Unit-III

Forces

→ In high speed machinery such as IC Engines, centrifugal pumps, steam & gas turbines it is very essential that all the rotating & reciprocating parts should be completely balanced as far as possible.

→ If these parts are not balanced properly, the dynamic forces are set up.

→ These forces not only increase the loads on bearings but also stresses in various members, and produce unpleasant & even dangerous vibrations.

→ The phenomenon of uneven distribution of mass about rotors rotating centre line is called as Unbalance

Causes of unbalance

1) in accuracy in the casting
2) " " " machining of the parts.
3) Due to slight variation in density of material.

Types of unbalance

1) Static unbalance
2) Dynamic "
3) Couple "

Effects of unbalance

1) These forces increase the stresses in various members of machine

2) increase loads on bearings

3) These forces produce dangerous vibrations

Balancing: It is the technique of correcting & Eliminating unwanted inertia forces & moments of rotating & Reciprocating mass and is achieved by changing the location of mass centre.

→ Balancing is the process of designing & modifying machinery so that unbalance is reduced to an acceptable level (or) completely Eliminated.

→ In general problem of balancing relates to correction & elimination of disturbing effects arise due to masses whose centre of mass do not lie in axis of rotation.

Objectives of Balancing

1) The centre of gravity of system remains stationary during complete revolution of crank shaft.

2) acceleration of different moving parts to be balanced.

Types of Balancing

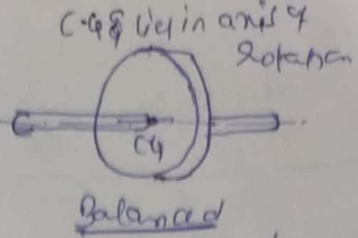
1) Static balancing
2) Dynamic balancing.

Static balancing: It is the balancing of force due to action of gravity. (or) A body is in static balance when centre of gravity in in axis of rotation.

Dynamic balancing: It is the balance due to action of inertia force. A body is in dynamic balance when the resultant moments of couple which involved in acceleration of different moving parts is equal to zero.

Reference Plane: In actual practice, it may not be always possible to balance rotating mass in plane of rotation. It can be balanced by mass acting in a different plane by transfering the rotation of mass in the plane in which balance mass is put. Such type of plane is called Reference Plane.

Balancing of Rotating masses

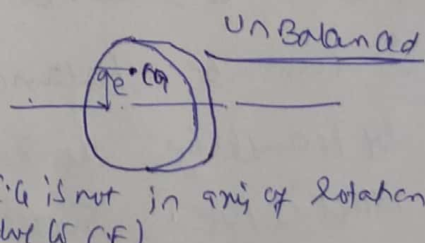


→ When a mass rotates along a circular path it experience a centripetal acceleration which generates radially inward force. An equal & opposite force called centrifugal force acts radially outwards, which is the main source of unbalancing of rotor system. The magnitude of centrifugal force is constant but direction changes with rotation of mass.

→ In revolving rotor centrifugal force remains balanced as long as centre of mass of rotor lie in axis of rotation of shaft. If this doesnot happen, there is an eccentricity & a unbalance force is produced.

→ In order to prevent effect (unbalance) of centrifugal force another mass is attached to the opposite side of the shaft, at a position to eliminate the effect of centrifugal force of first mass.

→ The process providing second mass in order to counter attack the effect of centrifugal force of first mass is called Balancing of rotating mass



Cases arise in Problems of balancing of rotating masses.

1) Balancing of Single rotating mass by a Single mass rotating in Same plane

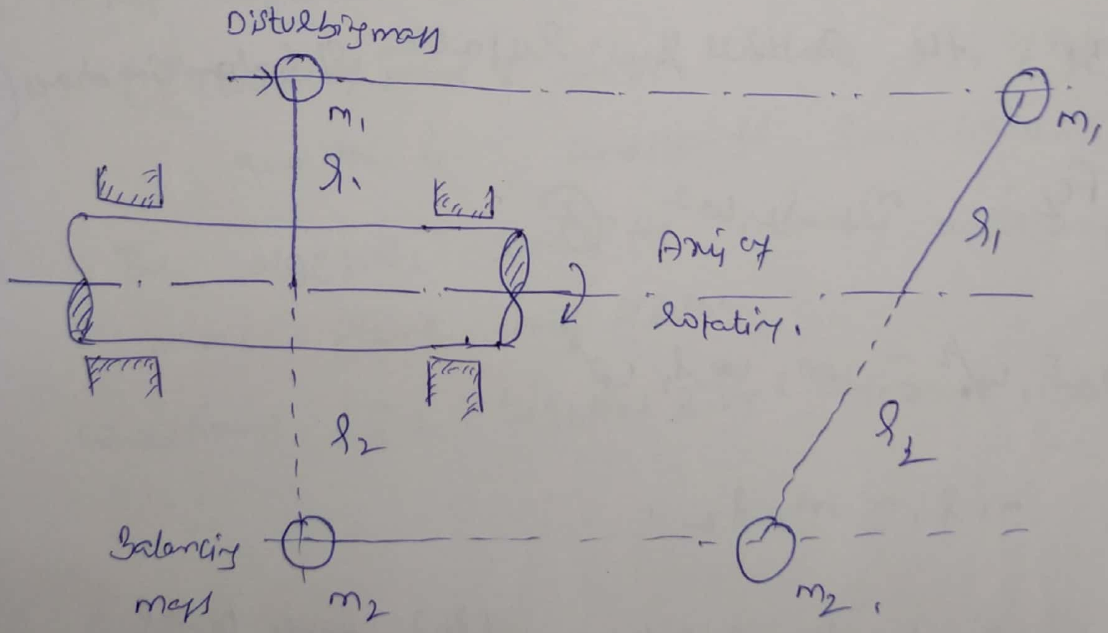
2) Balancing of Single rotating mass by two masses rotating in Same Plane

3) Balancing of Several masses rotating in Same Plane

4) Balancing of Several masses rotating in different plane.

Case 1

Balancing of Single rotating mass by a Single Mass rotating in same plane.



→ Consider a disturbing mass m_1 is attached to a shaft rotating with angular velocity ω rad/sec

→ Let r_1 be the radius of rotating mass (m_1)

$\therefore$ Centrifugal force exerted by mass m_1 on shaft is

$$F_{c_1} = m_1 r_1 \omega^2 \quad (1)$$

Which C.F acts radially outwards & thus bending moment on the shaft. To counter attack the effect of this force, a balancing mass (m_2) is attached in the same plane of rotation of that disturbing mass (m_1) such that C.F due to two masses are equal & opposite.

Let r_2 be the radius of rotation of balancing mass (m_2)

$$F_{c_2} = m_2 r_2 \omega^2 \quad (2)$$

$$m_1 r_1 \cancel{\omega^2} = m_2 r_2 \cancel{\omega^2}$$

$$m_1 r_1 = m_2 r_2 .$$

In general radius of rotation (r_2) made larger in order to reduce mass (m_2)

Balancing of a Single Rotating mass by two masses rotating in different planes

There are two possibilities while attaching two balancing masses

1) The plane of disturbing mass lie in between planes of two balancing masses

2) The plane of disturbing mass may be left or right side of two planes containing the balancing masses.

→ In order to balance a single rotating mass by two masses rotating in different planes which are parallel to plane of rotation of disturbing mass, they satisfy the following condition

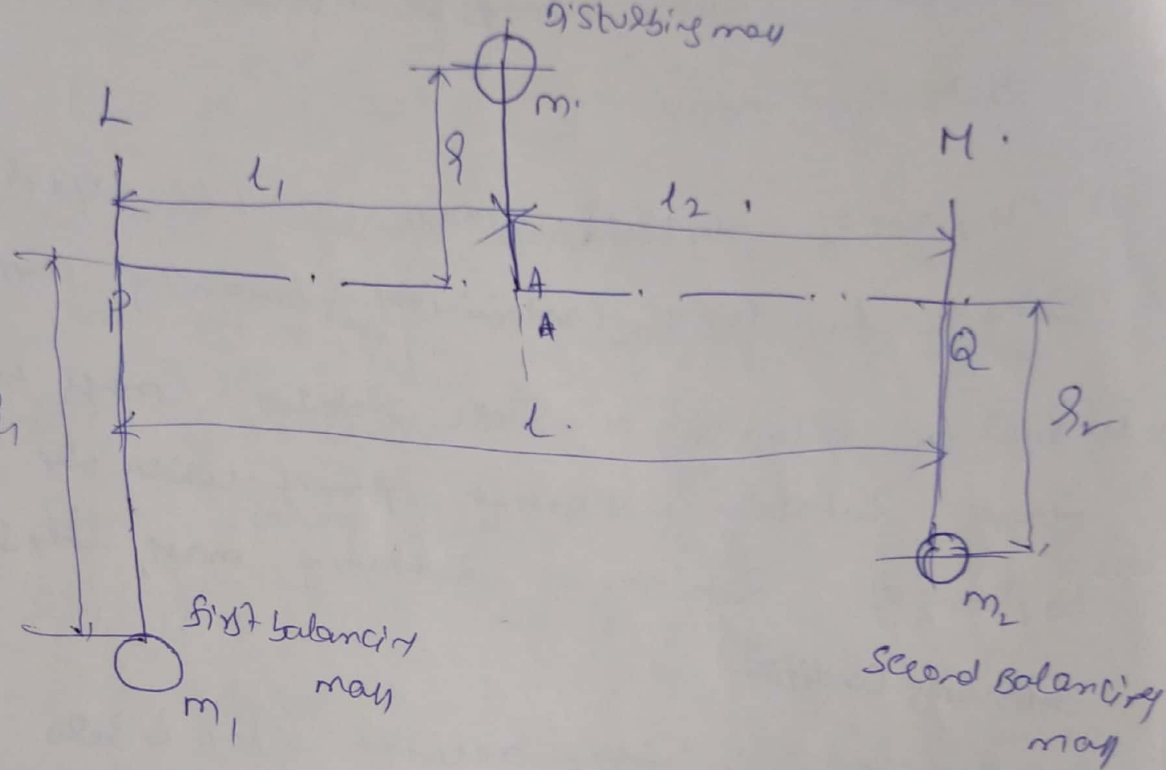
1) The net dynamic force acting on shaft is zero i.e (centre of mass of shaft lie on axis of rotation and this is condition for static balancing)

2) The algebraic sum of the moments about any point in the plane must be zero.

conditions 1 & 2 together give dynamic balancing.

Case 1)

Plane of disturbing mass lies in between the planes of the two balancing masses.



l = distance between the planes L & M

l_1 = " " " A & L

l_2 = " " " A & M.

r = radius of rotation of mass m in plane A

r_1 = " " " " m_1 in plane L

r_2 = " " " " m_2 in plane M

Centrifugal force exerted by mass 'm' in the plane 'A' is

$$F_c = m r \omega^2 \quad -(1)$$

CF exerted by mass m_1 in plane L

$$F_{c1} = m_1 r_1 \omega^2 \quad -(2)$$

CF exerted by mass m_2 in plane M

$$F_{c2} = m_2 r_2 \omega^2 \quad -(3)$$

For perfect balancing the net force acting on shaft = 0

$\therefore$ CF of disturbing mass = CF of balancing mass

$$F_c = F_{c1} + F_{c2}.$$

$$m r \omega^2 = m_1 r_1 \omega^2 + m_2 r_2 \omega^2$$

$$m r = m_1 r_1 + m_2 r_2 \quad -(4)$$

(for static balance) Eqn (4) must be satisfied.

Let P be the point of intersection of plane L & axis of rotation
Q " " " " plane M & axis of rotation

→ In order to find magnitude of balancing force in plane L, take moments about Q

Similarly take moments about P

$$F_{c1} \times l = F_c \times l_2.$$

$$m_1 r_1 \omega^2 \times l = m r \omega^2 \times l_2$$

$$m_1 r_1 l = m r l_2.$$

$$\boxed{m_1 r_1 = m r \frac{l_2}{l}} \quad -(5)$$

$$F_{c2} \times l = F_c \times l_1$$

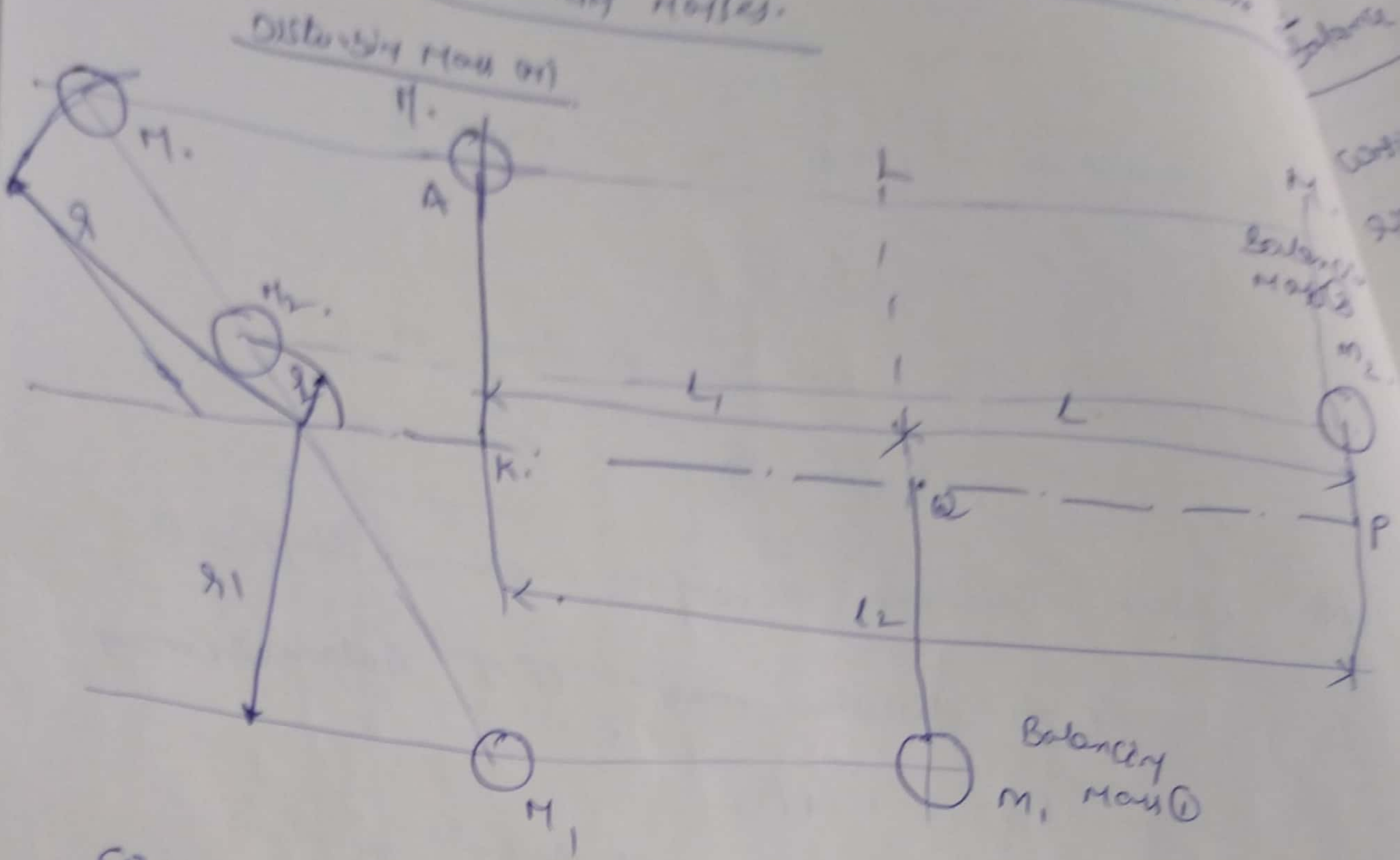
$$m_2 r_2 \omega^2 \times l = m r \omega^2 l_1$$

$$\boxed{m_2 r_2 = m r \frac{l_1}{l}} \quad -(6)$$

Eqn 5, 6, satisfied by dynamic balancing.

Case - 2

Plane of disturbing mass lie on one end of plane of balancing masses.



Consider a mass 'm' lie in plane 'K' & balancing mass in plane L, & M.

For perfect balancing the following condition must be satisfied.

$$F_c + F_{c_2} = F_{c_1}$$

$$m r \omega^2 + m_2 r_2 \omega^2 = m_1 r_1 \omega^2$$

$$\boxed{m r + m_2 r_2 = m_1 r_1}$$ - for static balancing.

In order to find balancing force in plane (L) take moments about (P)

$$F_{c_1} \times l = F_c \times l_2$$

$$\boxed{m_1 r_1 = m \cdot r \frac{l_2}{l}}$$

To find balancing force in (M) take moments about (Q)

$$F_{c_2} \times l = F_c \times l_1$$

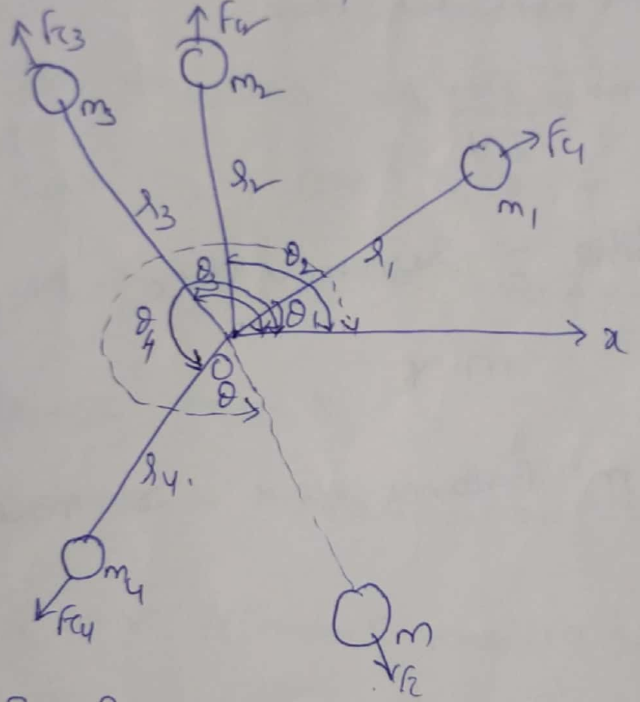
$$m_2 r_2 \omega^2 l = m r \omega^2 l_1$$

$$\boxed{m_2 r_2 = m \cdot r \frac{l_1}{l}}$$

Balance of Several Masses Rotating in Same Plane

→ Consider four masses of magnitude m_1 m_2 m_3 m_4 which are rigidly attached to a shaft which is revolving at uniform angular speed 'ω' radius at distance r_1 r_2 r_3 r_4 from axis of rotation of shaft.

→ let θ_1 θ_2 θ_3 θ_4 be the angle of these masses with horizontal line. let these mass rotate about an axis 'O' and perpendicular to plane of paper.



The magnitude & position of balancing mass may be found out analytically or graphically

Analytical Method

1) find the centrifugal force exerted by each mass on the rotating shaft.

2) Resolve the obtained centrifugal forces horizontally & vertically & find their sum ie ΣH, ΣV

Sum of horizontal components

$\Sigma H = 0$

$= m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + \ldots$

$\Sigma V = 0 \quad m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + \ldots$

3) Find Magnitude of resultant centrifugal force by

$F_c = \sqrt{\Sigma H^2 + \Sigma V^2}$

4) If θ is the angle, which the resultant force makes with horizontal then

$\tan\theta = \frac{\Sigma V}{\Sigma H}$

5) Balance force = resultant force

$F_c = \quad m \cdot r$

m = Balance Mass r = radius of rotation

Scanned by CamScanner

Balancing of Several M   * Plane

Problem:-

four Masses $m_1\ m_2\ m_3\ m_4$ having radius of 200, 150, 250, 300mm are 200, 300, 240, 260kg. The angle between successive masses are 45°, 75°, 135° respectively. Find the magnitude of direction of balance mass required if radius of rotation is 200mm.

Sol:- $\theta_1 = 0$ $\theta_2 = 45$ $\theta_3 = 45+75 = 120$ $\theta_4 = 120+135$
$= 255°$

$m_1 = 200$ $m_2 = 300$ $m_3 = 240$ $m_4 = 260kg$

$r_1 = 200 = 0.2m$ $r_2 = .15m$ $r_3 = 0.25m$ $r_4 = 0.3m$

radius of rotat. of ' ' ma mass $(m) = 0.2m$

let m = Balancing mass kg

θ = angle made by balancing mass to m_1

wkt $CF \propto mr$ (mass of radius of rotation of each part)

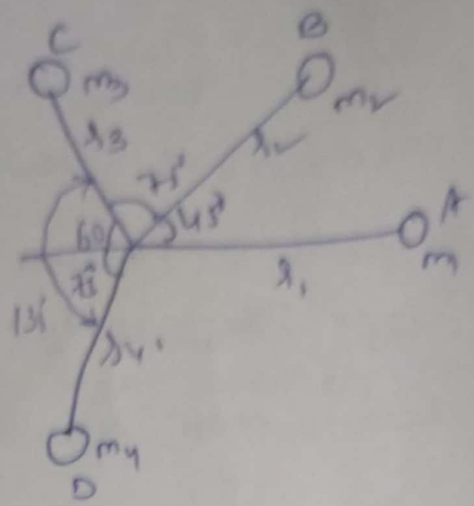
$m_1 r_1 = 200 \times 0.2 = 40kg\text{-}m$ $m_3 r_3 = 60kgm$

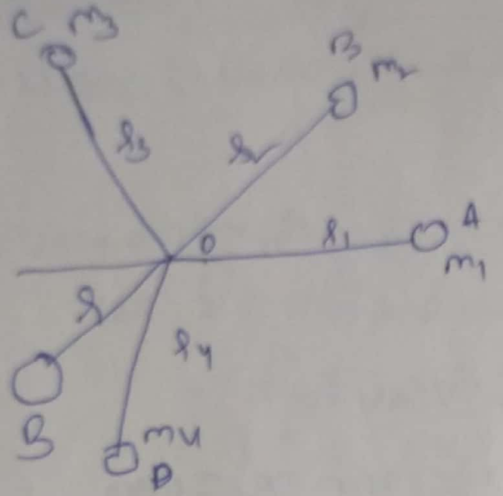
$m_2 r_2 = 300 \times 0.15 = 45kg\text{-}m$ $m_4 r_4 = 78kgm$

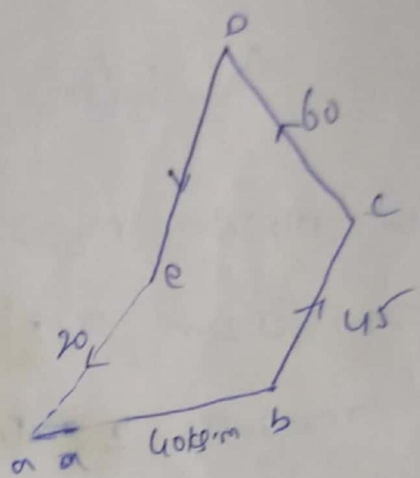
balancing force $mr = 0.2m\ kg.m$

we can solve this graphically & analytically

Graphical M







Draw ab parallel to

draw bc " to OB

cd " to OC

de " OD

Thus ae represents resultant force = 20 cm

wkt $m \cdot r = ae$

$m \times 0.2 = 20$ $\quad m = \frac{20}{0.2} = 115$ kgm

By measurement we find angle $\theta = 201$

Analytical Method

Resolve the forces vertically we get

$\Sigma F_V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4$

$= 40 \sin 0 + 45 \sin 45 + 60 \sin 120 + 78 \sin 225$

$= 28.6 \text{ kgm}$

$\Sigma F_H: \quad m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_4 \cos\theta_4$

$= 40 \cos 0 + 45 \cos 45 + 60 \cos 120 + 78 \cos 225$

$= 38.6 \text{ kgm}$

$F = \sqrt{F_H^2 + F_V^2} = \sqrt{(28.6)^2 + (38.6)^2} = 48.04 \text{ kg-m}$

WKT $mr = 48.04$

$m = \dfrac{48.04}{0.2} = 240.2 \text{ kg}$

$\tan\theta = \dfrac{\Sigma F_V}{\Sigma F_H} = \dfrac{28.6}{38.6} = 0.7409$

$\theta = \tan^{-1}(0.7409) = 36.53$

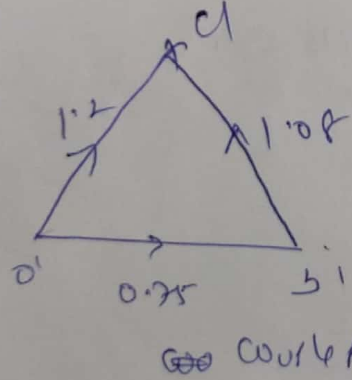
$\therefore$ Direction from $A = \theta + 180$

$= 36.53 + 180 = 216.53$

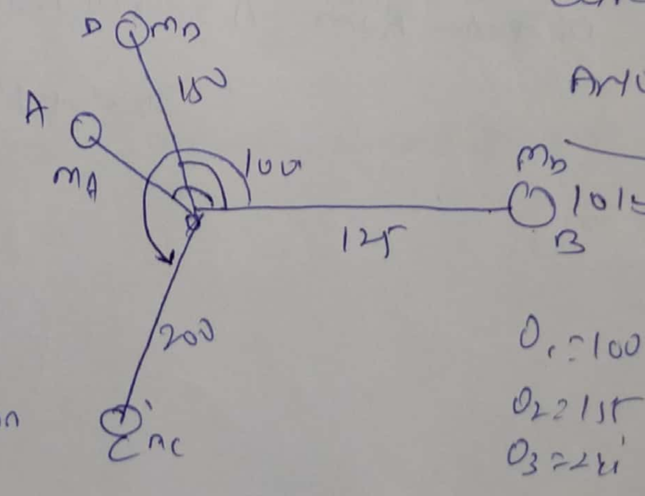
(2) A four masses carried by rotating shaft at radii 100, 125, 200, 150 respectively. The planes in which masses revolve are spaced 600mm apart & mass B, C, D are 10 kg, 5 kg, 4 kg respectively. Find required mass A & relative angular setting of four masses so that shaft shall be in complete balance.

S.No	Plane	Mass	radius	CF mr kg-m	Distance from plane (A) l (m)	Couple mrl kg-m²
1	A (RP)	mA	0.1	0.1 mA	0	0
2	B	10	0.125	1.25	0.6	0.75
3	C	5	0.2	1	1.2	1.2
4	D	4	0.15	0.6	1.8	1.08

Angular setting of mass C, D is obtained by drawing couple polygon from above data in couple assume position of 'B' is in Horizontal direction



Couple polygon



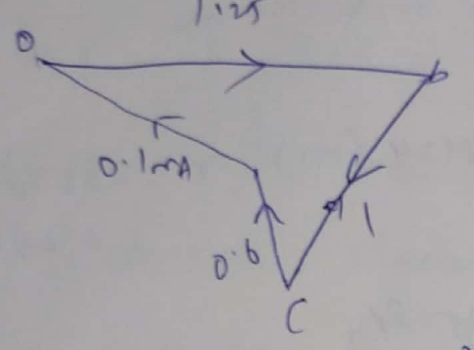
Angular position of masses

$\theta_1 = 100$

$\theta_2 = 155$

$\theta_3 = 24$

force Polygon



$0.1\,m_A = 0.7$ kg·m^2

$m_A = 7$ kg.

Measure angle $OA = 115°$

(3) A disc is rotating about a vertical Spindle, has the following masses A, B, C, of Magnitude 2.5, 3.5, 5.0 kg mounted an angle of 0°, 60°, 150° at a distance of 260, 300, 225 mm Determine the Magnitude and angular Position of the balance Mass that Should be Placed at 262.5 mm from centre of rotation. Determine the unbalanced force on the Spindle when disc is rotating at 250 rpm

Sol:

Mass	θ	distance (mm)	Magnitude
A	0	260	2.5 kg
B	60	300	3.5 kg
C	150	225	5 kg

$m_1 r_1 = 2.5 \times 0.26 \quad \theta_1 = 0 \quad = 0.65 \text{ kg m}$

$m_2 r_2 = 3.5 \times 0.3 \quad \theta_2 = 60 \quad = 1.05 \text{ kg m}$

$m_3 r_3 = 4.5 \times 0.25 \quad \theta_3 = 150° \quad = 1.125 \text{ kg m}$

Radius of rotating balance mass (m), $r = 262.5$
$= 0.2625 \text{ m}$

Rotation speed of balance mass = 250 rpm

Resolve the forces vertically

$\Sigma F_V = 0.65 \sin 0 + 1.05 \sin 60 + 1.125 \sin 150$
$= 0.01037$

$\Sigma F_H = 0.65 \cos 0 + 1.05 \cos 60 + 1.125 \cos 150$
$= 0.6499$

F = Resultant force $F = \sqrt{\Sigma F_V^2 + \Sigma F_H^2}$

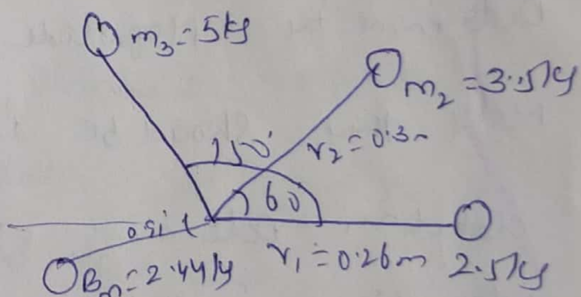
$= \sqrt{(0.01037)^2 + (0.6499)^2} = 0.6410 \text{ kg m}$

W.K.T

$m \cdot r = F$

$m \times 0.2625 = 0.6410$

$m = 2.44 \text{ kg}$



W.K.T $\tan\theta = \dfrac{\Sigma F_V}{\Sigma F_H} = \dfrac{0.01037}{0.6499} \quad \theta = 0.91°$

Direction from A = 0.91 + 180 = 180.91°

Magnitude of resultant force $= m r \omega^2$

$= 2.44 \times 0.2625 \times \left(\dfrac{2\pi \times 250}{60}\right)^2$

$= 43.885$

A shaft carries four rotating masses A, B, C & D in this order along its axis. Radius of rotation of A, B, C are 12, 15, 14, 18 cm. Magnitudes of 15, 10, 8, A, C, D are 15, 10, 8. The plane of rotation of A & B is 15 cm & B & C is 18 cm. The angle between A & C is 90°. If the shaft is in complete dynamic balance, determine

1) angle between radius of A, B, D.

2) The distance between planes of rotation C & D

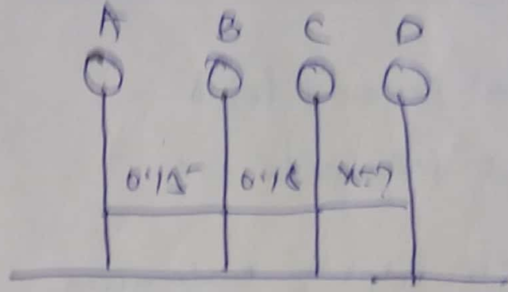
3) The balance mass B.

Sol: $m_A \times r_A = 15 \times 0.12$ kg m

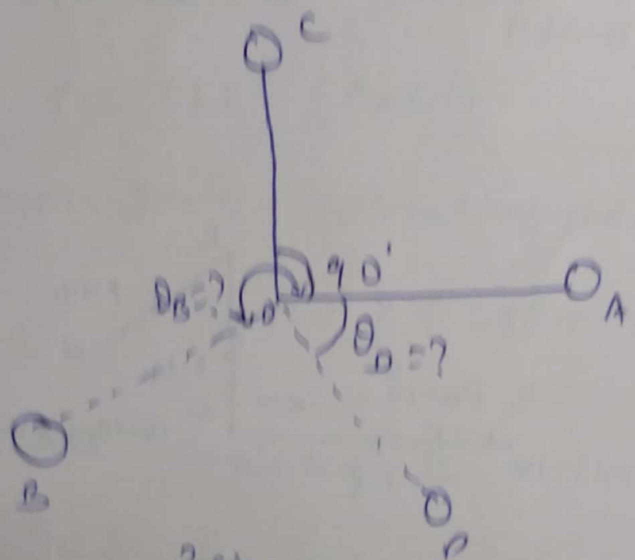
$m_B r_B = m_B \times 0.15$ kg m

$m_C r_C = 10 \times 0.14$ kg m

$m_D r_D = 8 \times 0.18$ kg m.



Position of planes



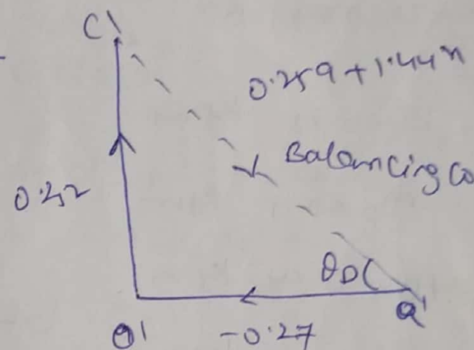
Position of masses

Based on Position of Planes, & masses — Before Taking

Plane	mass kg	Radius (m)	cf/ω^2 (m r) kg·m	distance from reference plane	couple/ω^2 (kg·m^2)
A	15 kg	0.12	1.8	−0.15	−0.27
B	m_B	0.15	0.15 m_B	0	0
C	10	0.14	1.4	0.18	0.252
D	8	0.18	1.44	0.18+x	0.252+1.44x

Force Polygon — Couple Polygon

Draw couple Polygon



oa' Parallel to OA

a'c' Parallel to OC

Join a'c' = Balancing couple

$\theta_D = 58^\circ$ in cw from A

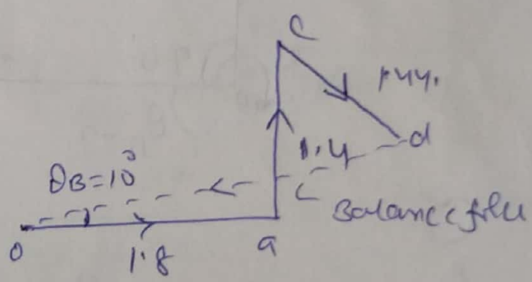
By measurement a'c' = 0.369

a'c' = 0.259 + 1.44x

0.369 = 0.259 + 1.44x

x = 0.076 m.

Draw force Polygon



Oa Parallel to OA

from a draw ac Parallel to OC

from c draw cd " to OD.

By measurement od = 3.15

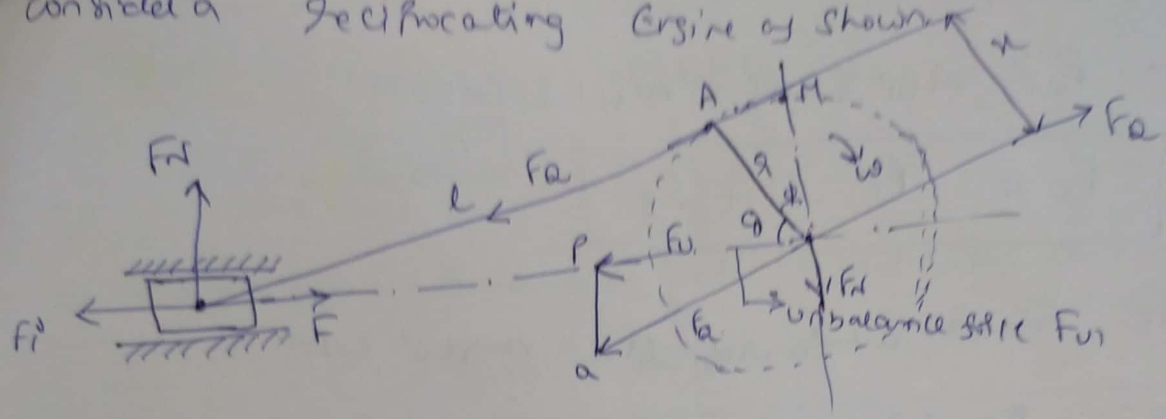
$m_B r_B = 3.15$

m_B = 21 kg.

$\theta_B = 19^\circ$ from mass 'A'

Balancing of Reciprocating masses

consider a reciprocating Engine as shown



let F_Q = force in connecting rod

F_N = force on cylinder walls

F = acceleration force

m = mass of reciprocating parts

ω = angular speed of the crank

l = length of connecting rod

$n = l/r$ (ratio)

acceleration of reciprocating mass of slider crank

$$a = \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

force required to accelerate reciprocating mass

$$F = ma$$

$F = F_U$ $\quad F = m\omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$

$$\boxed{F_U = m\omega^2 r \cos\theta + m\omega^2 r \frac{\cos 2\theta}{n}}$$ = unbalance force

$F_i = -F$ (inertia force is equal & opposite to acceleration force)

$F_i = -F = F_U$

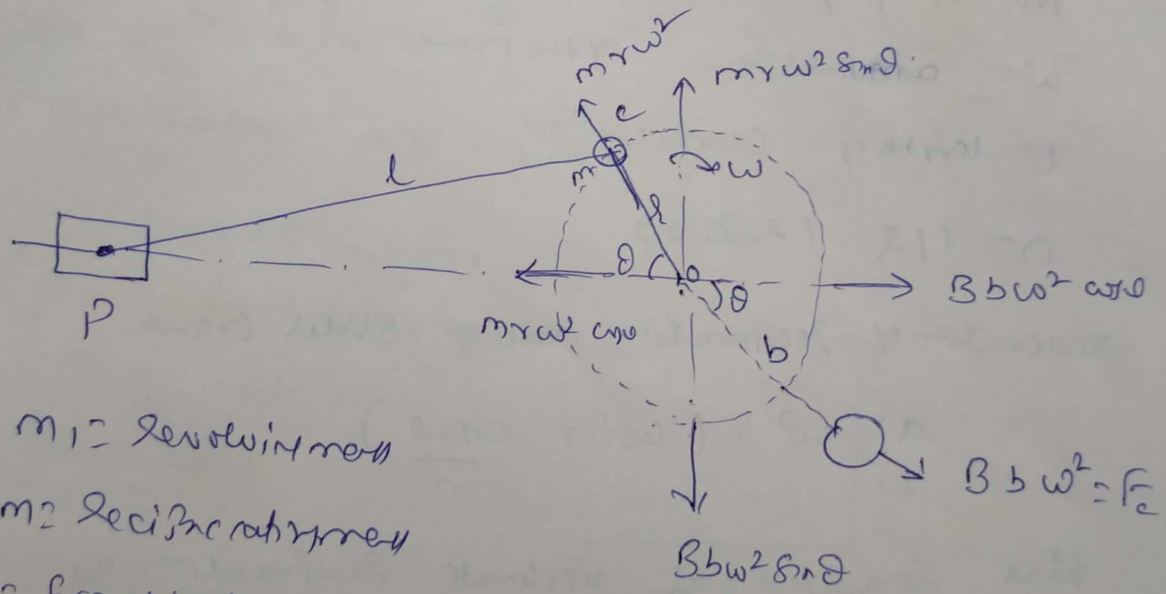
$\therefore F_U = m\omega^2 r \cos\theta + m\omega^2 r \frac{\cos 2\theta}{n}$

$$\boxed{F_U = F_P + F_S}$$

F_P = Primary unbalanced force $= mr\omega^2 \cos\theta$

F_S = Secondary , , $= mr\omega^2 \frac{\cos 2\theta}{n}$

Partial Balancing of unbalanced primary force in a reciprocating Engine.



Let

m_1 = revolving mass

m_2 = reciprocating mass

c = fraction to be balanced $= \frac{1}{2}$ (min)

B = Balancing mass)

b = radius of Balance mass

r_1 = radius of revolving mass

r_2 = radius of crank

Primary unbalanced force acts along OP = $m r \omega^2 \cos\theta$

This is balanced by balancing mass 'B' at radius 'b'

Placed diametrically opposite to crank pin 'C'.

$\therefore$ CF due to mass 'B' = $B b \omega^2 \cos\theta$

Primary force is Balanced if $B b \omega^2 \cos\theta = m r \omega^2 \cos\theta$

i.e $B \cdot b = m \cdot r$.

$\boxed{\text{Primary force is completely Balanced if } B \cdot b = m r}$

But CF produced due to revolving mass 'B' has a

vertical component $B b \omega^2 \sin\theta$ remains unbalanced.

The max value of this force = $B b \omega^2$ when $\theta = 90°, 270°$

→ in the first case Primary force acts along line of

Stroke.

in second case Unbalanced force acts perpendicular to line of Stroke.

The max value remains same in both cases = $B b \omega^2$

So the above method of balancing only changes the direction of unbalanced force from line of stroke to perpendicular to the line of stroke.

As a compromise let a fraction 'c' of reciprocating mass is balanced such that

$$B.b = c m r.$$

∴ unbalanced force along line of stroke

$$= m r \omega^2 \cos\theta - B b \omega^2 \cos\theta$$

$$= m r \omega^2 - c m \omega^2 r \cos\theta$$

$$= m r \omega^2 \cos\theta (1 - c)$$

unbalanced force ⊥r to line of stroke $= c m \omega^2 r \sin\theta$

∴ Resultant unbalanced force at any instant

$$= \sqrt{[m r \omega^2 \cos\theta (1-c)]^2 + [c m r \omega^2 \sin\theta]^2}$$

$$\boxed{\therefore RUF = m r \omega^2 \sqrt{(1-c)\cos^2\theta + c^2 \sin^2\theta}}$$

i) Balancing mass is required to balance revolving & reciprocating mass then

$B.b = c m r$

~~$B.b = c(m + m_1) r$~~

$B.b = c m r + c m_1 r_2$

$B.b = (c m + m_1) r \quad (r_1 = r)$

$= c m r + m_1 r_1 \quad (\text{if } r_1 \neq r)$

(13)

Problems

1) Single Cylinder Engine rotor Speed 250 rpm Stroke = 350 mm Mass of reciprocating parts = 60 kg Mass of revolving parts at 175 mm radius is 40 kg is 2/3 of reciprocating parts and all revolving parts are to be balanced. find
1) Balancing mass required at 400 mm radius
2) Residual unbalanced force when the crank has rotated $60°$ from TDC.

Sol:

$N = 250$ rpm

$r = \frac{\text{Stroke}}{2} = \frac{350}{2} = 175$ mm $= 0.175$ m

mass of reciprocating parts $(m) = 60$ kg

mass of revolving parts $m_1 = 40$ kg at 175 mm

Fraction of tobe balanced $(c) = 2/3$

Required radius $(b) = 400$ mm $= 0.4$ m

Angle from TDC $= \theta = 60$

1) Balancing mass

$B \cdot b = (cm + m_1) r$

$B \times 0.4 = \left(\frac{2}{3} \times 60 + 40\right) 0.175$

$B = 35$ kg

2) Residual unbalanced force

$Ruf = m r \omega^2 \sqrt{(1-c)^2 \cos^2\theta + c^2 \sin^2\theta}$

$= 60 \times \left(\frac{2\pi \times 250}{60}\right)^2 \times 0.175 \sqrt{\left(1-\frac{2}{3}\right)^2 + \cos^2 60 + \left(\frac{2}{3}\right)^2 \sin^2 60}$

$= 2398.86$

Scanned by CamScanner

Problem ⑧

A Single cylinder horizontal engine has crank 187.5mm & connecting rod 825mm. The revolving parts are equivalent to 55kg at crank radius & mass of piston & gudgeon pin is 45kg. The connecting rod having mass equal to 57.5kg & its mass centre is located 262.5mm from crank pin centre. The revolving balanced masses are fixed to the extension of crank webs at radius 21.25mm. to balance the revolving parts & half of reciprocating parts. Neglect the obliquity of connecting rod. Find

a) Balancing mass at 75mm b) Residual unbalanced force at 300 rpm.

Sol: Crank radius = r = 187.5mm = 0.1875m

Length of connecting rod l = 825mm = 0.825m

Mass of revolving parts = 55kg = m_1

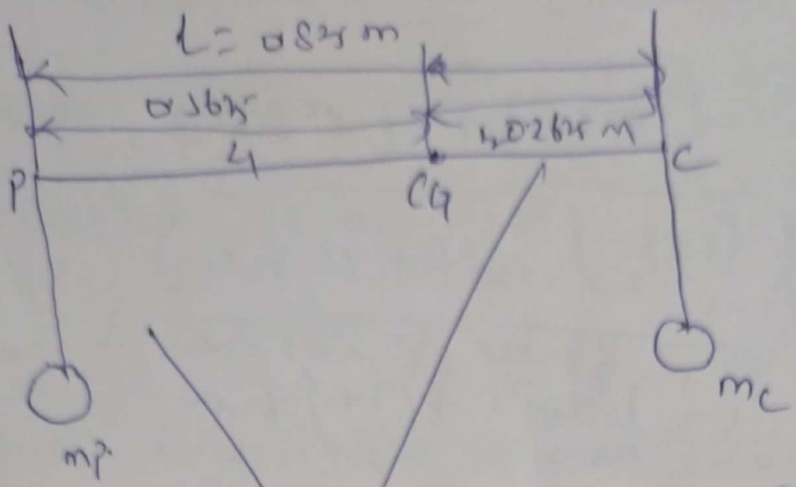
Mass of reciprocating parts = 45kg (m)

Mass of connecting rod = 57.5kg

Crank web radius = 21.25mm = 0.02125m.

Speed (n) = 300 rpm.

Fraction to be balanced = $c = 1/2$



As mass of connecting rod is given replace connecting rod into two equivalent masses, place one mass at crank pin & other at gudgeon pin.

$m_p \times l = m_G \times l_2$

$m_p = \frac{m_G \times l_2}{l} = \frac{57.5 \times 0.2625}{0.525}$

$m_c \times l = m_p \times l_1$

$m_c = \frac{m_p \times l_1}{l} = \ldots$

i) Balance mass at radius 215 mm

$B \cdot b = (c\,m + m_1)\,r$ m = reciprocating

$B \times 0.215 = (\frac{1}{2}\,8.45 + 55)\,0.1875$

$B = \_\_\_$

$$RUF = mr\omega^2 \sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}$$

$$= mr\omega^2 \sqrt{\left(1-\frac{1}{2}\right)^2\cos^2\theta + c^2\sin^2\theta}$$

$$= mr\omega^2 \sqrt{\left(\frac{1}{2}\right)^2\cos^2\theta + \left(\frac{1}{2}\right)^2\sin^2\theta}$$

$$= \frac{1}{2} mr\omega^2 = \frac{1}{2}\times 45\times 0.1875\times\left(\frac{2\pi\times 500}{60}\right)^2$$

$$= \_\_\_\_ N$$

~~Partial~~ Balancing of Locomotive Engines

Locomotive engines are two types

1) Coupled 2) Uncoupled.

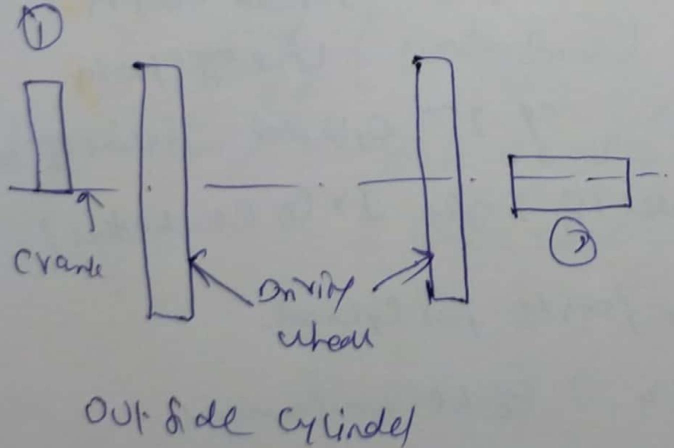
→ if two or more pair of wheels coupled together to increase adhesive force between wheel & track it is called coupled locomotives. otherwise it is an uncoupled locomotives

→ Most of locomotives have two cylinders of same dimension placed symmetrically either inside or outside the wheels.

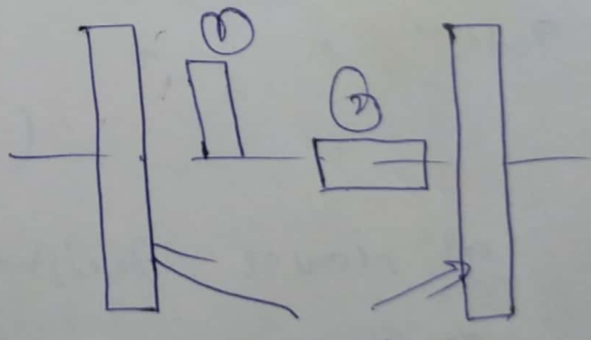
Locomotives are two types depending on position of cylinders

1) out cylinder locomotive 2) Inside cylinder locomotive

Outside cylinder locomotive: when two cylinders are placed outside the driving wheel one on each side it is outside cylinder locomotive



Outside cylinder locomotive



(Inside cylinder locomotive)

Effect of partial Balancing of two cylinder locomotive

Due to partial Balancing of reciprocating parts there is 1) unBalanced force along line of stroke

2) " " perpendicular to line of stroke. (Hammer blow)

The unBalanced primary force along of line of stroke produce

1) Variation of tractive force along of line of stroke

2) Swaying couple

3) Hammer blow

Variation of tractive force

The resultant unbalanced force due to two cylinder along line of stroke is called Tractive force.

The variation of tractive force is caused by unbalanced primary force along the line of stroke.

θ = angle of inclination of crank of 1st cylinder with line of stroke

$90+\theta$ = " " " of 2nd cylinder with line of stroke

∵ (cylinder 1 & 2 are ⊥r to each other)

m = Mass of reciprocating parts per cylinder.

c = Fraction of " " to be balanced

ω = angular velocity of crank   r = Radius of each crank

Effect of Partial Balancing two cylinder locomotive

Due to Partial Balancing of Reciprocating parts there is 1) UnBalanced force along line of stroke

2) " " Perpendicular to line of stroke. (Hammer blow)

The unBalanced Primary force along of line of stroke produce

1) Variation of tractive force along of line of stroke

2) Swaying couple

3) Hammer blow

Variation of tractive force

The Resultant unbalanced force due to two cylinder along line of stroke is called Tractive force.

The variation of tractive force is caused by unbalanced primary force along the line of stroke.

θ = angle of inclination of crank of 1st cylinder with line of stroke

90+θ = " " " of 2nd cylinder with line of stroke

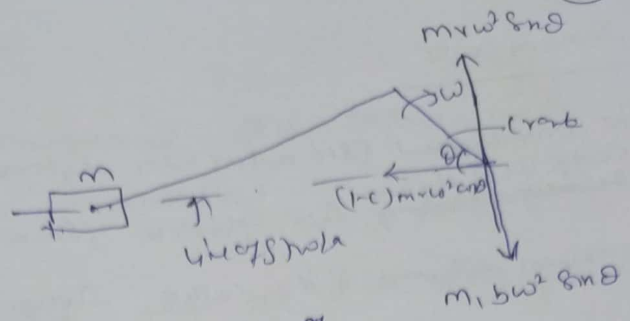
∵ (cylinder 1 & 2 are ⊥r to each other)

m = Mass of Reciprocating parts per cylinder.

C = Fraction of " " to be balanced

ω = angular velocity of crank   r = Radius of each crank

(16)



wkt unbalanced force along line of stroke for 1st cylinder

$= (1-c)\, mr\omega^2 \cos\theta$

Similarly unbalanced force along line of stroke for 2nd cylinder

$= (1-c)\, mr\omega^2 \cos(90+\theta)$

$\therefore$ Tractive force = Resultant unbalanced force along LOS

$F_T = (1-c)\, mr\omega^2 \cos\theta + (1-c)\, mr\omega^2 \cos(90+\theta)$

$F_T = (1-c)\, mr\omega^2 (\cos\theta - \sin\theta)$

F_T is max & min when $\cos\theta - \sin\theta$ is max & min

$\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0$

$= -\sin\theta - \cos\theta = 0$

$= -\sin\theta = \cos\theta = 0$

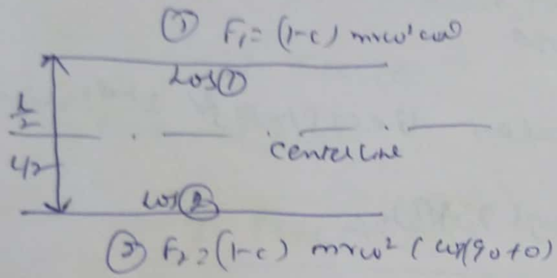
$-\tan\theta = 1$

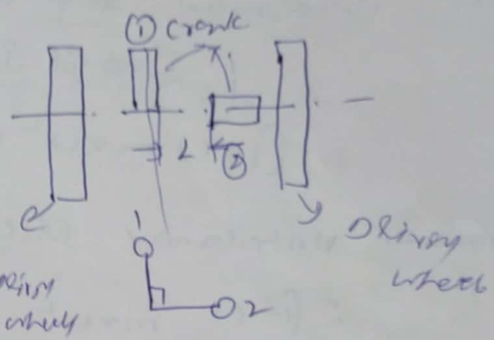
$\tan\theta = -1 \quad \theta = 135 \text{ or } 315°$

$F_T = (1-c)\, mr\omega^2 (\cos 135 - \sin 135) = \boxed{\pm\sqrt{2}(1-c)\, mr\omega^2 = F_T}$

Swaying couple:

The unbalanced force acting at a distance 'l' between ~~the lines of stroke of~~ centre line of 2 cylinders two cylinders form a couple in horizontal direction & it is called swaying couple





$$F_1 \times \frac{l}{2} = F_2 \times \frac{l}{2}$$

$$(1-c)\, m r \omega^2 \cos\theta \, \frac{l}{2} \mp (1-c)\, m r \omega^2 \cos(90+\theta)\, l/2 = 0$$

$$(1-c)\, m r \omega^2 \left[\cos\theta + \cos(90+\theta)\right] l/2 = 0$$

$$(1-c)\, m r \omega^2 \left[(\cos\theta + \sin\theta)\right] l/2$$

Couple will be max when $\cos\theta + \sin\theta$ is max,

$$\frac{d}{d\theta}(\cos\theta + \sin\theta) = 0$$

$$-\sin\theta + \cos\theta = 0$$

$$\tan\theta = 1$$

$$\theta = 45 \text{ or } 225^\circ$$

$$\boxed{\therefore \text{Max swaying couple} = \pm \frac{1}{\sqrt{2}} (1-c)\, m r \omega^2 l}$$

when $\theta = 45°$

Max swaying couple

$$= (1-c)\, m r \omega^2 (\cos 45 + \sin 45)\, l/2$$

$$= \frac{1}{\sqrt{2}} (1-c)\, m r \omega^2 l.$$

$\theta = 225$

$$(1-c)\, m r \omega^2 (\cos 225 + \sin 225)\, l/2$$

$$= -1/\sqrt{2}\, (1-c)\, m r \omega^2 l.$$

Hammer Blow

The max value of unbalanced force perpendicular to line of stroke is called Hammer blow.

WKT Unbalanced force along the ⊥r to line of stroke due to balancing mass m_1 at radius 'b' is $= m_1 b \omega^2 \sin\theta$.

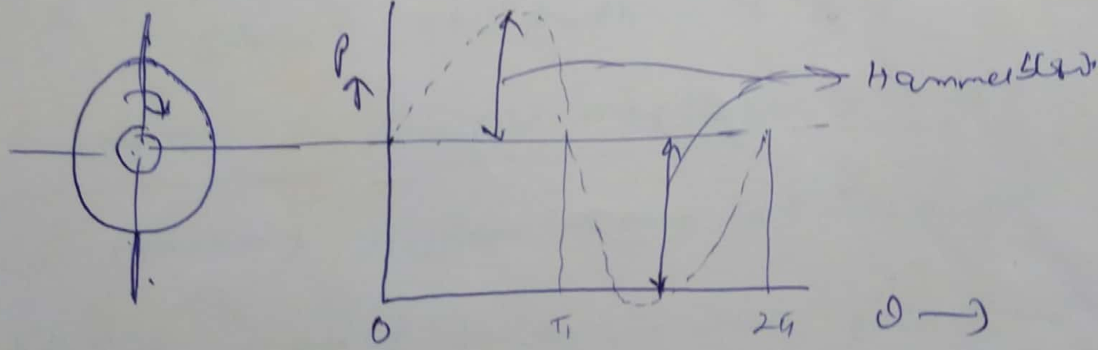
This force will be max when $\theta = 90°$ or $270°$

$\therefore$ Hammer blow $= m_1 b \omega^2 \sin 90$ or $m_1 b \omega^2 \sin 270$

$= m_1 b \omega^2$ or $-m_1 b \omega^2$.

$= \pm m_1 b \omega^2$.

The Effect of hammer blow is to cause the variation of pressure between the wheel & rail.



The variation is shown for one revolution of wheel

Let P = static wheel load (downwards)

$\therefore$ Net pressure between wheel & rail

$= P \pm m_1 b \omega^2$

if $(P - m_1 b\omega^2)$ is -ve wheel will be lifted from rail

∴ limiting condition in that that the wheel does not lift from rail is given by

$$P - m_1 b\omega^2 = 0$$

$$P = m_1 b\omega^2$$

$$\boxed{\omega = \sqrt{\frac{P}{m_1 b}}}$$ permissible angular speed so that wheel does not lift from rail

Problems on Balancing of Locomotives

The following data refers to two cylinder locomotive cranks at 90°

Reciprocating mass per cylinder = 300 kg

Crank radius = 0.3 m   Driving wheel dia = 1.8 m

distance between wheel central planes = 1.55 m

Find: 1) The fraction of reciprocating mass to be balanced if hammer blow is not to exceed 46 kN at 96.5 km/hr

2) Tractive force   3) max swaying couple.

Sol: m = Mass of reciprocating parts = 300 kg

Crank radius = r = 0.3 m

Driving wheel dia = 1.8 m   R = 0.9 m

Distance between cylinder lines, (l) = 0.65 m

Hammer blow = 46 kN

Speed = 96.5 kmph = 26.8 m/s

1) Fraction of reciprocating mass to be balanced

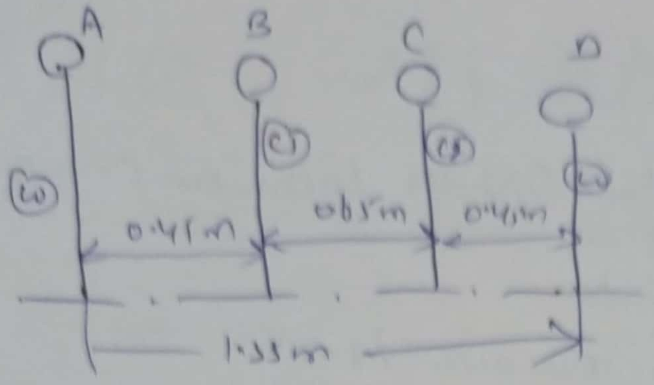
c = fraction of reciprocating mass to be balanced

B (or) m_1 = mass of balancing mass placed at each of driving wheel at radius 'b'

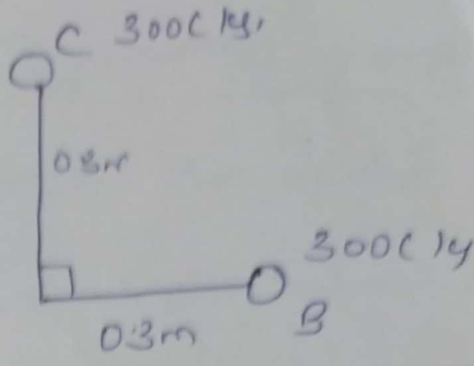
∴ Mass of reciprocating parts to be balanced = cm

= 300c kg.

~~B.b = (...)~~   ~~...~~



(Position of Plane)



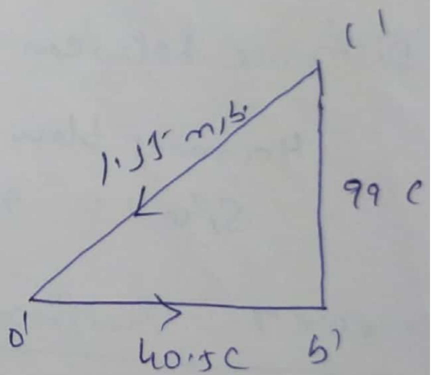
(Position of crank)

Assume A as Ref Plane

Plane	Mass	Radius	cf	Distance from Ref plane	Couple
A	m_1	b	$m_1 b$	0	0
B	300c	0.3	90c	0.45	40.5c
C	300c	0.3	90c	1.1	99c
D	m_1	b	$m_1 b$	1.55	$1.55 m_1 b$

Draw Couple Polygon

From Polygon



$$1.55 m_1 b = \sqrt{(99c)^2 + (40.5c)^2}$$

$$m_1 b = \frac{107c}{1.55} = 69c$$

$\therefore$ Angular Speed $\omega = \frac{v}{r} = \frac{26.8}{0.9} = 29.8$ rad/sec.

$\therefore$ Hammer blow $= B \cdot b\,\omega^2$

$$46 \times 10^3 = 69c \times (29.8)^2$$

$$\boxed{c = 0.751}$$

Sol:

Distance between cylinder lines $(l) = 0.7$ m

Mass of rotating parts $m_1 = 150$ kg

Mass of reciprocating parts $m_2 = 180$ kg

Fraction of reciprocating mass $(c) = 2/3$

Crank speed $(N) = 300$ rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 31.42 \text{ rad/sec}$$

$\therefore$ Mass of rotating parts to be balanced per cylinder at crank pin

$$m = m_b = m_c = m_1 + c\, m_2 = 150 + \frac{2}{3} 180 = 270 \text{ kg}$$

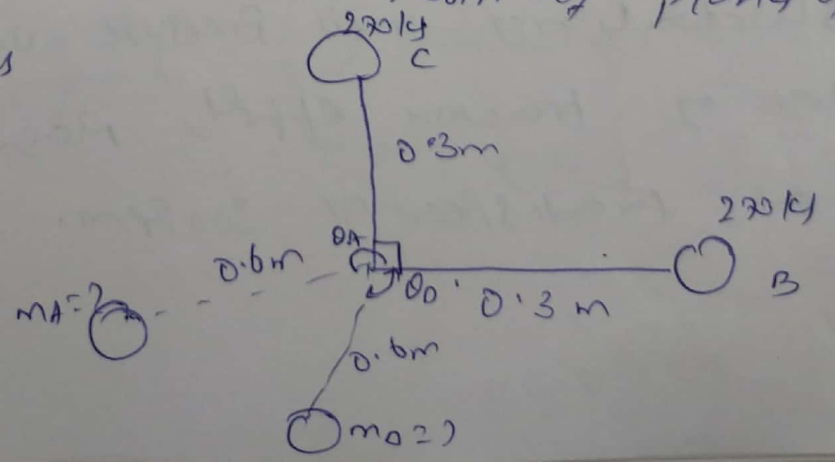
Let m_A, m_D Magnitudes of Balancing masses

θ_A, θ_D Angular position of balancing masses m_A & m_D from the first crank B.

Magnitude & angular position of Balance Mass Determined by

Graphical Method

1) Draw space diagram to show position of planes of wheels and cylinders



2) Variation in tractive force

$$F_{T_1} = \pm\sqrt{2}(1-c)\, m r \omega^2$$

$$= \pm\sqrt{2}(1-0.751)\, 300 \times 0.08 \times 69.8)^2 \times 0.3$$

$$= 28.14\,kN$$

3) Max swaying couple $= \pm \frac{1}{\sqrt{2}}(1-c)\, m r \omega^2$

$$= \frac{0.65}{\sqrt{2}}(1-0.751)\, 300 \times 0.3\,(29.8)^2$$

$$= 9.148\,kN\text{-}m$$

(2) An inside cylinder locomotive has its cylinder centre lines 0.7 m apart & has stroke 0.6 m. The rotating mass per cylinder are equivalent to 150 kg at crank pin & reciprocating mass per cylinder is 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles. The whole of rotating & $\frac{2}{3}$ of reciprocating masses are to be balanced by masses placed at radius of 0.6 m. Find magnitude & direction of balancing masses.

Find fluctuation of rail pressure under one wheel, variation of tractive effort, magnitude of swaying couple at crank speed of 300 rpm.

Fluctuation in Rail pressure & hammer Blow

Balancing mass for rotating mass 'D'

$$= \frac{m_1}{m} \times 105$$

$m = m_1 + Cm_2$

$= 150 + \frac{2}{3} \times 180 = 270$

$$= \frac{150}{270} \times 105 = 58.3\,kg$$

Balancing mass for reciprocating mass 'B'

$$= \frac{Cm_2}{m} \times 105$$

$$= \frac{2}{3} \times \frac{180}{270} \times 105 = 46.6\,kg$$

$\therefore$ hammer Blow $= B\,b\,\omega^2 = 46.6 \times 0.6 \times (31.42)^2$

$= 27602\,N$

Tractive force

$$F_T = \pm \sqrt{2}(1-c)\, m_2 r \omega^2$$

$$= \pm \sqrt{2}\left(1-\frac{2}{3}\right) 180 \times (31.42)^2 \times 0.3$$

$$= \pm 25127\,N$$

Swaying couple:

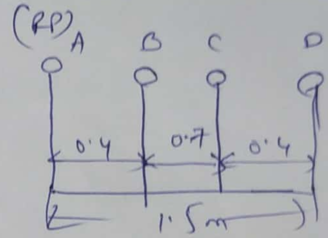
$$\frac{l}{\sqrt{2}}(1-c)\, m_2 \omega^2 r$$

$$= \frac{0.7}{\sqrt{2}}\left(1-\frac{2}{3}\right) 180 \times (31.42)^2 \times 0.3$$

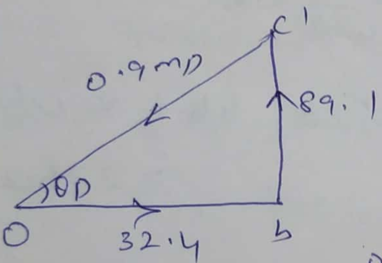
$$= 8797\,N\text{-}m$$

2) Tabulate the data & assume plane 'A' as Ref plane given

Plane	(m) mass kg	(r) Radius m	CF kg-m	(l) Distance from Ref plane m	kg-m² Couple mrl
A (RP)	m_A	0.6	$0.6m_A$	0	0
B	270	0.3	81	0.4	32.4
C	270	0.3	81	1.1	89.1
D	m_D	0.6	$0.6m_D$	1.5	$0.9m_D$



3) Draw couple polygon



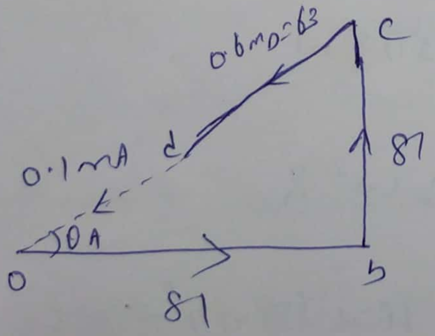
$0.9m_D$ = vector oc'

$0.9m_D = 94.5$ kg m²

$m_D = 105$ kg , $\theta_D = \underline{250^\circ}$ By measurement

$0.6m_D = 0.6 \times 105 = 63$

4) Draw force polygon



$0.1m_A$ = vector od

$m_A = 105$ kg .

$\theta_A = \underline{200^\circ}$ By measurement

Balancing of Multicylinder inline Engines

Consider a shaft consisting three equal cranks with unsymmetrical spacing. These three untwal reciprocating masses are transferred at crank pin

Then these masses equivalent

Primary force $= \Sigma m r \omega^2 \cos\theta = 0$
Primary couple $= \Sigma m r \omega^2 l \cos\theta = 0$ } Primary Balancing

Secondary force $= \Sigma m r \omega^2 \frac{\cos 2\theta}{n} = 0$
" couple $= \Sigma m r \omega^2 \frac{\cos 2\theta}{n} \times l = 0$ } secondary Balancing

for graphical solution draw Primary force Polygon

The resultant force $m r \cos\theta \omega^2$ give Primary unbalance force on system.

when $\theta = 90^\circ$ unbalance force is zero
$\theta = 0^\circ$ " " is Max

if the force Polygon is closed the resultant force $= 0$ and system will be in Primary balance.

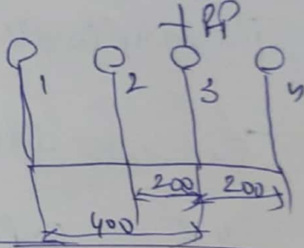
Note: closing side of secondary force polygon gives Max unbalanced secondary force
" " " couple " " " " secondary couple

Problem

① A four cylinder vertical Engine has crank 150 mm long. The plane of rotation of 1, 2nd, 4th crank are 400, 200, 200 mm respectively from the third crank & their reciprocating masses are 50 kg, 60 kg, 50 kg. Find the mass of reciprocating parts for 3rd cylinder and relative angular position of cranks in order that Engine is in complete primary balance.

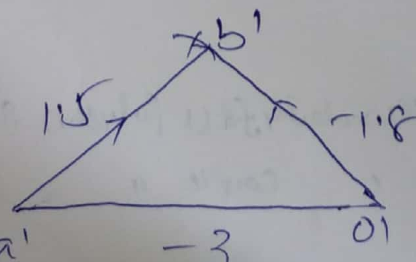
Sol: $r_1 = r_2 = r_3 = r_4 = 150 mm = 0.15 m$

$m_1 = 50 \quad m_2 = 60 \quad m_4 = 50 kg$

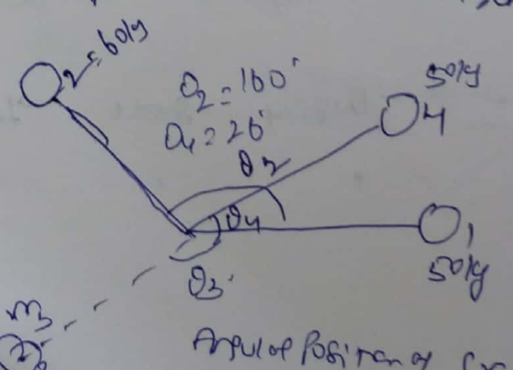


Plane	Mass (m)	Radius (r)	CF (mr)	Distance (L) from Ref plane	Couple (mrL)
1	50	0.15	7.5	-0.4	-3
2	60	0.15	9	-0.2	-1.8
3 (RP)	m_3	0.15	$0.15 m_3$	0	0
4	50	0.15	7.5	0.2	1.5

Angular position of crank 2 & 4 are obtained by drawing couple polygon. Assume position of crank '1' is horizontal.

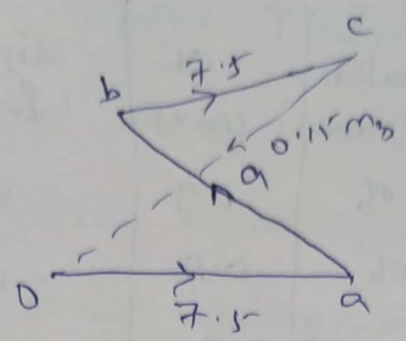


Couple Polygon



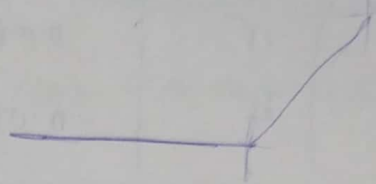
Angular Position of Crank

In order to find Mass of Third Cylinder (m_3) & angular Position draw force Polygon



$0.15\, m_3$ = vector oc

$m_3 = 60 kg$



(2) The crank & connecting rod of a 4 cylinder inline engine running at 1800 rpm are 60mm & 240mm. Cylinders are spaced 150mm apart. If the cylinders are ~~spaced~~ numbered 1 to 4 in sequence from one end, the cranks appear at intervals of 90° in an end view in the order 1-4-2-3. The reciprocating mass corresponding to each cylinder is 15kg. Find 1) Unbalanced primary & secondary forces if any 2) Unbalanced primary & secondary couples.

Sol:

$N = 1800$ rpm $\quad \omega = \frac{2\pi N}{60} = 188.52$ rad/sec

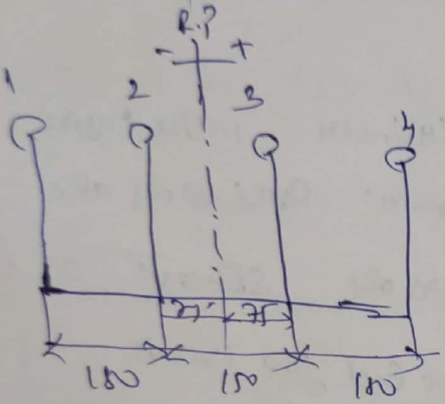
Crank radius $(r) = 60mm = 0.06 m$

length $(l) = 240mm = 0.24m$

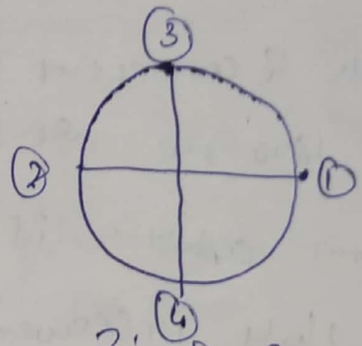
$m = 15 kg$

1) Unbalanced Primary & Secondary forces

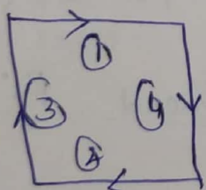
Plane	mass (kg)	Radius (r) m	CF (mr^2)	distance from ref plane	Couple (mrl)
1	15	0.06	0.9	−0.225	−0.2025
2	15	0.06	0.9	−0.075	−0.0675
3	15	0.06	0.9	+0.075	+0.0675
4	15	0.06	0.9	+0.225	+0.2025



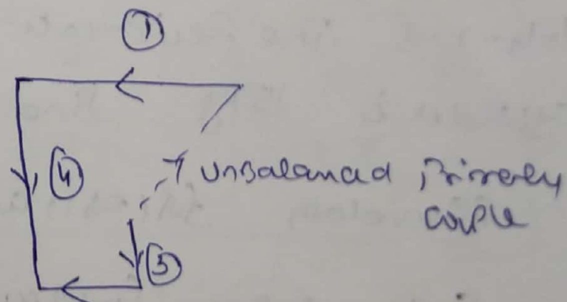
Cylinder, plane position



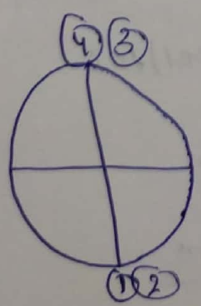
Primary Crank Position



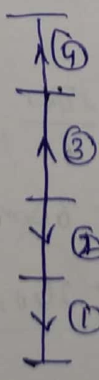
Primary force Polygon



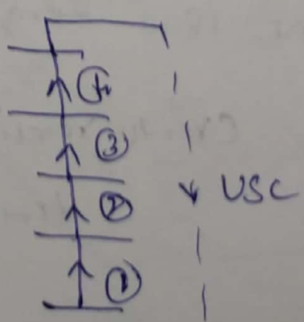
Primary Couple Polygon



Secondary crank position



Secondary force Polygon



Secondary Couple Polygon

(27)

A four crank engine has the two cranks set at 120° to each other & their reciprocating masses are 400 kg. The distance between the plane of rotation of adjacent cylinder 450, 750, 600 mm. If engine is to be complete primary balance find the reciprocating masses and their relative angular position for each of inner crank. If the length of crank is 300 mm, length of each connecting rod = 1.2 m speed of rotation 240 rpm. What is the max secondary unbalance force.

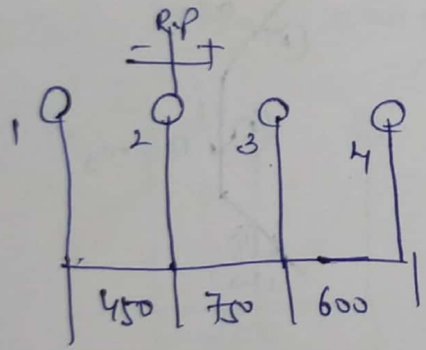
Sol: Given $m_1 = m_4 = 400$ kg $r = 300$ mm $= 0.3$ m $l = 1.2$ m

$N = 240$ rpm. $\omega = \frac{2\pi N}{60} = 25.14$ rad/sec

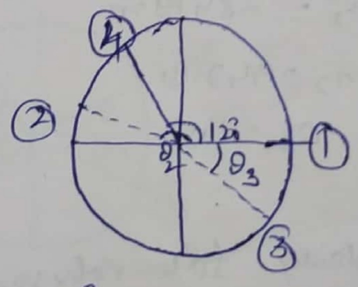
Let m_2 m_3 reciprocating mass for inner crank 2, 3 respectively

θ_2 θ_3, angular position of cranks 2, 3 w.r.t crank (1)

Take (2) as Ref Plane

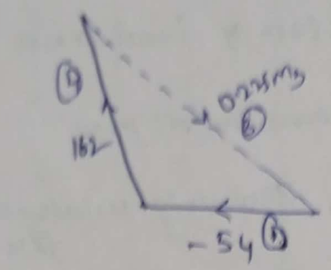


Position of planes



Primary crank position

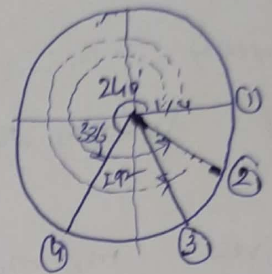
Plane	Mass (m) kg	Radius (r) (m)	CF (mr) kg m	(l) Distance from Ref plane (m)	Couple kg m² (mrL)
1	400	0.3	120	−0.45	−54
2 (RP)	m_2	0.3	$0.3 m_2$	0	0
3	m_3	0.3	$0.3 m_3$	0.75	$0.225 m_3$
4	400	0.3	120	1.35	162



$0.225\, m_3 = 196$ kg·m

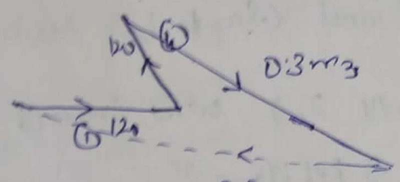
$m_3 = 871$ kg

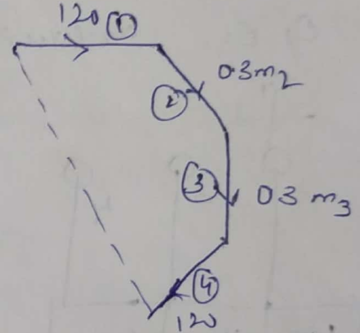
$\theta_3 = 326°$



Drawn primary force polygon

Secondary force Polygon





$0.3 m_2 = 284$ kg·m

$m_2 = 947.14$

$\theta_2 = 168°$

From secondary force polygon Max secondary unbalance force by measurement = 582 kg·m

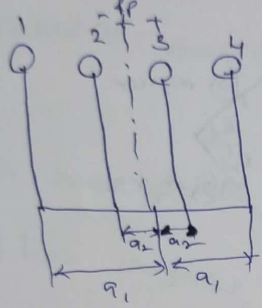
$\therefore$ Max secondary unbalanced force

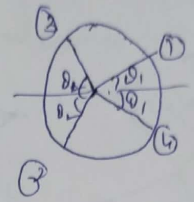
$$= 582 \times \frac{\omega^2}{n} = \frac{582 \times (25.14)^2}{1.2/0.3} = 91.96 \text{ kN}$$

$n = l/r$

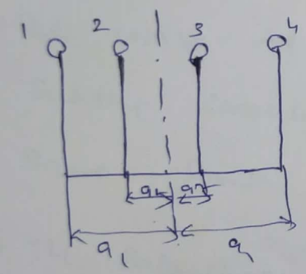
(2) Fig shows the arrangement of cranks in a four crank symmetrical engine in which the masses of the reciprocating parts at cranks (1) & (4) are equal to m_1 & cranks 2, 3 = m_2. Show that arrangement is balanced for primary forces & couples, secondary forces & couples provided that

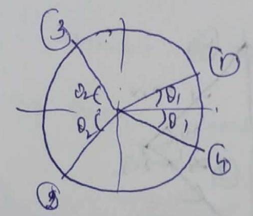
$$\frac{m_1}{m_2} = \frac{\cos\theta_1}{\cos\theta_2}, \quad \frac{a_1}{a_2} = \frac{\tan\theta_2}{\tan\theta_1} \quad \text{and} \quad \cos\theta_1 \cdot \cos\theta_2 = \frac{1}{2}$$





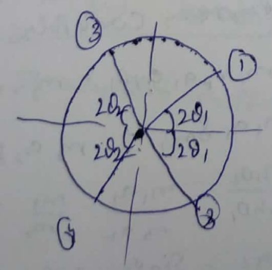
Sol: given mass of reciprocating parts at cranks 1, 4 = m_1
2, 3 = m_2





• Positioning planes

Primary crank position



Crank position from secondary force polygon

$R.S = m_1 r \cos 2\theta_1 = m_2 r \cos(180 - 2\theta_2)$

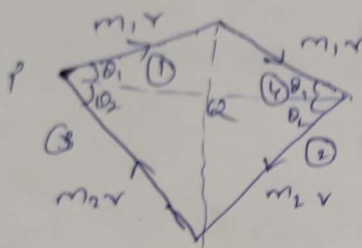
$m_1 r \cos 2\theta_1 = -m_2 r \cos 2\theta_2$

$$\frac{m_1}{m_2} = \frac{-\cos 2\theta_2}{\cos 2\theta_1} = \frac{-2\cos^2\theta_2 - 1}{2\cos^2\theta_1 - 1}$$

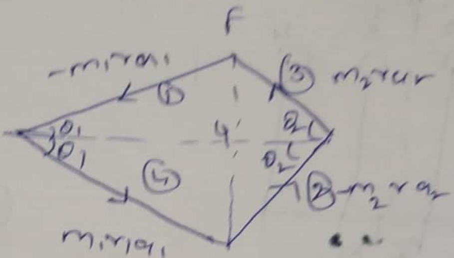
$$\frac{\cos\theta_2}{\cos\theta_1} = \frac{1 - 2\cos^2\theta_2}{2\cos^2\theta_1 - 1} \Rightarrow \cos\theta_1 \cos\theta_2 = 1/2$$

Plane	Mass	Radius	CF	distance from Ref Plane	Couple
1	m_1	r	$m_1 r$	$-a_1$	$-m_1 r a_1$
2	m_2	r	$m_2 r$	$-a_2$	$-m_2 r a_2$
3	m_2	r	$m_2 r$	$+a_2$	$+m_2 r_2 a_2$
4	m_1	r	$m_1 r$	$+a_1$	$+m_1 r a_1$

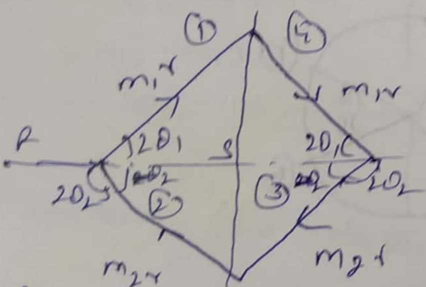
Primary force Polygon



Primary couple Polygon from Primary Crank Position



Secondary force Polygon from secondary crank Position



In order to balance Primary forces Primary force, couple Polygon must close

from Primary force polygon

$PQ: m_1 r \cos\theta_1 = m_2 r \cos\theta_2$

$\therefore \frac{m_1}{m_2} = \frac{\cos\theta_2}{\cos\theta_1}$

from Primary couple Polygon

$FG = m_1 r a_1 \sin\theta_1 = m_2 r a_2 \sin\theta_2$

$m_1 a_1 \sin\theta_1 = m_2 a_2 \sin\theta_2$

$\frac{\sin\theta_2}{\sin\theta_1} = \frac{m_1 a_1}{m_2 a_2}$ $\quad \frac{m_1}{m_2} = \frac{\cos\theta_2}{\cos\theta_1}$

$\frac{a_1}{a_2} = \frac{\sin\theta_2}{\sin\theta_1} \times \frac{\cos\theta_1}{\cos\theta_2} = \frac{\tan\theta_2}{\tan\theta_1}$

Balancing of Radial Engines

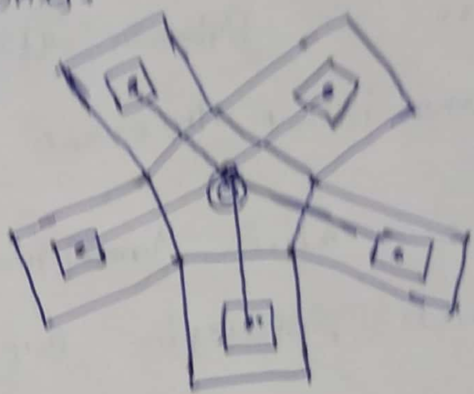
→ Radial Engine will have two & more than two cylinders arranged with their line of stroke.

→ Generally two cylinder Engines are called as V-Engines.

→ When there are more than two cylinders, then these engines are called as multicylinder Radial Engines.

Here angle between line of stroke of two adjacent cylinders $= \frac{360}{n}$

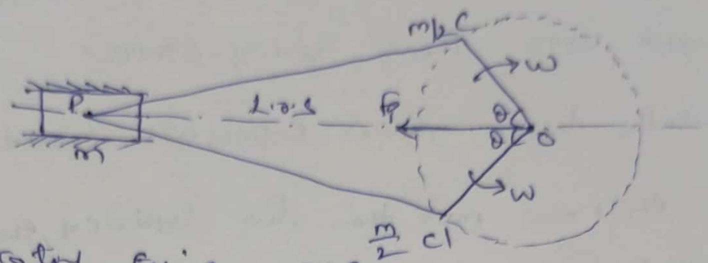
n = no of cylinders.



→ There will be one crank common to all cylinders such that connecting rod of each cylinder is connected to same crank.

→ The Balancing of V-Engines can be done either by graphical means by using direct & reverse crank method (or) by analytical method.

Direct and Reverse Crank method



→ Consider a reciprocating engine as shown.

let OC is primary direct crank rotating uniformly at ω rad/sec in c.w

→ at any instant the crank makes an angle θ with LOS.

→ OC' is mirror image of direct crank OC. (Reverse crank) which rotates in a.c.w direction.

→ Here primary force is $= mr\omega^2 \cos\theta$

→ It is assumed that half of mass $(\frac{m}{2})$ is fixed at C & C'

∴ centrifugal force acting on primary direct & reverse crank $= \frac{m}{2} r\omega^2$

∴ component of C.F acting ~~along LOS~~ on primary direct crank

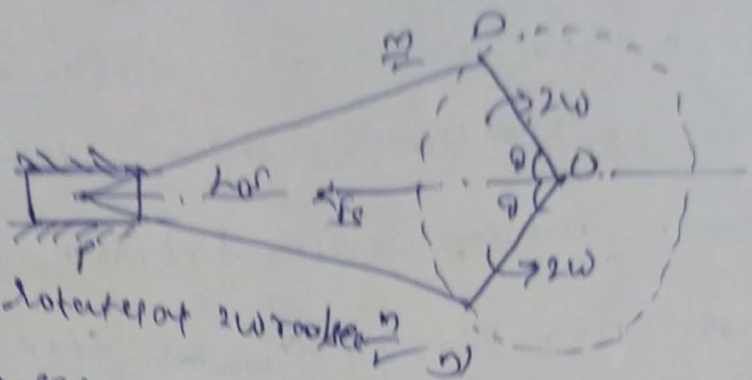
$$= \frac{m}{2} r\omega^2 \cos\theta$$

∴ Total component of CF acting along LOS $= 2\frac{m}{2} r\omega^2 \cos\theta$

$$= m r\omega^2 \cos\theta$$

∴ Primary force $\boxed{F_P = mr\omega^2 \cos\theta}$

Considering secondary forces



Crank OO' is

Secondary dilect crank rotates at 2ω rad/sec in CW. OO' is reverse crank rotate to at 2ω rad/sec in anti-CW

Secondary unbalanced force $\boxed{F_s = mr\omega^2 \frac{\cos 2\theta}{n}}$

Primary unbalanced force $F_p = mr\omega^2 \cos\theta$

$(F_p)_{max} = mr\omega^2 \quad \theta = 0 \ \& \ 180'$

$(F_s)_{max} = \frac{mr\omega^2}{n} \quad \theta = 0, 90, 180, 360$

Thus secondary force is max for four times in one revolution of crank.

$(F_s)_{max}$ = Max unbalanced secondary force $= \frac{mr\omega^2}{n}$

we see that, secondary unbalanced force is $\frac{1}{n}$ times the max primary unbalanced force.

Problem

(1) A three cylinder radial engine has axis at 120° to each other & their connecting rods are coupled to a single common crank. The stroke length is 100mm & length of each connecting rod is 150mm. Mass of reciprocating parts per cylinder is 2 kg. Determine the max primary & secondary force on engine running at 2400 rpm.

Sol:- Stroke $L = 100mm = 0.1m$

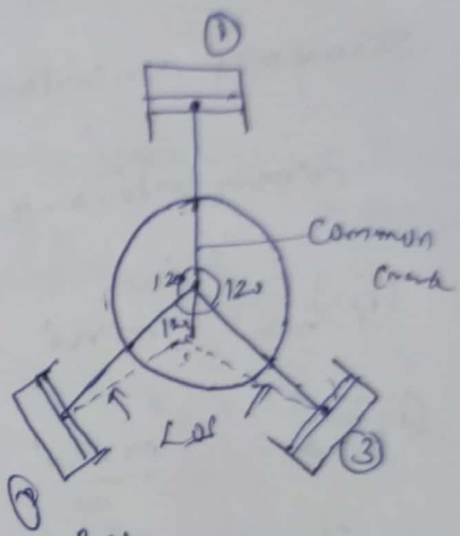
$r = \frac{L}{2} = 50mm = 0.05m$

Length of connecting rod $(l) = 150mm = 0.15m$

Mass of reciprocating parts per cylinder $= m = 2 kg$

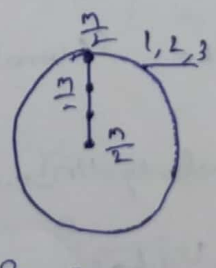
$N = 2400 rpm$

$\omega = \frac{2\pi N}{60} = 251.3 \, rad/s$

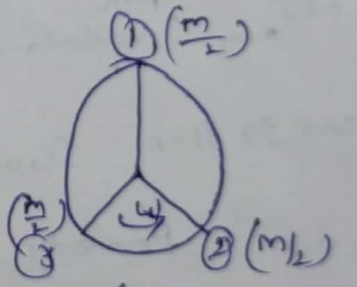


Position of 3 cylinders

Max Primary force of engine



Direct Primary Crank



Reverse Primary Crank

1) Since $\theta = 0$ for cylinder ① therefore both direct & reverse crank will coincide with common crank.

2) $\theta = \pm 120$ for cylinder ② i.e primary direct crank at 120° cw, reverse crank 120° acw from LoS of cylinder ② ∴

3) $\theta = \pm 240$ for cylinder ③ i.e 240° cw & 240° acw from LoS of cylinder ③.

∴ Max Primary force $= 3 \frac{m}{2} r \omega^2$

$= 3(\frac{1}{2})\, 0.05 \times (251.33)^2$

$= 9.47 kN$.

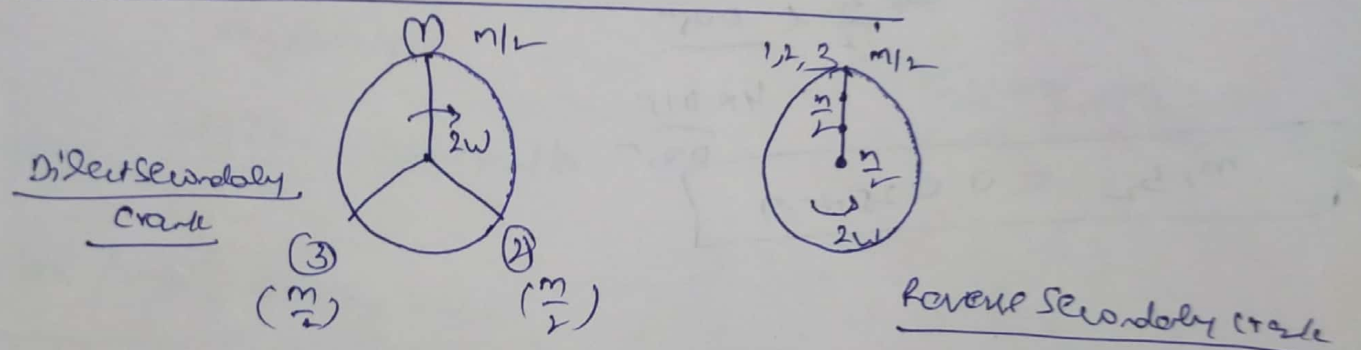
The max Primary force may be balanced by mass attached diametrically opposite to crank

∴ $m_1 b = 3 \frac{m}{2} \cdot r$;

$= 3 \times \frac{2}{2} \times 0.05$

$$\boxed{m_1 b = 0.15 \ N\cdot m}$$

Max secondary force acting on Engine



1) $\theta = 0$ $2\theta = 0$ for cylinder ① ∴ both direct crank & reverse crank will coincide with common crank

2) $\theta = \pm 120$, $2\theta = \pm 240$, for cylinder ② ie 240° cw, for secondary direct crank, 240° acw for secondary reverse crank

3) $\theta = \pm 240$, $2\theta = \pm 480$ ie for cylinder ③ secondary direct crank 480° & 120 cw, reverse crank 480 & 120° acw

$\therefore$ Max secondary force $= 3 \frac{m}{2} \times (2\omega)^2 \left(\frac{r}{4n}\right)$

$n = \frac{l}{r}$

$= 3 \times \frac{2}{2} \times (2 \times 251.33)^2 \times \frac{0.05}{4 \times \left(\frac{0.15}{0.05}\right)}$

$= 3105\,N$

The max secondary force may be balanced by mass m_2 at radius b_2 attached diametrically opposite to crank pin rotating acw at twice the crank speed

$m_2 b_2 = 3 \frac{m}{2} \frac{r}{4n}$

$= \frac{3 \times 2}{2} \times \frac{0.05}{4 \times \frac{0.15}{0.05}}$

$\boxed{m_2 b_2 = 0.0125\,N\text{-}m}$