Unit-II

Governors

→ flywheel which minimizes fluctuation of speed with in the cycle but it cannot minimize fluctuations due to load variation.

→ flywheel doesn't have any control over mean speed of the engine.

→ To minimize fluctuations in mean speed due to load variation governor is used.

→ When the load on an engine increases its speed decreases therefore it become necessary to increase the supply of working fluid. On the other hand when the load on the engine decreases its speed increases and hence less working fluid is required.

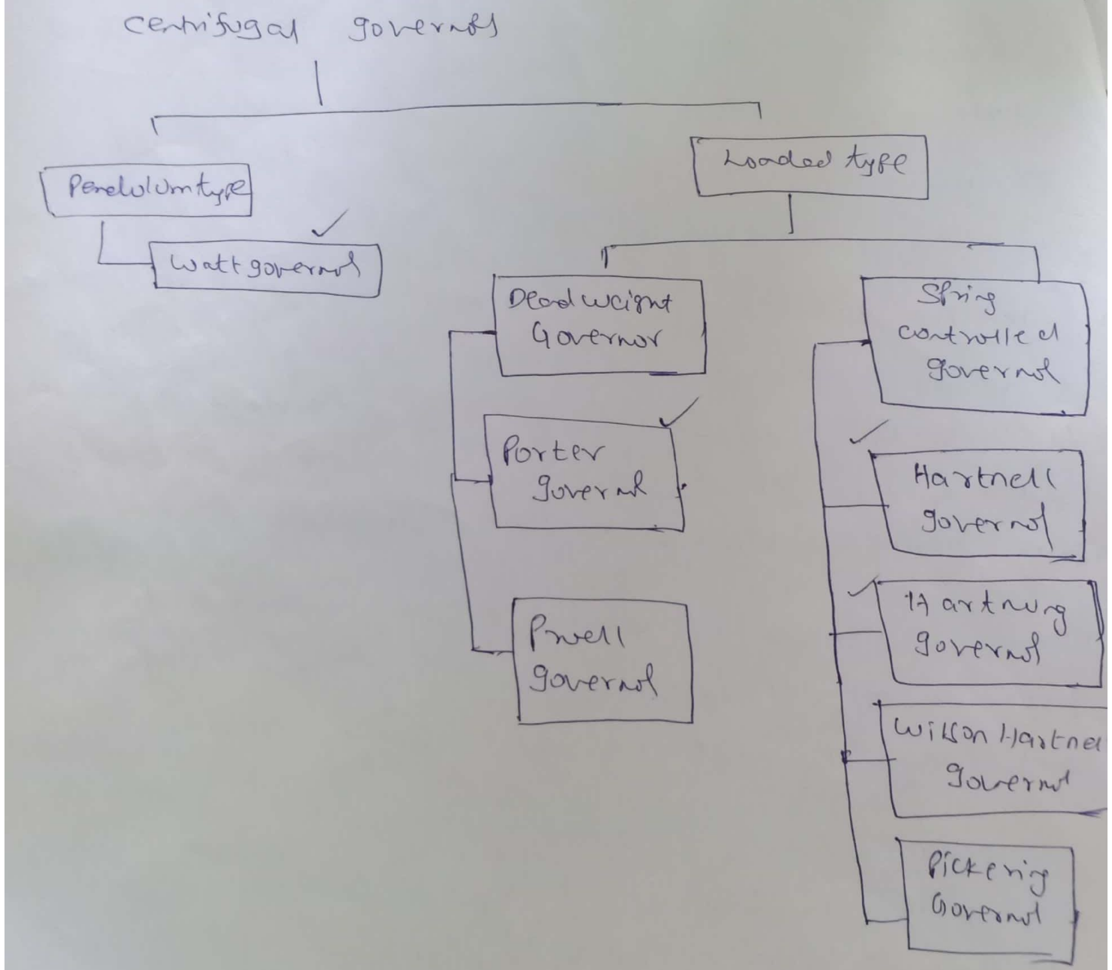
→ When there is load variation & speed variation then governor operates a regulatory control and adjust the fuel supply to maintain the constant speed.

Objectives

1) Classify governors
2) Analyze different type of governors
3) Know characteristics of governors
4) Know stability of spring controlled governors
5) Compare different type of governors.

Types of Governors.

1) centrifugal governor
2) Inertia governor



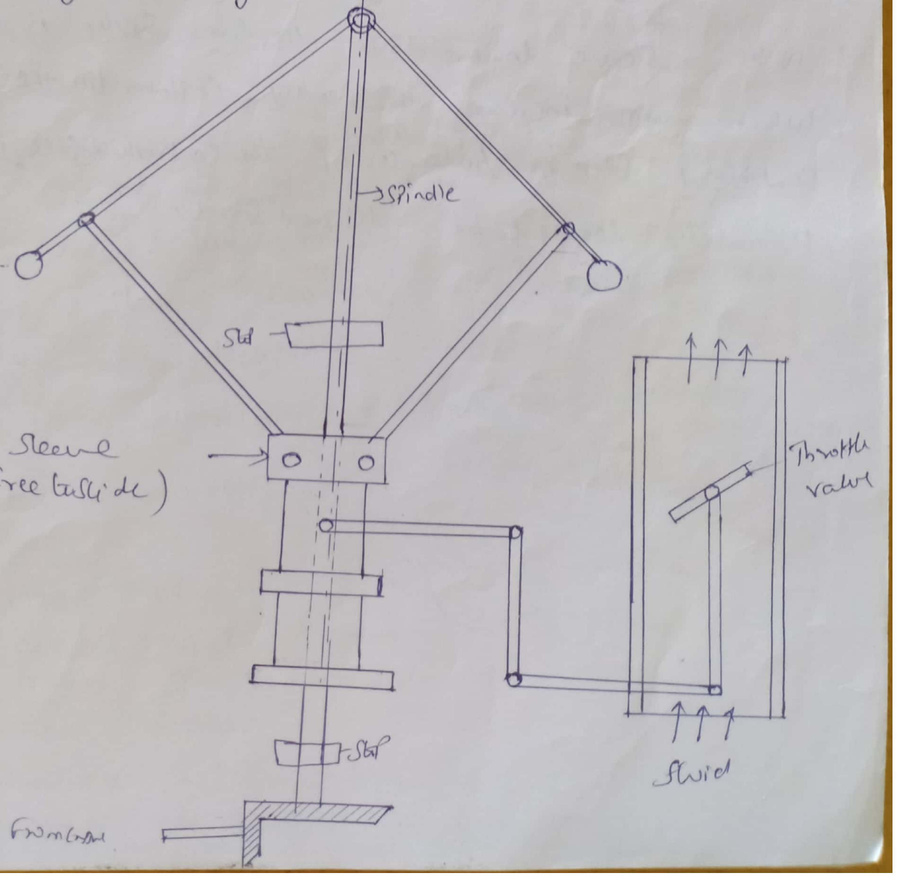
Centrifugal governor

The Shaft of a Primemover is connected to the governor Shaft by mean of gear or belt drive.

→ Modern governors get their connection by some electronic devices.

→ In a Centrifugal governor, two balls are fixed through the arms to the Shaft of the governor

→ The balls thus revolve with Shaft, giving rise to centrifugal force acting radially outwards.



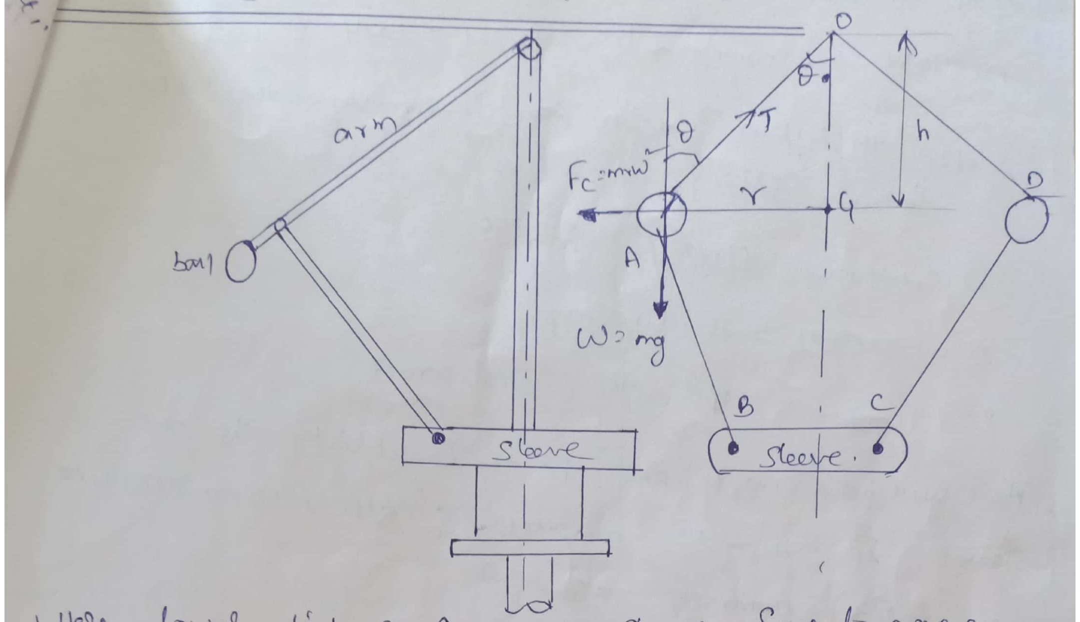
→ The centrifugal force is balanced by controlling force acting radially inwards which is provided by dead weight, a spring or both.

→ When the Engine running at designed speed, the balls of governor revolve at uniform speed; with which the two centrifugal & controlling force are balanced.

→ When the load on the Engine increases, its speed decreases and the balls of the governor move inwards thus lowering the sleeve. and motion of sleeve is transferred to the throttle valve.

→ As the sleeve lowers down, the fuel supply is enhanced which will increase the speed of Engine till the centrifugal force is balanced by the controlling force, which ultimately leads to the designed speed.

Pendulum type Simple Watt Governor.



→ Here lower links are fixed to a sleeve free to move on Spindle driven by the engine.

→ As the Spindle rotates, the balls take up position depending upon the Speed of the Spindle, when the Speed increases the balls move outwards, due to centrifugal force and pull the Sleeve upwards, as a result the ~~speed~~ height (h) of the governor decreases.

→ When the speed decreases the balls move inwards, Sleeve move downwards and height (h) increases.

Let

m = Mass of each ball in kg

ω = angular velocity of arm, balls & Sleeve about the axis of Spindle (rad/s)

T = Tension in arm

r = Radius of rotation of ball

h = Height of the governor.

→ It is assumed that weight of arm, links, & sleeve as negligible as compared to weight of ball

→ Now the system is in equilibrium under the action of following forces

1) centrifugal force $F_c = m r \omega^2$

2) weight of ball $W = mg$

3) Tension T in the upper arm.

There will be no tension in the lower links is the sleeve is assumed massless & also friction is neglected

Now taking moments about 'O'

$$F_c \times h = W \times \ell$$

$$h = \frac{W \cdot \ell}{F_c} = \frac{mg\ell}{mr\omega^2} = \frac{g}{\omega^2}$$

$$h = \frac{g}{\left(\frac{2\pi N}{60}\right)^2}$$

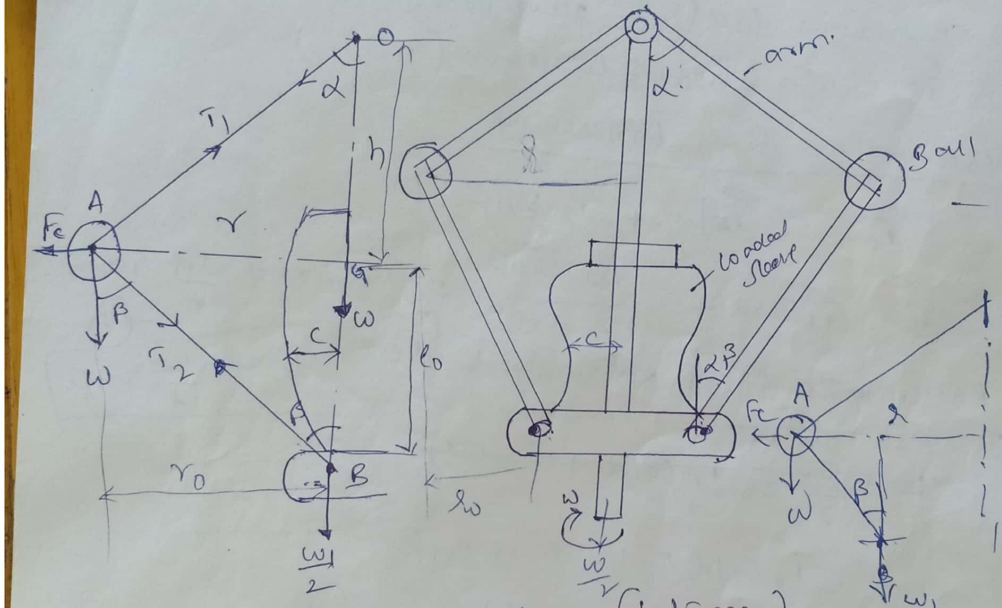
$$= \frac{9.81}{N^2} \times \left(\frac{60^2}{2\pi}\right)^2$$

$h = \frac{895}{N^2}$ or $h \propto \frac{1}{N^2}$

Porter governor

- Porter governor is the modified state of watt governor in which a central load is attached to the sleeve.

- The working is similar to watt governor



Let W_1 = Dead weight of sleeve (W = mg)

w = weight of each ball

~~c = distance of hinge 'B' from the axis~~

T_1 = Tension in upper arm

ω = angular speed of rotation

T_2 = Tension in lower arm

r = radius of ball rotation

~~r = r_0 + c r_0 = r − c.~~

Forces acting at hinge 'B' are

1) half of central load ω
2) Tension T_2 in lower arm
3) Reaction of the hinge

Resolving the forces vertically

$$\frac{w_1}{2} = T_2 \cos\beta \qquad \frac{GB}{AB} = \cos\beta \Rightarrow \frac{\frac{w}{2}}{T_2} = \cos\beta$$

$$T_2 = \frac{w_1}{2\cos\beta}$$

The ball is in Equilibrium under the action of following forces

1) centrifugal force (Fc) 2) weight of ball (w)

3) Tension in upper & lower arm (T_1 & T_2)

Resolving the forces horizontally

$$F_c = T_1 \overset{\sin}{\cancel{\cos}}\alpha + T_2 \sin\beta$$

$$F_c = T_1 \sin\alpha + \frac{w \sin\beta}{2\cos\beta} = T_1\sin\alpha + \frac{w_1}{2}\tan\beta$$

$$\boxed{F_c = T_1\sin\alpha + \frac{w}{2}\tan\beta} \quad ①$$

Resolving the forces vertically.

~~$w + \frac{w}{2} = T_1\cos\alpha + T_2\cos\beta$~~

$$T_1\cos\alpha = w + T_2\cos\beta$$

$$T_1\cos\alpha = w + \frac{w_1}{2} \quad ②$$

6 $$T_1\sin\alpha = F_c - \frac{w_1}{2}\tan\beta \quad ③$$

Eq 3 ÷ Eq ②

$$\frac{T_1\sin\alpha}{T_1\cos\alpha} = \frac{F_c - \frac{w_1}{2}\tan\beta}{w + \frac{w_1}{2}}$$

$$\left(w + \frac{w_1}{2}\right)\tan\alpha = F_c - \frac{w_1}{2}\tan\beta$$

$$\left(w+\frac{w_1}{2}\right) = \frac{F_c}{\tan\alpha} - \frac{w_1}{2}\frac{\tan\beta}{\tan\alpha}.$$

$$\frac{\tan\beta}{\tan\alpha} = q \qquad \tan\alpha = \frac{r}{h}.$$ Substitute

$$\left(w+\frac{w_1}{2}\right) = \frac{F_c}{(r/h)} - \frac{w_1}{2}\cdot q$$

$$w+\frac{w_1}{2} = \frac{h\,F_c}{r} - \frac{w_1}{2}\cdot q.$$

$$w+\frac{w_1}{2} = \frac{h\cdot m\cancel{r}w^2}{\cancel{r}} - \frac{w_1}{2}\cdot q$$

$$w+\frac{w_1}{2} = h\cdot m\omega^2 - \frac{w_1}{2}\,q$$

$$\frac{w+\frac{w_1}{2}+\frac{w_1}{2}\cdot q}{m\omega^2} = h$$

$$\frac{w+\frac{w_1}{2}(1+q)}{m\omega^2} = h.$$

$$\omega^2 = \frac{w+\frac{w_1}{2}(1+q)}{mh.}$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{mg+\frac{Mg}{2}(1+q)}{mh.}$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{m+\frac{M_1}{2}(1+q)}{m} \times \frac{g}{h}$$

$$N^2 = \frac{m+\frac{M_1}{2}(1+q)}{m} \cdot \frac{g}{h}\left(\frac{60}{2\pi}\right)^2 \qquad g = 9.81$$

$$= \frac{m+\frac{M_1}{2}(1+q)}{m} \times \frac{895}{h}$$

Length of arm = length of link

$\therefore \tan\alpha = \tan\beta = q = 1$

$$N^2 = \frac{m+M_1}{m} \times \left(\frac{895}{h}\right)$$

When the loaded sleeve moves up & down on the spindle; the frictional force acts on it in direction opposite to motion of sleeve.

$$\therefore N^2 = \frac{m+M\pm F}{m} \; \frac{895}{h} \qquad \tan\alpha = \tan\beta = q = 1$$

$$N^2 = m+\left(\frac{M}{2}\pm F\right)(1+q) \times \frac{895}{h}$$

length of arm # length of link.

Sleeve obliquity effect of arms we have

$a_1 = a_2 = a$

$b_1 = b_2 = b$

$$w + S_1 = \frac{2F_{c_1}a}{b}$$

$$w + S_2 = \frac{2F_{c_2}\,a}{b}$$

$$S_1 - S_2 = \frac{2a(F_{c_1} - F_{c_2})}{b}. \quad -①$$

Now ~~Sin θ~~ Tan θ $= \frac{c_1}{a} = \frac{h_1}{b}$

$$h_1 = \frac{c_1 b}{a}$$

Similarly $h_2 = \frac{c_2\,b}{a}.$

$$h_1 + h_2 = \frac{b(c_1 + c_2)}{a}.$$

lift of sleeve $h = h_1 + h_2 = \frac{b(r_1 - r_2)}{a}.$

$$S_1 - S_2 = Sh = \frac{S\,b(r_1 - r_2)}{a}. \quad -②$$

Compare for Eq ①

$$S = 2\left(\frac{a}{b}\right)^2\left(\frac{F_{c_1} - F_{c_2}}{r_1 - r_2}\right).$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{m + \frac{M_1}{2}(1+q)}{m} \times \frac{g}{h}.$$

$$N^2 = \frac{m + \frac{M_1}{2}(1+q)}{m} \cdot \frac{g}{h}\left(\frac{60}{2\pi}\right)^2 \qquad g = 9.81$$

$$= \frac{m + \frac{M_1}{2}(1+q)}{m} \times \frac{895}{h}.$$

Length of arm = length of link

$\therefore$ $\tan\alpha = \tan\beta = q = 1$

$$N^2 = \frac{m+M_1}{m} \times \left(\frac{895}{h}\right)$$

When the loaded sleeve moves up & down on the spindle; the frictional force acts on it in direction opposite to motion of sleeve.

$$\therefore N^2 = \frac{m + M \pm F}{m} \; \frac{895}{h} \qquad \tan\alpha = \tan\beta = q = 1$$

$$N^2 = m + \left(\frac{M}{2} \pm F\right)(1+q) \times \frac{895}{h}$$

length of arm
length of link.

Hartnell governor.

→ It is one of the Spring loaded type governors

→ A compressed spring is placed on the sleeve, so that it may exert some force on it.

→ Bell crank lever carry ball at one end and roller at other end. are pivoted to a pair of arms which rotate with spindle.

→ Each lever carries a ball at the end of vertical arm AB and roller at other end of horizontal arm

→ A helical compression spring provides equal downward force on the two rollers through sleeve.

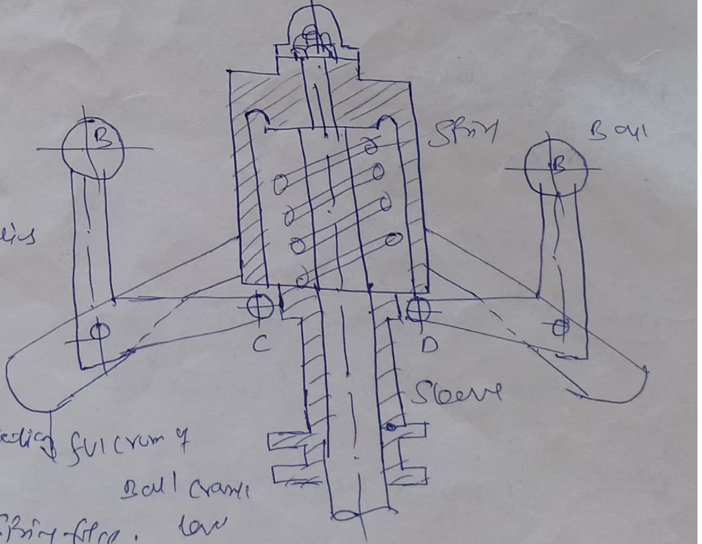
Let:

W_1 = weight of each ball

W = weight of Sleeve

$r_1\ r_2$ = Max, min radius of rotation of ball

$\omega_1\ \omega_2$ = max & min angular speeds of rotation



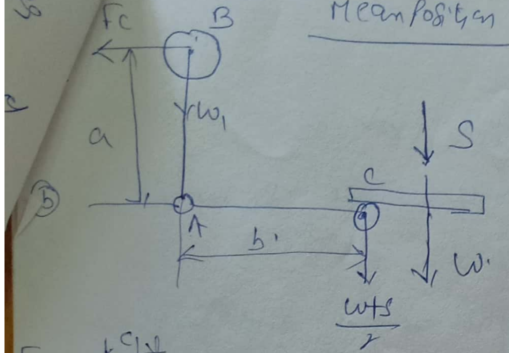
$S_1 S_2$ = Max, min Spring force.

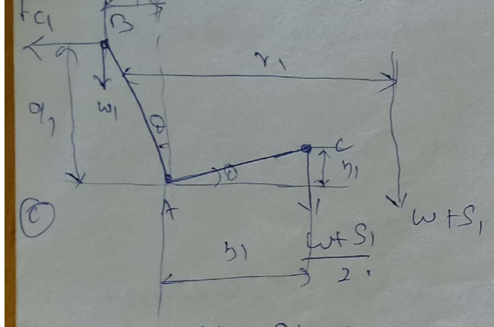
Fc_1, Fc_2 = centrifugal force at speeds ω_1, ω_2 respectively

S = Stiffness of Spring. , a, b = vertical, horizontal length of arm of bell crank lever.

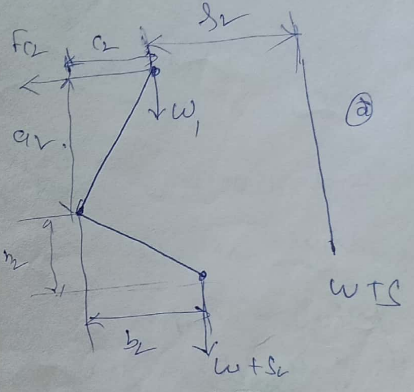
r = Radius of ball.

Mean Position





Max Speed



Min Speed

From b Take moments about A

$$Fc \times a = \frac{W+S}{2} \times b$$

$$W + S = \frac{2\,Fc \times a}{b'}$$

From c Take moments about 'A'

$$Fc_1 \times a_1 + w_1 \times c_1 = (W + S_1)\frac{b_1}{2}$$

From D $\quad Fc_2 \times a_2 - W_1 c_2 = (W + S_2)\frac{b_2}{2}.$

Now $\quad C_1 + C_2 = r_1 - r_2.$

$$r_1 - C_1 = l_2 + r_2$$

$$\boxed{r_1 - r_2 = c_1 + c_2}$$

Neglecting obliquity effect of arms we have

$$a_1 = a_2 = a$$

$$b_1 = b_2 = b$$

$$w + S_1 = \frac{2 F_{c_1} a}{b}$$

$$w + S_2 = \frac{2 F_{c_2} a}{b}$$

$$S_1 - S_2 = \frac{2a(F_{c_1} - F_{c_2})}{b} \quad -①$$

Now ~~Sinθ~~ Tanθ $= \frac{c_1}{a} = \frac{h_1}{b}$

$$h_1 = \frac{c_1 b}{a}$$

Similarly $h_2 = \frac{c_2 b}{a}$

$$h_1 + h_2 = \frac{b(c_1 + c_2)}{a}$$

A

lift of sleeve $h = h_1 + h_2 = \frac{b(r_1 - r_2)}{a}$

$$S_1 - S_2 = Sh = \frac{S\,b(r_1 - r_2)}{a} \quad -②$$

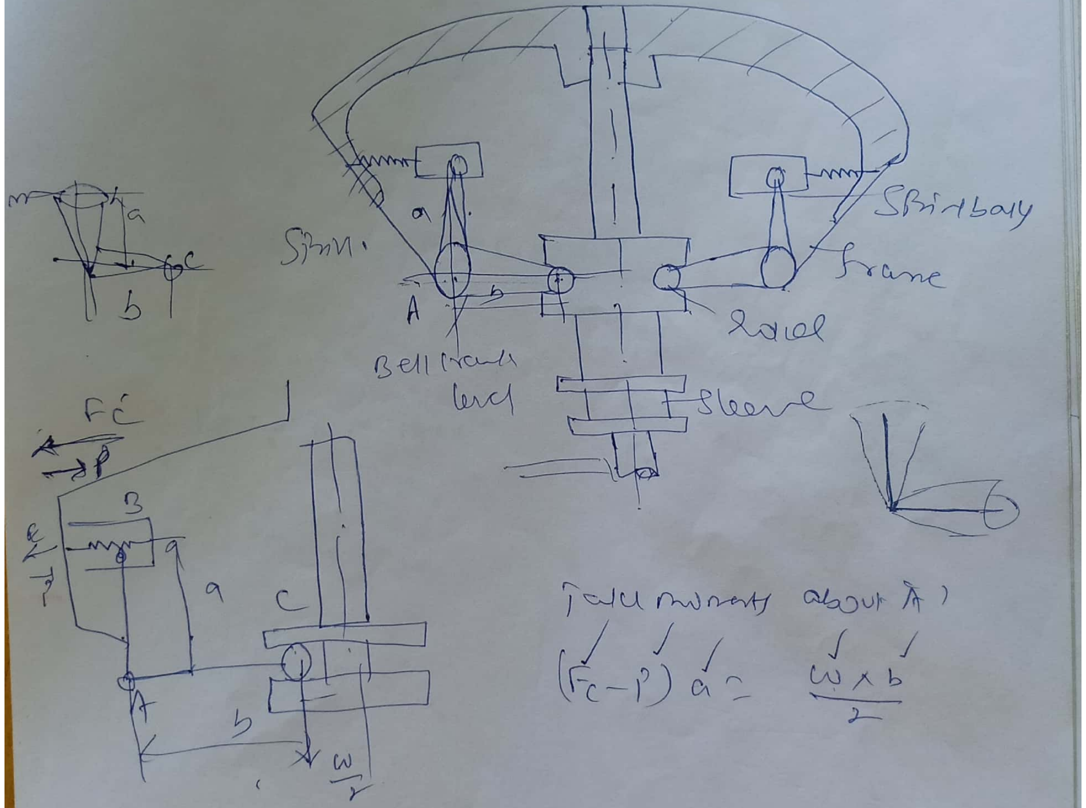
Compare for Eq ①

$$S = 2\left(\frac{a}{b}\right)^2 \left(\frac{F_{c_1} - F_{c_2}}{r_1 - r_2}\right).$$

Hartung governor

→ It is a Spring controlled governor in which vertical arms of bell crank lever are fitted with spring balls

→ The spring balls press against the frame of governor when the rollers at horizontal arm, press against sleeve



Take moments about A :

$$(F_c - P)\,a = \frac{W \times b}{2}$$

Problems

① A governor of hartnell type has equal balls of mass 3kg initially at radius of 200mm. The arms of bell crank lever are 110mm, vertically 150mm horizontally. Find the initial compression force on the spring if the speed for an initial ball radius of 200mm is 240 rpm & stiffness is 8 N/mm.

Sol:

Mass (m) = 3kg Radius (r) = 200m vertical (a) = 110 m

Speed (N) = 240 r/m Horizontal (b) = 150mm

Stiffness of spring (S) = 8 N/mm

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.13$ rad/sec

centrifugal force $F_c = m r \omega^2$

$= 3 \times 200 \times (25.13)^2 = 329.9$ N

$F_c \times a =$ ~~(w+S) b/2~~ $\frac{C \times b}{2}$

$379.9 \times 110 = \frac{C \times 150}{2}$ $C = \frac{w+S}{\cancel{2}}$

$C = 555.2$ N

∴ initial compression $= \frac{C}{S} = \frac{555.7}{8}$

$= 69.5$ mm

2 in Hartnell governor Mass of each ball 1kg, length of vertical arm (a) 100mm b = 50mm. Distance of fulcrum of each bell crank lever is 80mm from axis of rotation. Min extreme radius is 75mm, max extreme radius is 112.5mm. The max equilibrium speed 5% greater than min speed 360 rpm. Find 1) initial compression of spring,
2) Equilibrium speed corresponding to radius of rotation 100mm

Sol: $m = 1\,kg$ $a = 100$ $b = 50$ $r = 80\,mm$ $r_2 = 75$ $r_1 = 112.5$

$N_2 = 360\,rpm$

$$N_1 = \frac{5 \times 360}{100} = 378$$

Centrifugal force $F_{C1} = m r_1 \omega_1^2$

$$= 1 \times 112.5 \times \left(\frac{2\pi N_1}{60}\right)^2 = 175.5\,N$$

$$F_{C2} = m r_2 \omega_2^2 = 108.5\,N$$

$$r_1 - r_2 = 112.5 - 75 = 37.5\,mm$$

Spring Stiffness $S = 2\left(\frac{a}{b}\right)^2\left(\frac{F_{C1} - F_{C2}}{r_1 - r_2}\right)$

$$= 2\left(\frac{100}{50}\right)^2\left(\frac{175.4 - 108.5}{112.5 - 75}\right) = 14.69\,N/mm$$

let N = Speed in rpm at $r = 100\,mm$

$$F_C = m r \omega^2$$

$$125.4 = 1 \times 0.12\left(\frac{2\pi N}{60}\right)^2$$

$$N = 399.9\,rpm$$

Initial compression of Spring $= C/S$ $\frac{\text{load}}{\text{Stiffness}}$

$$F_c \times a = \frac{C \times b}{2}$$

$$m r \omega^2 \times 100 = \frac{C \times 50}{2}$$

$$1 \times 80 \times \left(\frac{2\pi \times \text{[illegible]}}{60}\right)^2 \times 100 = \frac{C \times 50}{2}$$

$$C = 561.1$$

$$\frac{C}{S} = \frac{561.1}{14.69} = 38.20\,\text{mm}$$

Problem

calculate the change in governor height of watt governors when speed is from 60 to 61 rpm

Sol: $N_1 = 60$ $N_2 = 61$

$$h_1 = \frac{g}{\omega^2} = \frac{895}{N_1^2} = \frac{895}{60^2} = 0.241m$$

$$h_2 = \frac{895}{N_2^2} = 0.24m$$

$h_1 - h_2 = 8mm$

2) length OA = 640mm and θ = 30° what is percent change in speed for 50 mm rise in level of balls.

Sol: OA = 640mm θ = 30°

initial height $h = OA \cos 30 = 640 \times \cos 0.866$
$= 554.2 mm$

final height $h' = h - 50 = 554 - 50$
$= 504.2 mm$

initial height + change in length = final height

$$h = \frac{g}{\omega^2} \qquad h' = \frac{g}{\omega'^2}$$

$$\omega_1^2 = \frac{g}{h'}$$

$$\frac{\omega'}{\omega} = \sqrt{\frac{g}{h'}} \times \sqrt{\frac{h}{g}} = \sqrt{\frac{h}{h'}} = \sqrt{\frac{554.2}{504.2}}$$
$= 1.048$

% change in speed

$$\frac{\omega' - \omega}{\omega} \times \omega = 4.48\%$$

3) A Porter governor has four arms 30cm. Pivoted elm hinged on axis of rotation while of lower arms at an offset 40mm from spindle axis weight of ball is 5kg & sleeve is 50kg. find equilibrium speed at radius of 16.5cm

Sol: $l = 30cm$ $w = 5kg$ $W = 50kg$ $c = 40mm$

$r = 165mm$

$h = (l^2 - r^2)^{0.5} = 264.2 mm$

$r_0 = r - c = 125 mm$

$h_0 = (l^2 - r_0^2)^{1/2} = 272.7 mm$

$\tan\alpha = r/h = 0.66$

$\tan\beta = \frac{r_0}{h_0} = 0.458$

$s = \tan\beta/\tan\alpha = 0.730$

$h = \frac{g}{\omega^2}\left(1 + (1+s)\frac{w}{2W}\right)$

$0.26 = \frac{9.81}{\omega^2}\left(1 + (1+0.730)\frac{50}{2\times5}\right)$

$\omega^2 = 431.18$

$\omega = 20.26 rad$

$N = 195.84 rpm$

A Porter governor has two balls of 50N weight each and a central load of 300N The arms are 300 mm long, pivoted on the axis. If max & min radius of rotation of the balls are 160mm, 140mm. find range of speed.

Sol'. w_1 = weight of ball = 50N   Central load (W) = 300N

$r_1 = 140mm$ (min) r_2 (max) $= 160mm$ $l = 300mm$ (length of arms)

$h_1 = \sqrt{(l^2 - r_1^2)} = (300^2 - 140^2)^{1/2} = 265mm$

$h_2 = (l^2 - r_2^2)^{1/2} = 300^2 - 160^2 = 253mm$

$$h = \frac{g}{\omega^2}\left(1 + \frac{W}{w_1}\right)$$

$$\omega_1^2 = \frac{g}{h_1}\left(1 + \frac{W}{w_1}\right) = \frac{9.81}{265}\left(1 + \frac{300}{50}\right)$$

$\omega_1 = 0.259$ rad/sec $N_1 = 24.5$ rpm

$\omega_2^2 = \left(1 + \frac{W}{w_1}\right)\frac{g}{h_2} \Rightarrow \omega_2 = 0.27$ rad/sec

$N_2 = 25.47$ rpm

Range of speed $N_2 - N_1 = 25.4 - 24.5 = 0.69$ rpm

⑤ A loaded type Porter governor has equal arms & links each 300mm long. the weight of each ball is 20N & central weight is 120N. when the ball radius is 150mm, valve is fully open. when radius is 180mm valve is closed. find the max speed & range of speed. If the max speed is increased by 25%, by an addition of weight to the central load, find its value.

ans)

Sol:- weight of each ball $(w) = 20N$

central load $W = 120N$

$(r_1)_{min} = 150$ $(r_2)_{max} = 180mm$ $l = 300mm$

$h_1 = (l_1^2 - r_1^2)^{1/2} = (300^2 - 150^2)^{1/2} = 259.8mm$

$h_2 = (l^2 - r_2^2)^{1/2} = (300^2 - 180^2)^{1/2} = 240mm$

$\omega_1^2 = \frac{g}{h_1}\left(1 + \frac{W}{w_1}\right) = \frac{9.81}{0.2598}\left(1 + \frac{120}{20}\right)$

$\omega_1^2 = 264.32$ $\omega_1 = 16.25$ rad/sec

$N_1 = 155.25$ rpm

$\omega_2^2 = 286.125$ $\omega_2 = 16.91$ rad/sec $N_2 = 161.47$ rpm

$N_2 - N_1 = 161 - 155 = 6.22$ rpm.

with 25% increase in max speed.

$N_2' = 161.47 \times 1.25 = 201.83$ rpm

$\omega_2' = \frac{2\pi N_2'}{60} = 21.35$ rad/sec

$\omega_2'^2 = \frac{g}{h_2'}\left(1 + \frac{W'}{w_1}\right)$

$(21.35)^2 = \frac{9.81}{0.24}\left(1 + \frac{W'}{20}\right)$

$W' = 198.65$ kg.

Flywheel

Turning Moment diagram also known as Crank effort diagram

It is graphical representation of turning moment & crank effort for various position of crank.

→ It is plotted as turning moment is at ordinate (y) crank angle is abscissa is at (x)

Turning moment diagram for a single cylinder double acting Steam Engine

→ WKT Turning moment on crankshaft

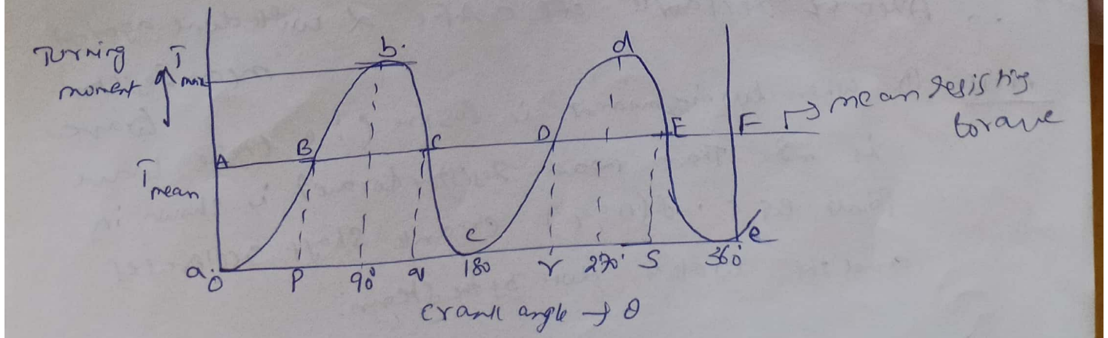
$$T = F_p \times r\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right]$$

F_p = piston effort

r = crank radius.

n = ratio of connecting rod length to radius of crank.

θ = angle turned by the crank from I.D.C (inner dead centre)



→ From the above we see that Turning moment (T) is zero when crank angle (θ) is zero & it is max when θ is 90° and again zero when crank angle is 180°

→ This is shown by the curve abc and it represents turning moment for outstroke. The curve cde is similar to abc & represents for inner stroke

→ Work done is product of turning moment and angle turned

∴ area of turning moment diagram represents workdone per revolution

→ in actual practice, the engine is assumed to work against the mean resisting torque. as shown by horizontal line AF

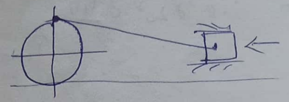
The height of aA represents mean height

→ It is assumed that work done by turning moment / revolution = work done against mean resisting torque

∴ Area of rectangle ~~aa~~ aAFe ∝ work done against mean resisting torque

<u>Note</u>: ① when turning moment is positive (i.e engine torque is more than mean resisting torque) is shown in points B & C. & (D & E) crank shaft accelerates and the work is done by the steam.

when the turning moments is negative (i.e ④
Engine torque is less than mean resisting torque). as shown in C & D. crank shaft retards & work is done on the steam.



⑤ T = Torque on crankshaft at any instant

T_{mean} = mean resisting torque

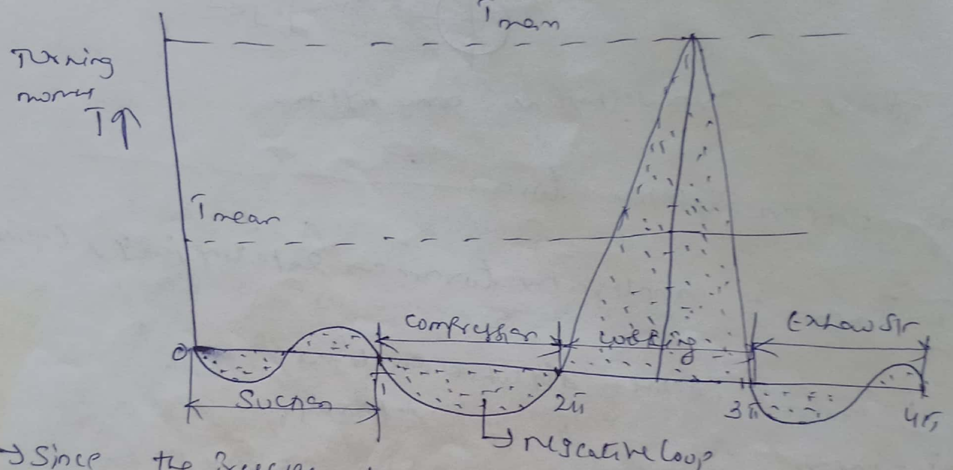
$T - T_{mean}$ = accelerating torque on rotating parts of Engine

$T - T_{mean}$ = positive flywheel accelerates

$T - T_{mean}$ = -ve " " retards.

Turning moment diagram for four stroke I·C Engine

→ We know in a four stroke cycle I C Engine there is one working stroke (ie power) after crank has turned through two revolution i.e 720° or 4π



→ Since the pressure inside cylinder is less than atmosphere pressure during Suction stroke, ∴ a negative loop is formed.

→ During compression, work is done on the gases therefore a large negative loop is formed

→ During Expansion fuel burns & work is done by the gases ∴ a large positive loop is formed

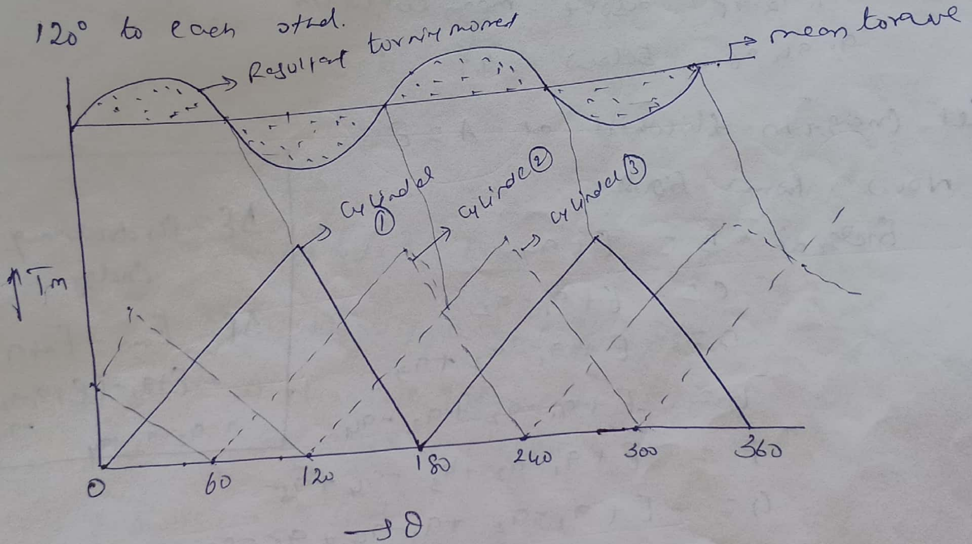
→ During Exhaust stroke, work is done on the gases ∴ a negative loop is formed.

Turning moment diagram for multi cylinder Engine .

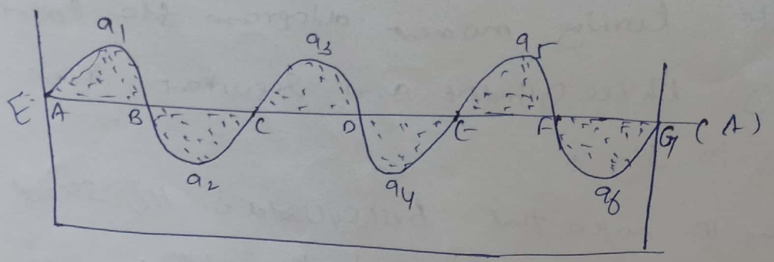
→ Seferate turning moment diagram for a Engine having three cylinder and resultant TMD is shown in fig .

→ It may be noted that first cylinder is HP, second cylinder medium pressure, third cylinder is low pressure .

→ The cranks in case three cylinders are placed at 120° to each other.



Determination of Max fluctuation of Energy



TMD for multi cylinder Engine is wave curve.
Horizontal line AG represents mean torque line.

let a_1, a_3, a_5 above mean torque line
a_2, a_4, a_6 below " "

let Energy in flywheel at $A = E$

Now from figure

Energy at $B = E + a_1$
$C = E + a_1 - a_2$
$D = E + a_1 - a_2 + a_3$
$E = E + a_1 - a_2 + a_3 - a_4$
$F = E + a_1 - a_2 + a_3 - a_4 + a_5$
$G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$.
= Energy at 'A' (i.e cycle repeats after G.)

ΔE = fluctuation of Energy
$\Delta E = E_{max} - E_{min}$
$= [E + a_1 - (E + a_1 - a_2 + a_3 - a_4]$
$= a_2 - a_3 + a_4$

let us now assume that greatest of these Energies is at 'B' & least at E

∴ Max Energy in fly wheel at 'B' $= E + a_1$

∴ Min Energy in flywheel at 'E' $= E + a_1 - a_2 + a_3 - a_4$.

∴ ΔE = fluctuation of Energy = Max Energy − Min Energy

efficient of fluctuation of Energy

$C_E = \dfrac{\text{Max fluctuation of Energy}}{\text{work done per cycle}}$

work done / cycle = $T_{mean} \times \theta$

θ = angle turned in Radians in one Revolution
= 2π = Steam Engine & 2S IC Engine
4π 4S IC Engine

$\therefore T_{mean} = \dfrac{P \times 60}{2\pi N} = \dfrac{P}{\omega}$

P is in watt

$\omega = \dfrac{2\pi N}{60}$ W.D can be obtained from following Relation

$\therefore$ work done per cycle = $\dfrac{P \times 60}{n}$

n = Number of working strokes per min

n = N, in case of Steam Engine, 2S IC Engine
= $\dfrac{N}{2}$ = 4S IC Engine.

coeff of fluctuation of Speed

→ The difference between Max & Min Speeds during a cycle is called Max fluctuation of Speed.

$\therefore$ The ratio of Max fluctuation of Speed to mean Speed is coeff of fluctuation of Speed

Let N_1, N_2 Max, Min rpm.

$N = \dfrac{N_1 + N_2}{2}$ $\therefore$ coeff of fluctuation of Speed $(C_S) = \dfrac{N_1 - N_2}{N}$, angular speed

$= \dfrac{2(N_1 - N_2)}{(N_1 + N_2)} = \dfrac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$

Interm of linear speed → $= \dfrac{2(v_1 - v_2)}{v_1 + v_2}$

Energy Stored in a Flywheel

Let m = Mass of flywheel in kg

k = Radius of gyration of flywheel in m

$I = mk^2$

N_1, N_2, Max, min speeds during cycle in rpm

ω_1, ω_2 " " angular speeds " " in rad/sec

N = Mean speed = $\frac{N_1+N_2}{2}$

ω = " angular speed = $\frac{\omega_1+\omega_2}{2}$

$C_s = \frac{N_1-N_2}{N}$ or $\frac{\omega_1-\omega_2}{\omega}$

$\Delta E = M R^2 \omega^2 C_s$

$= M v^2 C_s$.

k may be taken equal to radius of rim (R) if the thickness of rim << dia of rim

We know Mean K.E. of flywheel = $E_{mean} = \frac{1}{2} I \omega^2$

$= \frac{1}{2} m k^2 \omega^2$

As speed changes from ω_1 to ω_2 Max fluctuating energy ΔE occurs

ΔE = Max KE − Min KE

$\Delta E = \frac{1}{2} I \omega_1^2 - \frac{1}{2} I \omega_2^2$

$= \frac{1}{2} I (\omega_1^2 - \omega_2^2) = \frac{1}{2} I (\omega_1+\omega_2)(\omega_1-\omega_2)$

$\Delta E = \frac{1}{2} I \omega(\omega_1-\omega_2)$ $\frac{\omega_1+\omega_2}{2} = \omega$.

$\therefore \Delta E = I \omega(\omega_1-\omega_2) \times \frac{\omega}{\omega}$ × ÷ with ω

$\Delta E = I \omega^2 \left(\frac{\omega_1-\omega_2}{\omega}\right)$ $\frac{\omega_1-\omega_2}{\omega} = C_s$.

$\boxed{\therefore \Delta E = I \omega^2 C_s = m k^2 \omega^2 C_s}$

Problems

1) The mass of flywheel is 6.5 tons $k = 1.8$ m, It is found that from IMD fluctuating energy is 56 kN-m. Mean speed of engine is 120 rpm. find Max & min speeds

Sol: $m = 6.5t = 6500 kg$ $k = 1.8$ $\Delta E = 56$ kN m.
$= 56 \times 10^3$ N m

let N_1 N_2 max & min speeds

$$\Delta E = m k^2 \omega^2 q. \qquad q = \frac{N_1 - N_2}{N}.$$

$$\Delta E = m R^2 \omega^2 q = m v^2 q.$$

$$= \cancel{I} \times$$
$$\Delta E = I \omega(\omega_1 - \omega_2)$$

$$= I \times \frac{2\pi N}{60}\left(\frac{2\pi N_1}{60} - \frac{2\pi N_2}{60}\right)$$

$$= \frac{4\pi^2}{3600} I N (N_1 - N_2) \qquad N_1 - N_2 = N \times q.$$

$$\Delta E = \frac{\pi^2}{900} m k^2 N^2 q.$$

$$50 \times 10^3 = \frac{\pi^2}{900} \times 6500 \times 1.8^2 \times 120 (N_1 - N_2)$$

$$N_1 - N_2 = 2 \text{ rpm}.$$

We know, Mean speed (N) $= \frac{N_1 + N_2}{2}$

$$120 = \frac{N_1 + N_2}{2}$$

$N_1 + N_2 = 240.$

$N_1 = 121$ rpm $N_2 = 119$ rpm

③ The flywheel of a steam engine has ~~Se~~ $k = 1m$ $m = 25$

Starting torque steam engine is 1500 N-m. & may be assumed constant. Find 1) angular acceleration of flywheel.

2) Kinetic energy flywheel after 10 seconds from the start

Sol: $k = 1m$ $m = 2500 kg$ $T = 1500 N\text{-}m$

1) Angular acceleration of flywheel (α)

$$T = I\alpha.$$

$$1500 = 2500 \times 1^2 \times \alpha. \Rightarrow \alpha = 0.6 rad/sec^2$$

2) K.E of flywheel

ω_1 = Angular speed at rest = 0

ω_2 = " " after 10 sec

t = Time in sec

w.k.t $\omega_2 = \omega_1 + \alpha t$ $\boxed{v = u + at}$

$= 0 + 0.6 \times 10 = 6 rad/sec^2$

∴ K.E of flywheel $= \frac{1}{2} I \omega_2^2$

$= \frac{1}{2} \times 2500 \times 1^2 \times 6^2 = 45 kN\text{-}m$

A skw Engine develops 300kW at 90 rpm

coeff of fluctuation of energy is found from Tmd is 0.1 $\boxed{C_E = 0.1}$

fluctuation of speed is kept within ± ~~0.5~~ 1 %. of mean speed,

find weight of flywheel if k = ~~0~~ 2m

Sol: $P = 300kW$ $N = 900$ rpm $C_E = 0.1$ $k = 2m$

WKT $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 90}{60} = 9.426$ rad/s

$\omega_1 \omega_2$ = Max, min speeds

∵ fluctuation of speed is ± 0.5%. of mean speed

∴ Total fluctuation of speed $\omega_1 - \omega_2 = 1\%. \omega$

$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.01$ $WD = T\theta$

$= T \times 2\pi = \frac{P \times 60}{2\pi N} \times 2\pi$

WKT $WD/cycle = \frac{P \times 60}{N} = \frac{300 \times 10^3 \times 60}{90} = 200 \times 10^3 N\text{-}m$

∴ Max fluctuation of energy = WD/cycle × C_E

$\Delta E = I \omega^2 y$ $= 200 \times 10^3 \times 0.1$

$\Delta E = m k^2 \omega^2 y$

$20 \times 10^3 = m\, 2^2 \times 9.426^2 \times 0.01$

$m = 5630 kg.$

$0.5 = \frac{\omega_1 - \omega_2}{\left(\frac{\omega_1 + \omega_2}{2}\right)}$

$\omega_0 - 0.5 \frac{\omega}{2} = \omega_1 - \omega_2$

$\frac{1}{2} \frac{\omega}{2} = \omega_1 - \omega_2$

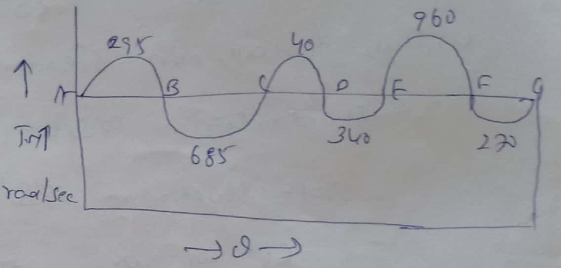
④ The turning moment diagram for a Petrol Engine is drawn to the following Scale. Turning moment 1 mm = 5 N-m. Crank angle 1 mm = 1° The turning moment diagram repeat it self at every half revolution of the Engine and the areas above & below the mean turning moment is taken in order are 295, 685, 40, 340, 960, 270 mm². The rotating parts are equivalent to a mass of 36 kg at a radius of gyration of 150 mm. find coeff of fluctuation of speed when engine runs at 1800 rpm

Sol:

$M = 36\ kg \quad K = 150\ mm$
$= 0.15\ m$

$N = 1800\ rpm.$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1800}{60} = 188.52\ rad/sec$



∴ turning moment Scale 1 mm = 5 N-m & Crank angle 1 mm = 1° = π/180 rad

∴ 1 mm² on turning moment diagram

work done = Area under the curve
= Tθ

$1\ mm^2 = 5 \times 1°$
$= 5 \times \pi/180 = \pi/36\ N\text{-}m$

Let the total Energy at A = E

∴ $E_B = E + 295$ (Max Energy)

$E_C = E + 295 - 685 = E - 390$

$E_D = E - 390 + 40 = E - 350$

$E_E = E - 350 - 340 = E - 690$ (Min Energy)

$E_F = E - 690 + 960 = E + 270$

$E_G = E + 270 - 270 = E$ = Energy at 'A'.

W.K.T Max fluctuation of Energy ΔE = Max Energy − min Energy
$= (E + 295) - (E - 690)$
$\Delta E = 985\ mm^2$

$\Delta E = 985 \times \pi/36 = 86\ N\text{-}m = 86J$

Let C_s = coeff of fluctuation of speed.

W.K.T $\Delta E = mk^2 \omega^2 C_s$

$86 = 36 \times (0.15)^2 \times (188.52)^2 \times C_s$

$C_s = 0.003$
$= 0.3\%$

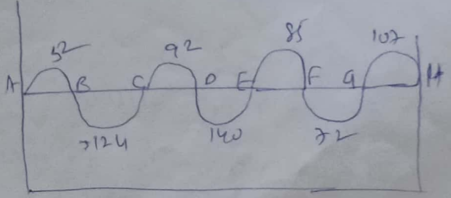
A TMD for a multicylinder Engine has been drawn to a scale 1mm = 600N-m vertically, 1mm = 3° horizontally. The intercepted area between output torque curve & mean resistance line taken in order from one end are as follow ⑦

+52, −124, +92, −140, +85, −72, +107 mm². Engine is running at 600 rpm. fluctuation of speed is not to Exceed ± 1.5% of mean speed. find mass of flywheel of radius 0.5 m.

Sol: $N = 600\,rpm$ $\omega = \frac{2\pi N}{60} = \frac{2\pi 600}{60} = 62.84\,rad/s$ $R = 0.5\,m$

$C_s = 1.5\%$ of mean speed

$\frac{\omega_1 - \omega_2}{\omega} = 1.5 \times \omega$ (fluctuation of speed)



$\omega_1 - \omega_2 = 3\%\,\omega$ (Total fluctuation of speed)

$\omega_1 - \omega_2 = 0.03\omega$

$\therefore \frac{\omega_1 - \omega_2}{\omega} = C_s = 0.03$

Turning moment scale 1mm = 600 N-m

crank angle scale 1mm = 3°

$\therefore 1mm^2 = 600 \times 3 \times \pi/180 = 31.42\,N\text{-}m$

∴ let total energy at A = E

$E_B = E + 52$

$E_C = E + 52 - 124 = E - 72$

$E_D = E - 72 + 92 = E + 20$ (Max Energy)

$E_E = E + 20 - 140 = E - 120$

$E_F = E - 120 + 85 = E - 35$

$E_G = E - 35 - 72 = E - 107$ (min Energy)

$E_H = E - 107 + 107 = E = A$

$\Delta E = (E + 20) - (E - 107)$

$\Delta E = 172\,mm^2 = 172 \times 31.42 = 5404\,N\text{-}m$

$\Delta E = mR^2\omega^2 C_s$

$5404 = m \times 0.5^2 \times 62.84^2 \times 0.03$

$m = \frac{5404}{29.6} = 183\,kg$