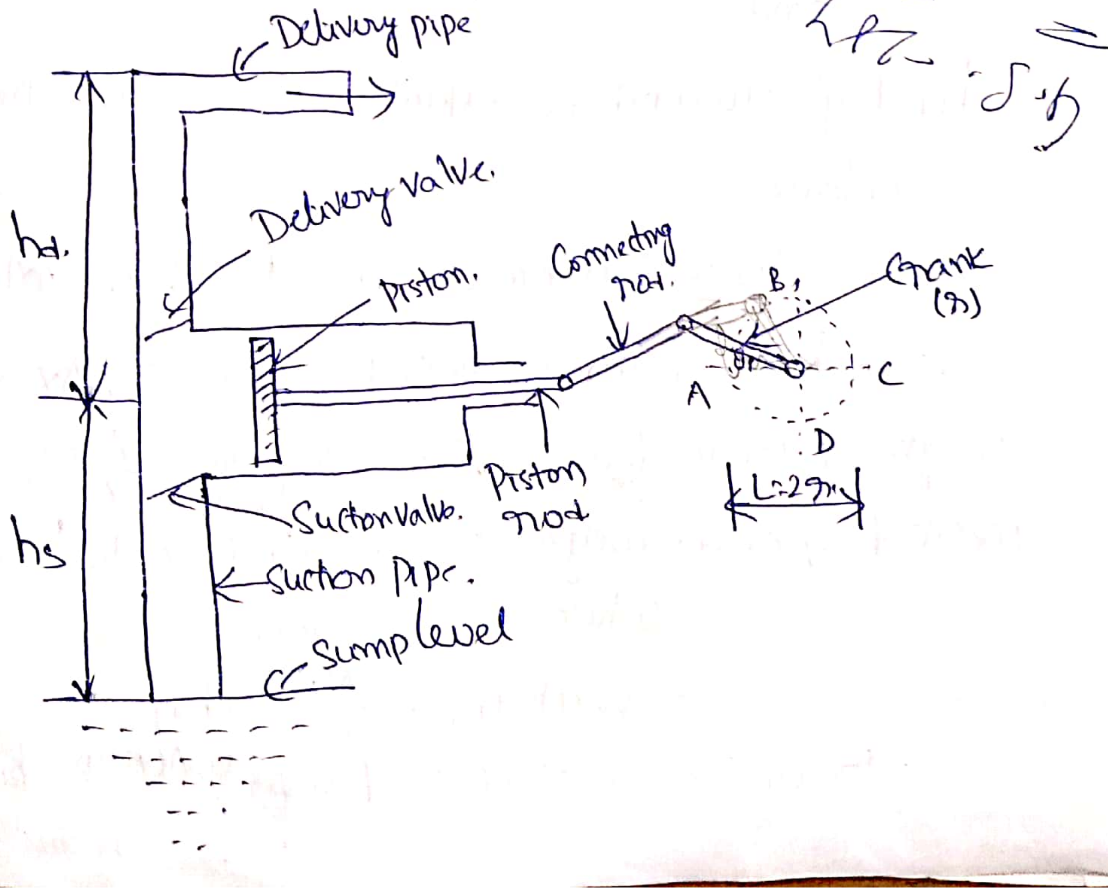
Unit-2

Reciprocating Pumps

Pumps are defined as the hydraulic machines which convert the mechanical energy into hydraulic energy which is mainly in the form of pressure energy.

If the mechanical energy is converted into hydraulic energy by means of a centrifugal force acting on the liquid, the pump is known as centrifugal pump. But if the mechanical energy is converted into hydraulic energy (or) pressure energy by sucking the liquid into a cylinder in which piston is reciprocating (moving backward and forward) which exerts thrust on the liquid and increases its hydraulic energy the pump is known as reciprocating pump.

Main Parts of a Reciprocating Pump-



① A Cylinder with a piston, piston rod, connecting rod and a crank.
② Suction pipe ③ Delivery pipe ④ Suction valve and ⑤ Delivery valve.

Working Principle of a Reciprocating Pump

Discharge, workdone and power required to drive reciprocating pump:

(i) Single acting pump

Consider a single acting reciprocating pump as shown in fig

Let D: Diameter of the cylinder

A: Cross-sectional area of the piston/cylinder: $\frac{\pi}{4}D^2$ m^2

r: Radius of crank, m

N: Speed of crank, r.p.m

L: length of the stroke (= 2r), m

h_s: height of the centre of the cylinder above the liquid surface m and

h_d: height to which the liquid is raised above the centre of the cylinder, m

∴ Volume of liquid sucked in during suction stroke = A×L (or) Discharge liquid

∴ Discharge of the pump Q per second: $Q = A \times L \times N/60$

weight of water delivered/second $W = w \times Q = \frac{wALN}{60}$ $= \rho g \times Q = \frac{\rho g ALN}{60}$

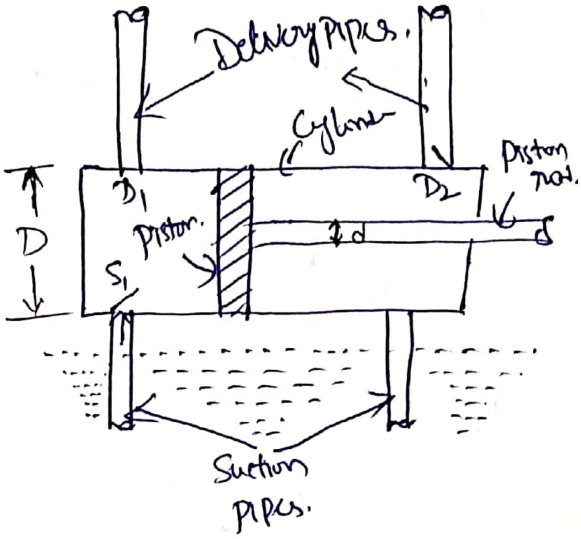
work done/second = weight of water lifted × total height through which liquid is lifted.

$= W \times (h_s + h_d) = \frac{wALN}{60}(h_s + h_d)$

∴ Power required to drive pump = $\frac{wALN}{60000}(h_s + h_d)$ kw

$= \frac{\rho g ALN}{60000}(h_s + h_d)$ kw

Working Principle of a Double-acting reciprocating Pump

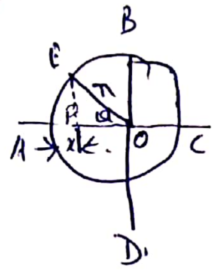


Variation of Velocity or accelaration in The Suction and delivery Pipe due to accelaration of The piston

ω: Angular Speed, A: Area of Cylinder a: Area of The pipe l: length of pipe
r: radius of Crank
$\theta = \omega t$ = Angle turned by Crank in radians in time t

$x = AF = AO - FO = r - r\cos\theta = r - r\cos\omega t$

$V = \frac{dx}{dt} = \frac{d}{dt}\left[r - r\cos(\omega t)\right] = 0 - r(-\sin\omega t)\times\omega = \omega r\sin\omega t$



from Continuity equation

$V\times A = v\times a$

velocity of water in pipe

$v = \frac{V\times A}{a} = \frac{A}{a}\times V = \frac{A}{a}\times r\omega\sin\omega t$

$\therefore$ Accelartion of water in pipe $= \frac{dv}{dt} = \frac{d}{dt}\left(\frac{A}{a} r\omega\sin\omega t\right) = \frac{A}{a}\omega^2 r\cos\omega t$

$\therefore$ Mass of water in pipe = ρ x volume of water in Pipe

= $\rho\times$ Area x length $= \rho\times a\times l = \rho a l$

$\therefore$ force required to accalarate water in pipe

= Mass x accalaration

$= \rho a l\times\frac{A}{a}\omega^2 r\cos\omega t$

$\therefore$ Intensity of Pressure due to accelaration

$= \frac{\text{Force required to accelarate}}{\text{Area of pipe}}$

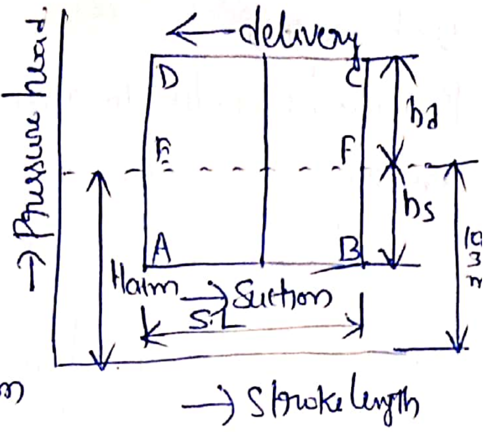
$= \frac{\rho a l\times\frac{A}{a}\omega^2 r\cos\omega t}{a} = \rho l\times\frac{A}{a}\omega^2 r\cos\theta$

$\therefore$ Pressure head $= h_a = \frac{\text{Intensity of Pressure}}{\text{weight density}} = \frac{\rho l\times\frac{A}{a}\omega^2 r\cos\theta}{\rho g} = \frac{l}{g}\times\frac{A}{a}\omega^2 r\cos\theta$

$h_{as} = \frac{l_s}{g}\times\frac{A}{a_s}\omega^2 r\cos\theta$, $h_{ad} =$

Indicator diagram:

The graph between pressure head in the cylinder and stroke length of a piston for one complete revolution of crank under ideal conditions is known as ideal indicator diagram.



EF represents atmospheric pressure head equal to 10.3 m of water.

During the suction stroke pressure head is constant and is equal to suction head which is below the atmospheric head (Hatm) by a height of h_s.

During the delivery stroke, the pressure head in the cylinder is constant and equal to delivery head (h_d) which is above the atmospheric head (Hatm) by a height of h_d.

Thus for one complete revolution of crank, the pressure head in the cylinder is represented by the diagram A-B-C-D-A. This diagram is known as ideal indicator diagram.

we know that,

$$\text{work done by the pump/sec} = \frac{\rho g L A N}{60}(h_s + h_d)$$

$$= K \times L\,(h_s + h_d) \quad \left(\because K = \frac{\rho g A N}{60} = \text{Const}\right)$$

$$\propto L\,(h_s + h_d) \quad \text{--- (1)}$$

But from fig, Area of Indicator diagram is

$$AB \times BC = AB \times (BC + CF) = L \times (h_s + h_d) \quad \text{--- (2)}$$

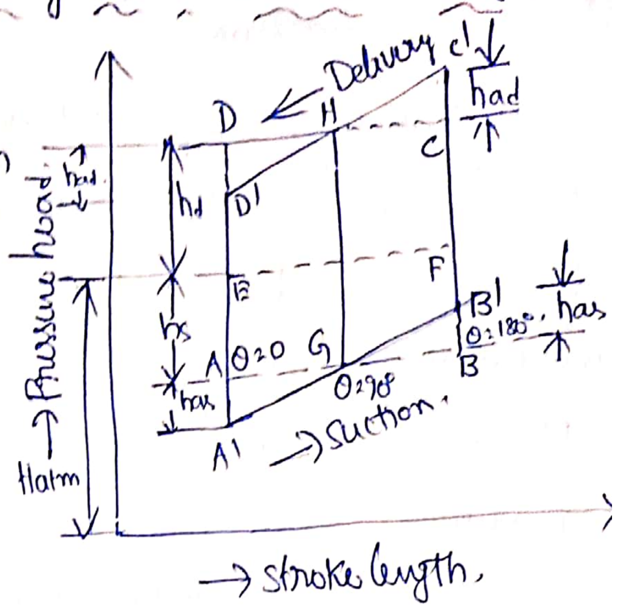
So from equations (1) & (2) we get.

work done by the pump $\propto$ Area of Indicator diagram.

Effect of Accelaration in Suction and delivery Pipes on Indicator diagram:

We know that,

Pressure head due to accelaration in the Suction and pipe is given by.



$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos\theta$$

when $\theta = 0°$; $\cos\theta = 1$; $h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$

when $\theta = 90°$; $\cos\theta = 0$; $h_{as} = 0$

when $\theta = 180°$; $\cos\theta = -1$; $h_{as} = -\frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$

Thus the pressure head inside the Cylinder during Suction Stroke will be not equal to 'h_s' as was the case for ideal indicator diagram, but it will be equal to the sum of h_s and h_{as}. At the beginning of Suction stroke $\theta = 0°$, h_{as} is +ve and hence the pressure head in the Cylinder will be $(h_s + h_{as})$ below the atmospheric pressure head. At the middle of Suction Stroke $\theta = 90°$ and $h_{as} = 0$ and hence Pressure head in the Cylinder will be h_s below the atmospheric pressure head. At the end of Suction Stroke $\theta = 180°$ and h_{as} is -ve and hence the pressure head in the Cylinder will be $(h_s - h_{as})$ below the atmospheric pressure head. For Suction stroke, the indicator diagram will be shown by $A'GB'$. Also the area

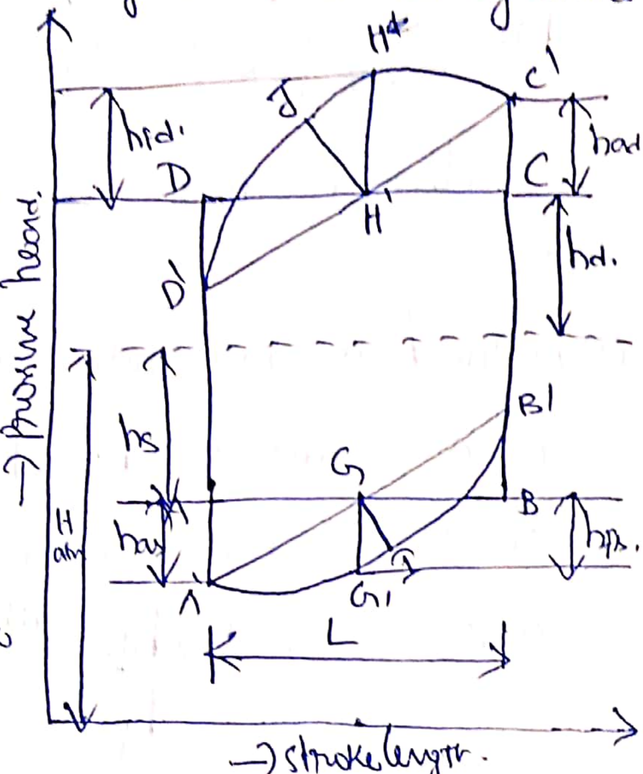
$A'AG$ = Area of BGB'.

Similarly the Indicator diagram for delivery stroke can be drawn. At the beginning of delivery stroke h_{ad} is +ve and hence the pressure head in the Cylinder will be $(h_d + h_{ad})$ above the atmospheric pressure head. At the middle of delivery stroke $h_{ad} = 0$ and hence pressure head in the Cylinder is equal to h_d above the atmospheric Pressure head. At the end of delivery stroke h_{ad} is -ve and hence pressure in Cylinder will be $(h_d - h_{ad})$ above the atmospheric Pressure head. And thus the indicator diagram for delivery stroke is represented by the line $C'HD'$. Also the area of $CC'H$ = Area of $DD'H$.

It is now clear that due to accelaration in Suction and delivery Pipe, the indicator diagram has changed from ABCD and A'B'C'D'. But the area of Indicator diagram ABCD = Area of A'B'C'D'. We know workdone by pump ∝ to area of Indicator diagram. Hence the workdone by the pump on the water remains same.

Effect of Acceleration and friction in Suction and delivery pipes on Indicator diagram :-

The Pressure head in the Cylinder during Suction and delivery strokes will changes as given below.



We know That

$$h_a = \frac{l}{g} \times \frac{A}{a} \omega^2 r \cos\theta \; (\because \theta = \omega t)$$

for Suction Pipe $h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos\theta$

(i) At The beginning of Suction stroke

$\theta = 0 \Rightarrow \cos 0 = 1 \Rightarrow h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$

But $h_{fs} = 0$. Thus The Pressure head in the Cylinder will be $(h_s + h_{as})$ below the atmospheric Pressure head.

(ii) At The middle of Suction Stroke $\theta = 90° \Rightarrow \cos 90 = 0 \Rightarrow h_{as} = 0$

But $h_{fs} = \frac{4 \times f \times l_s}{2g \times d_s}\left[\frac{A}{a}\omega r\right]^2$. Thus The Pressure head in The Cylinder will be $(h_s + h_{fs})$ below the atmospheric Pressure head.

(iii) At The end of Suction Stroke $\theta = 180° \Rightarrow \cos 180 = -1 \Rightarrow h_{as} = -\frac{l_s}{g} \times \frac{A}{a_s}$ but $h_{fs} = 0$

Thus The Pressure head in The Cylinder will be $(h_s - h_{as})$ below The atmospheric head

(iv) At The beginning of delivery Stroke $\theta = 0 \Rightarrow \cos 0 = 1 \Rightarrow h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r$

But $h_{fd} = 0$. Thus The Pressure head in The Cylinder will be $(h_d + h_{ad})$ above The atmospheric head.

(v) At the middle of delivery stroke $\theta = 90 \Rightarrow \cos 90 = 0 \Rightarrow h_{ad} = 0$ But $h_{fd} = \frac{4 f l_d}{2 g d}\left[\frac{A}{a}\omega r\right]^2$

Thus The Pressure head in The Cylinder is $(h_d + h_{fd})$ above atm. pres. head

(vi) At The end of delivery stroke $\theta = 180° \Rightarrow \cos 180 = -1 \Rightarrow h_{ad} = -\frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r$

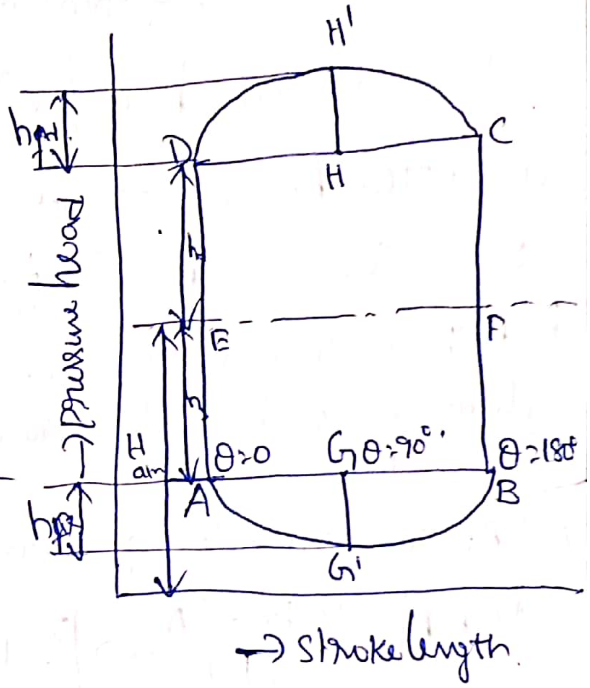
But $h_{fd} = 0$. Thus The Pressure head in The Cylinder is $(h_d - h_{ad})$

Effect of friction in suction and delivery pipes on Indicator diagram:

The loss of head due to friction in suction and delivery pipes is given by

$$h_{fs} = \frac{4fl_s}{d_s \times 2g}\left[\frac{A}{a_s}\, r\omega \sin\theta\right]^2 \text{ and } h_{fd} = \frac{4fl_d}{d_d \times 2g}\left[\frac{A}{a_d}\, r\omega \sin\theta\right]^2$$

It is clear from the above equation that h_{fs} and h_{fd} are parabolic with θ



(i) At the beginning of suction stroke

$\theta = 0 \Rightarrow \sin\theta = 0 \Rightarrow h_{fs}$ & $h_{fd} = 0$

(ii) At the middle of suction stroke

$\theta = 90 \Rightarrow \sin 90 = 1 \Rightarrow h_{fs} = \frac{4fl_s}{2gd_s}\left[\frac{A}{a_s} r\omega\right]^2$

$h_{fd} = \frac{4fl_d}{2gd_d}\left[\frac{A}{a_d} r\omega\right]^2$

(iii) At the end of suction stroke

$\theta = 180 \Rightarrow \sin 180 = 0 \Rightarrow h_{fs}$ and $h_{fd} = 0$

The areas of parabolas AGB and CHD represents the workdone against friction in suction and delivery pipes.

Now area $AGB = AB \times \frac{2}{3} GG' = L \times \frac{2}{3} h_{fs}$, where $h_{fs} = \frac{4 \times f \times l_s}{2gd_s}\left(\frac{A}{a_s} \omega r\right)^2$

Similarly area $CHD = CD \times \frac{2}{3} HH' = L \times \frac{2}{3} h_{fd}$ where $h_{fd} = \frac{4 \times f \times l_d}{2gd_d}\left(\frac{A}{a_d} \omega r\right)^2$

As the variation of h_{fs} and h_{fd} with θ is parabolic in nature the Indicator diagram during suction and delivery strokes with friction in suction and delivery pipes is shown in the above fig.

Problems

① A Single-acting reciprocating pump has a stroke length of 15cm. The suction pipe is 7metres long and the ratio of the suction diameter to plunger diameter is 3/4. The water level in the sump is 2.5metres below the axis of the pump cylinder. and the pipe connecting the ~~pu~~ sump and pump cylinder is 7.5cm diameter. If the crank is running at 75r.p.m. Determine the pressure head on the piston;

(i) In the beginning of suction stroke (ii) in the end of suction stroke

(iii) In the middle of suction stroke, Take coefficient of friction as .001

Ans $h_{as} = 5.87 \cos\theta$. $h_{fs} = 0.208 \sin^2\theta$

② The diameter and length of a single-acting reciprocating pump are 12cm and 20cm respectively. The lengths of suction and delivery pipes are 8m and 25m respectively and their diameter are 7.5cm. If the pump is running at 40r.p.m and suction and delivery ~~Pipe~~ heads are 4m and 14m resp. find the pressure head in the cylinder,

(i) At the beginning of suction and delivery strokes
(ii) At the middle " " " " "
(iii) At the end " " " " "

Take $H_{atm} = 10.3$, $f = 0.009$

Ans $h_{as} = 3.66 \cos\theta$; $h_{ad} = 11.44 \times \cos\theta$; $h_{fs} = 0.225 \sin^2\theta$; $h_{fd} = 0.703 \sin^2\theta$.

(i) 264m : 35.74 (ii) 6.075m ; 425.003m ; (iii) 9.96m ; 12.86m.

Area of the Indicator diagram $A'G'B'C'H'D'$.

$= $ Area of $A'G^*B'C'H^*D'$ + Area of Parabola $A'G^*B'$ + Area of $C'HD'$

But Area of $A'G'B'C'H'D' =$ Area of ABCD $= (h_s+h_d) \times L$

Area of Parabola $A'GB' = A'B' \times \frac{2}{3} G'I = \frac{2}{3} \times (A'B' \times G'I)$

$= \frac{2}{3} \times (AB \times GG')$

$= \frac{2}{3} \times L h_{fs}$.

Area of Parabola $C'HD' = C'D' \times \frac{2}{3} H'J = \frac{2}{3}(C'D' \times H'J)$

$= \frac{2}{3}(CD \times H'H) = \frac{2}{3} L \times h_{fd}$

$\therefore$ Area of Indicator diagram $= (h_s+h_d) \times L + \frac{2}{3} \times L \times h_{fs} + \frac{2}{3} \times L \times h_{fd}$

$= \left((h_s+h_d) + \frac{2}{3} \times h_{fs} + \frac{2}{3} h_{fd}\right) \times L$.

But we know that work done by pump is proportional to the area of Indicator diagram.

$\therefore$ work done by pump per second $\propto (h_s + h_d + \frac{2}{3} h_{fs} + \frac{2}{3} h_{fd}) \times L$

$= KL(h_s + h_d + \frac{2}{3} h_{fs} + \frac{2}{3} h_{fd})$

where K = Constant of Proportionality

$= \frac{\rho g A N}{60}$ — for single-acting

$= \frac{2 \rho g A N}{60}$ — for double-acting

$\therefore$ work done / Second for a single acting

$= \frac{\rho g A N}{60} \times L (h_s + h_d + \frac{2}{3} h_{fs} + \frac{2}{3} h_{fd})$

$= \frac{2 \rho g A N}{60} \times L (h_s + h_d + \frac{2}{3} h_{fs} + \frac{2}{3} h_{fd})$ for double-a

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Maximum Speed of a Reciprocating Pump-

Maximum Speed of a reciprocating Pump Can be determined from the fact That the Pressure in the Cylinder during Suction and delivery stroke, should not fall below The Vapour Pressure of The liquid, flowing Through Suction and delivery pipes. If The Pressure head in The Cylinder is below The Vapour pressure, The dissolved gases will be liberated from the liquid and Cavitation will takes place. Also the Continuous flow of liquid will not exist which means Separation of liquid will takes place. The Pressure at which Separation takes place is Known as a Separation pressure and the head Corresponding to Separation pressure is Known as Separation Pressure head. It is denoted by h_{sep}. For water, The limiting value of Separation pressure head (h_{sep}) is 7.8 m below The atmospheric Pressure head (or) 10.3-7.8 = 2.5 m abs, The Separation may take place during Suction and delivery Strokes.

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Air vessels:

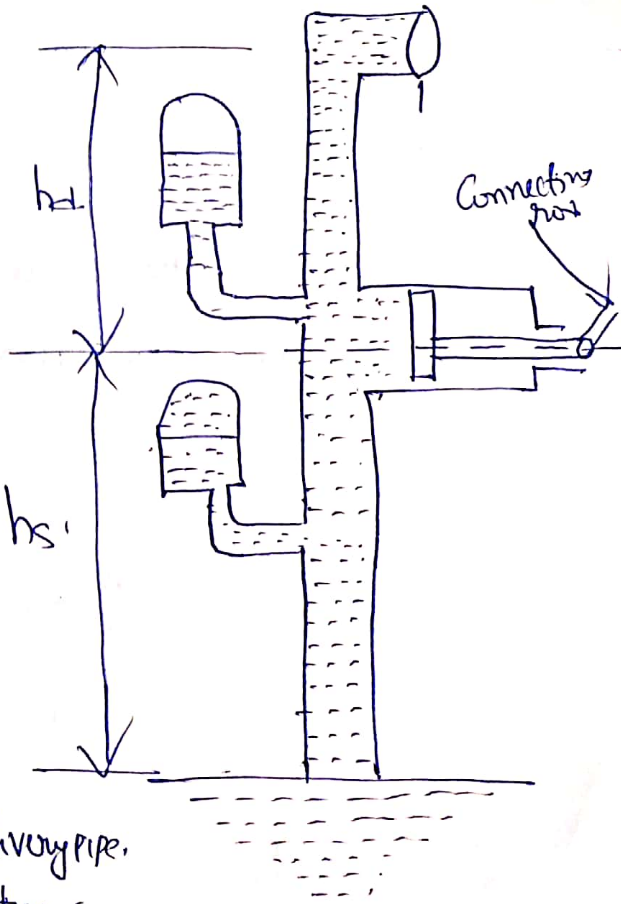
An air vessel is a closed chamber containing compressed air in the top portion and liquid (or) water at the bottom of the chamber. At the base of the chamber there is an opening through which the liquid ~~is~~ may flow into the vessel (or) out from the vessel. When the liquid enters the air vessel, the air gets compressed further and when the liquid flows out the vessel, the air will expand in the chamber.

An air vessel is fitted to the suction pipe and to the delivery pipe at a point close to the cylinder of single acting reciprocating pump

(i) To obtain a continuous supply of liquid at uniform rate

(ii) To save a considerable amount of work in overcoming the frictional resistance in the suction and delivery pipes, and

(iii) To run the pump at high speed without separation.



Let A = Cross-sectional area of cylinder

a = Cross sectional area of suction (or) delivery pipe

l_d = length of delivery pipe beyond the air vessel.

l_d' = length of delivery pipe between cylinder and air vessel.

l_s' = length of suction pipe " " and air vessel.

l_s = length of suction pipe below air vessel.

h_{ad} = Pressure head due to acceleration in delivery pipe.

h_{as} = " " " " " " suction "

h_{fd} = loss of head due to friction in delivery pipe beyond air vessel.

h_{fd}' = loss of head due to friction in delivery pipe between cylinder and air vessel.

h_{fs} = loss of head due to friction in suction pipe below the air vessel

h_{fs}' = " " " " " " " " " between cylinder and air vessel.

The effect of accelaration will be observed only in the lengths l_d' and l_s' which may be made very small by fitting air vessels very close to the cylinder. The velocity of flow of water in the length l_d and l_s will be equal to mean velocity of flow.

$$Q = \frac{ALN}{60}$$

$$\therefore \text{Mean velocity } \bar{V} = \frac{\text{Discharge}}{\text{Area of pipe}} = \frac{Q}{a} = \frac{ALN}{60a}$$

$$= \frac{AL}{60a} \times \frac{60\omega}{2\pi}$$

$$= \frac{A}{a} \times L \times \frac{\omega}{2\pi} = \frac{A}{a} \times 2r \times \frac{\omega}{2\pi}$$

$$= \frac{A}{a} \times \frac{\omega r}{\pi}$$

The velocity of water in the suction (or) delivery pipes for the lengths l_s' and l_d' due to accalaration and retardation of the piston is

$$v = \frac{A}{a} \omega r \sin \omega t$$

$$= \frac{A}{a} \omega r \sin \theta$$

$$(\because \theta = \omega t).$$

(i) Pressure head in the Cylinder during delivery stroke

The pressure head due to accelaration in the delivery pipe of length l_d' is given by

$$h_{ad}' = \frac{l_d'}{g} \times \frac{A}{a_d} \omega^2 r \cos\theta \quad -(1)$$

Loss of head due to friction in the delivery pipe for lengths l_d' is given as

$$h_{fd}' = \frac{4 f l_d' \times V^2}{d_d \times 2g}$$

where for delivery pipe $V = \frac{A}{a_d} \omega r \sin\theta$

$$\therefore h_{fd}' = \frac{4 f l_d'}{d_d \times 2g} \times \left(\frac{A}{a_d} r \omega \sin\theta\right)^2 \quad -(2)$$

loss of head due to friction in delivery pipe for the lengths beyond the air vessel (i.e length l_d).

$$h_{fd} = \frac{4 f l_d \times (\bar{V}_d)^2}{d_d \times 2g}$$

where $\bar{V}_d$ = Mean Velocity of delivery pipe $= \frac{A}{a_d} \times \frac{\omega r}{\pi}$

$$h_{fd} = \frac{4 f \times l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2$$

(i) At the beginning of delivery stroke $\theta = 0°$; $\sin\theta = 0$; $\cos\theta = 1$ and hence

Total pressure head $= (h_d + h_{ad}' + h_{fd}' + h_{fd}) +$ velocity head at the outlet of velocity

$$= h_d + h_{ad}' + h_{fd}' + h_{fd} + \frac{\bar{V}_d^2}{2g}$$

$$= h_d + \frac{l_d'}{g} \times \frac{A}{a_d} \omega^2 r + 0 + \frac{4 f l_d}{d_d \times 2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2}{2g}$$

$$= h_d + \frac{l_d'}{g} \times \frac{A}{a_d} \omega^2 r + \frac{4 f l_d}{d_d \times 2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 \quad -(3)$$

(ii) At $\theta = 90°$; $\sin 90 = 1$; $\cos 90 = 0$.

$$= h_d + h_{ad} + h_{fd}' + h_{fd} + \frac{\bar{V}_d^2}{2g}$$

$$= h_d + 0 + \frac{4fl_d'}{d_d \times 2g} \times \left(\frac{A}{a_d}\omega r\right)^2 + \frac{4fl_d}{d_d \times 2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 \quad (4)$$

(iii) At $\theta = 180°$; $\sin 180 = 0$; $\cos 180 = -1$

$$= h_d - \frac{l_d'}{2g} \times \frac{A}{a_d}\omega^2 r + \frac{4fl_d}{d_d \times 2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 \quad (5)$$

In equations (3) (4) and (5) the quantities

$$\left(\frac{l_d'}{g} \times \frac{A}{a_d}\omega^2 r\right) \text{ and } \left[\frac{4fl_d'}{d_d \times 2g}\left(\frac{A}{a_d}\omega r\right)^2\right]$$ are very small and can be neglected

7/10/14.

Work saved by fitting air vessel:

By fitting air vessel the loss of head due to friction in suction and delivery pipe is reduced. This reduction in head loss saves a certain amount of energy, which can be calculated by finding the work done against friction without air vessel and with air vessel. The difference of two gives the saving in workdone.

(i) Work done against friction without air vessel: Consider a single acting reciprocating pump without any air vessels on the pipes. The velocity of flow through the pipes is given by

$$v = \frac{A}{a} r\omega \sin\theta.$$

and loss of head due to friction is given by $h_f = \frac{4fl}{2gd}\left[\frac{A}{a} r\omega \sin\theta\right]^2$

The variation of h_f with θ is parabolic in nature and hence indicator diagram for the loss of head due to friction in pipes will be a parabola. The work done by pump against friction per stroke is equal to the area of the

Indicator diagram due to friction.

∴ work done by pump per stroke against friction.

$w_1 =$ Area of parabola $= \frac{2}{3} \times$ Base $\times$ Height

$$= \frac{2}{3} \times L \times \left[\frac{4fl}{2gd}\left(\frac{A}{a}\omega r\right)^2\right] \quad (\because \text{Height} = h_f \text{ at } \theta = 90^\circ).$$

$$= \frac{2}{3} \times L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a}\omega r\right)^2 \quad \text{—— (1)}$$

(ii) Work done against friction with air vessel: By fitting an air vessel to the pump, the velocity of flow through pipes (except for lengths l_1 and l_2 which may be considered negligible) is uniform and equal to mean velocity of flow.

$$\bar{V} = \frac{A}{a} \times \frac{\omega r}{\pi}$$

∴ loss of head due to friction with air vessel is given as

$$= \frac{4fl \times \bar{V}^2}{d \times 2g} = \frac{4fl}{2gd} \times \left(\frac{A}{a} \times \frac{\omega r}{\pi}\right)^2$$

The head loss due to friction with air vessel is independent of θ and hence indicator diagram will be a rectangle

∴ work done by pump per stroke against friction.

$W_2 =$ Area of rectangle $=$ Base $\times$ Height

$$= L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a} \times \frac{\omega r}{\pi}\right)^2$$

$$= \frac{1}{\pi^2} \times L \times \frac{4fl}{2gd} \times \left(\frac{A}{a}\omega r\right)^2 \quad \text{—— (2)}.$$

The work given by equation (2) is less than the work given by equation (1). Hence by fitting an air vessel work is saved.

(iii) Work saved in a single acting reciprocating pump: Hence, saving in work done per stroke is obtained by subtracting equation (2) from (1):

$\therefore$ Work saved per stroke $= W_1 - W_2$

$$= \frac{2}{3} L \times \frac{4fl}{2gd} \times \left(\frac{A}{a}\omega r\right)^2 - \frac{1}{\pi^2} \times L \times \frac{4fl}{2gd}\left(\frac{A}{a}\omega r\right)^2$$

$$= L \times \frac{4fl}{2gd}\left(\frac{A}{a}\omega r\right)^2 \left(\frac{2}{3} - \frac{1}{\pi^2}\right) \quad - (3)$$

The percentage of work saved per stroke

$$= \left(\frac{W_1 - W_2}{W_1}\right) \times 100 = \frac{L \times \frac{4fl}{2gd} \times \left(\frac{A}{a}\omega r\right)^2 \left[\frac{2}{3} - \frac{1}{\pi^2}\right]}{\frac{2}{3} \times L \times \frac{4fl}{2gd} \times \left(\frac{A}{a}\omega r\right)^2} \times 100$$

$$= \frac{\frac{2}{3} - \frac{1}{\pi^2}}{\frac{2}{3}} \times 100 = 84.8\%$$

(iv) Work saved in double acting reciprocating pump: The work lost in friction per stroke in case of double acting reciprocating pump without air vessel is the same as given in case of single-acting reciprocating pump. Hence it is given by

$$W_1 = \frac{2}{3} \times L \times \frac{4fl}{2gd}\left(\frac{A}{a}\omega r\right)^2$$

When the air vessel is fitted to the pipe near the cylinder, the mean velocity of flow $\bar{V}$ for double acting is given by.

$$\bar{V} = \frac{\text{Discharge}}{\text{Area of pipe}} = \frac{Q}{a} = \frac{2ALN}{60a}$$

$$= \frac{2A \times 2r}{60a} \times \frac{60\omega}{2\pi} \quad \left(\because L = 2r \text{ and } N = \frac{60\omega}{2\pi}\right)$$

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$$= \frac{2A}{a} \times \frac{\omega r}{\pi}$$