Fluid Dynamics

UNIT I

Introduction:-

→ Matter can be distinguished by the physical form of its existence.

→ These forms are known as phases, are called solid, liquid and gas.

→ liquid, gaseous phases are commonly referred as fluid, because of common characteristics exhibited by liquids and gases. Where as solids differs with liquids and gases.

→ Very strong inter molecular attractive forces exists in solids due to spacing of molecules and movement of molecules are extremely small in solids.

→ Inter molecular forces are weaker in liquids and extremely small in gases due to spacing molecules and movement of molecules are large in gases, smaller in liquid.

→ Liquids have the property of taking the shape of container where as gases occupies the full volume of container.

→ The solids, liquids and gases exhibit different characteristics due to their different ~~characterist~~ molecular structure.

Fluid :-

The word fluid means a substance having particles which readily change their relative positions.

A fluid may be defined as a substance which deform continuously under action of shear stress regardless of its magnitude. The time rate of deformation of fluid will however depends the magnitude of shear strain.

Scanned with CamScanner

In Nature we find three substances of matter like solids, liquids and gases.

→ The difference in properties and their behaviour can be studied by their molecular properties.

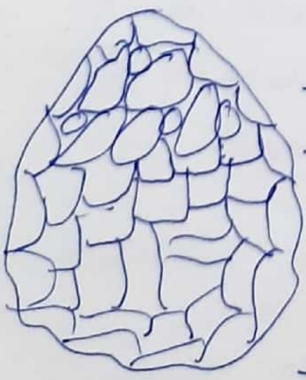
→ If substances of three fluid states like solids liquids, gases are kept out with out any container surrounding them, at a place on the ground and seen after some time, it can be observed that solid remains as it placed, while the liquid floods all over the ground and gases mix up with the atmosphere.

→ By this we can understand that inorder to move the position of solid some external force required, but liquids and gases do flow without much of external force acting on them.

→ Fluid can be defined as "Any (object) substance which has the capability to flow is called fluid".
and due to this reason liquids and gases can be termed as fluids.

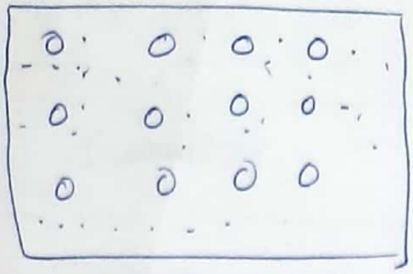
Substances of matter

Solids Liquids gases



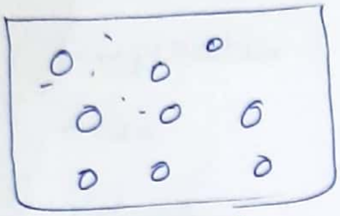
→ molecules closely packed
→ Intermolecular attraction force is high (force of attraction b/w Molecules)
→ due to this forces solid have rigid shape.
→ It has shape and volume

Solids



Liquids, molecules placed equidistant

→ molecules are loosely packed and are free to move
→ Inter molecular attractive (cohesive) force is comparatively low.
→ It has volume but no definite shape.
→ Incompressible fluid.



Gases, molecules placed random.

→ Molecules very loosely packed and are free to fly
→ Inter molecular attraction force (cohesive) is lower than liquids.
→ As gases have wide gap in between their molecules and hence are compressible.
→ Compressible fluid.
→ It don't have neither shape nor volume

* (liq & gas)

→ Solids and fluids have the similarities that these two substances follow similar kind of law's like <u>Hooke's law</u> and <u>Newton's law respectively</u>.

→ The solids obeys <u>Hook's law</u> in which stress ∝ strain and constant of proportionality is called <u>modulus of elasticity</u> (E)

→ Fluids obeys <u>Newton's law of viscosity</u>, where in shear stress is directly proportional to velocity gradient or strain rate and constant of proportionality is called as <u>Absolute viscosity (μ)</u>.
or
Coefficient of viscosity
or
Dynamic viscosity.

$$\tau \propto \left(\frac{du}{dy}\right)$$ — velocity gradient

Shear stress
$$\tau = \mu \frac{du}{dy}$$

$$\mu = \tau / \frac{du}{dy}$$

UNITS USED IN F.D:-

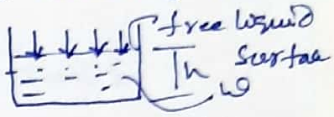
Quantity	S.I. Units	Symbol.
Mass	Kilogram	Kg
Length	Metre	m
Time	Second	s
Temperature	Kelvin	K
Area	Square meter	m^2
Volume	Cubic metre	m^3
Acceleration	meter per Second Square	m/s^2
Force	Newton	N
Moment of Force	Newton Metre	N-m
Pressure	Newton per Square Metre	N/m^2 or Pascal Pa
Discharge	Metre Cube per Sec	m^3/s
Density	Kilogram per Meter Cube	kg/m^3
Specific weight	Newton per Cubic metre	N/m^3
Dynamic Viscosity	Newton-Sec per Square metres	$\frac{N-Sec}{m^2}$
Kinematic Viscosity	Metre Square per Second	m^2/s
Dynamic Viscosity	Newton-Sec per Square metres	$\frac{N-Sec}{m^2}$
Surface Tension	Newton per Metre	N/m
Work-Energy	Joule	J or N-m
Power	watt	$W = J/s$.

PROPERTIES OF FLUIDS:-

Every fluid has certain characteristics by means of which its physical condition may be described. Such characteristics are called properties of fluid.

Pressure:- (P)

When a liquid contained in a vessel, it exerts normal forces on the surface of vessel.



This normal force per unit area of surface is called as Intensity of pressure or simply pressure.

The intensity of pressure increases with depth of liquid at the rate of specific weight of liquid i.e the pressure intensity at a depth 'h' below the free surface is equal to product of specific weight (w) and depth (h)

$$P = w\cdot h = \frac{W}{V}\cdot h = \frac{mg}{V}\cdot h$$

$$\boxed{P = \rho g h}$$

gravity $= g$

density $= \rho$

pressure head $= h$

$P =$ Pressure

Pressure Head (h):-

The vertical height of free surface above any point in a liquid is known as pressure head at that point

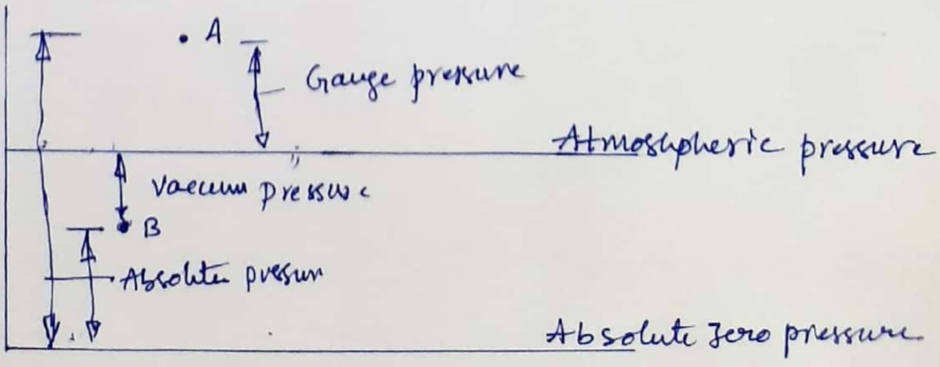
$$P = wh \Rightarrow h = P/w.$$

at A

$P_{abs} = P_{atm} + P_{gauge}$

at B

$P_{abs} = P_{atm} - P_{vacu}$



Incompressible & Compressible fluids:-

→ Change of density or volume of fluid with applied pressure indicates whether it is compressible or incompressible.

→ Incompressible fluid is the ~~termed~~ applied where the change in density or volume with pressure is so small as to be [neglected] negligible.
Normally all liquids are incompressible fluids.

→ Compressible fluid is the term applied where the change in density or volume with pressure is considerable.
Normally all gases are compressible fluids.

→ A solid has "volume and shape", a liquid has "volume but no shape", a gas has neither.

Continuum :-

Although fluids consist of discrete molecules, analysis of fluid flow problems made by a concept that treats fluid as a continuous media. All voids or cavities which may occur in the fluid are ignored.

The physical properties of the fluid are then continuous from point to point.

In other words, the fluid properties are treated to be same at any point and identical in all directions.

A continuous and homogeneous fluid medium is called continuum.

Types of fluids :- Classification :-

The fluids may be classified into the following 5 types.

i) Ideal fluid ii) Real fluid iii) Newtonian fluid

iv) Non-Newtonian fluid v) Ideal plastic fluid

Ideal fluid :-

A fluid which is incompressible and is having no viscosity, no surface tension is known as ideal fluid.

Ideal fluid is only an imaginary fluid as all the fluids, which exist have some viscosity.

Real fluid :-

Real fluids are those which occur in nature and posses the properties like viscosity, surface tension and compressibility.

In Actual practice all fluids are real fluids.

Newtonian fluid :-

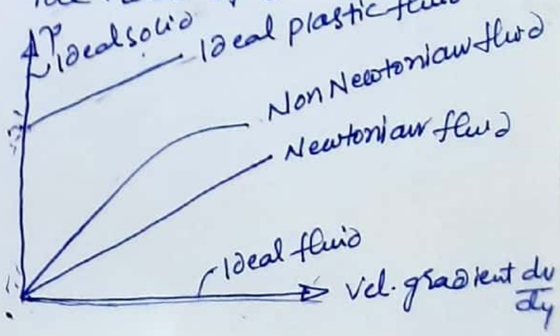
A real fluid in which shear stress is directly proportional to rate of shear strain (velocity gradient) is known as Newtonian fluid.

Non-Newtonian fluid :-

A real fluid in which shear stress is not directly proportional to the rate of shear strain is known as Non-Newtonian fluid.

Ideal plastic fluid :-

A fluid in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain



Surface Tension :-

The property of liquid surface to exert a tension is called Surface tension.

$$\tau = \frac{\text{surface energy}}{\text{unit surface area}} = \frac{N\text{-}m}{m^2} = N/m.$$

Fluid properties :-

1) Density / Mass density / specific mass (ρ) :-

Mass density of a fluid is its mass per unit volume.

$$\rho = \frac{m}{V} = \frac{Kg}{m^3} = \frac{mass}{volume}.$$

2) Weight density / specific weight (w) :- (γ)

Specific weight of a fluid is its weight per unit volume.

$$w = \frac{weight}{volume} = \frac{W}{V} = \frac{m \cdot g}{V} = \rho g = \frac{N}{m^3} \left(\frac{kg}{m^3} \times \frac{m}{s^2}\right)$$

$$w = \rho g. \quad \left[P = \rho RT \Rightarrow \rho = \frac{P}{RT} \therefore w = \frac{P}{RT} g \right]$$

NOTE :- Specific weight of water at $4^\circ c$ is $w = 1000 \times 9.81 = 9810 \, N/m^3$

w decreases with increase in temp. and

w increases with increase in pressure $\quad w = \rho g$

3) Specific ~~weight~~ Gravity / Relative density (S) :-

It is defined as ratio of specific weight of liquid to the specific weight of standard liquid

(or)

It is the ratio of density of liquid to the density of standard fluid

$$S = \frac{w_{liquid}}{w_{water}} = \frac{\rho_{liquid}}{\rho_{water}} \quad \text{NO UNITS}$$

Specific Volume:- (ϑ)

It is the volume of fluid to unit mass of fluid

$$\vartheta = \frac{\text{Volume}}{\text{mass}} = \frac{V}{m} = \frac{1}{e} = m^3/kg$$

(5) Compressibility :- (Z)

Under the application of external forces, fluids undergo change in volume and this property by virtue of which fluids undergo change in volume under the action of external pressure is called compressibility.

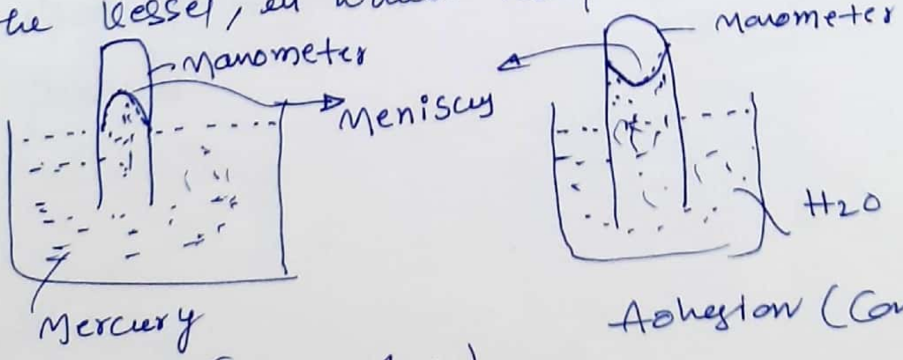
$$Z = \frac{\frac{dv}{V}}{P}$$

(6) Cohesion & Adhesion :-

Some liquids have the tendency to attract the intermolecules while other liquid molecules have the tendency to stick to the boundaries of vessel, in which they are contained.

The tendency of liquid molecules to attract the intermolecules is called 'Cohesion'.

The tendency of liquid molecules to stick to the boundaries of the vessel, in which they are contained is called "Adhesion"



Cohesion (Convex shape)

Adhesion (Concave shape)

Scanned with CamScanner

Surface Tension :-

The cohesive forces between liquid molecules are responsible for the phenomenon called surface tension.
The molecules at the surface donot have other like molecules on all sides of them and consequently they are attracted more strongly to those directly associated with them on the surface.
This forms a surface 'film in tension'. and is difficult to move small particles through the surface.
Hence Surface tension is the force required to break surface film by unit length

8) Viscosity :-

→ Viscosity is a property of fluid by which it offers resistance to shear & angular deformation.

→ The resistance to flow because of internal friction is called viscous resistance, and the property which enables the fluid to offer resistance to relative motion b/w adjacent layers is called the viscosity of fluid.

→ Viscosity of fluids is due to cohesion and interaction b/w particles.

Classification of fluid flow:-

According to different considerations fluid flows may be classified as

1) Steady flow:-

Motion of a fluid is said to be steady when the fluid (properties) parameters at any point in the flow field remain constant with respect to time; the parameters may however be different at different cross sections of the flow passage. Velocity, pressure, density, temp - - etc are fluid properties.

$$\frac{\partial V}{\partial t} = 0 ; \frac{\partial P}{\partial t} = 0 ; \frac{\partial \rho}{\partial t} = 0 ; \frac{\partial T}{\partial t} = 0$$

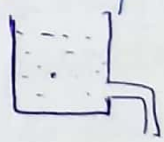
Ex:- Water flowing through a pipe at constant rate.

2) Unsteady flow:-

Motion of a fluid is said to be unsteady when the fluid parameters at any point in the flow field change with time

$$\frac{\partial V}{\partial t} \neq 0 ; \frac{\partial P}{\partial t} \neq 0 ; \frac{\partial \rho}{\partial t} \neq 0 ; \frac{\partial T}{\partial t} \neq 0$$

Ex:- Liquid flowing under gravity out of an opening in the bottom of a vessel.

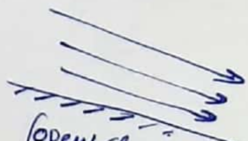


3) Uniform flow:-

Motion of a fluid is said to be uniform when the flow parameters like P, V, ρ, T - - etc remain constant throughout the flow field (section to section) at any given time.

$$\frac{\partial P}{\partial s} = 0 ; \frac{\partial T}{\partial s} = 0 ; \frac{\partial \rho}{\partial s} = 0 , \frac{\partial V}{\partial s} = 0$$


(flow b/w parallel plate)


(open channel flow)

Ex:- Flow of liquid through a long pipe of const diameter

Non-Uniform flow:-

Motion of a fluid is said to be Non-Uniform when the flow properties like P, V, ρ, T .. etc are changing from one section to another section.

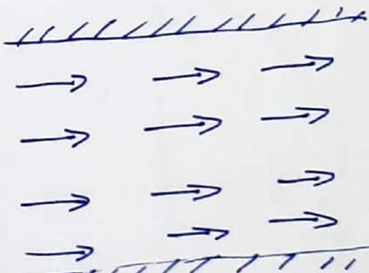
$\frac{\partial P}{\partial s} \neq 0$, $\frac{\partial V}{\partial s} \neq 0$, $\frac{\partial \rho}{\partial s} \neq 0$ (space rate of change of flow parameters $\neq 0$)

Ex:- Flow of fluid through a tapering pipe.

5) Laminar flow:- (Viscous flow) (Stream line flow)

A laminar flow is characterised by smooth flow of lamina of fluid over another.

Fluid elemements move in well defined paths and they retain the same relative position at successive c/s's of flow passage.



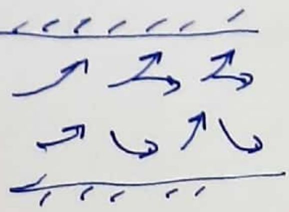
This type of flow occurs generally in smooth pipes when the velocity of flow is low and also liquids having high viscosity.

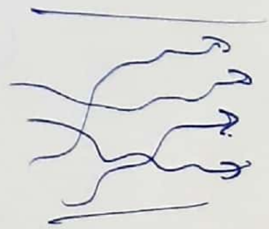
Ex:- Flow of thick oil through a small tube.

6) Turbulent flow:- (Non-Viscous flow)

In turbulent flow the fluid elements move in erratic & unpredictable paths (zig-zag) and their paths also cross each other. The fluid particles are subjected to fluctuating transverse velocities so that the motion is eddying. The random eddying motion is called turbulence.

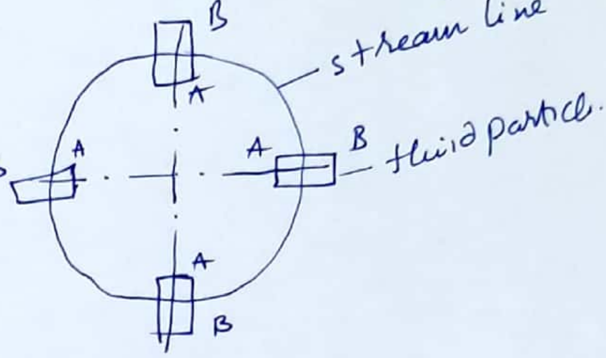
Ex:- Flow in rivers at the time of floods





Rotational flow:-

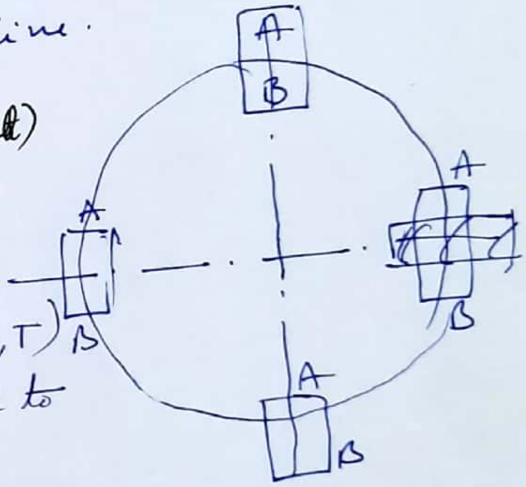
A rotational flow exist when the fluid particles rotate about their own mass centers while it is moving along stream line.



Ex: Rotary motion of liquid juice in fruit juice mixer.

8. ## Irrotational flow:-

A flow is said to be irrotational when the fluid particle doesnot rotate about ~~the~~ its masscenter while it is moving along stream line.



Ex:- carriages of feeris wheel (giant wheel)

One Dimensional Flow:-

In this flow the fluid parameters (V, P, T) remain const. through out any c/s normal to the flow direction

2-D flow:-

In this flow the fluid parameters vary along two direction

3-D flow: In this flow the fluid parameter vary along 3-direction

	Steady	Unsteady
1-D	$V = f(x)$	$V = f(x, t)$
2-D	$V = f(x, y)$	$V = f(x, y, t)$
3-D	$V = f(x, y, z)$	$V = f(x, y, z, t)$

Continuity Equation :- (1D)

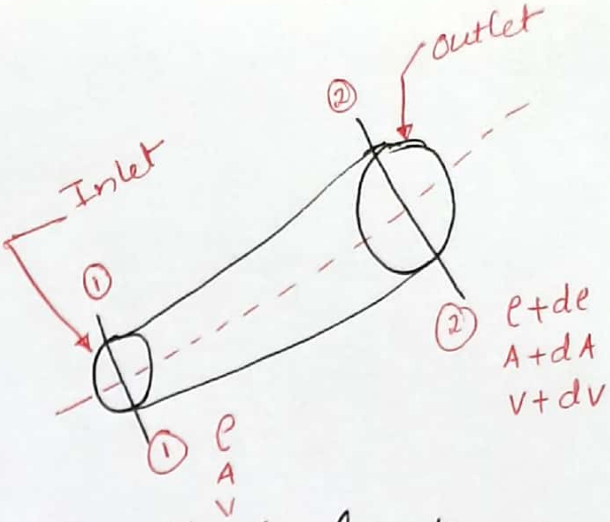


Fig:- Steady flow through Stream tube.

- Consider flow of an ideal fluid through a stream tube.
- Since no flow takes place across the streamlines, the fluid must enter and leave the tube only at the end sections.
- At the inlet the fluid properties are A, V, ρ and at the exit $A+dA$, $V+dV$, $\rho+d\rho$.

- The mass of fluid entering the stream tube at section ①-① during time interval 'dt' is given by $\rho AVdt$
- During the same time interval the mass of fluid leaving at section②-② is given by $(\rho+d\rho)(A+dA)(V+dV)\,dt$.

$\therefore$ The fluid mass accumulates between 2 sections is given by

$$dm = \rho AVdt - (\rho+d\rho)(A+dA)(V+dV)dt$$

$$dm = \rho AVdt - (\rho A + \rho dA + A d\rho)(V+dV)dt$$

$$dm = \rho AVdt - (\rho AV + \rho A dV + \rho V dA + AV d\rho)dt$$

$$dm = -(\rho A dV + \rho V dA + AV d\rho)dt$$

$$\frac{dm}{dt} = -(\rho A dV + \rho V dA + AV d\rho)$$

for steady flow, $\frac{dm}{dt} = 0 \Rightarrow$ $AVd\rho + \rho V dA + A\rho dV = 0$

dividing with 'ρAV' on both side

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

$$d(\rho AV) = 0 \Rightarrow \rho AV = \text{Const}$$

for incompressible flow $\rho = \text{Const} \Rightarrow AV = \text{Con}$ for 2 sec $\Rightarrow$ $\boxed{A_1V_1 = A_2V_2}$

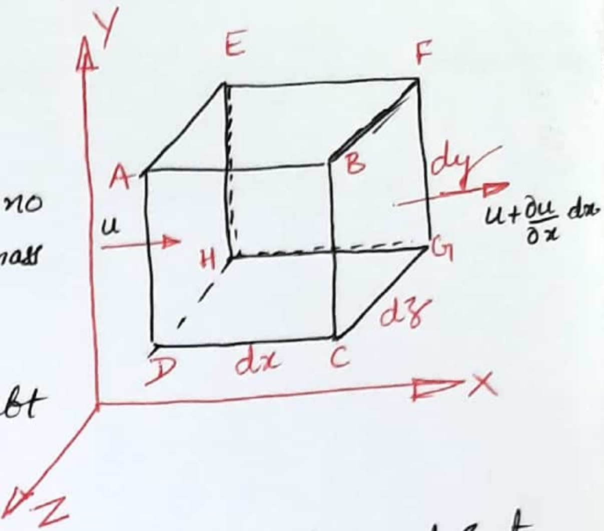
Law of Conservation of mass:-

→ Mass of an isolated system is neither created or nor destroyed by chemical reactions or physical transformations

→ Accordingly mass of products in chemical reaction must be equal to mass of reactants.

Continuity Equation in Cartesian Co-ordinates or 3D or diff. form:—

→ Consider the flow of continuos fluid through an elementary parallelopiped of dimensions dx, dy & dz.



→ Let the fluid is continuos both in space (no voids occur in fluid) and time (the fluid mass is neither created nor destroyed).

→ Let 'u' represents velocity of flow at left hand face ADHE (along X-dir).

→ The flow velocity changes in the direction of X-axis and rate of change is given by $\frac{\partial u}{\partial x}$

∴ change of velocity through distance dx will be $\frac{\partial u}{\partial x}.dx$

→ Thus the velocity of fluid at surface BCGF, a distance dx apart from surface ADHE, will be $\left(u+\frac{\partial u}{\partial x}dx\right)$.

→ Likewise for compressible fluid, there will also be a change in density of fluid passes through parallelopiped.

The mass of fluid entering at the surface ADHE during time interval dt,

$$\text{Fluid influx} = \text{density} \times \text{Area} \times \text{Vel} \times \text{time}$$
$$= \rho\, u\, dy\, dz\, dt$$

During the same time interval the mass of fluid leaving from BCGF is

$$\text{Fluid efflux} = \rho u\, dy\, dz\, dt + \frac{\partial}{\partial x}(\rho u\, dy\, dz\, dt).dx$$
$$= \left[\rho u + \frac{\partial}{\partial x}(\rho u)dx\right] dy\, dz\, dt$$

Scanned with CamScanner

→ The gain in mass due to flow in X-dir is given by difference between the fluid influx and fluid efflux.

$\therefore$ Mass accumulated due to flow in X-dir =

$$\rho u\,dy\,dz\,dt - \left[\rho u + \frac{\partial}{\partial x}(\rho u)\,dx\right] dy\,dz\,dt$$

$$= -\frac{\partial}{\partial x}(\rho u)\,dy\,dz\,dt\,dx$$

Like wise

mass accumulated in Y-dir = $-\frac{\partial}{\partial y}(\rho v)\,dx\,dz\,\partial t\,dy$

Z-dir = $-\frac{\partial}{\partial z}(\rho w)\,dx\,dy\,dz\,dt$.

Total or net gain in fluid mass = $-\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\,dy\,dz\,dt$

Rate of increase of mass with in the parallelopipe is = $\frac{\partial m}{\partial t}\,dt$

$$= \frac{\partial}{\partial t}(\rho \times vol)\,dt = \frac{\partial \rho}{\partial t}(dx\,dy\,dz)\,dt$$

According to principle of Conservation of mass, the total gain in mass equals the time rate of increase of mass in parallelopipe.

$$\therefore\ -\left[\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w\right] \cancel{dx\,dy\,dz\,dt} = \frac{\partial \rho}{\partial t}\,\cancel{dx\,dy\,dz\,dt}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

for steady flow $\frac{\partial \rho}{\partial t} = 0$ & for incompressible $\rho = $ Const

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3D) \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2D)$$

Acceleration of a fluid particle:- (Eulerian Method)

→ In this method attention was focussed on the motion and properties of different fluid particles as they pass fixed points in flow.

→ The observer remains stationary and observes what happen at some particular point.

→ Let x, y & z be space coordinates at time t. then the components of velocity vector are functions of these space-coordinates and time. Symbolically $\vec{V} = ui + vj + wk$.

$\vec{V}(x,y,z,t) = u(x,y,z,t)\bar{i} + v(x,y,z,t)\bar{j} + w(x,y,z,t)\bar{k}$.

$$u = u(x, y, z, t)$$
$$v = v(x, y, z, t)$$
$$w = w(x, y, z, t)$$

→ The velocity components for the field can also be represented by scalar equations
$$u = \frac{dx}{dt}; \quad v = \frac{dy}{dt}; \quad w = \frac{dz}{dt}$$

→ Since velocity is function of both position & time,

Accelaration $a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt} + \frac{\partial u}{\partial t}\frac{dt}{dt}$

(substantial or Material = Total)

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

→ In above eqn's $\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}$ & $\frac{\partial w}{\partial t}$ terms are called "Local Accn". the fluid particles are accelared locally because of a change in flow with time at each point.

→ The other terms $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$, $u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z}$, $u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$ are called "Convective accn". the fluid particles are accelared by the convective act of moving from one position to another where velocity is different

$$\vec{a} = \vec{a_x}i + \vec{a_y}j + \vec{a_z}k$$

P-①

The Velocity distribution for 3-D flow is given by

$\vec{V} = ax\,i + ay\,j - 2az\,k$. UNIT-I

Find the equ. of stream line passing through the position vector $\vec{r} = 2i + 2j + 4k$

Sol: For a given flow field velocity components are $u = ax$, $v = ay$, $w = -2az$

and we are to find equ of streamlines passing through (2, 2, 4).

The equ for streamline in 3-D = $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

Substitute u, v & w in above

$$\frac{dx}{ax} = \frac{dy}{ay} = \frac{dz}{-2az}$$

Consider first two parts & integrate $\int \frac{dx}{x} = \int \frac{dy}{y}$

$$\Rightarrow \log_e x = \log_e y + \log_e c_1$$

$$\Rightarrow x = y c_1$$

the stream line is passing through $x = 2, y = 2 \Rightarrow c_1 = 1$

$$\therefore x = y$$

Consider first & third parts and integrate $\int \frac{dx}{x} = -\frac{1}{2} \int \frac{dz}{z}$

$$\log_e x = -\frac{1}{2} \log_e z + \log_e c_2$$

$$\log_e x = \log_e z^{-1/2} + \log c_2$$

$$\Rightarrow x = z^{-1/2} \cdot c_2 \Rightarrow x = \frac{c_2}{\sqrt{z}}$$

but $x = 2, z = 4 \Rightarrow c_2 = 4$

$$\therefore x = \frac{4}{\sqrt{z}} \Rightarrow \boxed{x = y = \frac{4}{\sqrt{z}}}$$

P-② In a steady fluid flow, the velocity components are

$u = 2Kx$, $v = 2Ky$, $w = -4Kz$.

Find the eqn of stream line passing through $(1, 0, 1)$

Sol:- $y = 0$ and $z = \frac{1}{x^2}$

eqn of streamline

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$$\frac{dx}{2kx} = \frac{dy}{2ky} = \frac{dz}{-4kz}$$

$$\frac{dx}{2kx} = \frac{dy}{2ky} \Rightarrow \int \frac{1}{x}\,dx = \int \frac{1}{y}\,dy$$

$\log x = \log y + \log C_1 \Rightarrow x = yC_1 \rightarrow ①$

$C_1 = \frac{x}{y} = \infty$

$\Rightarrow \boxed{y = 0}$

$$\frac{dx}{2kx} = \frac{dz}{-4kz} \Rightarrow \int \frac{1}{x}\,dx = -\frac{1}{2}\int \frac{1}{z}\,dz$$

$\log x = -\frac{1}{2} \log z + \log c_2$

$x = \frac{c_2}{\sqrt{z}} \rightarrow ②$

$\Rightarrow c_2 = 1 \Rightarrow \boxed{x = \frac{1}{\sqrt{z}}}$

(P 3) for the velocity field given by $\vec{q}(x,y,z,t) = 10xy\,i + 5x^2 j + (t^2x + z)k$

find the velocity and acc^n of fluid particle at position

$\vec{q}(x,y,z) = (i + 2j + 3k)$ when time $t = 1$

Sol:- The velocity components are

$u = 10xy \quad v = 5x^2 \quad w = t^2x + z$

From position vector co-ordinates of fluid particles are

$x = 1; \; y = 2; \; z = 3$

Hence the velocity components at $(1,2,3)$ and $t = 1$

$u = 20, \quad v = 5, \quad w = 4$ units.

$\therefore$ The velocity vector at $(1,2,3)$ and $t = 1$ is $= 20i + 5j + 4k$

Resultant velocity $= \sqrt{20^2 + 5^2 + 4^2} = 21$ units.

→ Acc^n in x-direction is $a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$

$a_x = 10xy(10y) + 5x^2(10x) + (tx^2+z)0 + 0$

$a_x = 100xy^2 + 50x^3$

at $(1,2,3)$, $t = 1$, $\quad a_x = 450$ units.

Acc^n in y-direction is $a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$

$a_y = 200$ units

Acc^n in z-direction is $a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$

$a_z = 26$ units.

Total Acc^n / Material Acc^n / Substantial Acc^n $= \vec{a} = a_x i + a_y j + a_z k$

$\vec{a} = 450i + 200j + 26k$

Resultant Acc^n $= \sqrt{a_x^2 + a_y^2 + a_z^2} = 493.13$ units

(P4) An idealised flow is given by $\vec{V} = 2x^3 i - 3x^2 y j$

Is the flow steady or unsteady?

Is it two or three dimensional?

Make calculations for velocity, local acc^n and convective acc^n of a fluid particle in this flow field at point $P(x,y,z) = (2,1,3)$.

Sol:- for steady flow $\frac{dv}{dt} = 0$

the velocity components are $u = 2x^3$ $v = -3x^2 y$

at $P(x,y,z) = (2,1,3)$, Vel. components $u = 16$ units, $v = -12$ units

Velocity Vector $= \vec{V} = 16i - 12j$

Resultant Vector $= \sqrt{16^2 + -12^2} = 20$ units

The acc^n in x-direction $=$ $a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial u}{\partial t}$

$= 2x^3 (6x^2) + (-12)0 + 0$

$a_x = 12x^5$

$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial v}{\partial t}$

$= 2x^3(-6xy) + (-3x^2y)(-3x^2)$

$= -12x^4 y + 9x^4 y$

$= -3x^4 y$

local acc^n results from unsteadiness of flow.

$\vec{a} = 12x^5 i - 3x^4 y j$

at $P(2,1,3)$ $\vec{a} = 384i - 48j$

$a = \sqrt{384^2 + -48^2} = 387\ m/s^2$

P5) The flow velocity from the base to tip of a nozzle of length 'l' can be prescribed by the relation $V = 3t\left[\frac{1-x}{2l}\right]^{-2}$.
where x is the dist. from the nozzle base, t = time in sec from the commencement of flow and V is the flow vel. in m/s.
At time $t = 2$ sec, det. local accn, convective accn and substantial accn at a section midway b/w base and tip of nozzle of length 1m. Assume 1-D flow.

Soln: for 1-D flow through nozzle $\vec{a} = V\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t}$

local accn $= \frac{\partial V}{\partial t} = 3\left[\frac{1-x}{2l}\right]^{-2}$

at $x = 1/2$, $l = 1$: $a = 3\left[\frac{1}{2}\right] = 1.5$ m/s

Convective accn $= V \cdot \frac{\partial V}{\partial x} = 3t\left[\frac{1-x}{2l}\right]^{-2} 3t\,(+2)\left[\frac{1-x}{2l}\right]^{-3}$

$= 18t^2\left[\frac{1-x}{2l}\right]^{-5}$

at $t = 2$ sec, $l = 1$, $x = 1/2$ $=$

Substantial accn = local + Convective

Scanned with CamScanner

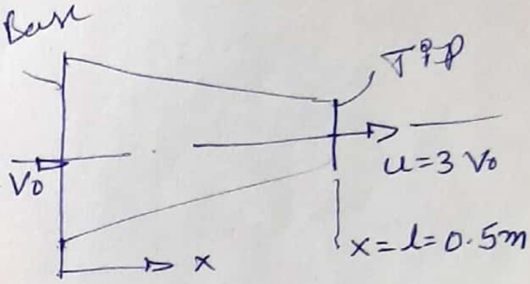
base to ..

P6 The 1-D steady flow through a converging nozzle is stated to have linear velocity distribution $u = u(x)$ with velocities;

$u = V_0$ at nozzle base and $u = 3V_0$ at the nozzle tip.

Setup expⁿ for accⁿ as a general function of distance 'x' from the nozzle base.

If the nozzle has a length of 500mm and velocity $V_0 =$ 5 m/s, make calculations for the accⁿ at the base & tip of nozzle.



Sol:- At a distance x from the nozzle base

$$u(x) = V_0 + \left(\frac{3V_0 - V_0}{l}\right)x$$

$$= V_0\left[1 + \frac{2x}{l}\right]$$

for 1-D, $\vec{a} = u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$

$$= V_0\left[1 + \frac{2x}{l}\right]\frac{2V_0}{l} = \frac{2V_0^2}{l}\left(1 + \frac{2x}{l}\right)$$

At nozzle base $x = 0$, $a = \frac{2(5)^2}{0.5} = 100 \, m/s^2$

$V_0 = 5$ m/s

$l = 0.5$ m

At nozzle tip $x = 0.5m$ $a = 300 \, m/s^2$

$V_0 = 5$ m/s

$l = 0.5$ m

P8) Given that $u = xy$, $v = 2yz$. Examine whether these velocity components represent two or 3-D incompressible flow, if 3-D, determine the third component.

Sol:- To satisfy 2-D. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

but it is $\neq 0$ $(y + 2z \neq 0)$

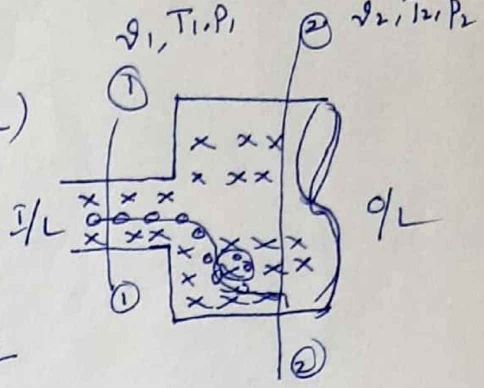
for 3-D $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \Rightarrow \frac{\partial w}{\partial z} = -y - 2z$

$w = -yz - z^2 + f(x, y, t)$

Fluid Kinematics:-

Flow Approaches:- Flow Visualization Techniques:-

Flow visualization (Study/Analysys) can be done by using 2 methods.



1. Lagrangian Approach:- (Theoritical Approach)

In this approach the observer moves along the fluid particles and fluid particles may change in shape, size and state.

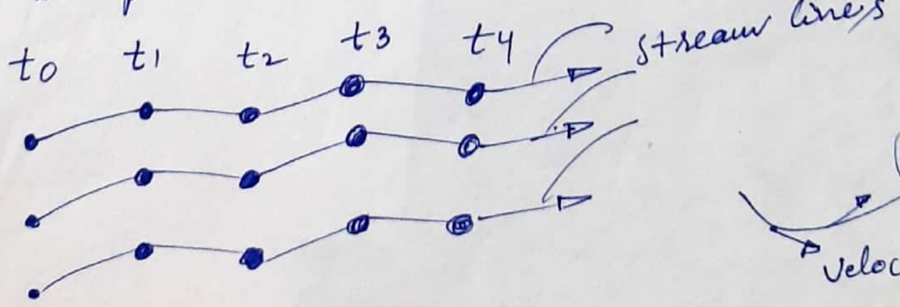
2. Eulerian Approach (Practical Approach)

In this approach the observer is stationary at predefined fixed locations and observe the properties of flow.

There are four basic line ~~properties~~ patterns used to describe the flow.

(a) Stream line:-

It is the imaginary line (path) traced by a given particle. Tangent drawn to the stream line at any point gives the direction of velocity vector at the point and at the instant. It is a Eulerian approach.



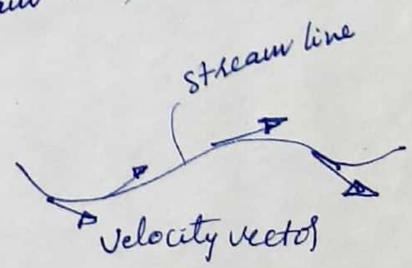


fig:- Concept of Stream line

Scanned with CamScanner

→ Let during time interval dt, a fluid particle travel a distance ds along the stream line.

→ Further let dx, dy & dz be the components of displacement 'ds' along x, y & z axis

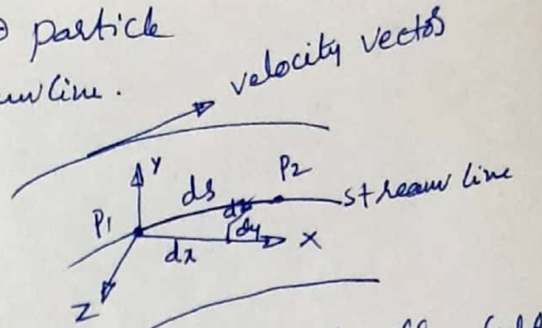


fig → stream line in flow field

→ Let u, v, w be the velocity of particle in x, y & z direction, this implies that fluid particles has traversed distance

dx along x-axis with u
dy along y-axis with v
dz along z-axis with w

during the same time interval dt.

Therefore the velocities components are

$$u = \frac{dx}{dt} \; ; \; v = \frac{dy}{dt} \; \& \; w = \frac{dz}{dt}$$

Hence, the equation of general stream line is given by

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

For one dimensional flow along x-plane $\frac{dx}{u} = \text{const.}$

2-D flow along x-y plane $\frac{dx}{u} = \frac{dy}{v} \Rightarrow \frac{dx}{dy} = \frac{u}{v}$

$$\frac{dy}{dx} = \frac{v}{u}$$

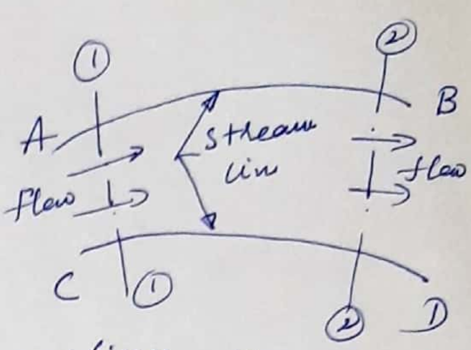
i.e the slope of a plane stream line equals the ratio of velocity components.

Characteristics of Streamlines:-

→ Stream lines donot cross, otherwise the fluid particle will have two velocity vectors at the point of intersection and that is physically impossible. Streamlines may, however intersect at points where velocity is zero.

→ Stream line spacing varies inversly as the velocity; Convergent of stream lines in any particular direction shows accelarated flow, in that direction.

This point can be clearly understood by considering a portion of fluid flow between 2 stream lines AB & CD

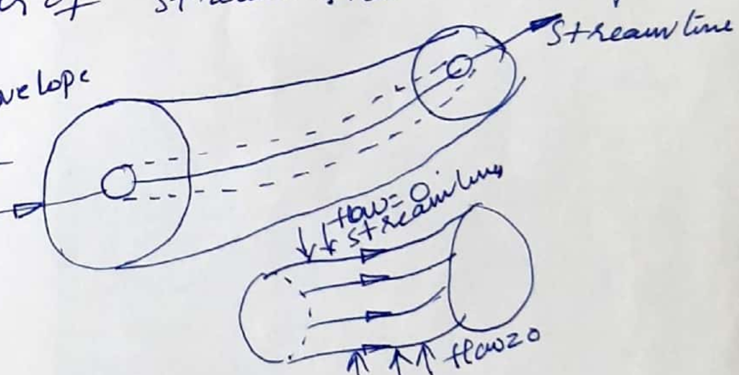


Since no fluid can penetrate the stream lines, the flow passing through each of Sections ①-① & ②-② would be same. Cross sectional area ①-① is smaller than ②-②. According to continuity equation, the flow velocity at section ①-① is greater than ②-②

Stream Tube:-

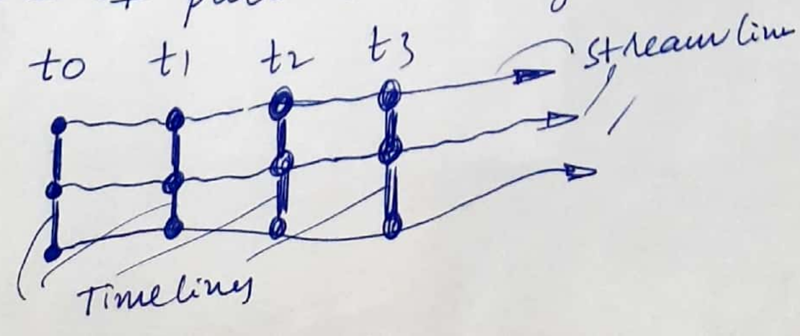
A grouping of neighbouring of stream lines forming a cylindrical passage with an elementary area of c/s is called stream filament. A number of stream filaments comprises a stream tube.

→ It is an enclosing ~~envelope~~ surface envelope formed by a set of streamlines ~~such that~~ it indicates visualization flow from ①-① to ②-②



(b) Time line:-

A time line is a line traced formed by a set of particles. Time lines do not show trajectories, rather they capture the line of particles at a given instant of time

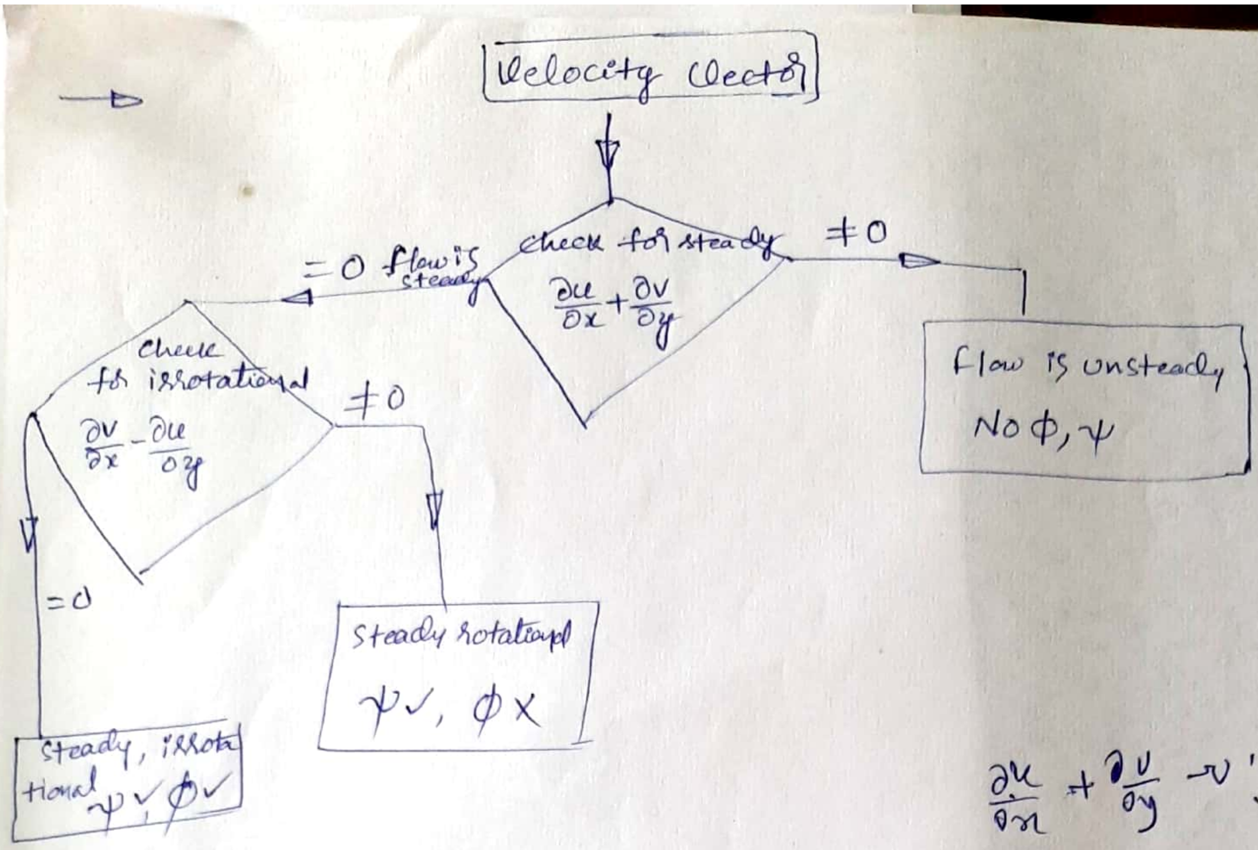


Georgia Tech College of Engineering

Fluid Lines

- Streamline
  - everywhere tangent to instantaneous velocity vector
- Pathline
  - actual path traversed by a fluid packet
- Streakline
  - locus of fluid packets that have all previously passed through a specific point in the flow
- Timeline
  - set of fluid packets that form a line at some instant in time

Fluids & properties - UNIT-1



$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \rightarrow v$!

$u = -\frac{\partial \phi}{\partial x}$

Rotational Flow	Irrotational Flow
1. Algebraic Avg Rotation $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$	1. Rotation $= \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$
2. Circulation $= \int\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dA$	2. Circulation $= 0$
3. Vorticity $= \frac{\text{Circulation}}{\text{Area}}$	3. Vorticity $= 0$
4. potential function (ϕ) doesnot exists	4. potential function exists.

→ Velocity potential function (ϕ) :- {exists for irrotational flow}

$$u = \frac{-\partial\phi}{\partial x} \quad , \quad v = \frac{-\partial\phi}{\partial y}$$

→ Condition for irrotationality → $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$

→ Stream function (ψ) :-

$$u = \frac{\partial\psi}{\partial y} \quad , \quad v = \frac{-\partial\psi}{\partial x}$$

→ continuity equation for steady incompressible flow (Mass Balance eqn)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

*→ Condition of irrotationality is always satisfied by velocity potential function (ϕ)

*→ Stream function (ψ) always satisfies the continuity equation for steady incompressible flow.

Scanned with CamScanner

→ Divergence (div) - $\text{div}\, q = \nabla \cdot q$ where $\bar{q} = u\hat{i} + v\hat{j} + w\hat{k}$.

$$\nabla q = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$ → continuity equ or Mass Balance equation.

→ Curl (Rotation of vector) $= \nabla \times q$

$$\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times (u\hat{i} + v\hat{j} + w\hat{k})$$

$$\Leftarrow \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$= \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} - \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$$

$$= \omega_x \hat{i} - \omega_y \hat{j} + \omega_z \hat{k}$$

$\nabla \times q = 0$ - Irrotational Flow

$\nabla \times q \neq 0$ - Rotational Flow

$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$ → Irrotational flow (Potential Flow).

$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \neq 0$ → Rotational Flow

→ $\nabla(\phi \psi) = \phi \nabla \psi + \psi \nabla \phi$ — scalar function

→ $\nabla \cdot (\phi \cdot \bar{A}) = (\nabla \cdot \phi) \cdot \bar{A} + \phi(\nabla \cdot \bar{A})$ — vector function.

→ $\nabla \times (\phi \cdot \bar{A}) = (\nabla \phi) \times \bar{A} + \phi(\nabla \times \bar{A})$

→ Vorticity = 2 × Ang. Velocity = 2 × Rotation.

→ ω_z = Angular velocity $= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

→ Vorticity $= 2\omega_z = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

→ Equation for Stream Line

$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$ → 3D-flow

$v\,dx - u\,dy = 0$ → 2D-flow.

$\rightarrow \tau = m\left(\frac{\partial u}{\partial y}\right)^n$ where n = flow index
m = consistency index

For Newtonean Fluids $n=1$ $m = \mu$ where μ = Viscosity = $\frac{\tau \rightarrow \text{Shear Stress}}{\partial u/\partial y \rightarrow \text{Shear Strain}}$

For Dilatant Fluids $n > 1$

For psedoplastic $n < 1$

$\rightarrow$ Mach Number $M = \frac{C}{a}$ $C \rightarrow$ Flow Velocity, $a \rightarrow$ Sound Velocity

$\rightarrow$ Velocity & Acceleration Mathematical Relations :-

$u = u(x,y,z,t)$ $v = v(x,y,z,t)$ $\omega = \omega(x,y,z,t)$

$u = \frac{dx}{dt}$ $v = \frac{dy}{dt}$ $\omega = \frac{dz}{dt}$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial t} + \frac{\partial u}{\partial t}$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_x = \left[u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \omega\frac{\partial}{\partial z} + \frac{\partial}{\partial t}\right]u$$

llly

$$a_y = \left[u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \omega\frac{\partial}{\partial z} + \frac{\partial}{\partial t}\right]v$$

$$a_z = \left[\underbrace{u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \omega\frac{\partial}{\partial z}}_{\text{Convective Acc}^n} + \underbrace{\frac{\partial}{\partial t}}_{\text{Local Acc}^n}\right]\omega$$

$\rightarrow$ Gradient $= \nabla\phi =$

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\nabla\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$$

$$\bar{r} = \hat{i}x + \hat{j}y + \hat{k}z \Rightarrow dr = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

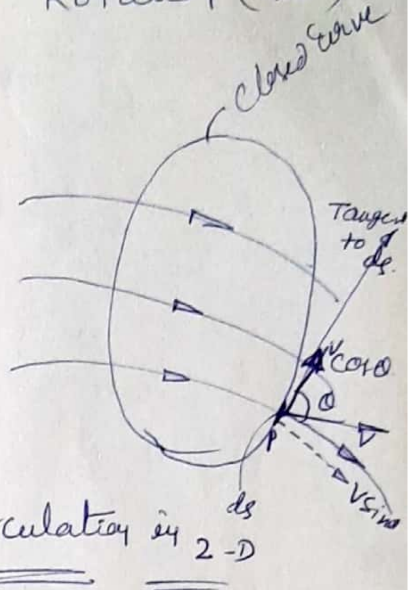
$$d\phi = \frac{\partial\phi}{\partial x}\cdot dx + \frac{\partial\phi}{\partial y}\cdot dy + \frac{\partial\phi}{\partial z}\cdot dz$$

$$d\phi = \nabla\phi\cdot dr$$

Circulation (Γ) & Vorticity (Ω) or (ξ) :- Rotation (ω)

Gamma — small omega

* Circulation is defined mathematically as the line integral of the tangential velocity about a closed path (Contour)



Circulation in 2-D

Thus $\Gamma = \oint V\cos\theta \cdot ds$

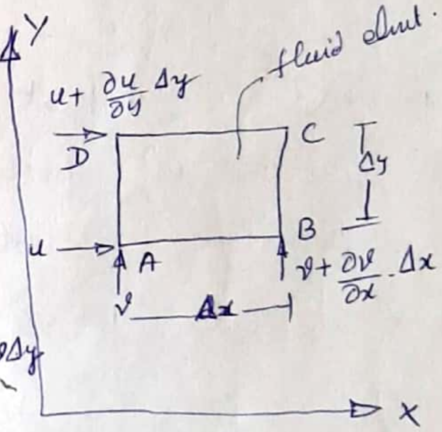
where V = Vel. in flow field at the elmt 'ds'

θ = Angle b/w V and tangent to the path at that point.

Circulation around regular curves can be obtained by integration

→ ~~Consid~~

Let us consider the circulation around an elementary box (fluid elmt ABCD) shown in fig



Starting from 'A' and proceeding A.C.W,

$$d\Gamma = u\,\Delta x + \left[v + \frac{\partial v}{\partial x}.\Delta x\right]\Delta y - \left[u + \frac{\partial u}{\partial y}\Delta y\right]\Delta x - v\Delta y$$

$$= \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\Delta x\,\Delta y$$

Irrotation flow Condition

* Vorticity is defined as the circulation per unit of closed area.

$$\Omega = \Gamma/A \Rightarrow \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}.$$

→ If a flow posses vorticity, it is called rotational.

Rotation (ω) is defined one-half of vorticity,

$$\omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

* The flow is irrotational if rotation 'ω' is zero.

For a 3-D flow, the rotation is possible about 3-axes

The exp^n for rotation ω_z, ω_x and ω_y can be obtained as

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right); \quad \omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right); \quad \omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\omega = \omega_x + \omega_y + \omega_z = \frac{1}{2}\left[\omega_x i + \omega_y j + \omega_z k\right]$$

Vorticity in x-direction $= \Omega_x = 2\omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}$

$$\Omega_y = 2\omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$$

$$\Omega_z = 2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

→ when the components of rotation or vorticity are zero then the flow is irrotational.

Problem (7) Given that $u = -4ax(x^2 - 3y^2)$ $v = 4ay(3x^2 - y^2)$
Examine whether these velocity components represents a physically possible 2-D flow; if so whether the flow is rotational or irrotational.

Sol:- For flow to exist it should satisfy continuity equ

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad ; \text{ it exist } \checkmark$$

To check rotation or irrotation; it is in x-y plane,

Rotation $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

if it is zero, the flow is irrotational.

Velocity Potential ϕ & Stream Function ψ :-

The Velocity potential is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction.

Mathematically it is defined as

$\phi = f(x, y, z, t)$ for unsteady

$\phi = f(x, y, z)$ for steady

$$u = -\frac{\partial \phi}{\partial x}, \quad v = -\frac{\partial \phi}{\partial y} \quad \& \quad w = -\frac{\partial \phi}{\partial z}.$$

where u, v & w are components of vel. in x, y & z directions.

The $-ve$ sign indicates that ϕ decreases with an increase in value of x, y and z. In other words it indicates that flow is always in the direction of decreasing ϕ.

For steady incompressible flow, continuity eqn

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

By substituting values of u, v & w

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$. This eqn is known as Laplace eqn.

The rotational components are

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right); \quad \omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \quad \& \quad \omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

By substituting u, v & w

$$\omega_x = \frac{1}{2}\left(-\frac{\partial^2 \phi}{\partial y \partial z} + \frac{\partial^2 \phi}{\partial z \partial y}\right] = 0$$

$\omega_y = 0$

$\omega_z = 0$

Thus if velocity potential (ϕ) satisfies the laplace eqn, it represents the possible steady, incompressible, irrotational flow. Often irrotational flow is known as "potential flow".

Equipotential Line:-

An equipotential line is one along which ϕ is const.

i.e for equipotential line ϕ = const

$$d\phi = 0$$

But $\phi = f(x, y)$ for steady flow

$$d\phi = \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} dy$$

$$d\phi = -u\,\partial x - v\,dy = -(u\,\partial x + v\,dy)$$

for equipotential $d\phi = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-u}{v}$$

where $\frac{dy}{dx}$ = Slope of equipotential line

Stream function (ψ):-

The stream function is defined as a function of space & time, such that partial derivative with respect to any direction gives the velocity component at right angles to this direction.

for 2D $\psi = f(x, y, t)$ — unsteady

$\psi = f(x, y)$ — steady

$$u = \frac{\partial \psi}{\partial y}, \quad v = \frac{-\partial \psi}{\partial x}$$

P.) A 2D flow is described by velocity components.

$u = 5x^3$, $v = -15x^2y$

Evaluate the stream fun, velocity and accⁿ at point $P(x=1, y=2)$

Sol: At $P(1,2)$, $u = 5\,m/s$, $v = -30\,m/s$

$\vec{V} = 5\hat{i} - 30\hat{j}$

Resultant velocity $= V = \sqrt{5^2 + -30^2} = 30.41\,m/s$

for steady flow

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 5x^3(15x^2) + 0$$

$$= 75x^5\,m/s^2$$

at $x=1, y=2$, $a_x = 75\,m/s^2$

further $a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 150\,m/s^2$

Stream fun

$$\psi = f(x,y)$$

$$d\psi = \frac{\partial \psi}{\partial x}\cdot dx + \frac{\partial \psi}{\partial y}\cdot dy$$

$$d\psi = -v\,dx + u\,dy = 15x^2y\,dx + 5x^3dy$$

$$\psi = 5x^3y + 5x^3y$$

P3) A flow is described by the stream fun $\psi = 3\sqrt{2}\,xy$.

Locate the point at which the velocity vector has a magnitude of 6 units and makes an angle of 145° with the x-axis

Sol $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$ $V = \sqrt{u^2+v^2} \Rightarrow 6 = \sqrt{}$

$$\tan 145 = \frac{v}{u}$$

$x = 1.158$, $y = 0.81$

Scanned with CamScanner

possible ...

$$\psi = f(x, y)$$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} \cdot dy$$

$$d\psi = -v dx + u dy$$

if ψ is const, $d\psi = 0 \Rightarrow u dy - v dx = 0$

$$\frac{dy}{dx} = \frac{v}{u}$$

To check for steady, incompressible flow

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, $\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0$ Hence Stream fun satisfied the continuity

To check for rotation, for irrotation $\omega_z = 0$

$$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial}{\partial x}\left(\frac{-\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right)$$

$$= \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

for irrotational it should satisfy Laplace eqn.

This is Laplace eqn, for irrotational it should satisfy Laplace eqn.

P) A stream fun is given by $\psi = 3x^2y + (2+t)y^2$

Find the velocity & det its value at a point defined by the position vector $\vec{r} = 1i + 2j - 3k$ when $t = 2$

Sol:

$$u = \frac{\partial \psi}{\partial y} = 3x^2 + (2+t)2y$$

$$v = -\frac{\partial \psi}{\partial x} = -6xy$$

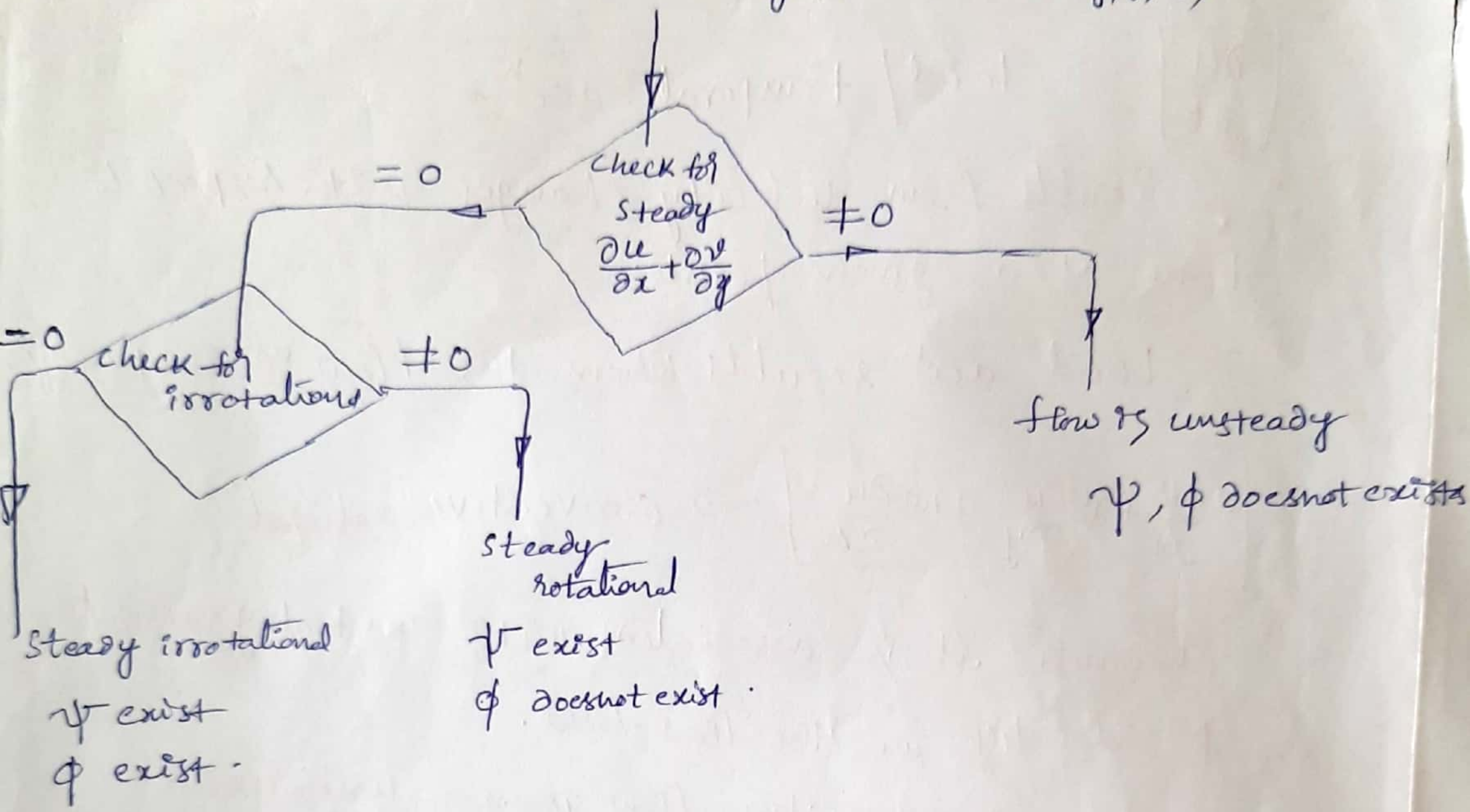
$$V = ui + vj$$

at $(1, 2, -3)$, $t = 2$, $V = \sqrt{19^2 + (-12)^2} = 22.47 \text{ m/s}$

$$\tan\theta = \frac{dy}{dx} = \frac{v}{u} = \frac{-12}{19} = -32.27^\circ = \theta$$

$\therefore$ The resultant velocity makes an angle 32.27° with x-axis

Given Velocity Vector = $f(x, y, z, t)$



	ϕ	ψ
$\frac{\partial}{\partial x}$	$-u$	$-v$
$\frac{\partial}{\partial y}$	$-v$	$+u$

$\boxed{\frac{\partial u}{\partial t}} \rightarrow$ local/ temporal accⁿ.

→ Results from velocity changes with respect to time at a given point.

→ Local accⁿ results when the flow is unsteady.

$\boxed{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}} \rightarrow$ convective/ ~~material~~

→ because it is associated with spatial gradient of velocity in the flow field.

→ It results when the flow is non-uniform i.e if the velocity changes along a stream line.

P) Check whether the following functions satisfy continuity and are valid potential fun's. A is numerical const

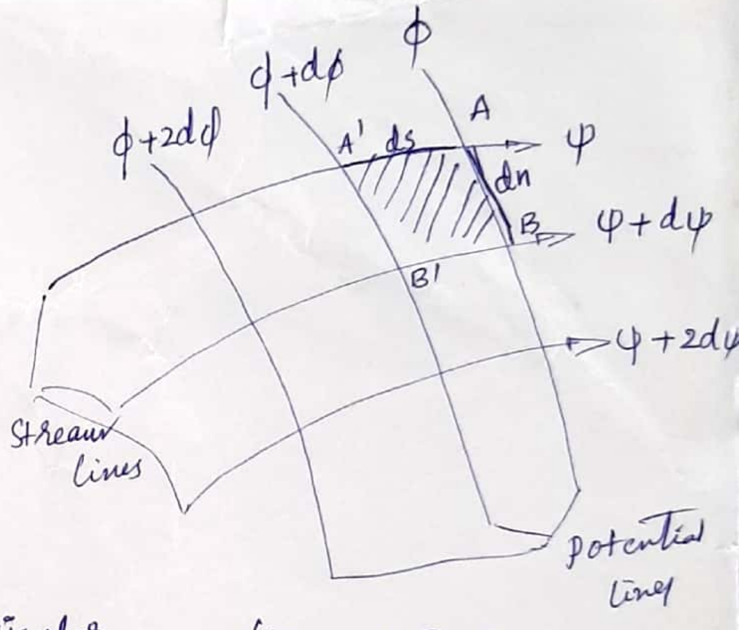
(i) $\phi = \frac{A}{2}(x^2 - y^2)$ (ii) $\phi = A(\cos x + \sin y)$ iii) $\phi = A \log_e xy$

Sol: It should satisfy laplace equ $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$

P) Check whether the flow defined by stream fun

$\psi = 2xy$ is irrotational? If so, Determine the corresponding velocity potential.

Flow NET or Net work :-



→ In a 2-D flow, complete visualisation of flow pattern can be obtained by plotting a series of stream lines and equipotential lines.

fig → 2-D flow net

→ Since these lines are perpendicular to each other, the resulting picture consists of a grid of quadrilaterals having 90° corners; the grid is called flow net or Net work.

Scanned with CamScanner