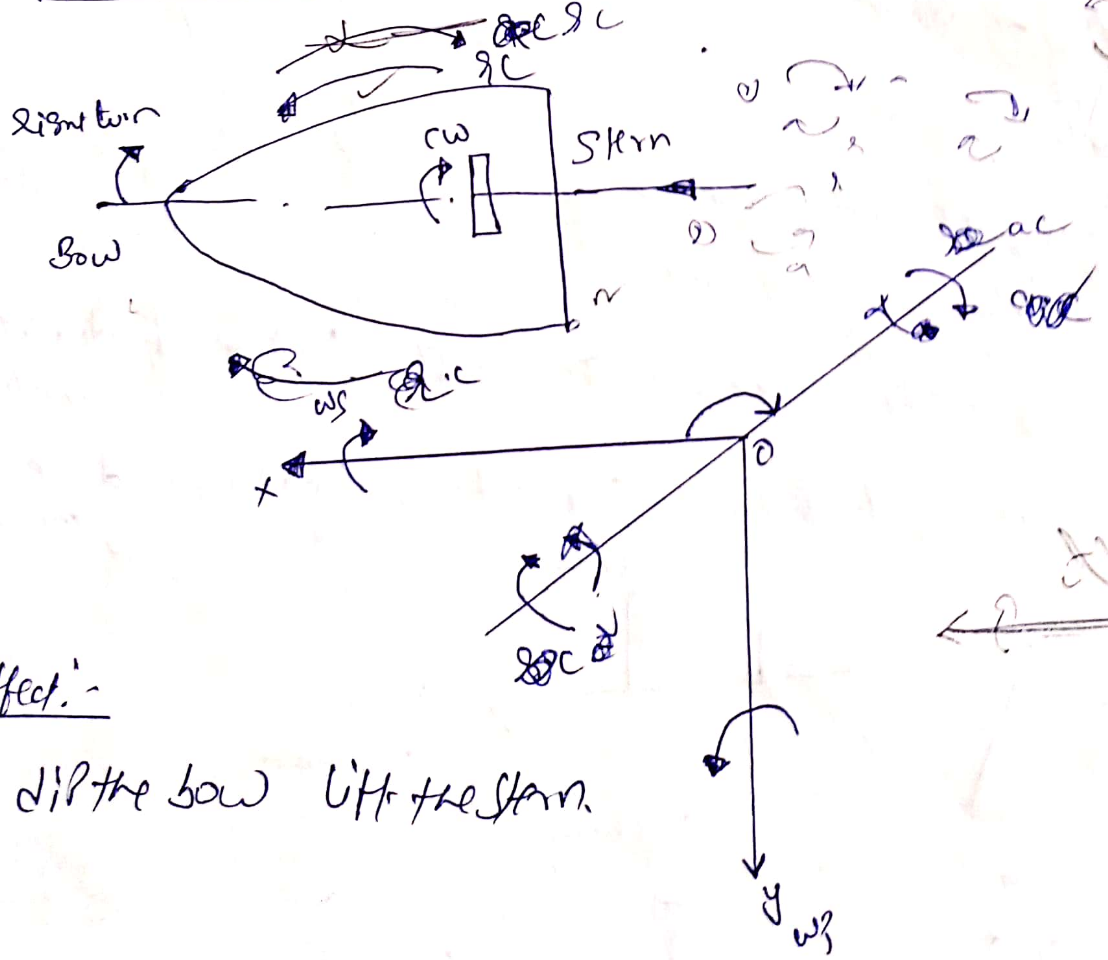
Gyroscopic effect on steering of ship

1) left turn with clockwise rotor.



Effect:- lift the Bow & dip or lower the stern or rear

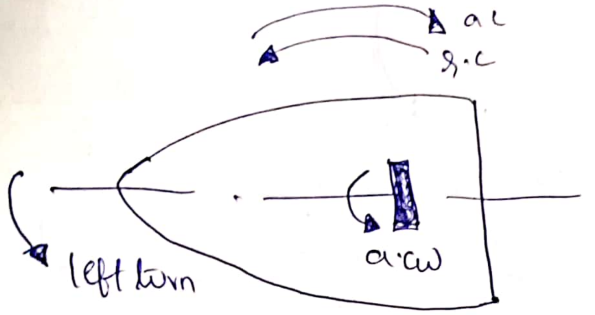
2) Right turn with clockwise rotor



Effect:-

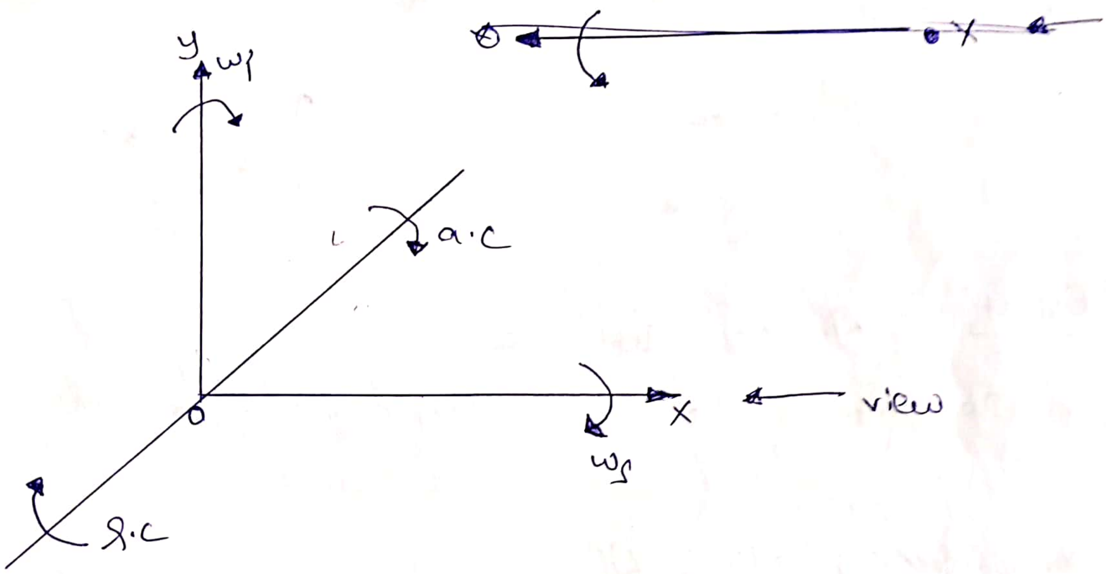
dip the bow lift the stern.

[L]eft turn with a.c.w rotor.

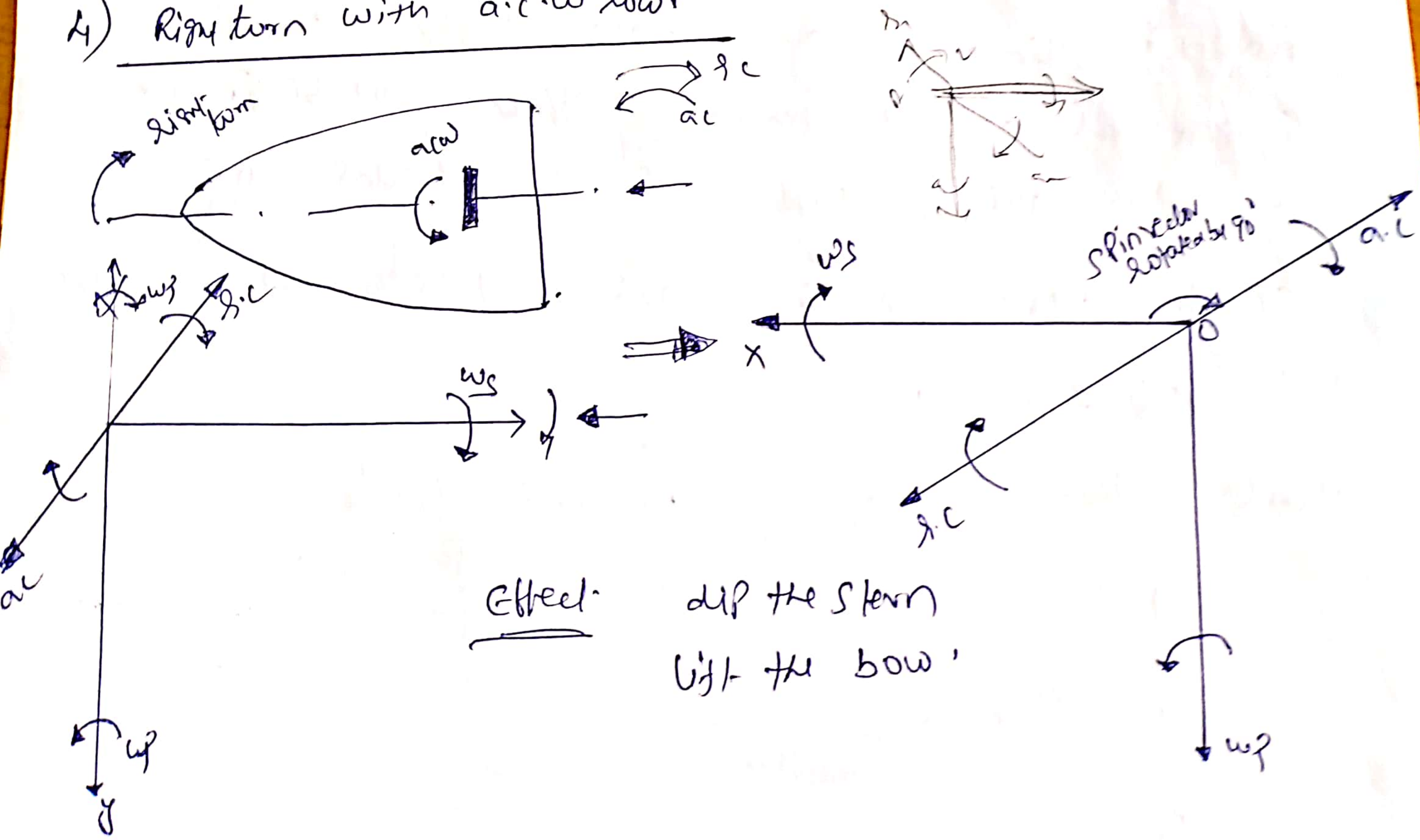


Effect

dip the bow
lift the stern

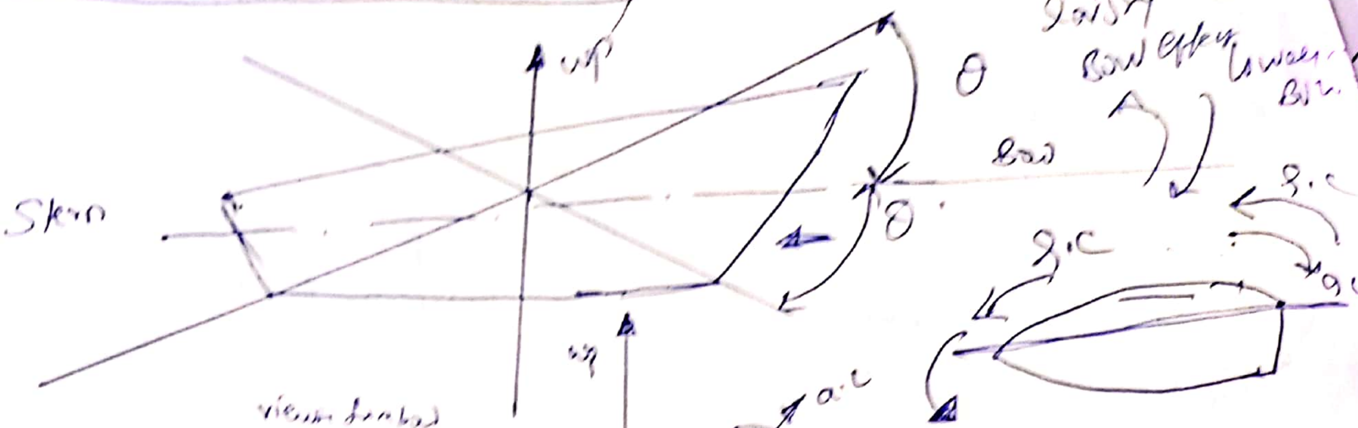


4) Right turn with a.c.w rotor.

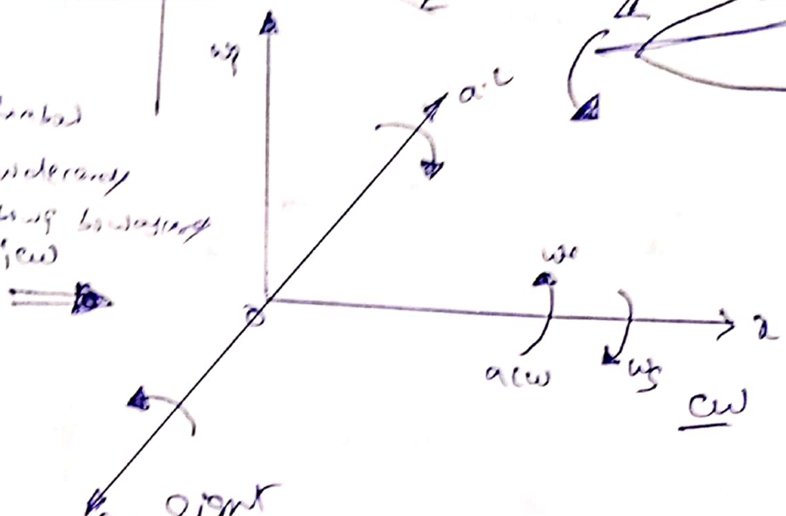


Effect. dip the stern
lift the bow.

Gyroscopic effect on Pitching of Ship



1) Rotor turns a.c.w & bow is raising
2) Rotor turns c.w view from bow raising



1) **Effect**: Ship moves ~~left~~ right side

2) When rotor rotates a.c.w Ship moves ~~right~~ left side

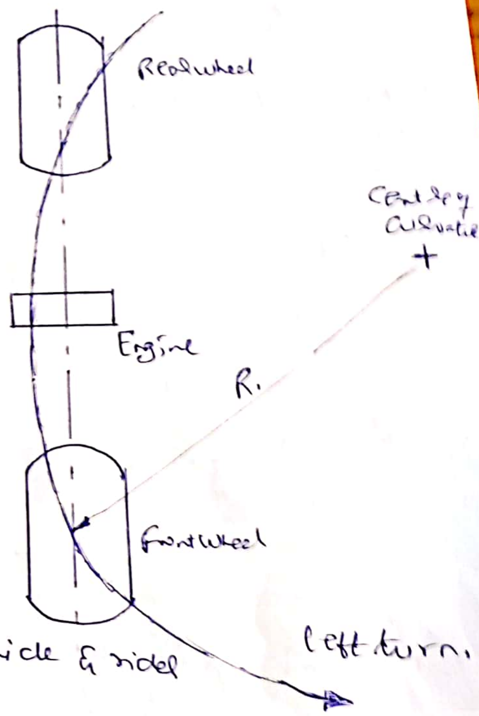
Gyroscopic effect on Rolling of Ship

Stability of two wheeler while taking a turn

Figure shows a two wheeler taking left turn over a curved path.

→ The vehicle is inclined to the vertical for equilibrium by an angle θ.

→ θ is called angle of heel.



Let

m = mass of vehicle & rider in kg

W = weight of vehicle & rider in N

$W = m \cdot g$

h = Height of centre of gravity of vehicle & rider

r_W = Radius of wheels.

R = Radius of track of curvature

I_W = Mass moment of Inertia of each wheel

I_E = Mass moment of inertia of rotating parts of Engine

ω_W = Angular velocity of wheels.

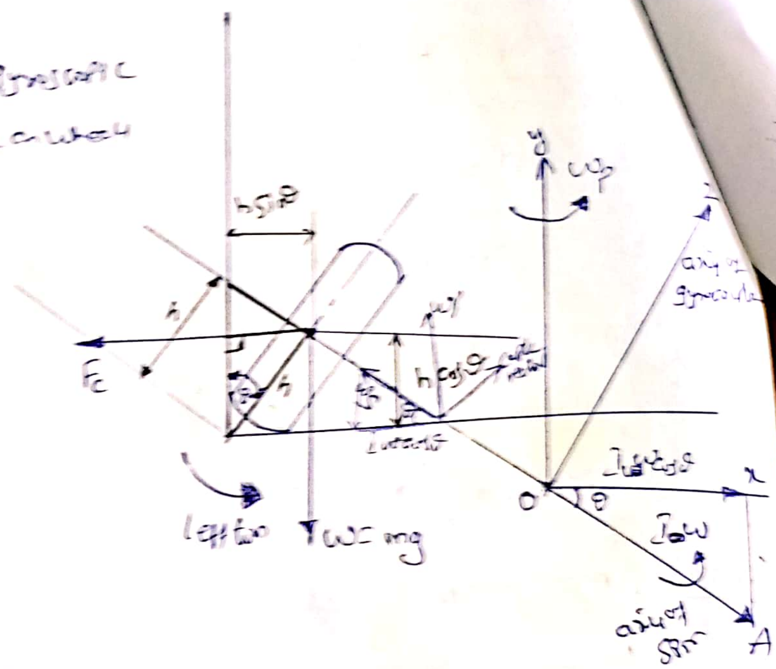
ω_E = Angular velocity of Engine rotating parts

G = Gear Ratio $= \dfrac{\omega_E}{\omega_W}$

V = Linear velocity of vehicle $= r_W \, \omega_W$

θ = angle of heel. It is the inclination of vehicle to the vertical for equilibrium.

let us consider the effect of gyroscopic couple & centrifugal couple on wheel



1) Effect of Gyroscopic couple

wkt

$$V = \omega_W r_W$$

$$G = \frac{\omega_E}{\omega_W}$$

$$\omega_E = G \times \omega_W$$
$$= G \times \frac{V}{r_W}$$

Angular momentum due to wheels $= 2 \times I_W \omega_W$

Angular momentum due to Engine $= I_E \omega_E$

$\therefore$ Total angular momentum $= 2 I_W \omega_W \pm I_E \omega_E$

$$H = 2 I_W \frac{V}{r_W} \pm I_E G \times \frac{V}{r_W}$$

$$H = \frac{V}{r_W}\left(2 I_W \pm G I_E\right)$$

Velocity of Precession $V_P = \omega_P = \cancel{\omega_P}$

$$V_P = R\,\omega_P \Rightarrow \omega_P = \frac{V_P}{R}.$$

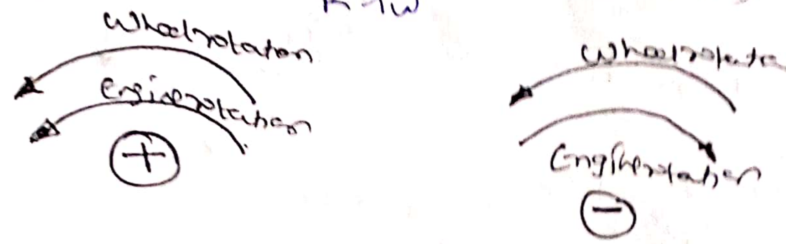
→ The axis of spin is inclined at angle θ (angle of heel) to the horizontal. Thus the angular momentum vector $I\omega$ due to spin is OA inclined to θ to OX & spin vector resolved along OX. Precession vector is vertical

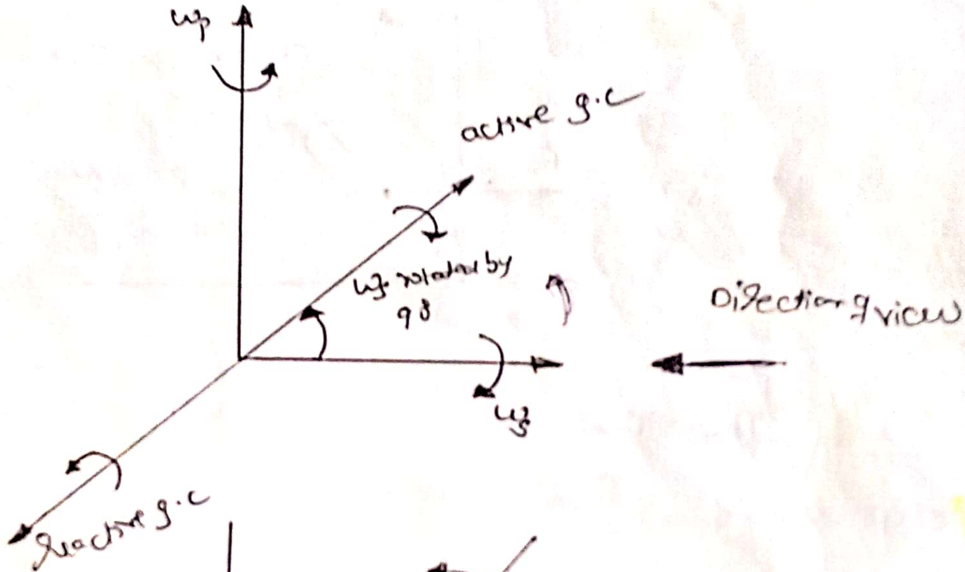
∴ Gyroscopic couple $C_g = I \omega \cos\theta \, \omega_p$.

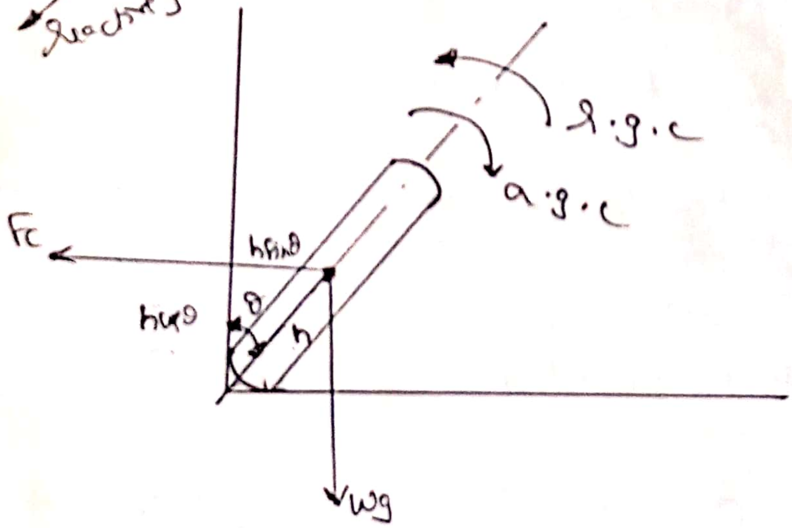
$$C_g = \frac{V}{R_w}(2I_w \pm G I_e)\cos\theta \, \frac{V}{R_w}$$

$$C_g = \frac{V^2}{R R_w}(2I_w \pm G I_e)\cos\theta$$

NOTE:







Effect: The gyroscopic couple will act the vehicle outwards. i.e in a.c.w when see from the front of two wheeler. (The couple tends to overturn the vehicle in outward direction)

Effect of centrifugal couple.

We have centrifugal force (F_c)

$$F_c = \frac{mv^2}{R}$$

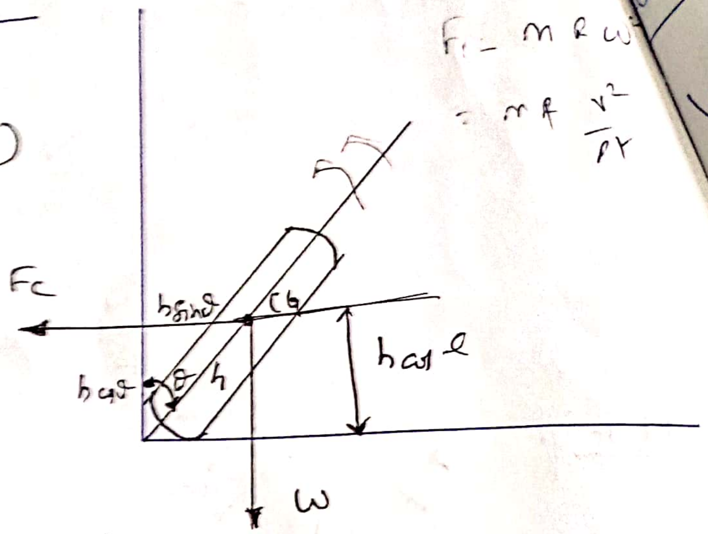
$\therefore$ centrifugal couple

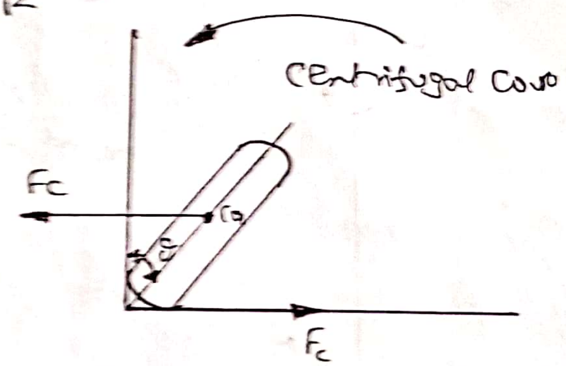
$$C_c = F_c \times h\cos\theta$$

$$C_c = \frac{mv^2}{R}\, h\cos\theta \leftarrow lgc$$

$F_i = mR\omega^2$

$= mR\frac{v^2}{Rr}$





Effect:- The centrifugal couple will act over the two wheeler outwards i.e in a.c.w direction when see from the front of two wheeler

(The couple tends to overturn the vehicle in outward direction)

$\therefore$ Total overturning couple $C = C_g + C_c$

$$\boxed{C = \frac{v^2}{Rr}(2I_w + G I_e)\cos\theta + \frac{mv^2}{R} h\cos\theta.}$$

raising
lowering

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Rising: Turn the bow towards to port i.e left hand

Lowering: turn the bow towards starboard i.e Right hand

$C = I\omega\omega_p$

$= 1000 \times 0.3^2 \times \frac{2\pi \times 1550}{60} \times 1 = 14608.4\,Nm$

2) Steering:
$C = I\omega\omega_p$

$= mk^2 \times \frac{2\pi N}{60} \times \frac{v}{R} = 1000 \times 0.5^2 \times \frac{2\pi \times 1550}{60} \times \frac{40 \times 1000}{3600 \times 200}$ $= 811.5\,Nm$

Effect: Raise the bow & dip the stern

3) Rolling
$C = I\omega\omega_p$
$= 1000 \times 0.3^2 \times \frac{2\pi \times 1550}{60} \times 0.5 = 2304.2\,Nm$

(4) A Ship is pitching through a total angle of 15^0, the oscillation may be taken as s.h.m and the complete period has 32 sec. The turbine has a mass of 500 kg & k = 450 mm N = 2000 rpm. find (G.C max) if rotor is turning CW when see from bow (front) How does the bow turn when falling. What is the max angular accelaration to which ship is subjected while pitching

Sol: Total angle of pitch $2\phi = 7.5^\circ$

$\phi = 7.5$; $\phi = (7.5 \times \pi/180)$ rad/sec

Time period $(t_p) = 32$ sec

$m = 500$ kg $k = 0.45$ m $N = 2000$ rpm

1) Pitching

$C_{max} = I \omega \omega_p$

$= mk^2 \times \frac{2\pi N}{60} \times \left(\phi \frac{2\pi}{t_p}\right)$

$= 500 \times 0.45^2 \times \frac{2\pi \times 2000}{60} \times 7.5 \times \pi/180 \times \frac{2\pi}{32}$

$= 545$ Nm

Effect: Turn the bow towards Right side

max angular acceleration

$$\alpha_{max} = -\phi \left(\frac{2\pi}{t_p}\right)^2$$

$\frac{d^2\theta}{dt^2} = -\phi \omega_1^2 \sin\omega_1 t = \alpha_1 = -7.5 \times \pi/180 \times \left(\frac{2\pi}{32}\right)^2$

$= -5.04 \times 10^{-3}$ rad/sec²

$\alpha_{max} = -\phi \omega_1^2$

$\boxed{\sin\omega_1 t = 1}$ for α_{max}

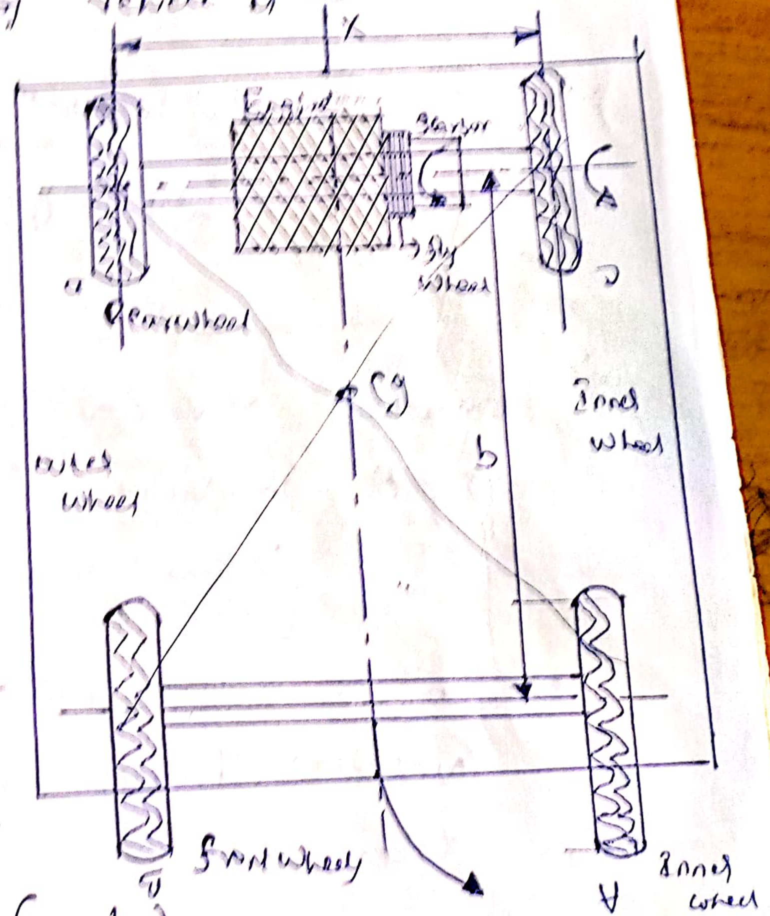
$\therefore \alpha_{max} = -\phi \omega_1^2$

Stability of four wheel vehicle while taking turn.

Consider a four wheel automotive vehicle. The engine is mounted at the rear with its crank shaft parallel to rear axle.

→ The centre of gravity of vehicle lies vertically upward above the ground where total weight of vehicle is acted upon.

Let



m: mass of vehicle in kg

W: weight of vehicle (N) = mg

h: Height of centre of gravity of vehicle (m)

r_w: Radius of wheels (m)

R: Radius of curvature (m)

I_w: Mass moment of inertia of each wheel kg m²

I_e: " " " " of engine kg m²

ω_w: Angular velocity of wheels (rad/s)

ω_e: Angular velocity of engine (rad/s)

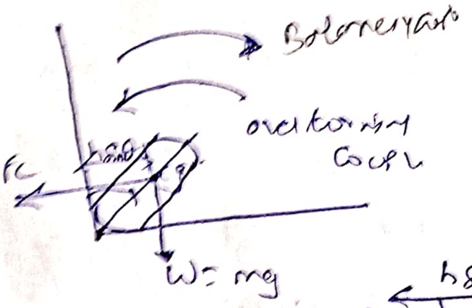
G: Gear ratio $= \frac{\omega_e}{\omega_w}$

v: Linear velocity of vehicle (m/s) $= \omega_w \times r_w$

x: wheel track (m) (width of track)

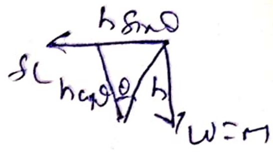
b: wheel base (m) (length of wheel base)

For vehicle to be in Equilibrium, The overturning couple should be equal to balancing couple acting in clockwise direction due to weight of vehicle & [illegible].



∴ $C = W \times h\sin\theta$

$C = mgh\sin\theta$



∴ For Stability, overturning couple = Balancing couple

$$\boxed{\frac{v^2}{R\,r_w}(2I_w + G I_E)\cos\theta + \frac{mv^2}{R}h\cos\theta = mgh\sin\theta}$$

∴ From the above Eqn

1) θ The value of (angle of heel) may be determined so that vehicle does not skid.

2) For the given value of θ the max speed of vehicle in the turn without skid may be determined.

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1) Reaction due to weight of vehicle

Assuming that weight of vehicle is equally distributed over four wheels. W, weight of vehicle distributed equally on 4 four wheels

$$W = \frac{mg}{4}$$

∴ force on each wheel acting downwards $= \frac{mg}{4} \downarrow = W$

Reaction force of road surface on wheel acting upward.

$$R_w = \frac{mg}{4} \uparrow$$

2) Effect of Gyroscopic couple due to wheel.

$$C_w = 4 I_w \omega_w \omega_p \qquad G = \frac{\omega_e}{\omega_w}$$

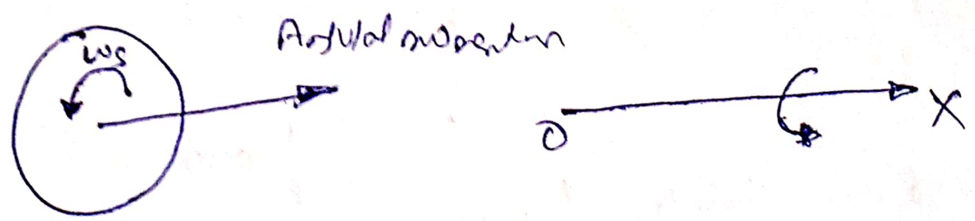
3) Effect of Gyroscopic couple due to Engine $C_E = I_E \omega_e \omega_p$

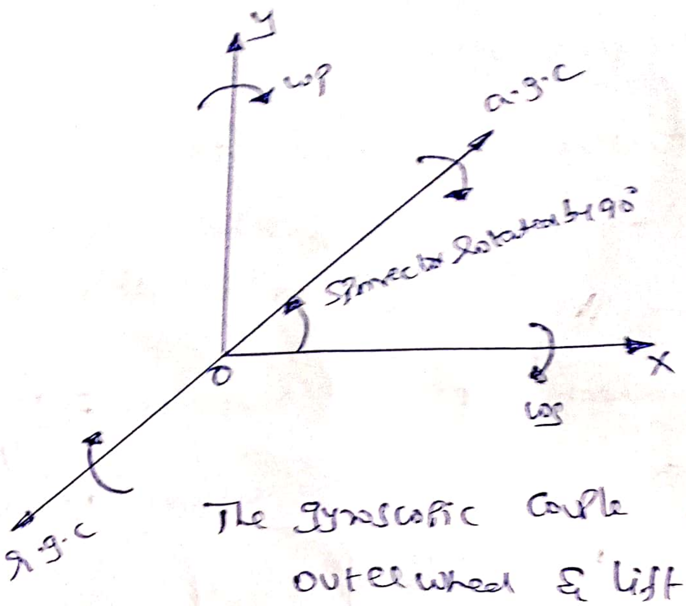
$$C_E = I_E \omega_w \omega_p . G .$$

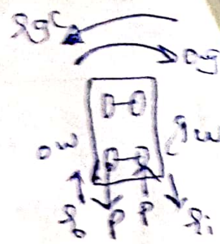
∴ Total gyroscopic couple $C_g = C_w + C_E$

$$C_g = \omega_w \omega_p (4 I_w \pm I_E G)$$

Assuming that vehicle takes a left turn, the reaction gyroscopic couple on vehicle acts in between outer & inner wheels

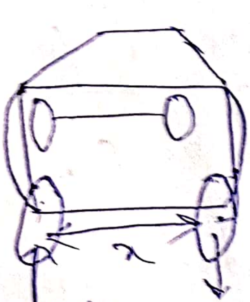






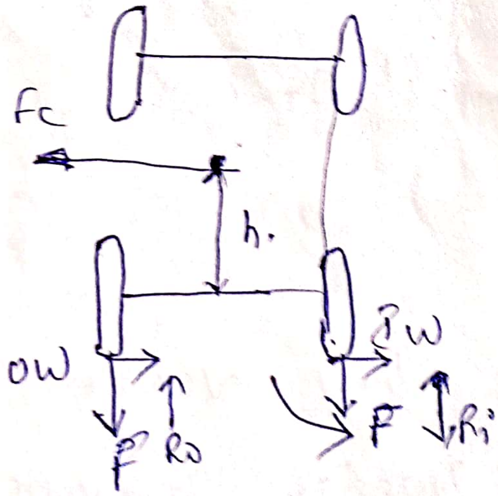
The gyroscopic couple tends to press the outer wheel & lift the inner wheel

Due to reactive gyroscopic couple. vertical reaction of road surface will be produced.



The reaction will be vertically upward on outer wheel
" " " " " downward on inner wheel

Effect of Centrifugal couple



when a vehicle moves in a curved path a centrifugal force acts on the vehicle in outward direction

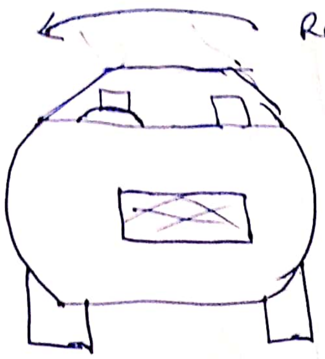
$$\therefore \quad F_c = \frac{mv^2}{R}$$

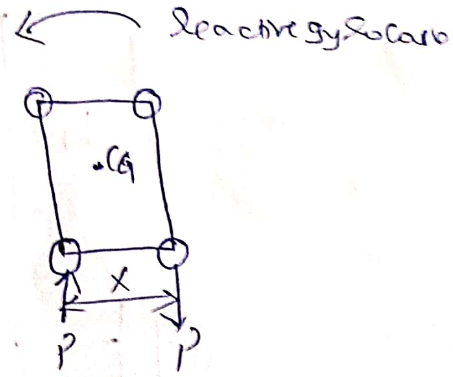
$$\therefore \text{ centrifugal couple } C_c = F_c \times h$$

$$= \frac{mv^2}{R} \times h$$

Centrifugal couple tends to press the outer wheel & lift inner wheel.

Reaction of outer & inner wheel are as follows.





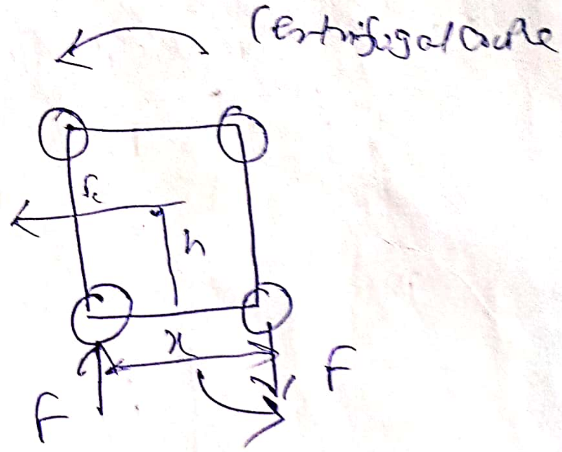
Magnitude of reaction of two outer/inner wheels (P) due to g.c

$\therefore \quad P \times X = C_{gc}$

$P = \frac{C_{gc}}{X}$

$\therefore$ Road reaction each outer/inner wheel

$$\boxed{\frac{P}{2} = \frac{C_{gc}}{2X}}$$

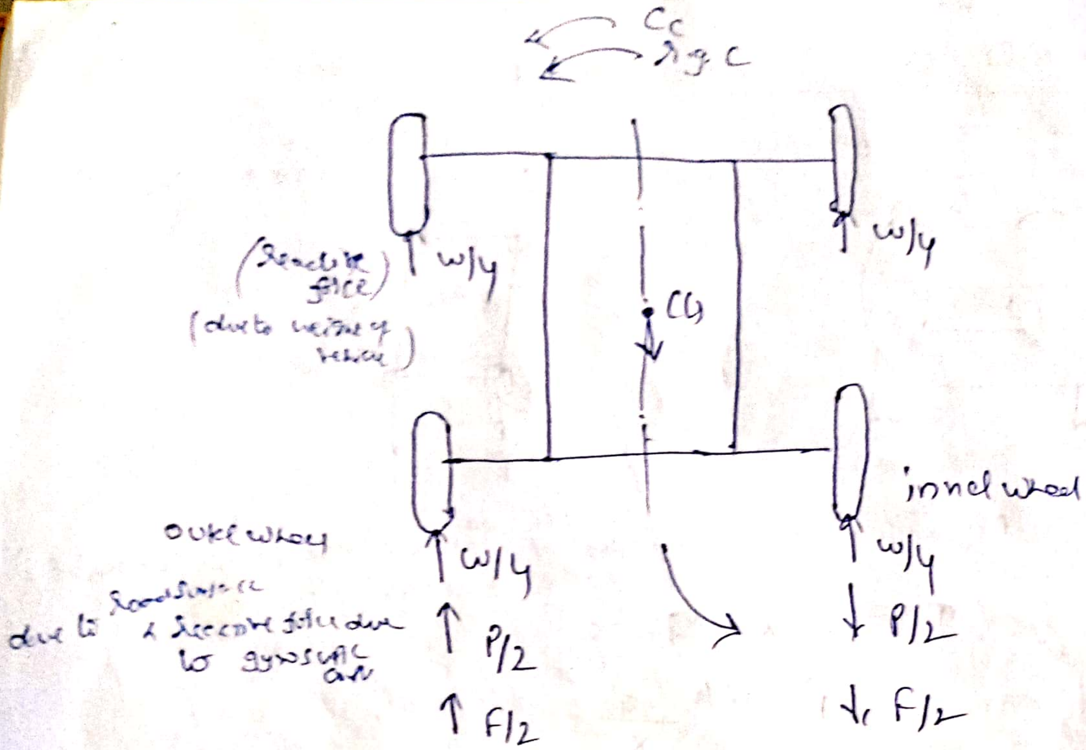


Effect of centrifugal couple

$\therefore \quad F \times x = C_{c}$

$F = \frac{C_{gc}}{X}$

$\boxed{\frac{F}{2} = \frac{C_{gc}}{2X}}$ reaction on each outer/inner wheel due to centrifugal couple



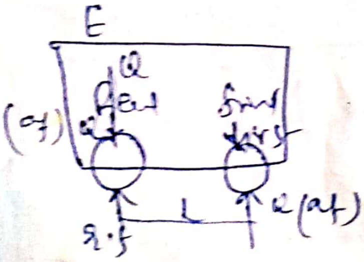
∴ Total vertical reaction at each outer wheel.

$$\boxed{P_o = \frac{w}{4} + \frac{P}{2} + \frac{F}{2}}$$

Total vertical reaction at inner wheel

$$\boxed{P_i = \frac{w}{4} - \frac{P}{2} - F/2}$$

Load due to Engine on wheels.



$Q \cdot L$ = g.c due to Engine

$Q \cdot L = I_e \, \omega_e \, \omega_p$

$$\frac{Q}{2} = \frac{I_e \, i \, \omega_w \, \omega_p}{2L}$$