

Problems on two wheels

D) Each road wheel of a motor cycle has a moment of inertia of $2 \text{ kg}\cdot\text{m}^2$. The rotating parts of the engine of the motor cycle has a moment of inertia $0.2 \text{ kg}\cdot\text{m}^2$. The speed of engine is 5 times the speed of wheel, and is in same sense. The mass of motor cycle with rider is 200 kg & C.G is 500 mm above ground level. The dia of wheel is 500 mm . The motor cycle is travelling at 15 m/sec on a curve of 30 m radius. find:

- 1) Gyroscopic couple, centrifugal couple, overturning couple, Balancing couple
in terms of angle of heel. 2) Angle of heel.

Sol: $I_w = 2 \text{ kg}\cdot\text{m}^2$ $I_e = 0.2 \text{ kg}\cdot\text{m}^2$ Gear ratio (i) = $\frac{\omega_e}{\omega_w} = 5$

$m = 200 \text{ kg}$. C.G = (h) = $500 \text{ mm} = 0.5 \text{ m}$.

Dia of wheel (D_w) = $500 \text{ mm} = 0.5 \text{ m}$ $R_w = 0.25 \text{ m}$.

$V = 15 \text{ m/sec}$ (R) Curve radius (R) = 30 m .

- 1) Gyroscopic couple:

$$C = \frac{v^2}{R R_w} (2 I_w + G I_e) \cos \theta$$

$$C = \frac{15^2}{0.25 \times 30} (2 \times 2 + 5 \times 0.2) \cos \theta$$

$$C = 150 \cos \theta$$

centrifugal couple

$$F_c = \frac{mv^2}{R}$$

$$C_c = F_c \times h \cos \theta$$

$$= \frac{mv^2}{R} \times h \cos \theta$$

$$= \frac{200 \times 15^2}{30} \times 0.5 \cos \theta = 750 \cos \theta$$

overturning couple = $C_g + C_c$

$$= 150 \cos \theta + 750 \cos \theta = 900 \cos \theta$$

Balancing couple = $mgh \sin \theta$

$$= 200 \times 9.8 \times 0.5 \sin \theta = 981.5 \sin \theta$$

2) Angle of heel

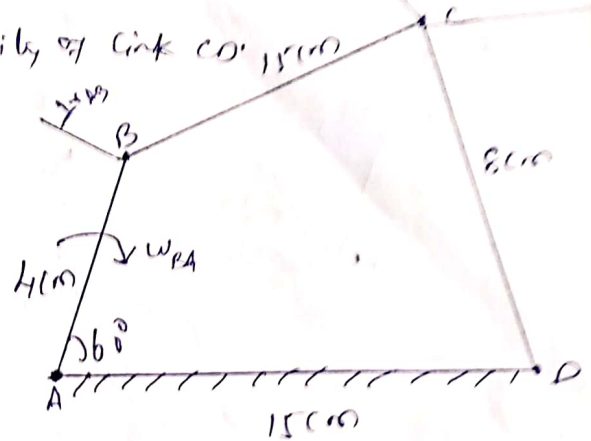
for equilibrium

$$\text{Balancing couple} = \text{overturning couple}$$

$$981.5 \sin \theta = 900 \cos \theta$$

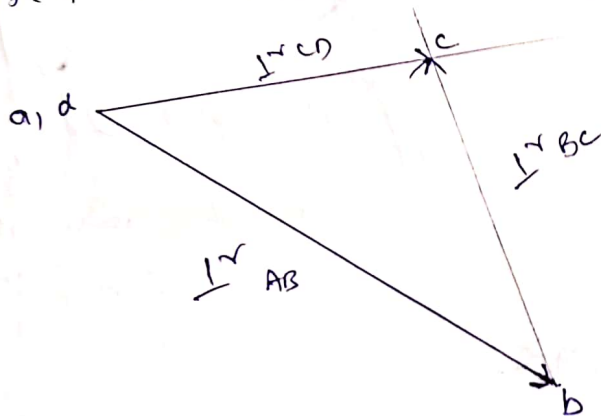
$$\tan \theta = \frac{900}{981} \quad \theta = 42.53^\circ$$

Four bar chain ABCD Link AD is fixed and is 15 cm long. The crank is AB is 4 cm long rotates at 1800 rpm clockwise about A. Link CD rotates about D & is 8 cm long. $\angle BAD = 60^\circ$. Find angular velocity of link CD.



$$\begin{aligned}
 v_B &= \omega \times r \\
 &= \omega_{BA} \times AB = 0.50 \text{ m/s} \\
 &= \frac{2\pi \times 120}{60} \times 4 = 50.24 \text{ cm/s}
 \end{aligned}$$

Choose a suitable scale $1 \text{ cm} = 20 \text{ m/s}$



$$\begin{aligned}
 v_{CB} &= \vec{bc} \\
 v_C &= \vec{ac} = v_{ca} = 38 \text{ cm/s}
 \end{aligned}$$

WKT

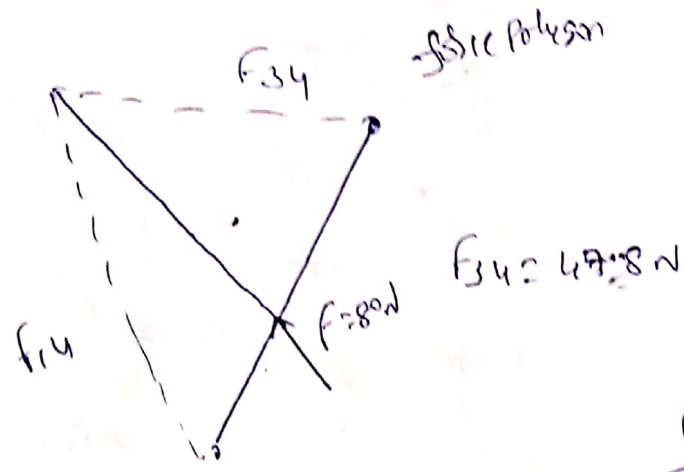
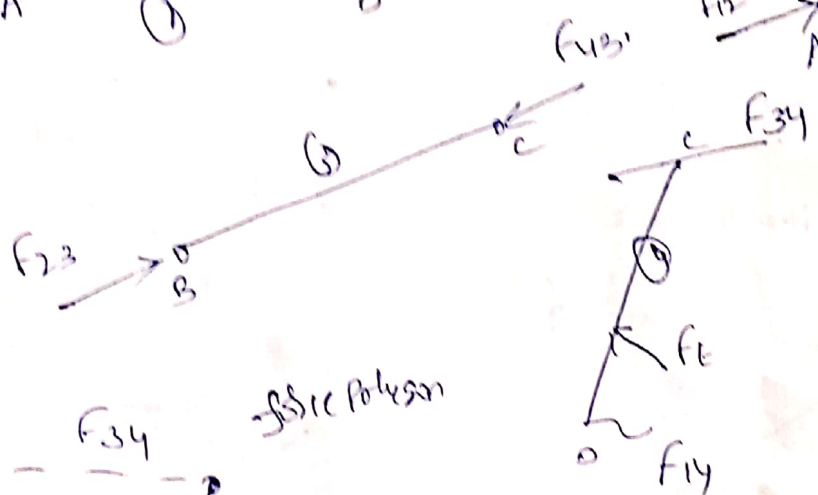
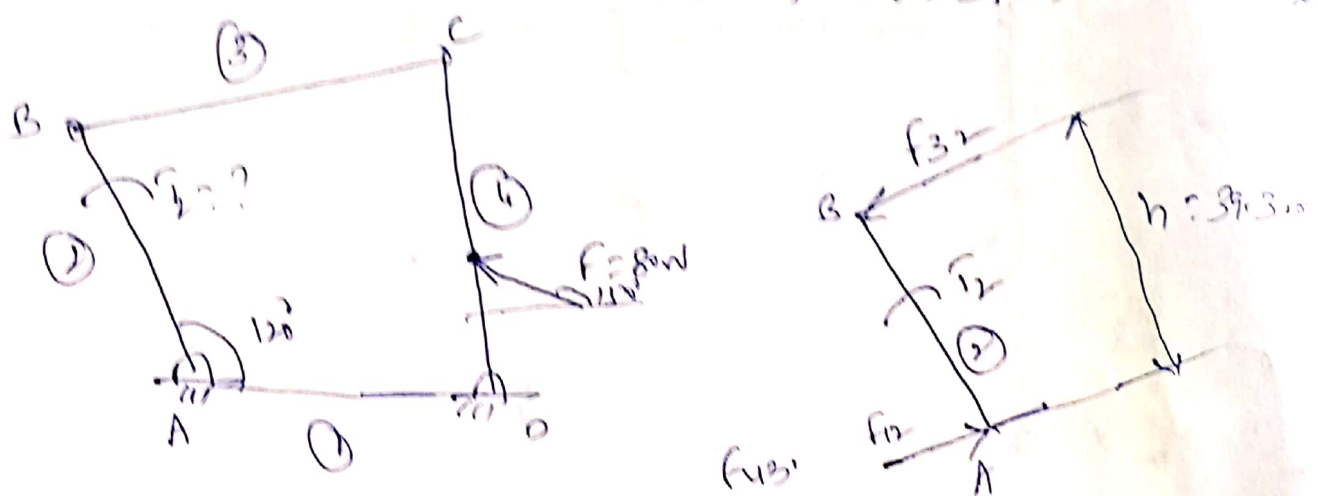
$$v = \omega r$$

$$v_{CD} = \omega_{CD} \times CD$$

$$\begin{aligned}
 \omega_{CD} &= \frac{v_{CD}}{CD} = \frac{38}{8} \\
 &= 4.75 \text{ rev/s}
 \end{aligned}$$

A fuel line mechanism is acted upon by force of 80 N

Determine torque T_2 $AO = 50 \text{ mm}$ $AB = 60 \text{ mm}$ $BC = 100 \text{ mm}$
 $OC = 25$ $OD = 37.5 \text{ mm}$



Scale $20 \text{ N} = 1 \text{ cm}$

$F_{34} = 47.8 \text{ N}$

$F_{43} = -F_{34} = -F_{23} = 47.8 \text{ N}$

$F_{23} = -F_{32} = 47.8 \text{ N}$

$-F_{32} = F_{12} = 47.8 \text{ N}$

$T_2 = 47.8 \times 37.5$
 $= 18.28 \text{ N}\cdot\text{mm}$

measure of force $F_{34} = 47.8 \text{ N}$

An airplane makes a complete half circle of 50 m radius
 towards left when flying at 200 km/hr. The mass of rotary engine
 and propeller is 400 kg with radius of gyration 300 mm. The
 engine runs at 3000 rpm counter clockwise when viewed from the
 rear. Determine gyroscopic couple & its effect on the aircraft.

Sol: Given $R = 50\text{ m}$
 $V = 200\text{ km/hr} = \frac{200 \times 1000}{3600} = 55.56\text{ m/s}$
 $M = 400\text{ kg}$
 $k = 300\text{ mm} = 0.3\text{ m}$ $n = 3000\text{ rpm}$

$C = I \omega \omega_p$
 $= m k^2 \left(\frac{2\pi n}{60} \right) \left(\frac{V}{R} \right) = 400 \times 0.3^2 \times \left(\frac{2\pi \times 3000}{60} \right) \left(\frac{55.56}{50} \right)$
 $= 12566.24\text{ Nm}$

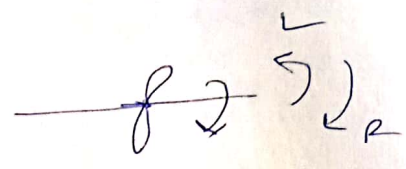
Effect opposite to the tail.

2) An airplane makes a complete half circle of 40 m radius
 towards left when flying at 177 km/hr. The mass of the
 rotary engine & propeller is 400 kg with radius of gyration 300 mm.
 The engine runs at 2500 rpm cw when viewed from the
 rear. Find gyroscopic couple and what is effect of airplane
 turn towards right instead of left.

Sol: (B) Column

$$V = 127 \text{ kN/m} = \frac{127 \times 1000}{3600} = 35.27 \text{ N/mm}$$

$m = 400 \text{ kg}$
 $k = 300 \text{ mm} = 0.3 \text{ m}$
 $n = 2500 \text{ rpm}$



C.S. I.W.W.P

$$I = \pi r^2 \frac{2 \sin \alpha}{\alpha} \frac{V}{P} = 400 \times 0.3^2 \times \frac{25 \times 2500}{60} \times \frac{35.27}{40} = 11453.7 \text{ N}\cdot\text{m}$$

left turn

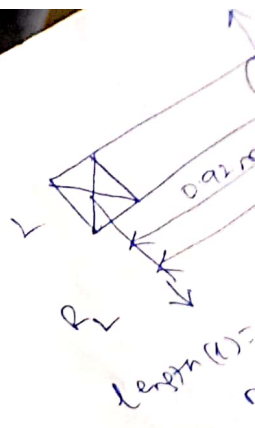
rotate nose up/left

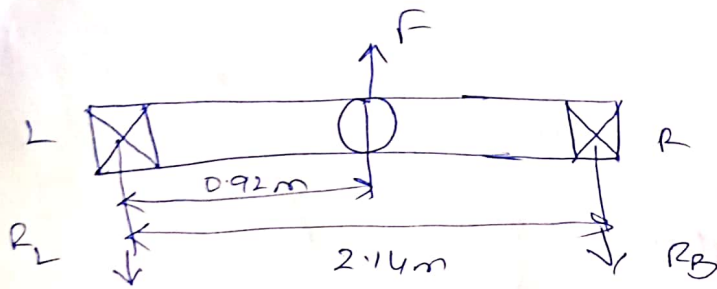
Right turn

(4/2)

rotate nose of rotor up

5) The rotor of a jet airplane engine is supported by two bearings 2.1m apart. The rotor assembly including compressor, turbine, shafts total mass of 68kg. If C.G. is situated at 0.92 m from left bearing & $I_{CG} = 0.229 \text{ m}^2$ determine the max bearing force on the airplane when it undergoes a pullout on a 1830m radius curve at a constant speed of 960 km/hr. Speed of engine rotor is 10000 rpm. Include the effect of centrifugal force due to pullout as well as gyroscopic effect





$$\text{Length } (l) = 2.14 \text{ m}$$

$$m = 688 \text{ kg}$$

$$(k) = 0.229 \text{ m}$$

$$R = 1830$$

$$v = 960 \text{ km/hr} = 266.6 \text{ m/s} \quad N = 10,000$$

1) Consider centrifugal force effect

$$F = \frac{mv^2}{R} = \frac{688 \times (266.6)^2}{1830} = 26733.33 \text{ N}$$

Take moments about left beam

$$R_B \times 2.14 = F \times 0.92$$

$$R_B = 11492.8 \text{ N} \text{ force on right beam due to CF}$$

$$R_L + R_B = F \Rightarrow R_L = 15240.53 \text{ N} \text{ force on LB due to CF}$$

2) Gyroscopic couple

$$C = I \omega \omega_p$$

$$= m k^2 \times \frac{2\pi N}{60} \times \frac{v}{R}$$

$$= 688 \times 0.229^2 \times \frac{2\pi \times 10,000}{60} \times \frac{960 \times 1000}{3600 \times 1830}$$

$$= 9385.9 \text{ N-m}$$

A40

$$C = F \times L$$

$$93859 = F \times 2.14$$

$$F = 4385.9 \text{ N force on each bearing due to gyroscopic effect}$$

∴ Resultant force on right bearing

$$= \sqrt{R_B^2 + F^2} = \sqrt{11492.8^2 + 4385.9^2}$$
$$= 12301.2 \text{ N}$$

Resultant force on left bearing

$$= \sqrt{R_L^2 + F^2} = \sqrt{15260.53^2 + 4385.9^2}$$
$$= 15859.10 \text{ N}$$

A turbine rotor of sea vessel having mass of 950 kg rotates at 1200 rpm CW while looking from stern. The vessel pitches with angular velocity 1.2 rad/sec. What will be the gyroscopic couple transmitted to the hull when the bow raises. $k = 300 \text{ mm}$

Sol. $m = 950 \text{ kg}$ $N = 1200$ $\omega = 1.2 \text{ rad/sec}$ $k = 0.3 \text{ m}$
 $I = mk^2 = 950 \times 0.3^2 = 85.5 \text{ kg m}^2$ $\omega_p = \frac{2\pi N}{60} = 125.6 \text{ rad/sec}$
 $C = I \omega \omega_p^2 = 85.5 \times 1.2 \times 125.6^2 = 12696.5 \text{ Nm}$

Q) A ship is propelled by a rotor turbine which has mass of 5000 kg and has a speed of 2100 rpm. $k = 0.5 \text{ m}$ rotates in CW when view from stern. Find gyroscopic effect in following conditions

- Ship runs at speed of 16 knots (1 knot = 1880 m/hr) if steers to the left in a curve of 600 m radius.
- The ship pitches 6° above & 6° below in horizontal position. The bow decays with its max velocity. The motion due to pitching is sinusoidal & the periodic time is 20 sec.
- The ship rolls at a certain instant has an angular velocity 0.03 rad/sec CW when view from stern.

Sol. $m = 5000 \text{ kg}$ $N = 2100 \text{ rpm}$ $k = 0.5 \text{ m}$ $v = 16 \text{ knots} = \frac{16 \times 1880}{3600} \text{ m/s}$
 $R = 600 \text{ m}$ $\phi = \text{pitching of ship } 6^\circ = 6 \times \frac{\pi}{180} \text{ rad}$
 Periodic time $T_p = 20 \text{ sec}$ $\omega_p = 0.03 \text{ rad/sec}$

1) Steering

$C = I \omega \omega_p^2 = 5000 \times 0.5^2 \times \frac{2\pi \times 2100}{60} \times \frac{16 \times 1880}{3600 \times 60}$

Effect: raise the bow & dip the stern = 28736 Nm

2) Pitching: $C = I \omega \omega_p$
 $= mk^2 \times \frac{2\pi n}{60} \times \theta \frac{2\pi}{180}$

$= 5000 \times 0.5^2 \times \frac{20 \times 2100}{60} \times 6 \times \frac{20}{20} \times \frac{\theta}{180} = 9043.49 \text{ Nm}$

Effect: tends to turn left hand

3) Rolling: $C = I \omega \omega_r = 5000 \times 0.1^2 \times \frac{20 \times 2100}{60} \times 0.03$

$= 8246.6 \text{ Nm}$
 In rolling no gyroscopic effect

3) The rotor of a marine turbine has a mass of 1000 kg & $k = 0.3 \text{ m}$ rotates at 1550 rpm CW when taken from low. determine $g.c$ & its effect on following cases

- When the ship pitches with an angular velocity of 1 rad/sec when the ship is rolling fully
- When the ship is moving at 40 km/hr & takes a right turn in circular path of 200 m radius
- When the ship rolls at a certain instant, it has an angular velocity of 0.5 rad/sec when view from stern.

Sol: $k = 0.3 \text{ m}$ $n = 1550 \text{ rpm}$ $m = 1000 \text{ kg}$

(ω_p) Pitching = 1 rad/sec

$v = 40 \text{ km/hr} = R = 200 \text{ m}$

(ω_r) Rolling = 0.5 rad/sec