Fluid Dynamics

Energies of (liquid) fluid in Motion:-

When the (liquid) fluid is in motion, the various energies possessed by a (liquid) fluid particle are

(i) pressure energy
(ii) kinetic energy
(iii) potential energy (or) Datum energy

Pressure energy:-

Pressure energy is the energy possessed by a liquid particle due to its hydrostatic pressure (p).

This pressure energy per unit weight of liquid is known as pressure head,

$$p = \rho g h = \frac{m}{V} \cdot g \cdot h = \frac{W}{V} \cdot h = w \cdot h$$

$$\therefore h = \frac{P}{w} \text{ m of fluid}$$

Kinetic Energy:-

The energy possed by a fluid particle due to its velocity is known as kinetic energy.

Kinetic energy per unit weight of liquid is known as velocity head.

If 'V' is the velocity of particle and 'g' is acceleration due to gravity, then

$$\text{Kinetic head} = \frac{V^2}{2g} \text{ m of fluid}$$

Potential / Datum Energy:-

Potential energy is one which is possessed by a particle due to its position with reference to any arbitary datum line.

potential energy per unit weight of fluid is known as potential head or Datum head.

If the fluid particle is 'Z' meters above the datum line, then Datum head = Z m of liquid.

→ The sum of pressure head and datum head i.e. $\left(\frac{p}{w}+Z\right)$ is known as piezometric head.

Total Energy :-

The sum of pressure energy, kinetic energy and datum energy is nothing but the total energy of a flowing fluid particle.

The total energy head is the sum of pressure head, Velocity head and datum head.

$$\text{Total energy head} = \left(\frac{p}{w} + \frac{V^2}{2g} + Z\right) \text{ m of fluid.}$$

→ For ideal fluids $\gamma = 0$,

$$u\,du + \frac{dp}{\rho} + g\,dy = 0$$

Euler's Equation Along stream line:-

→ Euler's eqn. of motion is established by applying Newton's second law of motion to a small element of fluid with in control volume.

→ The element has a mean c/s area dA, length ds.

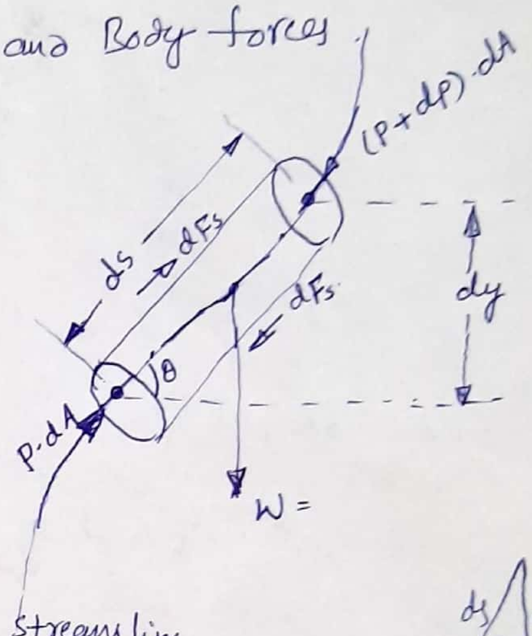
Motion of the element is influenced by;

Total forces acting on fluid element are

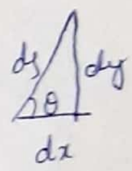
→ Normal forces, Tangential forces and Body forces

① Normal forces due to pressure :-

Let p and (p+dp) pressure intensities at the upstream and down stream face respectively



→ Net pr. force acting on the element in the direction of motion is then given by



$$p.dA - (p+dp)dA = -dp.dA$$

dy = ds.sin

② Tangential force due to viscous shear:-

If the fluid element has a perimeter 'dp', then shear force on the element is

$$dF_s = \tau.dp.ds$$

where τ is frictional surface force per unit area acting on the walls of stream tube

③ Body force such as gravity acting in the direction of gravitational field.

If ρ is density, then body force equals $\rho g.dA.ds$.

Its component in the direction of motion is $= \rho g.dA\,ds\sin\theta$
$= \rho g\,dA.dy$

→ The resultant force in the direction of motion must be equal to product of mass and acceleration in that direction

$$-dp\,dA - \tau\,dp\,ds - \rho g\,dA\,dy = \rho\,dA\,ds \cdot a_s$$

It may be recalled that the velocity of an elementary fluid particle along stream line is function of position & time

$$u = f(s,t)$$

$$du = \frac{\partial u}{\partial s} ds + \frac{\partial u}{\partial t} dt$$

$$\frac{du}{dt} = \frac{\partial u}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial u}{\partial t} \cdot \frac{dt}{dt}$$

$$a_s = u \cdot \frac{\partial u}{\partial s} + \frac{\partial u}{\partial t}$$

Assume steady flow, $\frac{\partial u}{\partial t} = 0$, partial differentials become total differentials

$$a_s = u \cdot \frac{du}{ds}$$

$$-dp\,dA - \tau\,dp\,ds - \rho g\,dA\,dy = \rho u\,dA\,du$$

dividing throughout with mass, $\rho\,dA\,ds$

$$-\frac{dp}{ds} \cdot \frac{1}{\rho} - \frac{\tau}{\rho} \frac{dp}{dA} - g\frac{dy}{ds} = u\frac{du}{ds}$$

$$\boxed{u\frac{du}{ds} + \frac{1}{\rho}\frac{dp}{ds} + g\frac{dy}{ds} = -\frac{\tau}{\rho}\frac{dp}{dA}}$$

which is Euler's equ. of motion

→ the term $u \cdot \frac{du}{ds}$ – convective accn, experienced by the fluid as it moves from a region of one velocity to another region of diff. vel, evidently it represents a change in K.E

→ $\frac{1}{\rho}\frac{dp}{ds}$ → force per unit mass caused by pr. distribution

→ $g \cdot \frac{dy}{ds}$ → " " " " " gravitational pull

→ $-\frac{\tau}{\rho} \cdot \frac{dp}{dA}$ → " " " " " friction

Bernoulli's theorem: Integration of Euler's Equation:-

Bernoulli's equation relates velocity, pressure and elevation changes of a fluid in motion.

The equation is obtained when the Euler's equation is integrated along streamline for constant density (incompressible) fluid.

Euler's equation

$$v\,dv + \frac{dp}{\rho} + g\,dy = 0$$

Integrating the above eqn, Assuming 'ρ' is const.

$$\frac{v^2}{2} + \frac{p}{\rho} + gy = \text{Const}$$

$$\Rightarrow \frac{v^2}{2g} + \frac{p}{w} + y = \text{const}$$

where $\frac{v^2}{2g}$ = Vel. head

$\frac{p}{w}$ = pressure head or static head

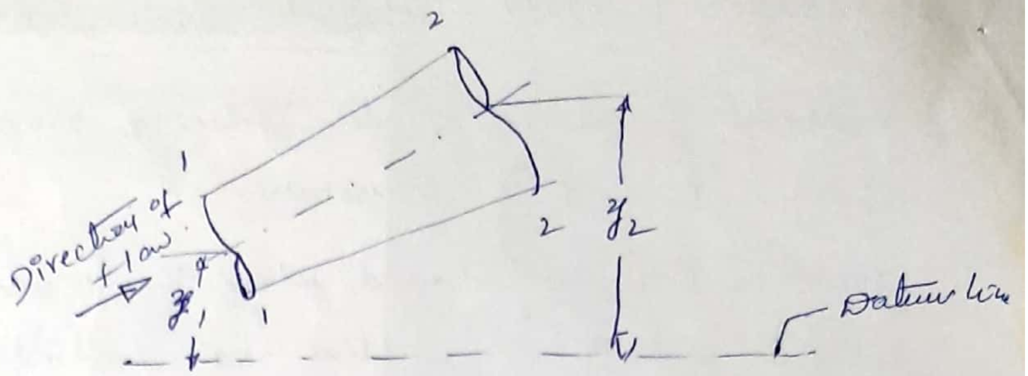
y = datum head / elevated head / potential head.

$$\therefore \frac{v^2}{2g} + \frac{p}{w} + y = H = \text{Total head / Hydrodynamic head}$$

Def:- "In a continuous flow of incompressible fluid, the total energy of a particle remains the same, while the particle moves from one point to another".

(or)

Bernoulli's theorem states that in a steady, ideal flow of incompressible fluid, the total energy at any point of fluid is const.



Bernoulli's equation is applied b/w two sections ① & ②

$$\frac{p_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{v_2^2}{2g} + z_2$$

where p_1, v_1, z_1 — Pressure, velocity & height above datum at ① & ②

If loss of energy due to friction is also taken into consider-ation,

$$\frac{p_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{v_2^2}{2g} + z_2 + h_L$$

h_L = loss of energy from ① - ②.

Limitations of Bernoulli's theorem :-

① Bernoulli's eqn has been derived on assumption that the velocity is uniform over the section, But actually the velocity of fluid particle is maximum at the centre of pipe and gradually decreases towards the pipe walls due to friction. Thus in Bernoulli's equation only mean velocity of liquid should be taken into account.

(2) Bernoulli's eqn has been derived by assuming that there is no loss of energy of the fluid particle while flowing, But actually energy losses occurs in liquid flows due to viscous and turbulence friction.

(3) Bernoulli's equation is applicable only for steady, incompressible and irrotational flows.

(4) Bernoulli's eqn has been derived under the assumption that the only forces acting on the liquid are gravity forces, pressure forces, but in actual practice some external forces always acting on the liquid.

Assumptions:-

→ flow is steady (No variation of fluid properties with respect to time)

→ flow is ideal. (No friction due to fluid viscosity)

→ flow is incompressible (No change in density)

→ flow is Continuous & velocity is uniform over the section.

(P1) Water is flowing with a velocity of 15 m/s under a Pr. of 300 K.Pa. If the height above the datum is 30m, calculated total energy per unit weight of water.

Sol. T.E per unit weight = K.E head + Pr. E head + Pot. E head.

$1\ Pa = 1\ N/m^2$

$$= \frac{V^2}{2g} + \frac{P}{w} + z$$

$$= \frac{15^2}{2\times 9.81} + \frac{300\times 10^3}{9810} + 30 = 72\ \frac{Nm}{N} / \frac{J}{N}$$

P2] A pipe 12.5 cm in dia is used to transport oil of relative density 0.75 under a pressure of 1 bar. If the total energy relative to a datum plane 2.5 m below the centre of pipe is 20 Nm/N. workout the flow rate of oil.

$1\,bar = 1\times10^5\,N/m^2$

Sol Energy per Newton of oil $= \frac{V^2}{2g} + \frac{P}{w} + y$

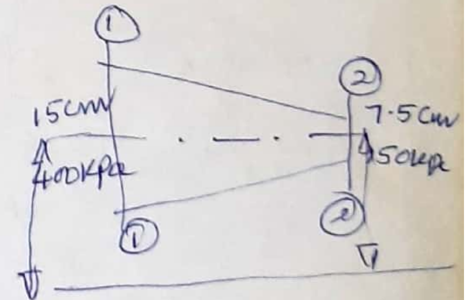
$$20 = \frac{V^2}{2\times9.81} + \frac{1\times10^5}{0.75\times9810} + 2.5$$

$$\Rightarrow V = 8.76\ m/s$$

Flow rate = Discharge = $Q = A \cdot V = 0.1075\ m^3/s$.

$\left(\text{Relative density} = \frac{\text{Specific weight of oil}}{\text{Specific weight of water}} \right)$

P3] A horizontal water pipe of dia 15 cm converges to 7.5 cm dia. If the pr at 2-sections are 400 kPa and 150 k.Pa respectively. calculate flow rate of water.



Sol:- Applying Bernoulli's eqn

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + y_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + y_2$$

pipe is horizontal $y_1 = y_2$

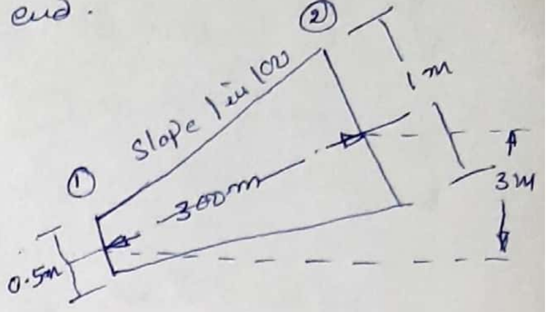
$$\frac{V_2^2 - V_1^2}{2g} = \frac{P_1 - P_2}{w} = \frac{(400-150)10^3}{9810} = 25.48$$

from Continuity eqn $A_1V_1 = A_2V_2 \Rightarrow V_2 = 4V_1$

$$V_1 = 5.77\ m/s,\ V_2 =$$

Flow rate $Q = A_1V_1$ or $A_2V_2 = 0.102\ m^3/s$.

P4) A pipe 300m long has a slope of 1 in 100 and tapers from 1m diameter at high end to 0.5m at the low end. Quantity of water flowing is 5400 lt/min. If the pr at high end is 70 K.pa, find pr. at lower end.



Sol:- $A_1 = 0.196 m^2$

$A_2 = 0.785 m^2$

Discharge $Q = 5400$ lt/min $= \frac{5400 \times 10^{-3}}{60} m^3/s$

from Continuity eqn

$Q = A_1V_1 = A_2V_2 \Rightarrow V_1 = 0.46 m/s$ &

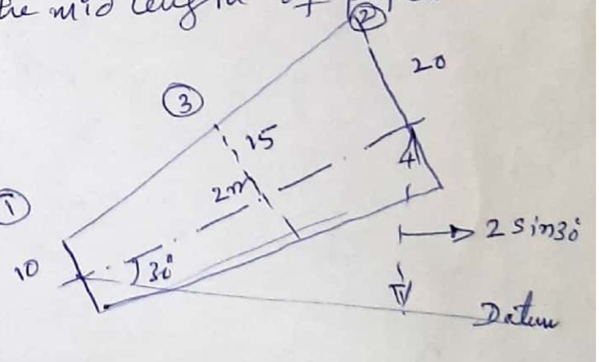
$V_2 = 0.115 m/s$

Applying Bernoulli's eqn b/w Sections ① & ②

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2 \quad ; Z_1 = 0, Z_2 = 3$$

$$\Rightarrow P_1 = 99330 N/m^2 = 99.33 K.Pa$$

P5] A 2m long pipeline tapers uniformly from 10cm dia to 20cm diameter at its upper end. The pipe Centre line slopes upwards at an angle of 30° to the horizantal and flow is from smaller to bigger cross-section. If the pr gauge installed at the lower & upper ends of the pipeline read 200 K.pa and 230 kPa respectively. Det. flow rate & fluid pressure at the mid length of pipeline. Assume No energy loss.



Sol: $Z_1 = 0$, $Z_2 = 2\sin 30 = 1$

from Bernoulli's eqn

$$\frac{200 \times 10^3}{9810} + \frac{V_1^2}{2g} + 0 = \frac{230 \times 10^3}{9810} + \frac{V_2^2}{2g} + 1$$

$$\Rightarrow \frac{V_2^2 - V_1^2}{2g} = 4.058 \rightarrow ①$$

from Continuity $A_1V_1 = A_2V_2 \Rightarrow \frac{V_1}{4} = V_2 \rightarrow ②$

② in ① $\Rightarrow$ $V_1 = 9.215 m/s$, $V_2 = 2.30 m/s$

$Q = A_1V_1 = 0.0723 m^3/s$, At the mid length, $D_3 = 1.5 m$, $V_3 = \frac{Q}{A_3} = 4.095 m/s$

Applying b/w ① & ③ $\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{w} + \frac{V_3^2}{2g} + Z_3 \rightarrow$ ③ $Z_1 = 0, Z_3 = 1$

$$\Rightarrow P_3 = 2.29 \times 10^5 N/m^2 = 2.29 \text{ bar}$$

P6] A 60cm dia pipeline carries oil (sp. gr = 0.85) at 82500 m³/day. The friction head loss is 8.5 m per 1000 m run. It is planned to place pumping stations every 20 km along the pipe. Make calculations for the pressure drop in kN/m² between pumping stations, between two adjacent pumping stations.

Sol Applying Bernoulli's eqn b/w two adjacent pumping stations

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2 + h_f$$

Since pipe line laid horizontal, and is of uniform c/s area

$z_1 = z_2$ & $V_1 = V_2$

$$\frac{P_1 - P_2}{w} = h_f \Rightarrow (P_1 - P_2) = w \times 8.5 \times 20$$

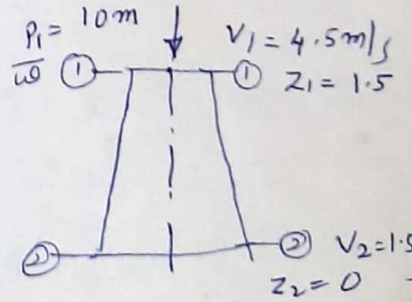
$$= (9810 \times 0.85) \times 170$$

$$P_1 - P_2 = 1417.55\ kN/m^2$$

P7] A conical tube 1.5 m long is fixed vertically with its smaller end upwards and it forms a part of pipe line. Water flows down the tube and measurements indicate that velocity is 4.5 m/s at the smaller end, 1.5 m/s at larger end and pressure head is 10 m of water at the upper end. Assuming that loss of head in tube is expressed as $0.3 \frac{(V_1 - V_2)^2}{2g}$, where V_1, V_2 are vel. at upper and lower end. Make cal. for Pr. head at lower end of conical tube.

Sol:

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2 + h_f$$



$$\frac{P_2}{w} = 12.2\ m \text{ of water.}$$

Applications of Bernoulli's equ :-

Bernoulli's equ is applicable in all problems of incompressible flow where there is involvement of energy considerations but here we shall discuss its applications in the following measuring devices.

1. Venturi meter 2. Orifice meter 3. Rotameter / Elbow Meter
4. Pitot tube

1. Venturimeter :-

It is an instrument used to measure the rate of discharge in a pipeline and is often fixed permanently at different sections of the pipeline to know discharges there.

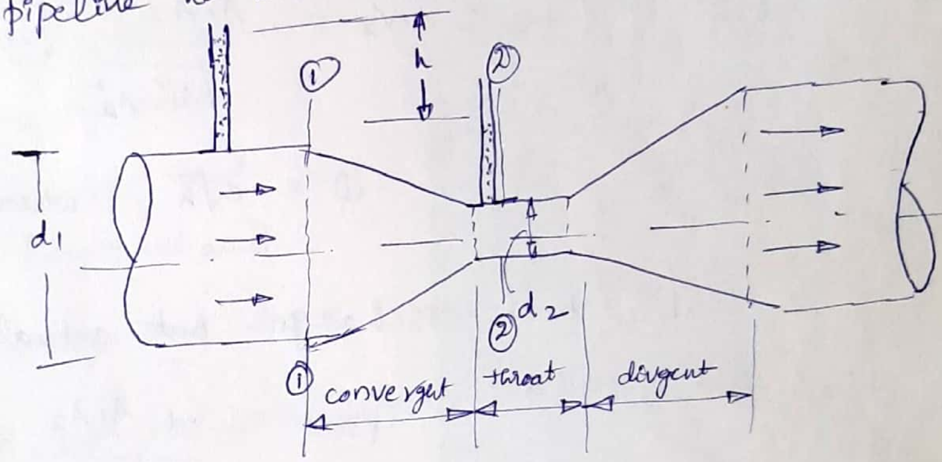


Fig shows a venturimeter fitted in horizontal pipe through which fluid is flowing.

let d_1, P_1, v_1 are dia, pressure and velocity of fluid at inlet (section ①)
d_2, P_2, v_2 " " " " section ②

Applying Bernoulli's equ at section ① & ②

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2 \text{, pipe is horizontal } Z_1 = Z_2$$

$$\frac{P_1 - P_2}{w} = \frac{V_2^2 - V_1^2}{2g}$$

But $\frac{P_1 - P_2}{w} = h$ = difference of pressures at section ① & ②

$$h = \frac{V_2^2 - V_1^2}{2g}$$

Applying continuity eqn at sections ① & ②,

$$A_1V_1 = A_2V_2 \Rightarrow V_1 = \frac{A_2V_2}{A_1}$$

$$h = \frac{V_2^2 - \left[\frac{A_2V_2}{A_1}\right]^2}{2g} = \frac{V_2^2}{2g}\left[1 - \frac{A_2^2}{A_1^2}\right]$$

$$h = \frac{V_2^2}{2g}\left[\frac{A_1^2 - A_2^2}{A_1^2}\right] \Rightarrow V_2^2 = 2gh\left[\frac{A_1^2}{A_1^2 - A_2^2}\right]$$

$$\Rightarrow V_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

Discharge, $Q = A_2V_2 = \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$

$$Q = C\sqrt{h} \quad \text{where } C = \frac{A_1A_2\sqrt{2g}}{\sqrt{A_1^2 - A_2^2}}$$

It is theoritical discharge. But actual discharge is less than theoritical discharge. $Q_{act} = \frac{C_d A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$.

where C_d = Coefficient of discharge, its value is less than unity.

$C_d = 0.96$ to 0.98.

Value of h by differential U-tube Manometer – manometer contains lighter liquid, heavier liquid is passing

$h = y\left[1 - \frac{S_{l.l}}{S_{f.l}}\right]$, reverse $h = y\left[\frac{S_{hl}}{S_{fl}} - 1\right]$

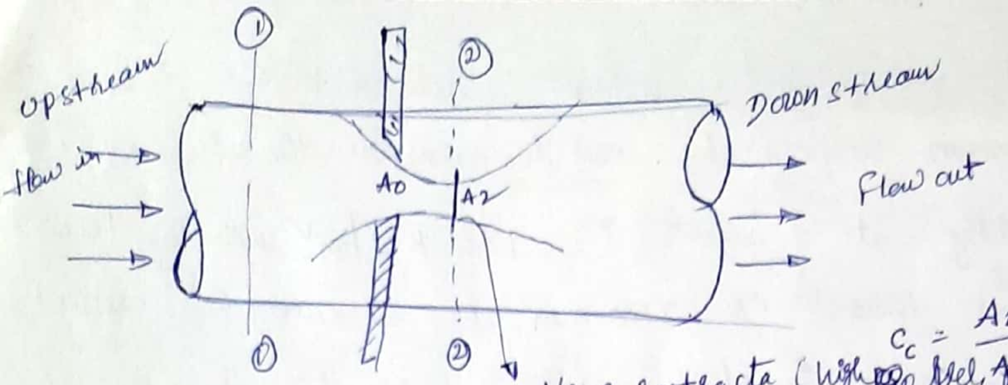
y – diff of lighter liquid col in U-tube

2. Orifice meter :-

Orifice meter is a device employed for measuring the discharge of fluid through a pipe. It also works on the same principle of Venturimeter.

It consists of a flat circular plate having a sharp edged hole (orifice) concentric with the pipe.

The dia of the orifice may vary from 0.4 to 0.8 times the dia of pipe.



$C_c = \frac{A_2}{A_0}$

Vena contracta (high vel, low pr)

Vena contracta is a point in fluid stream where the dia is least, jet is more or less horizontal.

$$Q = C_d \frac{A_0 A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

P] A horizontal venturimeter with I/L diameter 200 mm and throat diameter 100 mm is used to measure ~~rate of~~ flow of water. The pr at inlet is 0.18 N/mm² & vacuum pr at throat is 280 mm of mercury. Find the rate of flow. Cd = 0.98

Sol:- A_1 = Area at I/L $= \frac{\pi}{4} \times 0.2^2 = 0.0314\ m^2$

throat dia $D_2 = 100\ mm = 0.1\ m$

Area of throat $= A_2 = 0.00785\ m^2$

Pr at inlet $P_1 = 0.18\ N/mm^2 = 0.18 \times 10^6\ N/m^2$

$$\frac{P_1}{w} = \frac{0.18 \times 10^6}{\cancel{1000 \times 10}\ 9810 \times 9.8} = 18.3\ m$$

Vacuum pr at throat $= \frac{P_2}{w} = -0.28\ m$ of Hg

$-0.28 \times 13.6 = -3.8\ m$ of water

$C_d = 0.98$; $h = 18.3 - (-3.8) = 22.1\ m$

$$Q = C_d \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} = 0.165\ m^3/s.$$

P] A horizontal venturimeter with inlet & throat diameters 300mm & 100mm respectively used to measure the flow of water. The pr intensity at inlet is 130 KN/m² while the vaccum pr head at throat is 350 mm of mercury. Assuming 3 % of head is lost in b/w inlet & throat, find (i) Cd (ii) Rate of flow.

Sol

$A_1 = 0.07\,m^2$

$A_2 = 0.00785\,m^2$

$P_1 = 130\,KN/m^2$, $\frac{P_1}{w} = 13.25\,m$

$\frac{P_2}{w} = -350$ mm of Hg $= -0.35 \times 13.6$ m of water
$= -4.76\,m$

$h_1 = \frac{P_1}{w} - \frac{P_2}{w} = 18.01\,m$

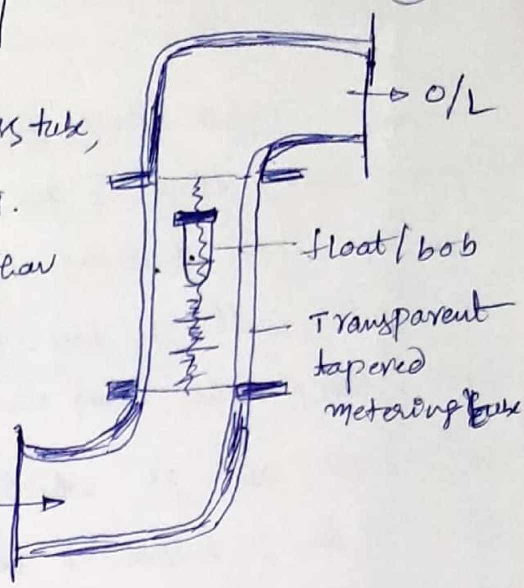
$h_f = 3\%\,h = \frac{3}{100} \times 18.01 = 0.54\,m$

$Cd = \sqrt{\frac{h - h_f}{h}} = 0.985$

$Q = \frac{Cd \cdot A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} = 0.146\,m^3/s$

3. Rotameter :-

→ It consist of a tapered metering glass tube, inside of which is a located float of meter. The spec. gravity of float material is higher than that of fluid to be measured.



→ On the surface of bob spherical slot are cut which cause it to rotate slowly about the axis of tube.

Working :- when the rate of flow increases the float rises in the tube and consequently there is an increase in annular area b/w bob and tube. Thus the float rides higher or lower depending on the rate of flow.

The discharge through a rotameter $Q = C_d \cdot A_{ann} \left[2 \cdot g \, V_{fl} (\rho_{fl} - \rho_f) / A_f \rho_f \right]^{1/2}$

Q = vol. flow rate

C_d = C.E.D

A_{ann} = Annular area b/w float & tube

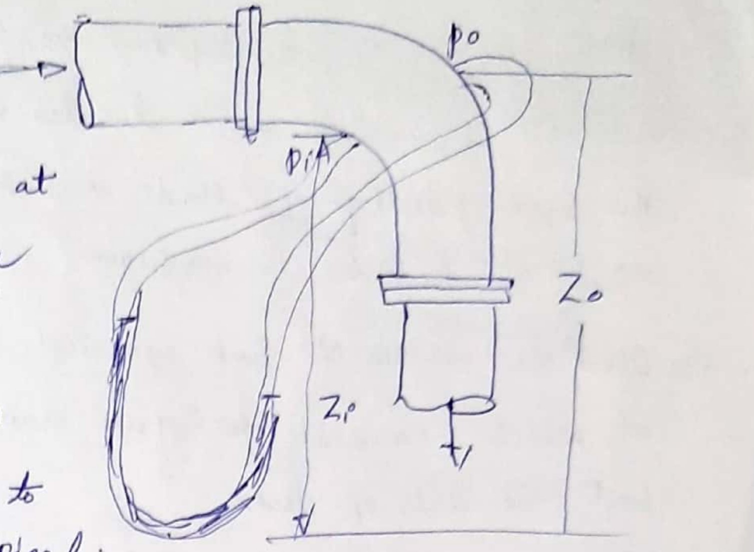
V_{fl} = vol of float

ρ_{fl} = Density of float matl

ρ_f = " " flowing fluid

A_f = max c/s area of fluid.

Elbow meter :-



when liquid flows around pipe bend, there is an increase in pr with radius i.e the pr. at the outer wall of bend is more than the pr at the inner wall.

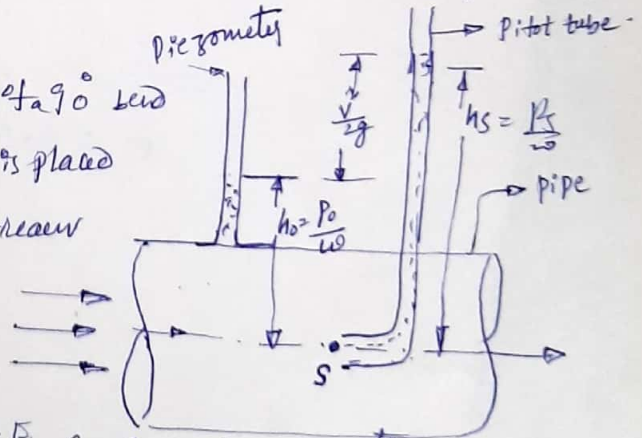
The diff of pr. which exist b/w outside & inside of bend is used to measurement of discharge in a pipe line

$$Q = C_d A \sqrt{2g} \sqrt{\left(\frac{P_o}{w} + Z_o\right) - \left(\frac{P_i}{w} + Z_i\right)}$$

$C_d = \sqrt{\frac{R_b}{D}}$ — R_b — Rad. of pipe bend, D — dia of pipe.

4) Pitot tube :-

pitot tube is one of the most accurate devices for velocity measurement. it works on the principle that if the velocity of flow at point becomes zero, the pressure there is increased due to conversion of K.E to pressure energy.



It consists of a glass tube in the form of a 90° bend of short length open at its both ends. It is placed in the flow with its bent leg directed upstream so that a stagnation point is created immediately in ~~the~~ front of opening.

the K.E at this point gets converted into P.E causing liquid ~~get~~ to rise in the vertical limb, to a height equal to stagnation pressure.

Applying Bernoulli's eqn,

$$\frac{P_o}{w} + \frac{V^2}{2g} = \frac{P_s}{w} \Rightarrow V = \sqrt{2g(h_s - h_o)} = \sqrt{2g \Delta h}$$

Impulse-Momentum Equation :-

Momentum of a fluid in motion :-

The time rate of change of momentum is proportional to impressed force and takes ~~force~~ place in the direction in which it acts.

Momentum is the product of mass and velocity of the body and represents energy of motion stored in a moving body.

Mathematically $F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\cdot\frac{dm}{dt}$

for a const fluid mass ; $dm = 0$ and therefore

$$F = m\cdot\frac{dv}{dt} \Rightarrow F\cdot dt = m\cdot dv \rightarrow (1)$$

~~The at~~

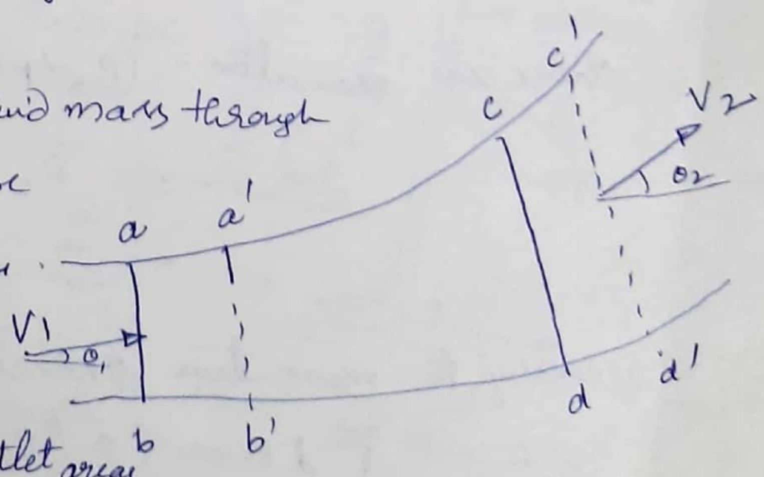
The quantity $F\cdot dt$ represents impulse of applied force.

The above equ is called Impulse-Momentum equ. It may be stated as

" The impulse due to force acting on a fluid mass in a small interval of time is equal to change in momentum of fluid mass".

→ This equ is used to find the force on pipe bends and transitions, force exerted by fluid jet on fixed and moving blades

Consider steady flow of fluid mass through a diverging stream tube lying entirely in x-y plane.



- Assume flow is uniform and normal to inlet and outlet areas.

$aa' = ds_1 = V_1 dt$

$dd' = ds_2 = V_2 dt$

Fluid properties at entrance V_1, ρ_1, at exit V_2, P_2

→ Under the effect of external forces on the stream, the mass of fluid in the region abcd shift to new position a'b'c'd' in time dt.

Because of gradual increase in flow area in the direction of flow, velocity of fluid mass and hence momentum is gradually reduced.

→ The area a'b'cd is common to both the regions abcd & a'b'c'd', it will not experience any momentum changes of fluid mass, in the area abb'a' and cdc'd' have to considered.

According to continuity equ

Fluid mass in region $abb'a' = cc'dd'$

$$(\rho_1 A_1 ds_1) = \rho_2 A_2 ds_2$$

momentum of fluid in $aba'b' = (\rho_1 A_1 ds_1) V_1 = (\rho_1 A_1 V_1 dt) V_1$

" " " $cdc'd' = (\rho_2 A_1 ds_2) V_2 = (\rho_2 A_1 V_2 dt) V_2$

change in momentum $= (\rho_2 A_2 V_2 dt) V_2 - (\rho_1 A_1 V_1 dt) V_1$

$$= \rho_2 Q_2 dt V_2 - \rho_1 Q_1 dt V_1$$

$$= \rho Q [V_2 - V_1] dt$$

$\rho = \frac{mg}{Vg} = \frac{w}{g}$

According to momentum principle,

$$F.dt = \rho Q (V_2 - V_1) dt$$

$$\Rightarrow F = \rho Q (V_2 - V_1) = \frac{wQ}{g} (V_2 - V_1)$$

This is basic momentum flux equ, $\frac{wQ}{g} = \rho Q$ = mass flow rate / sec = mass flux

$$F_x = \frac{wQ}{g}\left(V_2 \cos\theta_2 - V_1 \cos\theta_1\right)$$

$$F_y = \frac{wQ}{g}\left(V_2 \sin\theta_2 - V_1 \sin\theta_1\right)$$

The above eqn's represents force exerted by pipe bend on fluid mass.

But force exerted by fluid on pipe bend =

Example 6.51. *In a 45° bend a rectangular air duct of 1 m² cross-sectional area is gradually reduced to 0.5 m² area. Find the magnitude and direction of force required to hold the duct in position if the velocity of flow at 1 m² section is 10 m/s, and pressure is 30 kN/m².*

Take the specific weight *of air as 0.0116 kN/m³.* **[AMIE]**

Solution. Refer Fig. 6.49

Area at section '1' = 1 m²; area at section '2' = 0.5 m²

Velocity at section '1' $V_1 = 10$ m/s

Pressure at section '1', $p_1 = 30$ kN/m²

Sp. weight of air, $w = 0.0116$ kN/m³

As per continuity equation, $A_1V_1 = A_2V_2$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{1 \times 10}{0.5} = 20 \text{ m/s}$$

Discharge, $$Q = A_1V_1 = 1 \times 10 = 10 \text{ m}^3/\text{s}$$

Applying Bernoulli's equation at sections '1' and '2', we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

But $$z_1 = z_2$$

$$\therefore \quad \frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g}$$

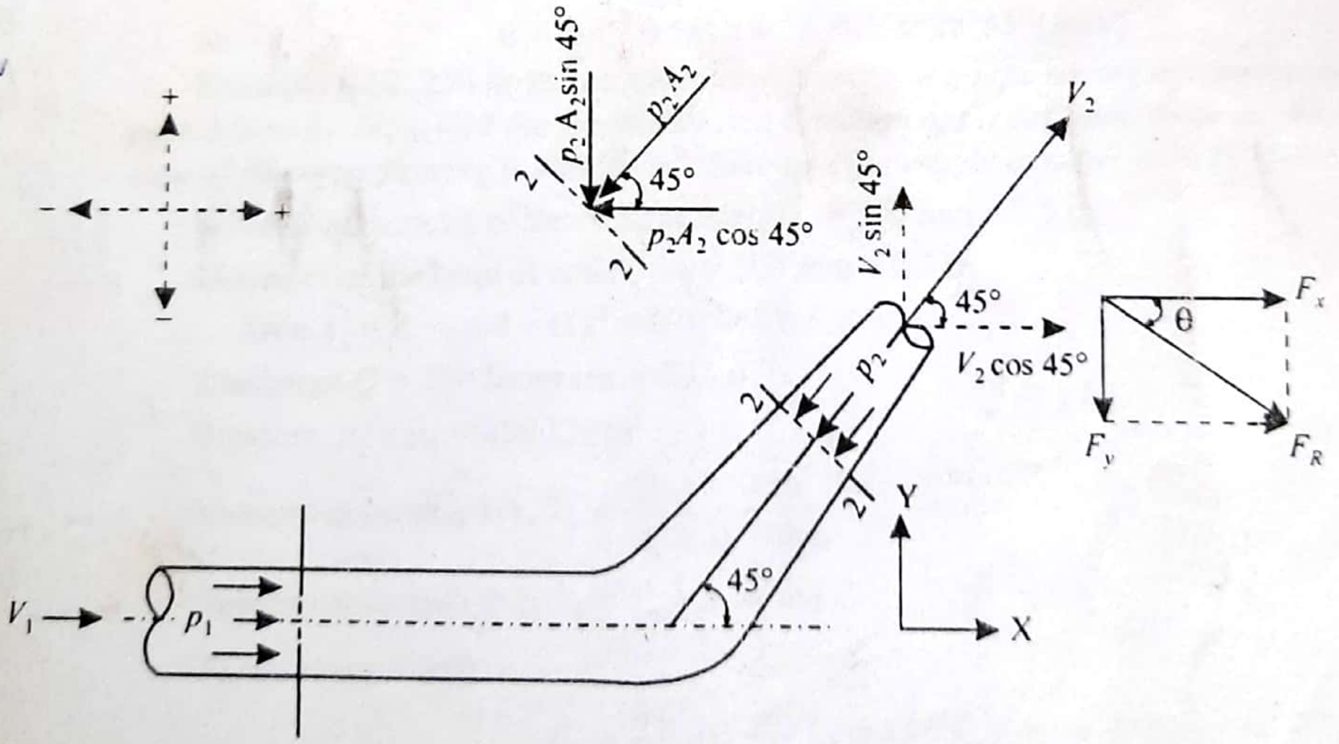


Fig. 6.49

$$\frac{30}{0.0116} + \frac{10^2}{2 \times 9.81} = \frac{p_2}{w} + \frac{20^2}{2 \times 9.81}$$

or $$2586 + 5.1 = \frac{p_2}{w} + 20.4$$

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or $$\frac{p_2}{w} = 2586 + 5.1 - 20.4 = 2570.7$$

or $$p_2 = 2570.7 \times 0.0116 = 29.82 \text{ kN/m}^2$$

Magnitude and direction of force (resultant) F_R:

Force along X-axis:

$$F_x = \frac{wQ}{g}(V_{1x} - V_{2x}) + (p_1A_1)_x + (p_2A_2)_x$$

where, $V_{1x} = 10$ m/s; $V_{2x} = V_2 \cos 45° = 20 \times 0.707 = 14.14$ m/s

$(p_1A_1)_x = p_1A_1 = 30 \times 1 = 30$ kN; $(p_2A_2)_x = -p_2A_2 \cos 45° = -29.82 \times 0.5 \times 0.707 = -10.54$ kN

$$\therefore \quad F_x = \frac{0.0116}{9.81} \times 10\,(10 - 14.14) + 30 - 10.54 = 19.41 \text{ kN } (\rightarrow)$$

Force along Y-axis:

$$F_y = \frac{wQ}{g}(V_{1y} - V_{2y}) + (p_1A_1)_y + (p_2A_2)_y$$

where, $V_{1y} = 0$; $V_{2y} = V_2 \sin 45° = 20 \times 0.707 = 14.14$ m/s

$(p_1A_1)_y = 0$; $(p_2A_2)_y = -p_2A_2 \sin 45° = -29.82 \times 0.5 \times 0.707 = -10.54$ kN

$$\therefore \quad F_y = \frac{0.0116 \times 10}{9.81}(0 - 14.14) + 0 - 10.54 = -10.71 \text{ kN } (\downarrow)$$

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(19.41)^2 + (10.71)^2} = $ **22.17 kN (Ans.)**

The direction of F_R with X-axis is given as

$$\tan\theta = \frac{F_y}{F_x} = \frac{10.71}{19.41} = 0.5518$$

or $$\theta = \tan^{-1} 0.5518 = 28.88° \text{ or } \mathbf{28°53'} \textbf{ (Ans.)}$$

Example 6.52. 250 *litres/sec. of water is flowing in a pipe having a diameter of 300 mm. If the pipe is bent by 135°, find the magnitude and direction of the resultant force on the bend. The pressure of the water flowing is 400 kN/m². Take specific weight of water as 9.81 kN/m³.* [AMIE]

Solution. Diameter of the bend at inlet, $D_1 = 300$ mm $= 0.3$ m

Diameter of the bend at outlet, $D_2 = 300$ mm $= 0.3$ m

$\therefore$ Area $A_1 = A_2 = \pi/4 \times 0.3^2 = 0.07068$ m²

Discharge $Q = 250$ litres/sec $= 0.25$ m³/s.

Pressure, $p_1 = p_2 = 400$ kN/m²

Velocity at section 1-1, $V_1 = \dfrac{Q}{A_1} = \dfrac{0.25}{0.07068} = 3.54$ m/s

Velocity at section 2-2, $V_2 = V_1 = 3.54$ m/s $\quad (\because A_1 = A_2)$

Force along X-axis:

$$F_x = \frac{wQ}{g}[V_1 - (-V_2 \cos 45°)] + p_1A_1 + p_2A_2 \cos 45°$$

$$= \frac{9.81 \times 0.25}{9.81}[3.54 - (-3.54 \times 0.707)]$$

$$+ (400 \times 0.07068) + (400 \times 0.07068 \times 0.707)$$

$$= 0.25 \times (3.54 + 3.54 \times 0.707) + 28.27 + 19.98$$

$$= 49.76 \text{ kN } (\rightarrow)$$

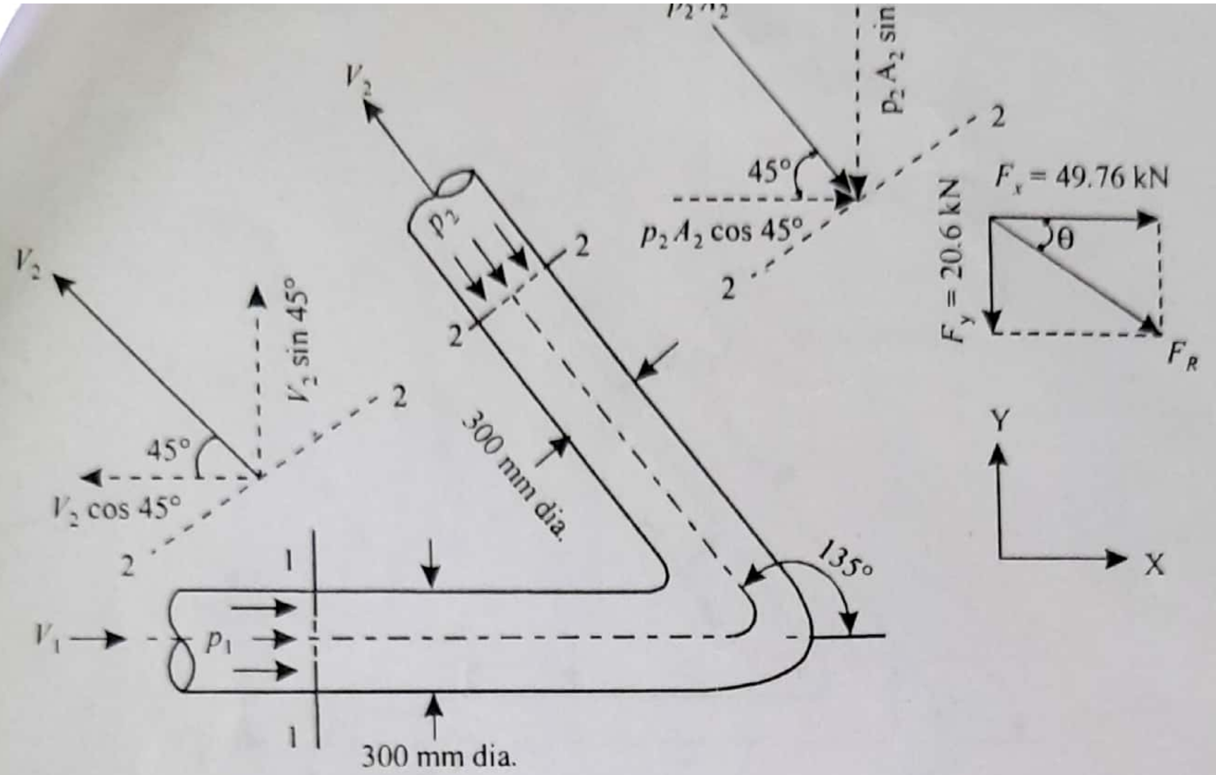

Fig. 6.50

Force along Y-axis:

$$F_y = \frac{wQ}{g}\,[0 - V_2 \sin 45°] - p_2 A_2 \sin 45°$$

$$= \frac{9.81 \times 0.25}{9.81}(0 - 3.54 \times 0.707) - 400 \times 0.07068 \times 0.707$$

$$= -0.625 - 19.98 = -20.6 \text{ kN}(\downarrow)$$

The *magnitude of the resultant force*,

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{49.76^2 + 20.6^2} = \mathbf{53.85\ kN\ \ (Ans.)}$$

The *direction of* F_R with X- axis is given as

$$\tan\theta = \frac{F_y}{F_x} = \frac{20.6}{49.76} = 0.414$$

$$\therefore \qquad \theta = \tan^{-1} 0.414 = \mathbf{22.5°\ (Ans.)}$$

Example 6.53. *360 litres per second of water is flowing in a pipe. The pipe is bent by 120°. The pipe bend measures 360 mm × 240 mm and volume of the bend is 0.14 m³. The pressure at the entrance is 73 kN/m² and the exit is 2.4 m above the entrance section.*

Find the force exerted on the bend.

Solution. Discharge through the pipe, Q = 360 litres/sec = 0.36 m³/s

Volume of bend = 0.14 m³

Diameter of the bend at 1-1, D_1 = 360 mm = 0.36 m

$\therefore$ Area $\qquad A_1 = \frac{\pi}{4} \times 0.36^2 = 0.1018 \text{ m}^2$

Diameter of the bend at 2-2, D_2 = 240 mm = 0.24 m

$\therefore$ Area, $\qquad A_2 = \frac{\pi}{4} \times 0.24^2 = 0.04524 \text{ m}^2$

Velocity at section 1-1, $V_1 = \dfrac{Q}{A_1} = \dfrac{0.36}{0.1018} = 3.54$ m/s

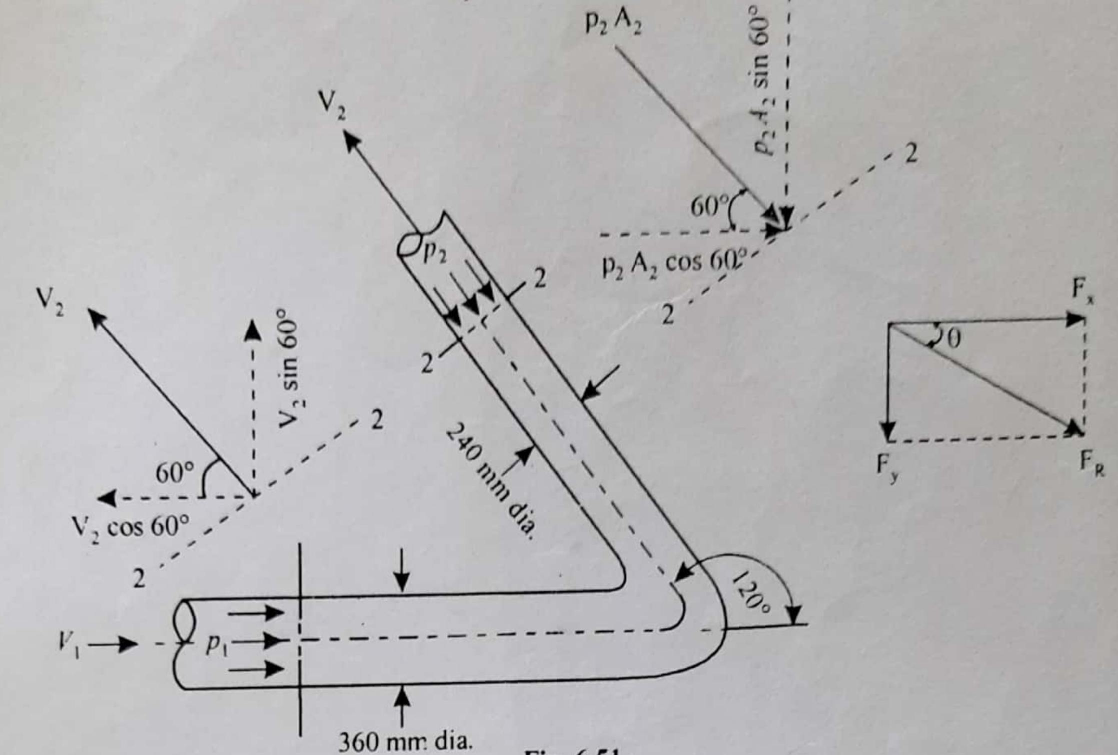

Fig. 6.51

Velocity at section 2-2, $V_2 = \dfrac{Q}{A_2} = \dfrac{0.36}{0.04524} = 7.96$ m/s

Considering a horizontal line through the section 1-1 as datum for elevation head and applying Bernoulli's equation to the sections 1-1 and 2-2, we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{72}{9.81} + \frac{3.54^2}{2 \times 9.81} + 0 = \frac{p_2}{w} + \frac{7.96^2}{2 \times 9.81} + 2.4 \qquad (\because p_1 = 72 \text{ kN/m}^2 \text{ ... } \textit{Given})$$

$$7.34 + 0.64 = \frac{p_2}{w} + 3.23 + 2.4$$

$$\therefore \quad \frac{p_2}{w} = 2.35 \text{ or } p_2 = 2.35 \times 9.81 = 23.05 \text{ kN/m}^2$$

Force along the X-axis:

$$F_x = \frac{wQ}{g}[V_1 - (-V_2 \cos 60°)] + p_1 A_1 + p_2 A_2 \cos 60°$$

$$= \frac{9.81 \times 0.36}{9.81}[3.54 - (-7.96 \times 0.5)] + 72 \times 0.1018 + 23.05 \times 0.04524 \times 0.5$$

$$= 0.36\,(3.54 + 3.98) + 7.33 + 0.52 = 10.55 \text{ kN } (\rightarrow)$$

Force along Y-axis:

$$F_y = \frac{wQ}{g}[0 - V_2 \sin 60°] - p_2 A_2 \sin 60° - \text{weight of water in the bend}$$

$$= \frac{9.81 \times 0.36}{9.81}(0 - 7.96 \times 0.866) - 23.05 \times 0.04524 \times 0.866 - 0.14 \times 9.81$$

$$= 0.36\,(-6.89) - 0.9 - 1.37 = -4.75 \text{ kN } (\downarrow)$$