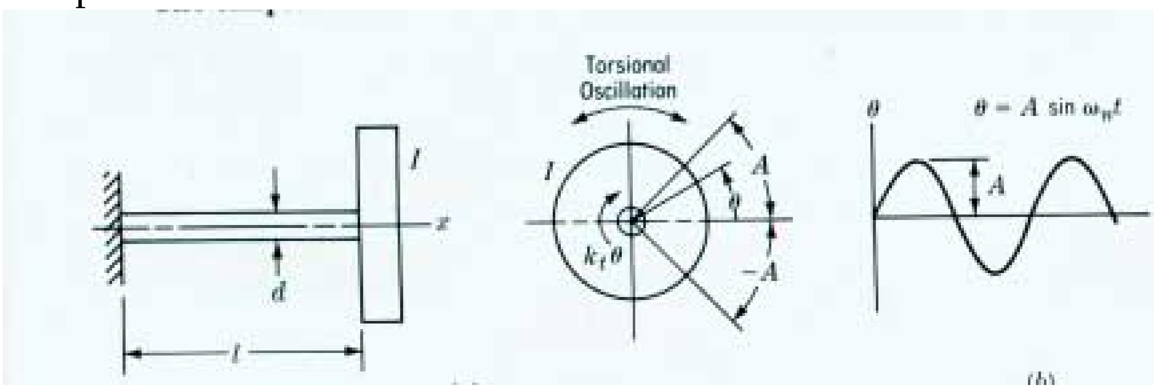
Torsional Vibrations:

---When a shaft is transmitting torque it is subjected to twisting of torsional deflection; and if there are cyclic variations in the transmitted torque the shaft will oscillate, that is twist and untwist.

Assumption: Mass moment of inertia of the disk is large compared with the mass moment of inertia of the shaft.



For the torque exerted by the rod:

$$T = I * \alpha$$

Therefore

$$-K_t\theta = I \frac{d^2\theta}{dt^2}$$

Where

K_t = torsional spring constant of the shaft

The negative sign is used because T is opposite in sense to θ.

$\therefore$

$$\frac{d^2\theta}{dt^2} + \frac{K_t}{I}\theta = 0$$

It is a homogeneous differential equation. Solution is

$$\theta = C_1 \cos\left(\sqrt{\frac{K_t}{I}}\quad t\right) + C_2 \sin\left(\sqrt{\frac{K_t}{I}}\quad t\right)$$

Boundary conditions:

(1) $\boldsymbol{\theta = 0}$ when $\boldsymbol{t = 0}$
(2) $\boldsymbol{\theta = A}$ when ***sqrt(Kt/I) * t = π/2***

∴ $\boldsymbol{C_1 = 0}$ and $\boldsymbol{C_2 = A}$

and the solution becomes

$$\theta = A \sin\left(\sqrt{\frac{K_t}{I}}\quad t\right)$$

This is a simple harmonic motion.

$$\omega_n = \sqrt{\frac{K_t}{I}}$$

Since $\boldsymbol{K_t = T/\theta}$ ***and*** $\boldsymbol{\theta = (TL)/(GJ)}$***, so*** $\boldsymbol{K_t=(JG)/L}$

Therefore

$$\omega_n = \sqrt{\frac{\pi\, d^4\, G}{32\, I\, L}}$$

Where

ω_n = natural frequency of torsional vibration, ***rad/s***

f_n = natural frequency in ***cycles/sec*** = ω_n /(2 π)

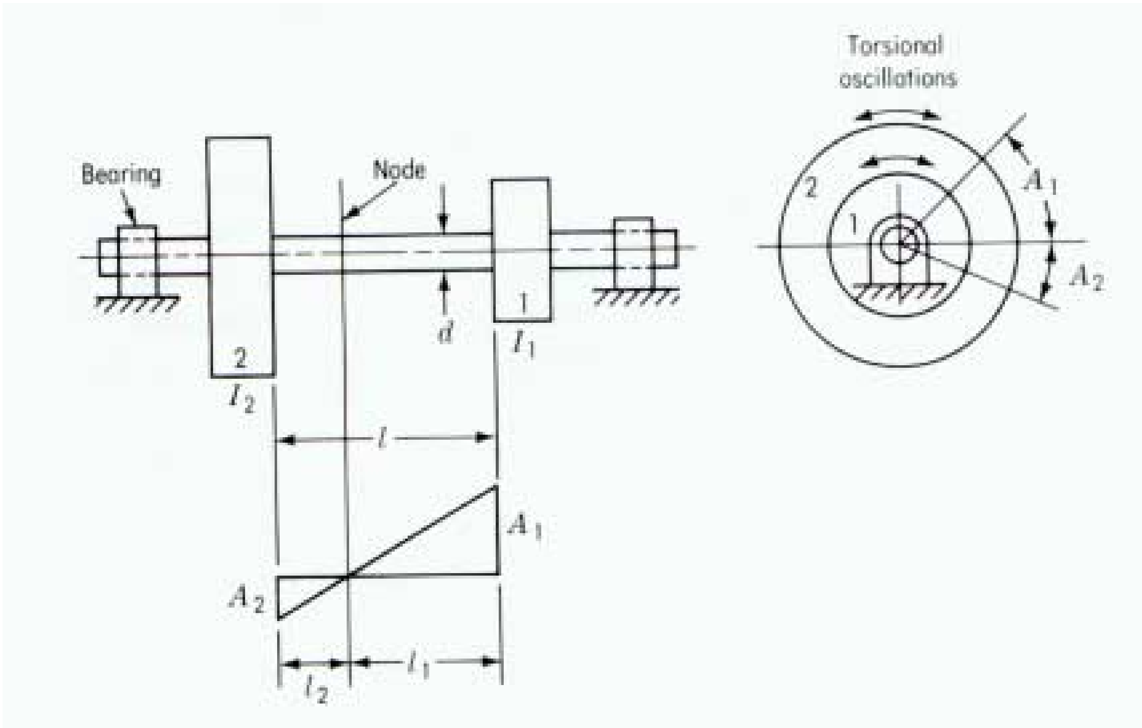
d = shaft diameter, ***m***

G = modulus of elasticity in shear for shaft material (***79.3*** x 10^9 ***Pa*** for steel)

I = mass moment of inertia of disk about the x-axis = Mr^2

L = shaft length, ***m***

<u>Torsional Vibration of a Shaft with Two Disks</u>



When disk 1 is rotating in a counterclockwise direction, disk 2 must be rotating in a clockwise direction. The natural frequency can be calculated be considering the system as composed of two single mass systems where the shaft consist of two lengths $\boldsymbol{l_1}$ and $\boldsymbol{l_2}$ and their ends meet at the plane of zero motion, or ***node***.

The frequency of the two masses is the same.

Since $\boldsymbol{K_t = (GJ)/L}$,

$$\omega_n = \sqrt{\frac{J\,G}{\ell_1\, I_1}} = \sqrt{\frac{J\,G}{\ell_2\, I_2}}$$

Thus

$$\ell_1 I_1 = \ell_2 I_2 = I_2 (\ell - \ell_1)$$

$\therefore$

$$\ell_1 = \frac{I_2 \ \ell}{I_1 + I_2}$$

(This gives the position of the node.)

therefore

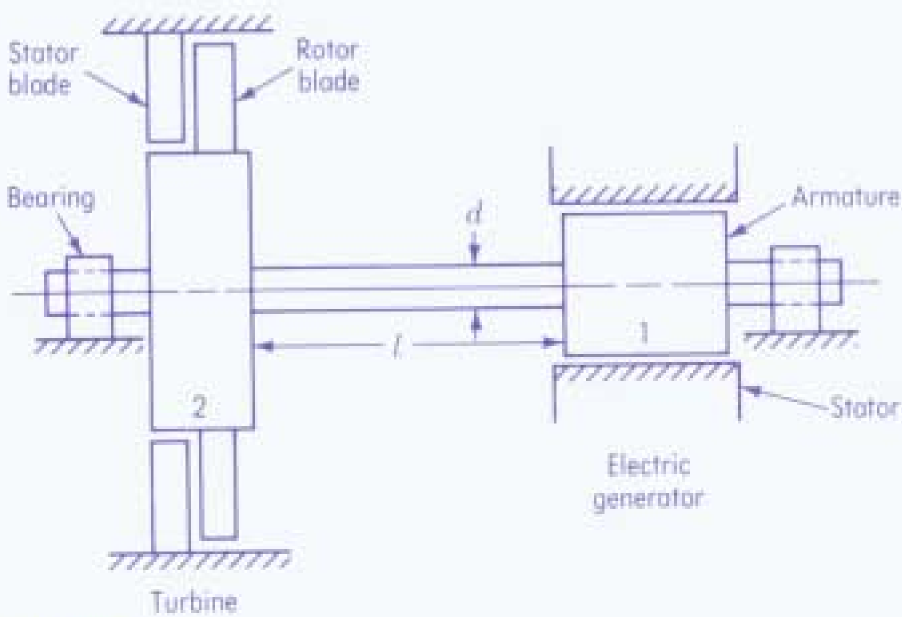
$$\omega_n = \sqrt{\frac{J\,G}{\ell} \ \frac{I_1 + I_2}{I_1\,I_2}} = \sqrt{K_t \ \frac{I_1 + I_2}{I_1\,I_2}}$$

Disk 1 and 2 will oscillate with amplitudes A1 and A2, respectively.

The node will be nearer the disk with the larger moment of inertia.

Examples:

(1) motor driving a generator
(2) turbine driving a generator
(3) motor driving a centrifugal pump
(4) radial aircraft engine driving a propeller


Stator
blade
Rotor
blade
Bearing
d
Armature
l
1
2
Stator
Electric
generator
Turbine
Figure 22-9

Example 22-3 In Fig. 22-9 there are 14 blades on disk 2 and the stator has the same number of blades. The mass of disk 2 is 9.43 kg, and it has a radius of gyration of 0.160 m. The armature has a mass of 4.72 kg and a radius of gyration of 0.0844 m. The diameter and length of the steel shaft which transmits torque from the turbine rotor to the armature are 52 and 257 mm, respectively. Determine the torsional critical speed n_c for the system due to the disturbance of the passing blades.

SOLUTION

$$k_t = \frac{JG}{l}$$

where $G = 79.3 \times 10^9$ Pa

$$J = \frac{\pi d^4}{32} = \frac{\pi(0.052)^4}{32} = 0.717\ 8 \times 10^{-6}\ \text{m}^4$$

$$k_t = \frac{(0.717\ 8 \times 10^{-6})(79.3 \times 10^9)}{0.257} = 221.5 \times 10^3\ \text{N} \cdot \text{m/rad}$$

$$I_1 = M_1 r_1^2 = 4.72(0.084\ 4)^2 = 0.033\ 62\ \text{kg} \cdot \text{m}^2$$

$$I_2 = M_2 r_2^2 = 9.43(0.160)^2 = 0.241\ 4\ \text{kg} \cdot \text{m}^2$$

$$f_n = \sqrt{k_t \frac{I_1 + I_2}{I_1 I_2}} = \sqrt{(221.5 \times 10^3)\frac{0.033\ 62 + 0.241\ 4}{0.033\ 62 \times 0.241\ 4}}$$

$$= \sqrt{7.506 \times 10^6} = 2\ 740\ \text{rad/s} \quad \text{or} \quad \frac{2\ 740 \times 60}{2\pi}$$

$$= 26\ 165\ \text{cycles/min}$$

Number of stator blades $N = 14$. Number of disturbances per minute $= Nn = 14n$. For resonance, $14n = f_n = 26\ 165$ cycles/min. Critical torsional speed $n_c = n = 26\ 165/14 = 1\ 869$ r/min.

Stepped Shafts

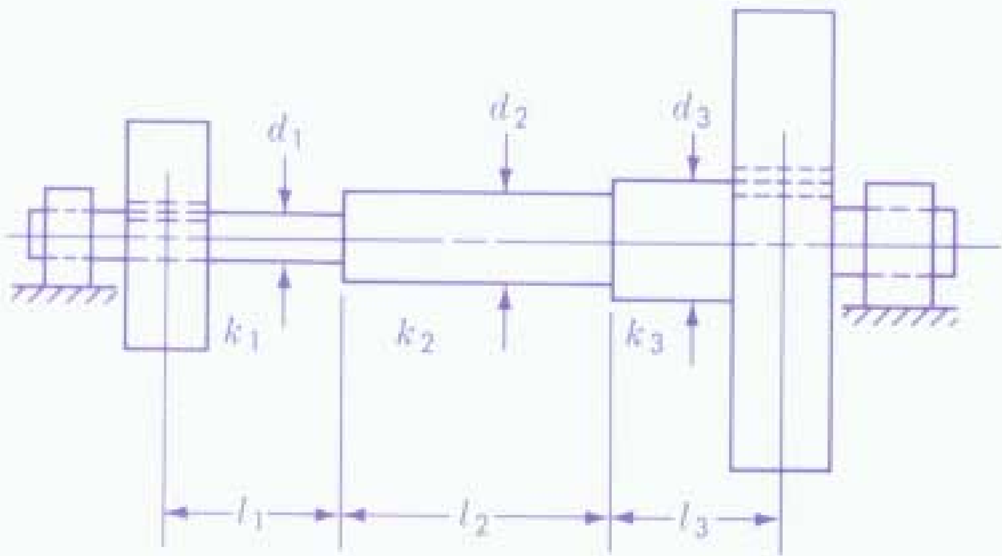


Figure 22-10

$$\theta_t = \theta_1 + \theta_2 + \theta_3 + \cdots + \theta_n$$

$$\frac{T}{k_t} = \frac{T}{k_1} + \frac{T}{k_2} + \frac{T}{k_3} + \cdots + \frac{T}{k_n}$$

$$\frac{1}{k_t} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \cdots + \frac{1}{k_n}$$

$$\frac{1}{k_t} = \Sigma \frac{1}{k}$$

Substitute K_t into equation for ω_n of two disks system.