Unit-3 Centrifugal pumps

Work done by the impeller (or) centrifugal Pump on liquid (Inlet and outlet velocity Triangles):

Assumption (a) The liquid enters the impeller at its centre and leaves at its outer Periphery

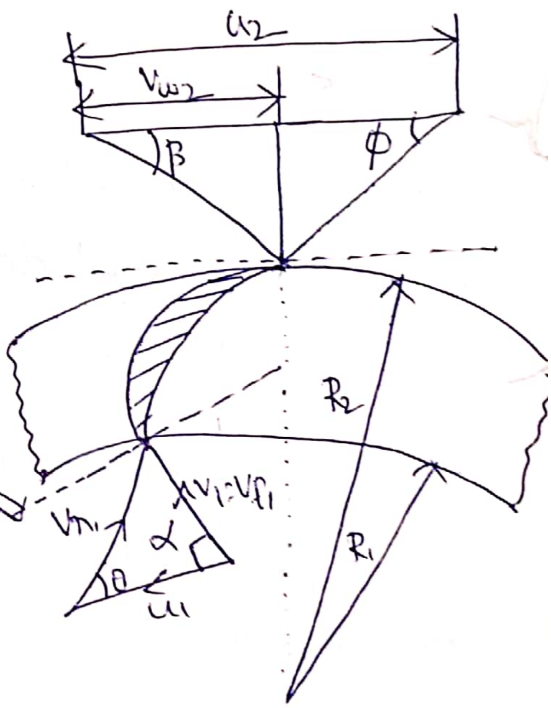
Assumptions

① Liquid enters the impeller eye in radial direction, The whirl component V_{w_1} (of the inlet absolute velocity V_1) is zero and The flow component V_{f_1} equals the absolute velocity itself (i.e. $V_{f_1} = V_1$); $\alpha = 90°$.

② No energy loss in the impeller due to friction and eddy formation.

③ No loss due to shock at entry

④ There is uniform velocity distribution in the narrow Passages formed between two adjacent vanes.



Let D_1 = Diameter of impeller at inlet ($R_1 = D_1/2$)

N = Speed of the impeller in r.p.m

ω = Angular Speed = $\frac{2\pi N}{60}$

u_1 = Tangential velocity of impeller at inlet

$= \frac{\pi D_1 N}{60} = \left(\frac{2\pi R_1 N}{60}\right) = \omega R_1$

D_2 = Diameter of impeller at outlet ($R_2 = D_2/2$)

u_2 = Tangential velocity of impeller at outlet

$= \frac{\pi D_2 N}{60} = \frac{2\pi R_2 N}{60} = \omega R_2$

V_1 = Absolute velocity of water at inlet.

V_{w_1} = Velocity of whirl at inlet

V_{r_1} = Relative velocity of liquid at inlet.

V_{f_1} = Velocity of flow at inlet

α = Angle made by absolute velocity (V_1) at inlet with the direction of motion van

θ = " " " relative velocity (V_{r_1}) " " " " " " "

V_{w2}, V_{r2}, V_{f2}, β and ϕ are the corresponding values at outlet while passing through the impeller. The velocity of whirl changes and there is a change of moment of momentum.

Torque on the impeller = Rate of change of moment of momentum

Moment of momentum at inlet = 0

" " " outlet $= \frac{W}{g}(V_{w2} R_2)$

$\therefore$ Torque $= \frac{W}{g}(V_{w2} R_2)$

Workdone/Second = Torque × angular velocity

$\therefore \frac{W}{g}(V_{w2} R_2) \times \omega = \frac{W}{g}(V_{w2} u_2) \quad (\because u_2 = \omega R_2)$

workdone/unit weight of liquid

$\therefore \frac{1}{g}(V_{w2} u_2) = \frac{V_{w2} u_2}{g}$.

The above equation has developed by assuming the flow is in radial direction. If it is not in radial direction then the equation is

~~To~~ workdone/second $= \frac{W}{g}(V_{w2} u_2 - V_{w1} u_1)$

workdone/unit weight $= \frac{1}{g}(V_{w2} u_2 - V_{w1} u_1)$

This equation is known as Euler momentum equation for centrifugal pump.

The term $\frac{1}{g}(V_{w2} u_2 - V_{w1} u_1)$ is referred to as Euler head H_e.

From outlet triangle, we have

$V_{r2}^2 = V_{f2}^2 + (u_2 - V_{w2})^2$ (or) $V_{f2}^2 = V_{r2}^2 - (u_2 - V_{w2})^2$ — ①

Also $V_{f2}^2 = V_2^2 - V_{w2}^2$ — ②

From ① & ② ~~But we have~~ $V_2^2 - V_{w2}^2 = V_{r2}^2 - (u_2 - V_{w2})^2 = V_{r2}^2 - (u_2^2 + V_{w2}^2 - 2u_2 V_{w2})$

$V_2^2 - V_{w2}^2 = V_{r2}^2 - u_2^2 - V_{w2}^2 + 2u_2 V_{w2}$

$u_2 V_{w2} = \frac{1}{2}(V_2^2 + u_2^2 - V_{r2}^2)$

Similarly $u_1 V_{w_1} = \frac{1}{2}(V_1^2 + u_1^2 - V_{r_1}^2)$

work done / sec / unit weight of liquid (on) the

$$= \frac{V_2^2 - V_1^2}{2g} + \frac{u_2^2 - u_1^2}{2g} + \frac{V_{r_1}^2 - V_{r_2}^2}{2g}$$

The first term $\left(\frac{V_2^2 - V_1^2}{2g}\right)$ represents the increase in kinetic energy

The second term $\left(\frac{u_2^2 - u_1^2}{2g}\right)$ " an increase in static pressure

The third term $\left(\frac{V_{r_1}^2 - V_{r_2}^2}{2g}\right)$ " the change in kinetic energy due to redu of flow relative to the impeller.

Manometric head: The head against which a centrifugal pump has to work is known as the manometric head. It is the head measured across the pump inlet and outlet flanges. It is denoted by H_{mano} and is given below.

(i) H_{mano} = Head imparted by the impeller to liquid − loss of head in the pump (i.e impeller and cas)

$$= \frac{V_{w_2} u_2}{g} - (h_{Li} + h_{Lc})$$

$$= \frac{V_{w_2} u_2}{g} \text{ (if loss of head in pump is 0).}$$

(ii) H_{mano} = H_{static} + loss in pipes + $\frac{V_d^2}{2g}$

$$= (h_s + h_d) + (h_{fs} + h_{fd}) + \frac{V_d^2}{2g}$$

h_s = Suction head

h_d = delivery head

h_{fs} = Frictional head loss in suction pipe.

h_{fd} = " " " " delivery "

V_d = velocity of liquid in " "

(iii) H_{mano} = Total head at outlet of pump − Total head at inlet of pump

$$= \left(\frac{P_2}{w}+\frac{V_2^2}{2g}+Z_2\right)-\left(\frac{P_1}{w}+\frac{V_1^2}{2g}+Z_1\right)$$

$\frac{P_2}{w}$ = Pressure head at outlet of pump = h_d.

$\frac{V_2^2}{2g}$ = velocity head at outlet of pump.

= velocity head in delivery pipe = $\frac{V_d^2}{2g}$.

Z_2 = Vertical height of the pump outlet from the datum line

Losses in Centrifugal Pump

1. Hydraulic losses - Shock at the entrance to and exit from impeller.
   losses due to friction
   Friction and eddy losses in guide vanes.
2. Mechanical losses
3. Leakage losses.

Efficiencies ① Manometric efficiency = $\frac{\text{Manometric head}}{\text{Head imparted by impeller to water}}$

$$= \frac{g \times H_m}{V_{w2} \times u_2}$$

Mechanical efficiency

$$\eta_m = \frac{\text{Power delivered by the impeller to the liquid}}{\text{Power input to the pump shaft } (P)}.$$

$$\eta_m = \frac{w(Q+q)(V_{w2}u_2/g)}{P}$$

$$= \frac{P - P_{mech.loss}}{P}$$

Overall efficiency

$$\eta_o = \frac{\text{Power output of the pump}}{\text{Power input to pump}} = \frac{wQH_{mano}}{P}$$

$$\eta_o = \eta_{mano} \times \eta_v \times \eta_m$$

$$= \frac{H_{mano}}{(V_{w2}u_2/g)} \times \frac{Q}{(Q+q)} \times \frac{w(Q+q)(V_{w2}u_2/g)}{P} = \frac{wQH_{mano}}{P}$$

Problems on Centrifugal Pump:

① A Centrifugal Pump is to discharge 0.118 m³/s at a speed of 1450 r.p.m against a head of 25m. The impeller diameter is 250mm, its width at outlet is 50mm and manometric efficiency is 75%. Determine the vane angle at the outer periphery of the impeller.

$u_2 = \frac{\pi D_2 N}{60} = 18.98$ m/s; $Q = \pi D_2 B_2 \times V_{f_2} \Rightarrow V_{f_2} = 3.0$ m/s. ; $\eta_{man} = \frac{g H_m}{V_{w_2} u_2}$

$V_{w_2} = 17.23$ m/s. ; $\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = 1.7143$. $\phi = \tan^{-1}(1.7143) = 59.74^\circ$

② A Centrifugal pump delivers water against a net head of 14.5 metres a design speed of 1000 r.p.m. The vanes are curved back to an angle of 30° with the periphery. The impeller diameter is 300mm and outlet width is 50mm. Determine the discharge of the pump if manometric efficiency is 95%

$u_2 = \frac{\pi D_2 N}{60}$; $\eta_{man} = \frac{g H_m}{V_{w_2} \times u_2} \Rightarrow V_{w_2} = 9.54$ m/s. $\tan\phi = \frac{V_{f_2}}{(u_2 - V_{w_2})} \Rightarrow V_{f_2} = 3.556$ m/s.

$Q = \pi D_2 B_2 \times V_{f_2} = 0.1675$ m³/s.

③ A Centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 r.p.m works against a total head of 40m. The velocity of flow through the impeller is constant and equal to 2.5 m/s. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500mm and width at outlet is 50mm, determine:

(i) Vane angles at inlet (ii) work done by impeller on water/sec (iii) η_{man}

(i) $\tan\theta = 0.191$; $\theta = 10.81^\circ$ (ii) $\frac{W}{g} V_{w_2} u_2 = \frac{\rho g Q}{g} V_{w_2} u_2$

$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} \Rightarrow V_{w_2} = 23.2$ m/s. Work done $= 119227.9$ N-m/s.

$\eta_m = \frac{g H_m}{V_{w_2} u_2} = 64\%$

Pressure rise in the impeller

By Applying Bernoulli's equation at the inlet and outlet of the impeller neglecting losses from inlet to outlet,

(Total energy)$_{inlet}$ = (Total energy)$_{outlet}$ − work done by impeller on water

$$\frac{P_1}{w}+\frac{V_1^2}{2g}+Z_1 = \left(\frac{P_2}{w}+\frac{V_2^2}{2g}+Z_2\right) - \text{work done by impeller on water/unit weight of water.}$$

$$\therefore \frac{P_2}{w}+\frac{V_2^2}{2g}+Z_2-\frac{V_{w_2}u_2}{g}$$ (Assuming flow to be radial at inlet).

If the inlet and outlet of the impeller are at the same height ($Z_1 = Z_2$)

$$\frac{P_1}{w}+\frac{V_1^2}{2g} = \frac{P_2}{w}+\frac{V_2^2}{2g}-\frac{V_{w_2}u_2}{g}$$

$$\left(\frac{P_2}{w}-\frac{P_1}{w}\right) = \frac{V_1^2}{2g}-\frac{V_2^2}{2g}+\frac{V_{w_2}u_2}{g} \quad -(1)$$

$\frac{P_2}{w}-\frac{P_1}{w}$ = Pressure rise in impeller

From Inlet Velocity Triangle, we have $V_1 = V_{f_1}$

From outlet " " "

$$\tan\phi = \frac{V_{f_2}}{u_2-V_{w_2}} \quad (\text{or}) \quad u_2-V_{w_2} = \frac{V_{f_2}}{\tan\phi}$$

$$V_{w_2} = u_2-\frac{V_{f_2}}{\tan\phi} = u_2-V_{f_2}\cot\phi$$

Also, $V_2^2 = V_{f_2}^2+V_{w_2}^2 = V_{f_2}^2+(u_2-V_{f_2}\cot\phi)^2$

$$= V_{f_2}^2+(u_2^2+V_{f_2}^2\cot\phi-2u_2V_{f_2}\cot\phi)$$

$$= V_{f_2}^2+V_{f_2}^2\cot\phi+u_2^2-2u_2V_{f_2}\cot\phi$$

Also $V_2^2 = V_{f_2}^2(1+\cot^2\phi) + u_2^2 - 2u_2V_{f_2}\cot\phi$

$$= V_{f_2}^2\operatorname{cosec}^2\phi + u_2^2 - 2u_2V_{f_2}\cot\phi$$

$(\because 1+\cot^2\phi = \operatorname{cosec}^2\phi)$

Substituting all the values in 1

$$\left(\frac{P_2}{w} - \frac{P_1}{w}\right) = \frac{V_{f_1}^2}{2g} - \frac{V_{f_2}^2\operatorname{cosec}^2\phi + u_2^2 - 2u_2V_{f_2}\cot\phi}{2g} + \frac{(u_2 - V_{f_2}\cot\phi)u_2}{g}$$

$$= \frac{1}{2g}\left[V_{f_1}^2 - V_{f_2}^2\operatorname{cosec}^2\phi - u_2^2 + \cancel{2u_2V_{f_2}\cot\phi} + 2u_2^2 - \cancel{2u_2V_{f_2}\cot\phi}\right]$$

$$= \frac{1}{2g}\left[V_{f_1}^2 + u_2^2 - V_{f_2}^2\operatorname{cosec}^2\phi\right]$$

Minimum Speed for Starting a Centrifugal Pump

If the pressure rise in the impeller is more than (or) equal to manometric head (H_m), the centrifugal pump will start delivering water. Otherwise, the pump will not discharge any water, through the impeller is rotating when impeller is rotating. The water in contact with the impeller is also rotating. This is the case of forced vortex. In case of forced vortex the centrifugal head (or) head due to pressure rise in the impeller.

$$= \frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g}$$

ωr_2 = Tangential velocity of impeller at outlet = u_2
ωr_1 = Tangential velocity of impeller at inlet = u_1

$\therefore$ Head due to pressure rise in the impeller = $\frac{u_2^2}{2g} - \frac{u_1^2}{2g}$.

The flow of water commence only if
Head due to pressure rise in the impeller $\geq H_m$ (or) $\frac{u_2^2}{2g} - \frac{u_1^2}{2g} \geq H_m$.

For minimum speed we must have $\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m$.

But we have $H_m = \eta_{man} \times \frac{V_{w_2} u_2}{g}$.

$$\therefore \frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \eta_{man} \times \frac{V_{w_2} u_2}{g}.$$

$$u_2 = \frac{\pi D_2 N}{60} \quad u_1 = \frac{\pi D_1 N}{60}$$

$$\therefore \frac{1}{2g}\left(\frac{\pi D_2 N}{60}\right)^2 - \frac{1}{2g}\left(\frac{\pi D_1 N}{60}\right)^2 = \eta_{man} \times \frac{V_{w_2} \times \pi D_2 N}{g \times 60},$$

Dividing by $\frac{\pi N}{g \times 60}$, we get $\frac{\pi N D_2^2}{120} - \frac{\pi N D_1^2}{120} = \eta_{man} \times V_{w_2} \times D_2$

$$\frac{\pi N}{120}(D_2^2 - D_1^2) = \eta_{man} \times V_{w_2} D_2$$

$$\therefore N = \frac{120 \times \eta_{man} \times V_{w_2} \times D_2}{\pi (D_2^2 - D_1^2)}.$$

Problems on minimum starting speed:

① The diameters of an impeller of a Centrifugal Pump at inlet and outlet are 30cm & 60cm resp. Determine the minimum starting speed of a Pump if it works against a head of 30m.

$$\frac{u_2^2}{2g}-\frac{u_1^2}{2g}=H_m \Rightarrow \left(\frac{\pi D_2 N}{2g}\right)^2-\frac{(\pi D_1 N)^2}{2g}=H_m$$

$$\Rightarrow \frac{1}{2g}(0.3141N)^2-\frac{1}{2g}(0.0157N)^2=30.$$

$$\therefore \frac{1}{2g}(0.3141N)^2-\frac{1}{2g}(0.0157N)^2=30.$$

$$N^2=795297.9 \Rightarrow N=891.8\ r.p.m.$$

② The diameters of an impeller of a Centrifugal Pump at inlet and outlet are 30cm and 60cm resp. The velocity of flow at outlet is 2.0m/s and vanes are set back at an angle of 45° at the outlet. Determine the minimum starting speed of a pump if η_{man} is 70%.

$$\therefore N_{min}=\frac{120\times\eta_{man}\times V_{w_2}D_2}{\pi(D_2^2-D_1^2)}=137.22\ r.p.m$$

$$u_2-V_{w_2}=2.0.$$

$$\tan\phi=\frac{V_{f_2}}{(u_2-V_{w_2})} \Rightarrow V_{w_2}=(0.0314N-2.0)$$

Specific Speed of a Centrifugal Pump:

The Specific Speed of a Centrifugal Pump is defined as the speed of a geometrically similar pump which would deliver one cubic metre of liquid per second against a head of one metre. It is denoted by N_s.

$$Q=\text{Area}\times\text{Velocity of flow}$$

$$=\pi D\times B\times V_f$$

$$Q\propto DBV_f \quad -①$$

We know that $B\propto D$

$$\therefore Q\propto D^2V_f \quad -②$$

$$u = \frac{\pi D N}{60} \propto DN. \quad -(3)$$

$$\therefore u \propto V_f \propto \sqrt{H_m}. \quad -(4)$$ ($\because u_2^2 - u_1^2 = H_m \Rightarrow u \propto \sqrt{H_m}$; $\tan\theta = \frac{V_{f1}}{u} \Rightarrow V_f \propto u_t$.)

$$\therefore \sqrt{H_m} \propto DN \text{ (or) } D \propto \frac{\sqrt{H_m}}{N}.$$

Substituting D in (2) we get

$$Q \propto \frac{H_m}{N^2} \times V_f.$$

$$\propto \frac{H_m}{N^2} \times \sqrt{H_m}$$

$$\propto \frac{H_m^{3/2}}{N^2}$$

$$Q = K \frac{H_m^{3/2}}{N^2}. \quad -(5)$$

where K = Constant of proportionality

If $H_m = 1$ and $Q = 1 m^3/s$ $N = N_s$.

$$\therefore 1 = K \frac{1^{3/2}}{N_s^2} = \frac{K}{N_s^2}$$

$$\therefore N_s^2 = K$$

Substituting value of K in (5) we get

$$Q = N_s^2 \frac{H_m^{3/2}}{N^2} \Rightarrow N_s^2 = \frac{QN^2}{H_m^{3/2}} \Rightarrow N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}.$$

Model testing of Centrifugal pump.

Before manufacturing large sized pumps. Their models which are in Complete Similarity with the actual pumps (prototypes) are made. The Complete Similarity between the model and actual pump will exist if the following Conditions are Satisfied.

(1) Specific Speed of model = Specific Speed of Prototype

$$(N_s)_m = (N_s)_p \text{ (or) } \left(\frac{N\sqrt{Q}}{H^{3/4}}\right)_m = \left(\frac{N\sqrt{Q}}{H^{3/4}}\right)_p.$$

② Tangential Velocity is given by $u = \frac{\pi DN}{60}$ also $u \propto \sqrt{H_m}$.

$$\sqrt{H_m} \propto DN$$

$$\therefore \frac{\sqrt{H_m}}{DN} = \text{Const.}$$

$$\left(\frac{\sqrt{H_m}}{DN}\right)_m = \left(\frac{\sqrt{H_m}}{DN}\right)_p.$$

③ we have $Q \propto D^2 \times V_f$

$$\propto D^2 \times D \times N$$

$$\propto D^3 \times N.$$

$$\frac{Q}{D^3 N} = \text{Const} \quad (\text{or}) \left(\frac{Q}{D^3 N}\right)_m = \left(\frac{Q}{D^3 N}\right)_p.$$

④ Power of the pump. $P = \frac{\rho g Q H_m}{75}$

$$P \propto Q H_m$$

$$\propto D^3 \times N \times H_m$$

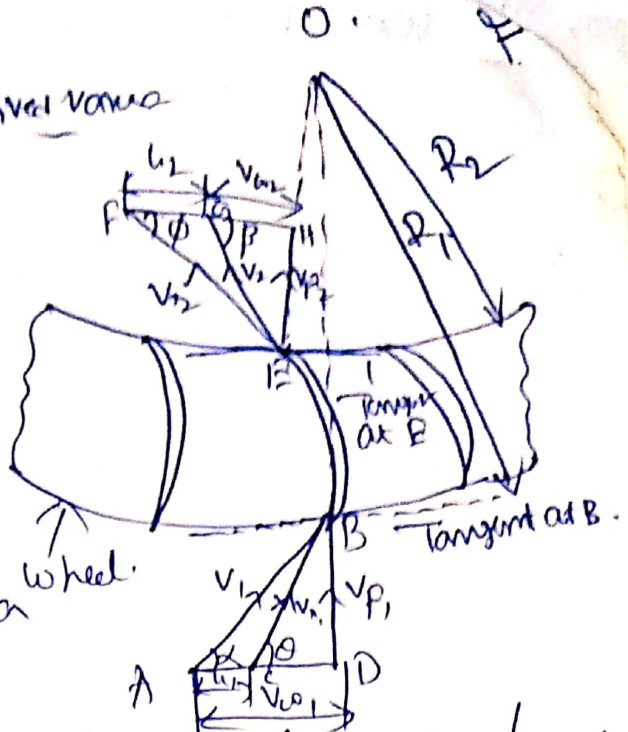
$$\propto D^3 N \times D^2 N^2$$

$$\propto D^5 N^3$$

$$\frac{P}{D^5 N^3} = \text{Const} \quad (\text{or}) \left(\frac{P}{D^5 N^3}\right)_m = \left(\frac{P}{D^5 N^3}\right)_p.$$

Centrifugal Pump workdone

force exerted on a Series of radial Curved vanes



R_1 = Radius of wheel at inlet of Vane

R_2 = " " " " outlet " "

ω = angular Speed of the wheel.

$\therefore u_1 = \omega R_1$; $u_2 = \omega R_2$

Mass of water striking / second for a Series of Vanes = $\rho a V_1$.

Momentum of water striking the vanes in tangential direction / sec at inlet = Mass / Sec $\times$ Component of V_1 in the tangential direction

$= \rho a V_1 \times V_{w_1}$ ($V_{w_1} = V_1 \cos\alpha$)

Momentum of water at outlet / sec $= \rho a V_1 \times (-V_{w_2}) = -\rho a V_1 V_{w_2}$

angular momentum / sec at inlet $= \rho a V_1 V_w R_1$

" " " " outlet $= -\rho a V_1 V_{w_2} \times R_2$,

Torque exerted by the water on the wheel =

T = Rate of change of momentum

= Initial angular momentum / sec − final angular moment

$= \rho a V_1 \times V_w \times R_1 - (-\rho a V_1 \times V_{w_2} \times R_2) = \rho a V_1 (V_{w_1} R_1 + V_{w_2} R_2)$

workdone / sec = Torque $\times$ angular velocity = $T \times \omega$

$\rho a V_1 (V_w R_1 \omega + V_{w_2} R_2 \omega) = \rho a V_1 (V_{w_1} u_1 + V_{w_2} u_2)$

If β is an obtuse angle $= \rho a V_1 (V_{w_1} u_1 - V_{w_2} u_2)$.

$\therefore$ General expression $= \rho a V_1 (V_{w_1} u_1 \pm V_{w_2} u_2)$

$\beta = 90$ $= \rho a V_1 (V_{w_1} u_1)$

(2) The workdone/sec per unit weight of water/second.

$$= \frac{\rho Q\left[V_{w_1}u_1 + V_{w_2}u_2\right]}{\rho Q \times g} = \frac{1}{g}\left(V_{w_1}u_1 \pm V_{w_2}u_2\right)$$

For Centrifugal Pump water enters the impeller radially which means absolute velocity of water at inlet is in the radial direction and here an angle $\alpha = 90^\circ$; $V_{w_1} = 0$.

Priming of a Centrifugal Pumps

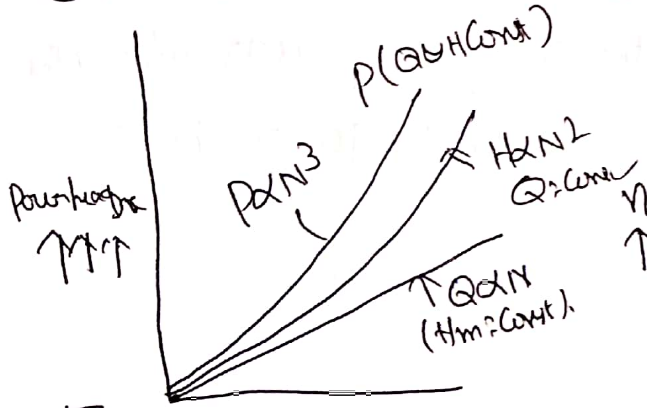
Priming of Centrifugal pump is defined as the operation in which, Suction pipe, Casing, a portion of the delivery pipe upto delivery valve is completely filled up from the outside source with the liquid to be raised by the pump before starting the pump. Thus the air from these parts of the pump is removed and these parts are filled with the liquid to be pumped.

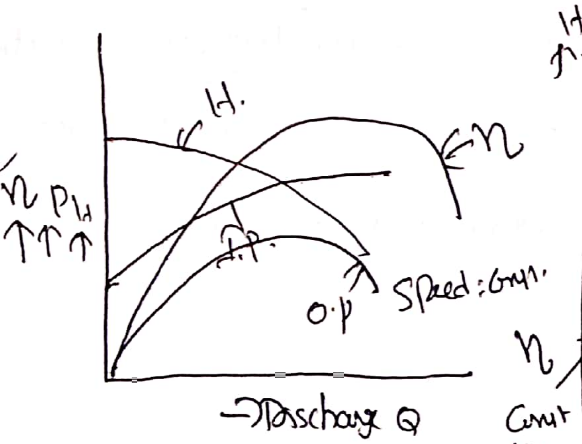
The workdone by the impeller per unit weight of liquid per sec is known as the head generated by the pump. The equation for workdone by the impeller is given as $\frac{1}{g} V_{w2} u_2$. This equation is independent of density of the liquid. This means that when pump is running in the air, the head generated in terms of metre

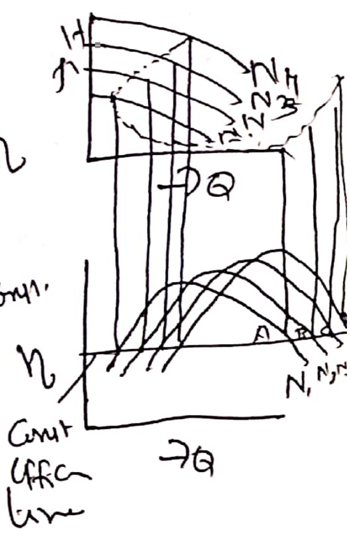
Characteristic Curves of Centrifugal pumps

characteristic Curves of Centrifugal pump are defined as those curves which are plotted from the results of a numbers of tests on the Centrifugal pump. These Curves are necessary to predict the behaviour and performance of the pump when the pump is running at different flow rate, head and speed.

① Main characteristic curves ② operating characteristic curve ③ Constant efficiency







$\frac{\sqrt{H_m}}{DN} = \text{Const}$ → Speed $\qquad \frac{P}{D^5 N^3} = \text{Const} \qquad \frac{Q}{D^3 N} = \text{Const}$

$H_m \propto N^2 \qquad P \propto N^3 \qquad Q \propto N$.

Cavitation :-

Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapour pressure and the sudden collapsing of these vapour bubbles in a region of high pressure. Formation of vapour bubbles of the flowing liquid takes place only whenever the pressure in any region falls below vapour pressure

Precautions against Cavitation :-

(i) The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure. If the flowing liquid is water then the absolute pressure head should not be below 2.5 m of water

(ii) The special materials (or) coatings such as aluminium-bronze and stainless steel, which are cavitation resistant materials, should be used.

Effects of Cavitation :-

(i) The metallic surfaces are damaged and cavities are formed on the surface

(ii) Due to sudden collapse of ~~water~~ vapour bubble, considerable noise and vibrations are produced

(iii) The efficiency of pump decreases due to cavitation.

Cavitation in Centrifugal pumps :- Cavitation may occur at the inlet of the impeller of the pump (or) at the suction side of the pumps where the pressure is considerably reduced