HMS

HMS Unit-I
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Classification of Hydraulic machines

① According to the head and quantity of water available
② According to the name of the originator
③ " " " action of water on moving blades
④ " " " direction of flow of water in the runner
⑤ " " " disposition of turbine shaft.
⑥ " " " Specific Speed N.

① According to the head and quantity of water available.
(i) Impulse turbine → Requires high head and small quantity of flow
(ii) Reaction turbine → Requires low head and high rate of flow

② According to the name of the originators.
(i) Pelton turbine → Lester Allen Pelton from California. It is an impulse type turbine and is used for high head and low discharge
(ii) Francis turbine → James Bichens Francis. It is a reaction type of turbine from medium high to medium low heads and medium small to medium large quantities of water
(iii) Kaplan turbine → Dr. Victor Kaplan. It is the reaction type of turbine for low heads and large quantities of flow

③ According to action of water on Blades.

Turbine →
- → Impulse turbine – Pelton turbine
- Reaction turbine →
  - → Francis turbine
  - → Kaplan and Propeller turbine

(Head → Diff. b/w water level at the reservoir and water level at tail race.)

(4) According to direction of flow of water in the runner:

(i) Tangential flow turbines (Pelton turbine)

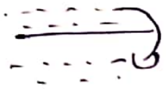
(ii) Radial flow turbine (no more used)

(iii) Axial flow turbine (Kaplan turbine)

(iv) Mixed (Radial and axial) flow turbine (Francis turbine)

(1) Tangential flow turbine → water strikes ~~tang~~ the runner tangential to the Path of rotation

(2) Axial flow turbine → water flows parallel to the axis of the turbine shaft. Kaplan turbine is an axial flow turbine



(3) mixed flow turbine → water enters the blades radially and comes out axially, parallel to the turbine shaft.

(5) According to the disposition of the turbine shaft:

Turbine shaft may be either vertical (or) horizontal. In modern practice Pelton turbines usually have horizontal shafts whereas the rest, especially the large units, have vertical shafts

(6) According to Specific Speed:

$$\text{Specific Speed } N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

N = The normal working speed,

P = Power output of the turbine

H = The net (or) effective head in m

(1) Low Specific Speed turbine

(2) Medium " "

(3) High " "

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Applications of I-m equation:

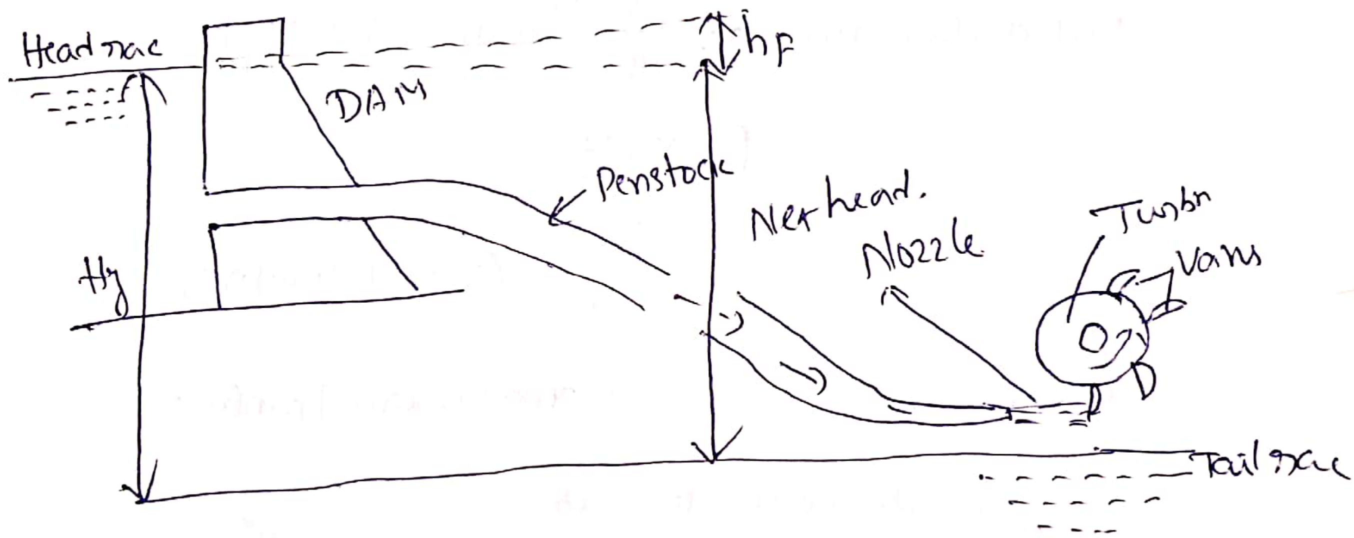
1. To determine the resultant force acting on the boundary of flow passage by a stream of fluid as the stream changes in direction, magnitude (or) both.

(i) pipe bends (ii) Reducers (iii) Moving vanes (iv) Jet propulsion etc.

2. To determine the characteristic of flow when there is an abrupt change of flow section.

(i) Sudden enlargement in a pipe (ii) Hydraulic jump in a channel etc.

layout of Hydroelectric Powerplant:



① **Gross head:** The difference between the head race level and tail race level when no water is flowing is known as Gross head. Hg

② **Net head:** It is also called as effective head, and is defined as the head available at the inlet of the turbine. when water is flowing from head race to turbine, a loss of head due to friction between the water and penstock occurs. other losses like loss due to bend, pipe fittings loss at the entrance of penstock etc. are very small in magnitude.

Hg : Gross head, $H_f = \frac{4fLV^2}{2gD}$ L: length of penstock

Impulse-momentum equation:

The Impulse-momentum equation is one of basic tool for the Solution of flow Problems. Its application leads to the Solution of Problems in fluid mechanics which cannot be solved by energy principle alone.

The momentum equation is based on the law of conservation of momentum (or) momentum principle:

The net force acting on a mass of fluid is equal to change in momentum of flow per unit time in that direction

As per newton's Second law of motion,

$F = ma$ (F→ force acting on fluid m→mass a→acceleration)

But accelaration $a = \frac{dv}{dt}$ (change of velocity)

$$F = m\frac{dv}{dt}$$

$$F = \frac{d(mv)}{dt} \quad (\because 'm' \text{ is constant})$$

This equation is known as momentum principle.

It can also be written as

$$F \cdot dt = d(mv)$$

This equation is known as Impulse-momentum equation.

"The impulse of a force F acting on a fluid mass 'm' in a short interval of time dt is equal to the change of momentum d(mv) in direction of force"

Efficiencies of a turbine

(a) Hydraulic efficiency (η_h) (b) Mechanical efficiency (η_m)
(c) Volumetric efficiency (η_v) (d) overall efficiency (η_o)

(a) Hydraulic efficiency (η_h):- It is defined as the ratio of power given water to the runner of a turbine to the power supplied by the water at the inlet of the turbine. The power at the inlet of the turbine is more and this power goes on decreasing as the water flows over the vanes of the turbine due to hydraulic losses as the vanes are not smooth. Hence the power delivered to the runner of the turbine will be less than the power available at the inlet of the turbine.

$$\eta_h = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}} = \frac{R.P}{W.P}$$

$$R.P = \frac{\frac{w}{g}\left[V_{w_1} \pm V_{w_2}\right] \times u}{1000} \text{ Kw} \quad \text{— for Pelton turbine}$$

$$= \frac{\frac{w}{g}\left[V_{w_1}u_1 \pm V_{w_2}u_2\right]}{1000} \text{ Kw} \quad \text{- for radial flow turbine}$$

$$W.P = \frac{w \times H}{1000} \text{ Kw}$$

(b) Mechanical efficiency (η_m) The power delivered by water to the runner of a turbine is transmitted to the shaft of the turbine. Due to the mechanical losses, the power available at the shaft of the turbine is less than the power delivered to the runner of a turbine. The ratio of power available at the shaft of the turbine to the power delivered to the runner is known as η_m

$$\eta_m = \frac{S.P}{R.P}$$

(c) Volumetric efficiency (η_v) The volume of the water striking the runner of a turbine is slightly less than the volume of the water supplied to the turbine, some of the volume of the water is discharged to the tail race without striking the runner of the turbine. Thus the

$$\eta_v = \frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to turbine}}$$

(d) Overall efficiency (η_o):-

$$\eta_o = \frac{\text{Volume available at the shaft of the turbine}}{\text{Power supplied at the inlet of turbine}}$$

$$= \frac{S.P}{W.P}$$

$$= \frac{S.P}{W.P} \times \frac{R.P}{R.P}$$

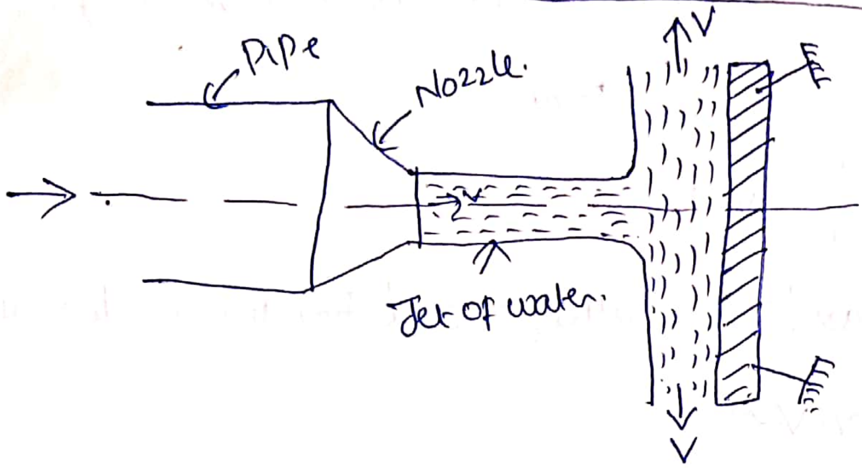
$$= \frac{S.P}{R.P} \times \frac{R.P}{W.P}$$

$$= \eta_m \times \eta_h$$

Shaft power is in kW

water " " "

Force exerted by the jet on a Stationary Vertical plate:



V
d
$a = \frac{\pi}{4} d^2$

The liquid comes out in the form of a jet from the outlet of a nozzle, which is fitted to a pipe through which liquid is flowing under pressure. If some plate, which may be fixed (or) moving, is placed in the path of the jet, a force is exerted by the jet on the plate. This force is obtained from Newton's second law of ~~motion~~ (or) ~~2nd~~ I-m equation. Thus impact of jet means the force exerted by the jet on a plate which may be stationary (or) moving.

1. Force exerted by the jet on a stationary plate
   - Plate is Vertical to jet
   - " " inclined " "
   - Plate is curved.
2. " " " " " " " " moving plate
   - plate is vertical to jet
   - plate is inclined
   - plate is curved.

The jet after striking the plate will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking, will get deflected through 90°. Hence the component of the velocity of jet in the direction of jet, after striking will be zero.

F_x = Rate of change of momentum in the direction of force

$= \dfrac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$

$= \dfrac{(\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{final velocity})}{\text{Time}}$

$= \dfrac{\text{Mass}}{\text{Time}} (I.V - F.V)$

= (Mass / Sec) x (Velocity of Jet before striking - velocity after striking)

$= \rho a v (V - 0)$

$= \rho a v^2$

① Find the force exerted by a jet of water of diameter 75mm on a stationary flat plate, when the jet strikes the plate normally with a velocity of 20m/sec ($\rho = 1000\ kg/m^3$)

$kg/m^3 \times m^2 \times m^2/sec^2 = \dfrac{kg \cdot m}{sec} = N$

② Water is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100mm and the head of water at the centre nozzle is 100m. Find the force exerted by the jet of water on a fixed vertical plate. The coefficient of velocity is given as 0.95

$V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 100} = 44.924\ m/sec$ $\quad C_v = \dfrac{\text{Actual velocity}}{\text{Theoretical velocity}}$

Actual velocity $V = C_v \times V_{th} = 0.95 \times 44.924 = 42.08\ m/sec$

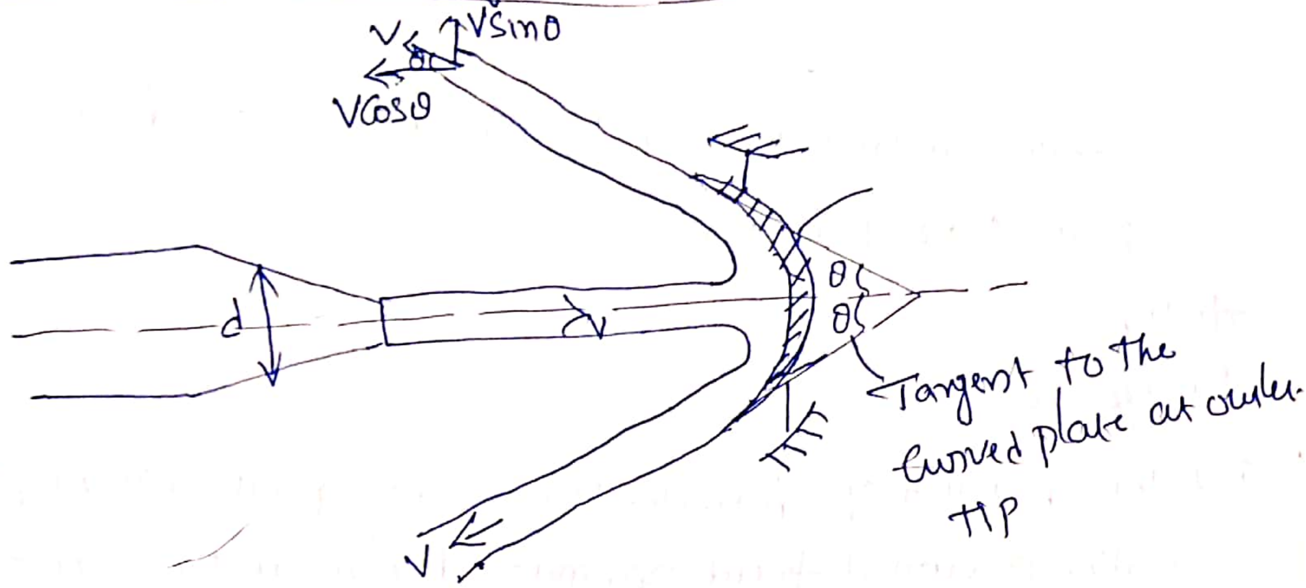
$F = \rho a v^2 = 13.9\ kN$

$F_n = \rho a v (v \sin\theta - 0)$

$F_x = F_n \cos(90° - \theta) = F_n \sin\theta$

$F_y = F_n \sin(90° - \theta) = F_n \cos\theta$

Force exerted on a Stationary Curved Plate



Jet Strikes the curved plate at the Centre:

Consider a fluid jet striking a stationary curved plate (smooth) at the Centre as shown in fig. The jet striking the plate comes out with the same velocity, in the tangential direction of the curved plate.

The velocity at the outlet of the plate can be resolved into the following two components:

(i) Component of velocity in the direction of jet $= -V\cos\theta$

(-ve sign indicates that the velocity at the outlet is in a direction opposite to that of the fluid jet coming out from nozzle)

(ii) Component of velocity perpendicular to the jet $= V\sin\theta$

Applying I.M equation, we have

force exerted by the jet (in the direction of jet)

$F_x = \rho a V (V_{1x} - V_{2x})$ (V_{1x} = Initial velocity in the direction of jet, V_{2x} = final velocity " " ")

$F_x = \rho a V (V - (-V\cos\theta))$

$= \rho a V^2 (1 + \cos\theta)$

Similarly, $F_y = \rho a v (V_{1y} - V_{2y})$

$= \rho a v (0 - V \sin\theta)$

$= -\rho a v^2 \sin\theta.$

-ve sign indicates that force is acting in the downward direct

Note: Angle of defection = 180 - θ

Problems

① A jet of water of diameter 50mm moving with a velocity of 40m/se strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of the jet, if the jet is deflected through an angle of 120° at the outlet of the curved plate.

$a = 0.001963 m^2$; $F_x = 4711.15N$

② A jet of water of diameter 75mm moving with a velocity of 30m/s strikes a curved fixed plate tangentially at one end at an angle of 30° to the horizontal. The jet leaves the plate at an angle of 20° to the horizontal. Find the force exerted by the jet on the plate in the horizontal & vertical directions.

$F_x = \rho a v^2 (\cos\theta + \cos\phi) = 7178.2N$; $F_y = \rho a v^2 (\sin\theta - \sin\phi) = 628.13N$.

③ A jet of water of diameter 50mm strikes a fixed plate in such a way that the angle between the plate & jet is 30°. The force exerted in the direction of jet is 1471.5 N. Determine the rate of flow of water.

$F_x = \rho a v^2 \sin^2\theta \Rightarrow V = 54.77 m/s$

$Q = A \times V$

$= 0.001963 \times 54.77 = 0.1075 m^3/s = 107.5 lit/s$

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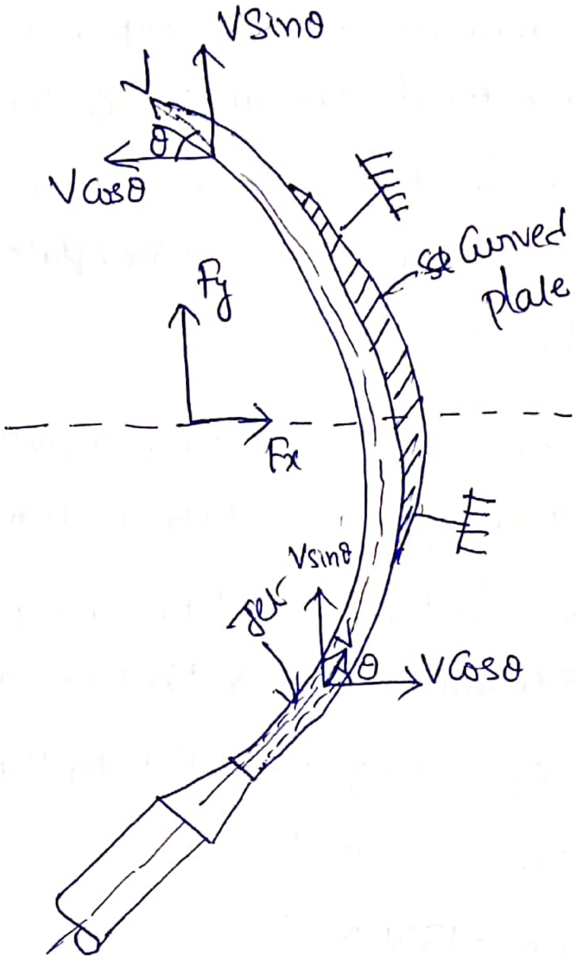
(ii) Jet strikes the curved plate at one end tangentially when the plate is symmetric



Let the jet strikes the curved plate at one end tangentially as shown in fig. Let the curved plate is symmetrical about x-axis. Then the angle made by the tangents at the two ends of the plate will be same.

Let V = Velocity of jet water

θ = Angle made by jet with x-axis at inlet tip of the curved plate

If the plate is smooth and loss of energy due to impact is zero, then the velocity of water at the outlet tip of the curved plate will be equal to ~~Zero~~ V.

The force exerted by the jet of water in the directions of x and y are

F_x = (mass/sec) × $[V_{1x} - V_{2x}]$

$= \rho a V[V\cos\theta - (-V\cos\theta)]$

$= 2\rho a V^2\cos\theta$

$F_y = \rho a V[V_{1y} - V_{2y}]$

$= \rho a V[V\sin\theta - V\sin\theta]$

$= \rho a V[0] = 0$

Problems on jet Striking The Curved Vane at Centre,

① A jet of water of diameter 40mm moving with a velocity 30m/sec, Strikes a Curved fixed Symmetrical plate at The Centre find the force exerted by the jet water in the direction of the jet if The jet is deflected through an angle of 120° at the outlet of The Curved plate

$A = 0.001256 m^2$; $F_x = 1695.6 N$

② A jet of water 20mm diameter and moving at 15m/sec strikes upon The Centre of a Symmetrical Vane, After impingement, the jet gets deflected Through 160° by the Vane. Presuming Vane to be Smooth, det. (i) The force exerted by jet on the vane and

(ii) The ratio of Velocity at outlet to That at inlet if actual reaction of The vane is 127N.

(i) $F = \rho a v^2 (1+\cos\theta) = 137.1 N$

(ii) $\frac{V_2}{V_1}$:

If the vane is not Smooth, then outgoing velocity at the Vane tips = less Than the incoming Velocity (i.e.) $\frac{V_2}{V_1} = K$ where $K<1$,

$$F = \rho a v^2 (1+K\cos\theta)$$

$$127 = 1000 \times \pi/4 (0.02)^2 \times 15^2 (1+K\cos 20°)$$

$$1+K\cos 20° = \frac{127}{1000 \times (\pi/4 \times 0.02^2) \times 15^2} = 1.796.$$

$$K = \frac{1.796-1}{\cos 20°} = 0.847$$

(iii) Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical:

When the curved plate is unsymmetrical about x-axis, then the angle made by the tangents drawn at the inlet and outlet tips of the plate with x-axis will be different.

Let θ = angle made by tangent at inlet tip with x-axis

ϕ = angle made by tangent at outlet tip with x-axis

The two components of velocity at inlet are

$$V_{1x} = V\cos\theta, \quad V_{1y} = V\sin\theta$$

The two components of velocity at outlet are

$$V_{2x} = -V\cos\phi, \quad V_{2y} = V\sin\phi.$$

∴ The force exerted by jet of water in the directions of x and y are

$$F_x = \rho a v[V_{1x} - V_{2x}] = \rho a v[V\cos\theta - (-V\cos\phi)] = \rho a v^2[\cos\theta + \cos\phi]$$

$$F_y = \rho a v[V_{1y} - V_{2y}] = \rho a v[V\sin\theta - V\sin\phi] = \rho a v^2[\sin\theta - \sin\phi]$$

Problems on above two cases.

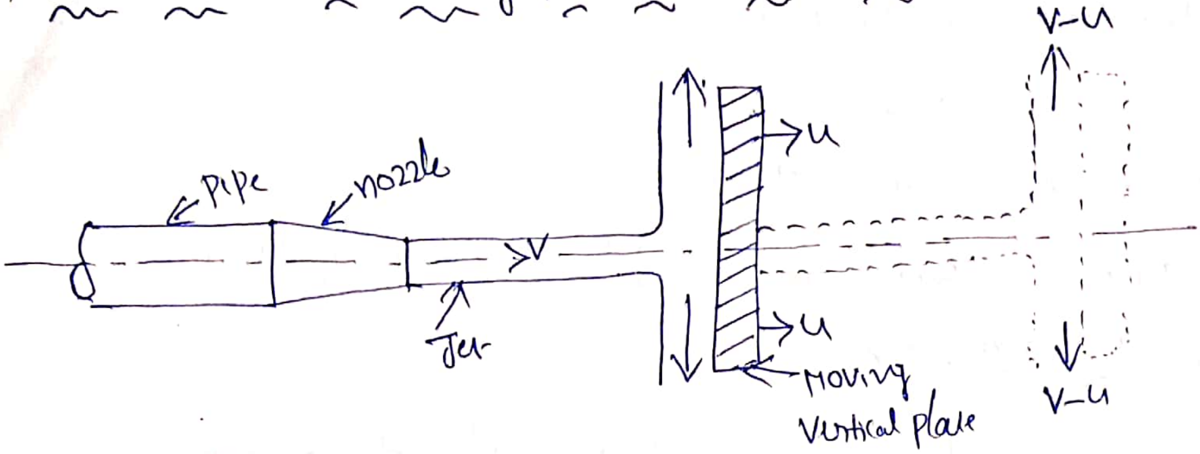
① A jet of water of diameter 75mm moving with a velocity of 30 m/s, strikes a curved fixed plate tangentially at one end at angle of 30° to the horizontal. The jet leaves the plate at an angle of 20° to the horizontal. Find the force exerted by the jet on the plate in the horizontal & vertical directions.

$F_x = 717.82$ N

$F_y = 628.13$ N.

force exerted by the jet on the moving plates

(i) force exerted on moving flat plate held normal to jet



The above figure shows a fluid jet striking a flat vertical plate moving with a uniform velocity away from the jet

Let V = absolute velocity of the jet

a = Cross-Sectional area of the jet and

u = Velocity of the flat plate held normal to the jet.

The relative velocity with which the jet strikes the plate is $(V-u)$

Mass of water striking the plate per second = $\rho a(V-u)$

∴ force exerted by the jet on the plate in the direction of jet,

F_x = Mass of water striking the plate/sec × (initial velocity with which water strikes - final velocity)

$F_x = \rho a(V-u)[(V-u)-0]$

$F_x = \rho a(V-u)^2$

work done = force × the distance through which the body moves in the direction of force/sec

∴ work done = $\rho a(V-u)^2 \times u$

(Note: work done = N-m/sec $\frac{N \cdot m}{sec}$ = watt

Problems:

① A nozzle of 60mm diameter delivers a stream of water at 24m/sec perpendic- to a plate that moves away from the jet at 6m/sec.

find (i) The force on the plate

(ii) The work done

(iii) The efficiency of the jet

Ans (i) 916N (ii) 5496 N.m/s (iii) K.E = $\frac{1}{2}mV^2 = \frac{1}{2}(\rho aV) \times V^2 = 19543.2$ N·m/s

$\eta_{jet} = \frac{\text{work done}}{K.E} = 28.1\%$

② A jet of water of diameter 10cm strikes a flat plate normally with a velocity of 15m/sec. The plate is moving with a velocity of 6m/s in the direction of the jet and away from the jet.

find (i) the force exerted by the jet on the plate

(ii) work done by the jet on the plate/second

(iii) Power of the jet in kw

(iv) Efficiency of the jet.

Ans (i) F = 636.17N (ii) 3817.02 N-m/s (iii) Power = $\frac{\text{work done/sec}}{1000}$ = 3.817kw

(iv) 28.8%

③ A nozzle of 50mm diameter delivers a stream of water at 20m/s to plate that moves away from the jet at 5m/sec

Find (i) The force on the plate

(ii) The work done and

(iii) The efficiency of the jet

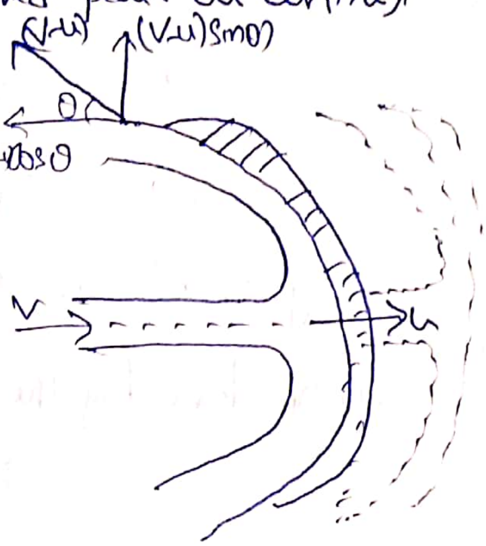
Ans (i) $F_x = 441.78$N

(ii) W.D = 2208.9 N-m/sec

(iii) η = 33.77% 28%

(ii) Force exerted on the curved plate when the plate is moving in the direction of jet (i.e. jet strikes the moving curved plate at centre).

Let a jet of water strikes a curved plate at the centre which is moving with a uniform velocity in the direction of the jet as shown in fig.



Let V = absolute velocity of jet

a = Area of jet

u = velocity of the plate in the direction of the jet

Relative velocity of the jet of water (or) the velocity with which jet strikes the curved plate = (V-u)

∴ If the plate is smooth and loss of energy due to impact of jet is zero, then the velocity with which the jet will be leaving the curved vane = (V-u)

This velocity can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of the jet.

Component of velocity in the direction of the jet :- $-(V-u)\cos\theta$

" " " ⊥er " " " " " :- $(V-u)\sin\theta$.

Mass of water striking the plate = ρ x a x velocity with which jet strikes the plate

$= \rho a(V-u)$.

∴ Force exerted by the jet of water on the curved plate in the direction of the jet.

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F_x = Mass striking per sec × [Initial velocity with which jet strikes the plate in the direction of jet − final velocity]

$= \rho a(V-u)\left[(V-u) - (V-u)\cos\theta\right]$

$= \rho a(V-u)^2\left[1+\cos\theta\right]$

$F_y = -\rho a(V-u)^2 (\sin\theta)$

work done by the jet on the plate / second

= F_x × distance travelled / sec in the direction of x.

$= F_x \times u = \rho a(V-u)^2[1+\cos\theta]\times u$

$= \rho a(V-u)^2 \times u[1+\cos\theta]$

Problems

① A jet of water of diameter 7.5 cm strikes a curved plate at its centre with a velocity of 20 m/sec. The curved plate is moving with a velocity of 8 m/sec in the direction of the jet. The jet is deflected through an angle of 165°. Assuming plate is smooth

find (i) force exerted on the plate in the direction of jet (ii) power of the jet & (iii) efficiency of the jet

Ans F_x = 1250.38 N, work done/sec = 10003.04 N-m/sec, Power = 10 kW, η = 56%.

13/8/14 (3A) Absent — 3, 5, 8, 10, 13, 15, 24, 26, 27, 28, 33, 34, 35, 39, 40, 42, 43, 44, 47, 50, 51, 52, 53, 55, 56, 58, 61, 301, 302, 303, 304, 305, (31, 52) ~~1129, 24, 32~~,

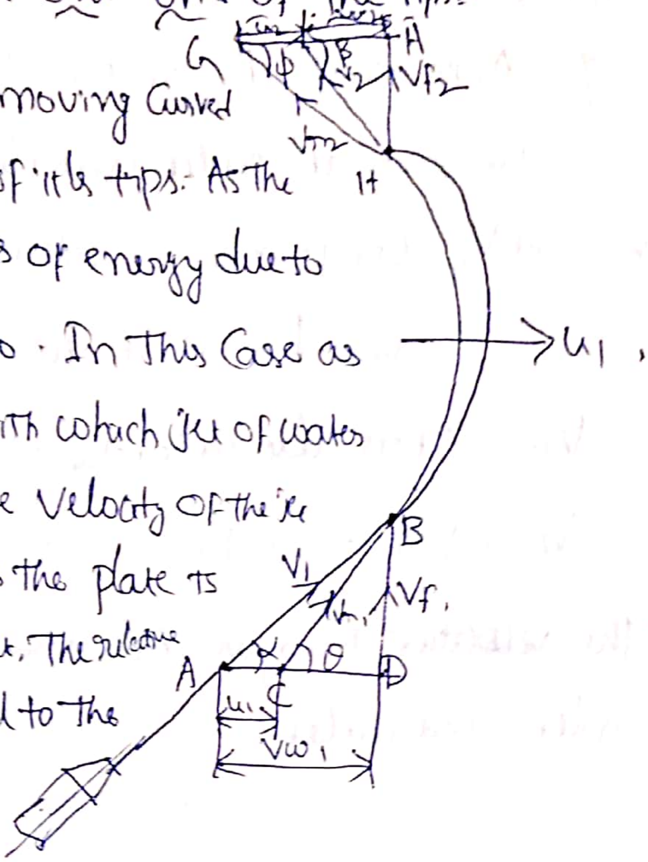
13/8/14 (3B)

Prsn : 62, 70, 76, 86, 93, 96, 98, 308, 91, 92, 310, 97, 115, 89, 112,

13/8/14 (3B)

Presentees : 62, 70, 81, 86, 89, 92, 96, 97, 115, 1074, 6309

Force exerted by a jet of water on an unsymmetrical moving curved plate when jet strikes tangentially at one end of the tips.



The jet of water striking a moving curved plate tangentially, at one end of its tips. As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero. In this case as plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of the jet with respect to the plate. Also as the plate is moving in different direction of jet, the relative velocity at inlet will be ~~only~~ equal to the vector difference of the velocity of the jet and velocity of plate at inlet.

Let V_1 = Velocity of the jet at inlet

u_1 : Velocity of the plate at inlet

V_{r_1} = relative velocity of the jet and plate at inlet

α = Angle between the direction of the jet and direction of motion of the plate, also called guide blade angle

θ : Angle made by the relative velocity (V_{r_1}) with the direction of motion at inlet, also called vane angle at inlet

V_{w_1} & V_{f_1} : The components of the velocity of the jet V_1, in the direction of motion and perpendicular to the direction of motion of vane respectively

V_{w_1} : It is also known as velocity of whirl at inlet

V_{f_1} = It is also known as velocity of flow at inlet

V_2 = Velocity of jet leaving the vane (or) velocity of jet at outlet of the vane

u_2 = Velocity of the vane at outlet

V_{r_2} : Relative velocity of the jet with respect to the vane at outlet

β : Angle made by the velocity V_2 with the direction of motion of vane at outlet

ϕ : Angle made by relative velocity V_{r_2} with the direction of motion of the vane at outlet and also called vane angle at outlet

V_{w_2} and V_{f_2} : Components of velocity V_2 in the direction of motion of vane and perpendicular to the direction of motion of vane at outlet.

V_{w_2} : It is also called the velocity of whirl at outlet

V_{f_2} : velocity of flow at outlet

The ~~velocities~~ triangles ABD and EGH are called the velocity triangles at inlet and outlet

B – 68, 83, 89, 93, 94, 96, 98, 116, 111, 116040, (Present)

A – 7, 22, 23, 28, 36, 43, 58, 11 29, 31, 52, 12, 306 (L) (Pr

2/8/14

B – 66, 68, 86, 93, 96, 308, 309, 313 (Present), 70, 76, 89, 94, 98, 108, 111, 116, 113

~~A~~ 56 A – 1, 3, 8, 10, 13, 14, 15, 17, 18, 21, 24, 25, 26, 27, 30, 31, 33, 34, 35, 36, 38, 39, 40, 42
43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 302, 303, 304, ~~329~~, 31

22/8/14

A → 5, 5, 9, 10, 12, 13, 15, 16, 21, 24, 25, 26, 27, 28, 30, 33, 34, 35, 36, 38, 39, 40, 42, 43, 45, 46, 47
49, 50, 51, 53, 55, 58, 302, 303, 304, 305, 307, (29, 31) (Absent) 9, 28, 49, 302, 307

B → 62, 66, 68, 89, 93, 98, 115, 309, 312, 313 (Present) ~~113~~, 86, 108, 70, 310, 116040, 113
308, 94, 81, 76

B → 62, 66, 68, 89, 93, 98, ~~1~~115, 309, ~~312~~, 313, 86, 108, 70, ~~310~~ 113, 308, 94, 81, 76 (Afternoon)

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Inlet velocity triangle

① Take any Point A and draw a line $AB = V_1$ (in magnitude), making an angle α with the horizontal line AD

② Draw a line $AC = u_1$ and join C to B, CB Then represents relative velocity (V_{r_1}) of the jet at inlet. If the loss of energy at inlet due to impact is zero, Then CB must be in tangential direction to The vane at inlet.

③ From B draw a ⊥er BD meeting The horizontal line AC Produced at D. Then BD represents The velocity of flow at inlet (V_{f_1}). AD represents The velocity of whirl at inlet (V_{w_1}). ∠BCD = θ = Vane angle at inlet.

outlet Velocity triangle

If The Vane surface is assumed to be smooth, The energy loss due to friction will be Zero and Thus $V_{r_1} = V_{r_2}$ will be in tangential direction to The Vane at outlet.

① Draw B'C' in The tangential direction of The Vane at outlet and cut $B'C' = V_{r_2}$

② From C' draw a line C'A' in The direction of Vane at outlet and equal to u_2 (The velocity of Vane at outlet). Join B'A'. Then B'A' represents The absolute velocity of The jet (V_2) at outlet in magnitude and direction.

③ From B' draw a ⊥er B'D' to meet The line C'A' Produced at D'. The B'D' and A'D' represent The velocity of flow (V_{f_2}) and velocity of whirl (V_{w_2}) at outlet respectively

④ ϕ = Angle of Vane at outlet, β = angle made by V_2 with the direction of motion of Vane at outlet

If Vane is smooth and is having velocity in the direction of motion at inlet and outlet equal, then $u_1 = u_2 = u$ and $V_{r_1} = V_{r_2}$

Mass of water striking the vane/second $= \rho a V_{r_1}$

$\therefore$ Force exerted in the direction of motion,

F_x = Mass of water striking the vane/sec × (Initial velocity with which the jet strikes in the direction of motion − final velocity).

$$= \rho a V_{r_1}[V_{r_1}\cos\theta - (-V_{r_2}\cos\phi)]$$

But $V_{r_1}\cos\theta = (V_{w_1} - u_1)$ and $V_{r_2}\cos\phi = (u_2 + V_{w_2})$

$$F_x = \rho a V_{r_1}[(V_{w_1} - u_1) - (-(u_2 + V_{w_2}))]$$

$$= \rho a V_{r_1}[V_{w_1} - u_1 + u_2 + V_{w_2}]$$

$$= \rho a V_{r_1}(V_{w_1} + V_{w_2}) \quad (\because u_1 = u_2) \quad —①$$

The equation ① is true only when β is an acute angle & when $\beta = 90°$, $V_{w_2} = 0$

The equation ① reduces to

$$F_x = \rho a V_{r_1}(V_{w_1})$$

If β is an obtuse angle, the expression for F_x will become

$$F_x = \rho a V_{r_1}(V_{w_1} - V_{w_2})$$

Thus in general F_x is written as

$$F_x = \rho a V_{r_1}(V_{w_1} \pm V_{w_2})$$

work done/sec by the jet on the vane

$$= F_x \times u = \rho a V_{r_1}(V_{w_1} \pm V_{w_2}) \times u.$$

$\therefore$ work done/sec per unit weight of fluid striking.

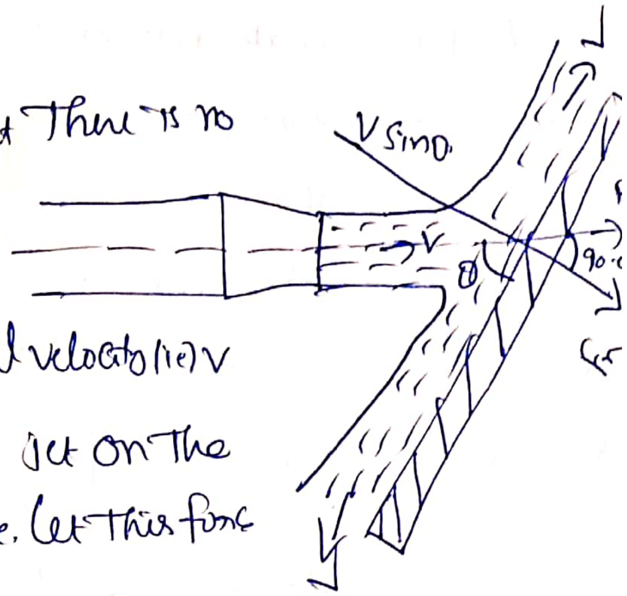
$$= \frac{\rho a V_{r_1}(V_{w_1} \pm V_{w_2}) \times u}{\text{weight of fluid striking}}$$

$$= \frac{\rho a V_{r_1}(V_{w_1} \pm V_{w_2}) \times u}{\rho a V_{r_1} \times g} = \frac{1}{g}(V_{w_1} \pm V_{w_2}) \times u.$$

Force exerted by jet on stationary inclined flat plate:

Let θ = angle between jet & plate

If the plate is smooth & assumed that there is no loss of energy due to impact of the jet, then jet will move over the plate after striking with a velocity equal to initial velocity (i.e.) V



Let us find the force exerted by the jet on the plate in the direction normal to the plate. Let this force is represented by F_n

$\therefore F_n = \text{mass}[\text{I.V in n direction} - \text{F.V in n direction}]$

$= \rho a V[V\sin\theta - 0] = \rho a V^2 \sin\theta$

The force can be resolved into two components, one in the direction of jet & other perpendicular to the direction of jet

$\therefore F_x$ = Component of F_n in the direction of flow

$= F_n \cos(90-\theta) = F_n \sin\theta = \rho a V^2 \sin^2\theta$

F_y = Component of F_n $\perp$ to flow

$= F_n \sin(90-\theta) = F_n \cos\theta = \rho a V^2 \sin\theta \cos\theta$

① A jet of water of diameter 75 mm moving with a velocity of 25 m/s strikes a fixed plate in such a way that the angle between the jet and plate is 60°. Find the force exerted by the jet on the plate (i) in the direction normal to the plate (ii) in the direction of the jet

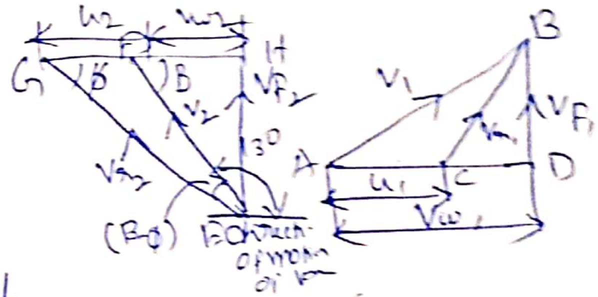
Sol: $d = 75$ mm $a = 0.004417 \, m^2$ $F_n = 2390.7$ N ; $F_x = 2070.4$ N.

Problems on unsymmetry plate (Inlet outlet velocity triangle)

① A Jet of water having a velocity of 20m/s strikes a curved vane, which is moving with a velocity of 10m/s. The jet makes an angle of 20° with the direction of motion of vane at inlet and leaves at angle of 130° to the direction of motion of vane at outlet. Cal. (i) vane angles so that water enters and leaves the vane without shock (ii) workdone/sec/unit weight of water striking.

Soln $V_1 = 20m/s$; $u_1 = 10m/s$; $\alpha = 20°$; $\beta = 180-130 = 50°$, $u_1 = u_2 = 10m/s$, $V_{r_1} = V_{r_2}$

(i) Vane angle: $\tan\theta = \frac{BD}{CD}$



$$= \frac{V_{f_1}}{AD-AC} = \frac{V_{f_1}}{V_{w_1}-u_1}$$

where $V_{f_1} = V_1 \sin\alpha = 6.84m/s$

$V_{w_1} = V_1\cos\alpha = 20\times\cos 20° = 18.794m/s$.

$\therefore \theta = 37.875°$.

$\sin\theta = \frac{V_{f_1}}{V_{r_1}}$ (or) $V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{6.84}{\sin 37.875°} = 11.14$

$V_{r_1} = V_{r_2} = 11.14m/s$.

Sine rule, we have $\frac{V_{r_2}}{\sin(180-\beta)} = \frac{u_2}{\sin(\beta-\phi)}$

$\frac{11.14}{\sin\beta} = \frac{10}{\sin(\beta-\phi)}$ (or) $\frac{11.14}{\sin 50°} = \frac{10}{\sin(50°-\phi)}$

$\sin(50°-\phi) = \frac{10\times\sin 50}{11.14} = 0.6876 = \sin 43.44$.

$50-\phi = 43.44$ (or) $\phi = 6.56°$.

(ii) workdone/sec/unit weight $= \frac{1}{g}(V_{w_1}+V_{w_2})\times u$

$\therefore$ workdone/unit weight of water $= \frac{1}{9.81}[18.794+1.067)\times 10$

$V_{w_2} = V_{r_2}\cos\phi - u_2$

$= 20.24$ N-m/N.

1617 cm
60, 62, 67, 72, 77, 78, 79, 85, 91, 92, 95, 97, 108, 322, 327, 12-23

② A jet of water having a velocity of 45m/s impinges without shock on a Series of vanes moving at 15m/s. The direction of motion of the vanes is inclined at 20° to that of jet. The relative velocity at outlet is 0.9 of that at inlet, and absolute velocity of water at exists is to be normal to motion of vanes. find

$180-\beta+\phi=180$

$x=\beta-\phi$

(i) Vane angles at inlet and outlet

(ii) work done on vanes per(N) of water supplied by the jet and

(iii) Hydraulic efficiency.

$\alpha=20°$

$V_{r_2}=0.9V_{r_1}$,

$V_1=45m/s$, $u_1=u_2=u=15m/sec$, $\alpha=20°$, $V_{r_2}=0.9V_{r_1}$,

Ans: $\theta=57.87°$, $29.42°$, workdone/N of water $=\frac{1}{g}(V_{w_1}u_1+V_{w_2}u_2)$

$\theta=29.42°$

$\phi=57.87°$

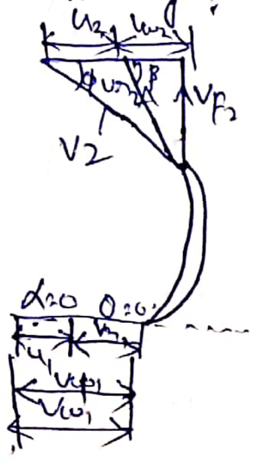
$=\frac{1}{g}(V_{w_1}u_1)=64.66\ \frac{N\text{-}m}{N\text{-}s}$ (or) $\frac{J/s}{N}$

$$\eta_{hyd}=\frac{W.D}{K.E\ \text{supplied by jet}}=\frac{64.66\ (\text{Per N})}{\frac{V_1^2}{2g}\ (\text{Per N})}=62.65\%.$$

③ A jet of 50mm diameter impinges on a Curved vane and is deflected through an angle of 175°. The vane moves in the same direction as that of jet with a velocity of 35m/s. If the rate of flow is 170lit/sec, Determine the Component of force on the vane in the direction of motion. How much would be the Power developed by the vane and what would be the water efficiency?

(since the jet of water moves in the same direction as that of vane, $\alpha=0=\theta$)

$V_1=Q/a=\frac{0.17}{0.001963}=86.6m/s$, $V_{r_1}=V_1-u_1=86.6-35=51.6m/sec$,



$\phi=180-175=5°$, Since the vane is Smooth ∴ $V_{r_2}=V_{r_1}=51.6m/sec$.

$V_{w_2}=V_{r_2}\cos\phi-u_2=16.4m/s$,

Power force exerted by the jet on vane $F=\rho a V_{r_1}(V_{w_1}+V_{w_2})=10432.9N$

workdone = force × velocity = 10432.9×35 = 36151 N-m/s (or) J/sec.

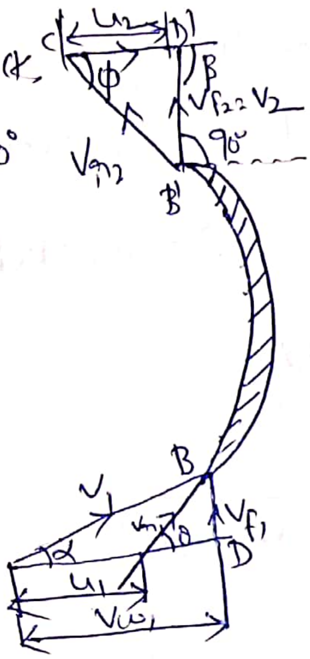
Power developed = 36151 J/s (or) W (or) 365.151 kW.

$\eta_{water}=\frac{W.D}{K.E}=\frac{365151}{\frac{1}{2}\rho Q V_1^2}=57.3\%$.

Scanned with CamScanner

Problems

① A jet of water having a velocity of 40m/s strikes a curved vane, which is moving with a velocity of 20m/s. The jet makes an angle of 30° with the direction of motion of vane at inlet and leaves at an angle of 90° to the direction of motion of vane at outlet. Draw the velocity triangle at inlet and outlet and determine the vane angles at inlet and outlet so that the water enters and leaves the vane without shock.



$V_1 = 40m/s$, $u_1 = 20m/s$, $\alpha = 30$, $\beta = 180-90 = 90°$

Here, we have $u_1 = u_2 = u = 20m/s$

Vane angles at ~~outlet~~ inlet

From ΔBCD we have

$$\tan\theta = \frac{BD}{CD} = \frac{BD}{AD-AC} = \frac{V_{f_1}}{V_{w_1} - u_1}$$

where, $V_{f_1} = V_1 \sin\alpha = 40 \times \sin 30 = 20m/sec$

$V_{w_1} = V_1 \cos\alpha = 40 \times \cos 30 = 34.64m/sec$

$u_1 = 20m/sec.$

$$\tan\theta = \frac{20}{34.64-20} = 1.366.$$

$$\theta = \tan^{-1}(1.366) = 53.79°$$

$V_1 = 10m/s$
$u_1 = 10m/s.$
$\alpha = 20°$; $\beta = 130°.$
Find θ & ϕ
$\theta = 35°$ $\phi = 6°$
$W.D = \frac{1}{g}(V_{w_1} + V_{w_2}) \times u$

Again from ABCD, we have

$$\sin\theta = \frac{V_{f_1}}{V_{r_1}} \text{ (or) } V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{20}{\sin 53.79°} = 24.78m/sec$$

But, $V_{r_2} = V_{r_1} = 24.78m/sec.$

Hence from $\Delta B'C'D'$, we have

$$\cos\phi = \frac{u_2}{V_{r_2}} = \frac{20}{24.78} = 0.8071 \Rightarrow \phi = \cos^{-1}(0.8071) = 36.18°$$

(2) A Jet of water having a velocity of 40m/s strikes a curved vane which is moving with a velocity 20m/s. The jet makes an angle of 30° with the direction of motion of vane at inlet and leaves at an angle of 90° to the direction of motion of vane at outlet. Draw the velocity triangles at inlet & outlet and determine the vane angles at inlet & outlet so that the water enters and leaves the vane without shock.

180-90

Sol: $V_1 = 40\,m/s$; $u_1 = 20\,m/s$; $\alpha = 30^\circ$; $\beta = 90^\circ$, $u_1 = u_2 = 20\,m/s$, $V_{r_1} = V_{r_2}$

Vane angles: $\tan\theta = \frac{BD}{CD} = \frac{V_{f_1}}{V_{w_1} - u_1}$

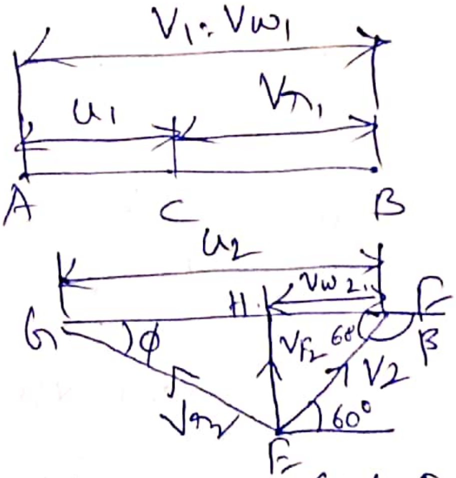
$V_{f_1} = \sin\alpha \cdot V_1 = 20\,m/s$; $V_{w_1} = \cos\alpha \cdot V_1 = 34.64\,m/s$; $\tan\theta = 1.366$; $\theta = 53.79^\circ$

$\cos\phi = \frac{u_2}{V_{r_2}}$; $V_{r_2} = V_{r_1}$; $\sin\theta = \frac{V_{f_1}}{V_{r_1}} \Rightarrow V_{r_1} = \frac{V_{f_1}}{\sin\theta} \Rightarrow V_{r_1} = 24.78$

$\cos\phi = \frac{20}{24.78} = 0.8071 \Rightarrow \phi = 36^\circ$

(3) A jet of water of diameter 50mm having a velocity of 20m/s strikes a curved vane which is moving with a velocity of 10m/s in the direction of the jet. The jet leaves the vane at an angle of 60° to the direction of motion of vane at outlet. Determine:
(i) The force exerted by the jet on the vane in the direction of motion
(ii) workdone/sec by the jet

Sol: As jet and vane are moving in same direction $\alpha = 0$



$\beta = 180 - 60 = 120^\circ$

$F_x = \rho a V_{r_1}[V_{w_1} - V_{w_2}] = 294.45\,N$.

$V_{r_1} = V_1 - u_1 = 10\,m/s$.

$V_{w_1} = V_1 = 20\,m/s$.

$\frac{EG}{\sin 60^\circ} = \frac{GF}{\sin(120-\phi)} \Rightarrow \frac{10}{\sin 60^\circ} = \frac{10}{\sin(120-\phi)}$

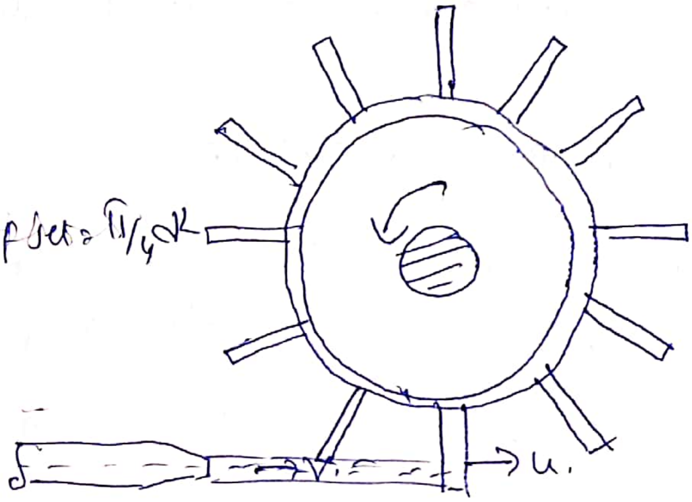
$60 = 120 - \phi \Rightarrow \phi = 60^\circ$

$V_{w_2} = u_2 - V_{r_2}\cos\phi = 5\,m/s$.

$\therefore$ W.D/s $= F_x \times u = 2944.5$ N-m/s

Force exerted by jet of water on a Series of Vanes:

The force exerted by a jet of water on a single moving plate which is not practically possible. This case is only a theoretical one. In actual practice a large number of plates are mounted on the circumference of a wheel at a fixed distance



Let V = velocity of jet

d = diameter of jet

a = Cross-sectional area of jet = $\pi/4 d^2$

u = velocity of vane

mass of water/sec = $\rho a V$

Jet strikes the plate with a velocity = (V-u).

After striking, velocity of jet becomes zero.

$F_x = \rho a V((V-u)-0) = \rho a V(V-u)$.

Workdone by jet on series of plates per second

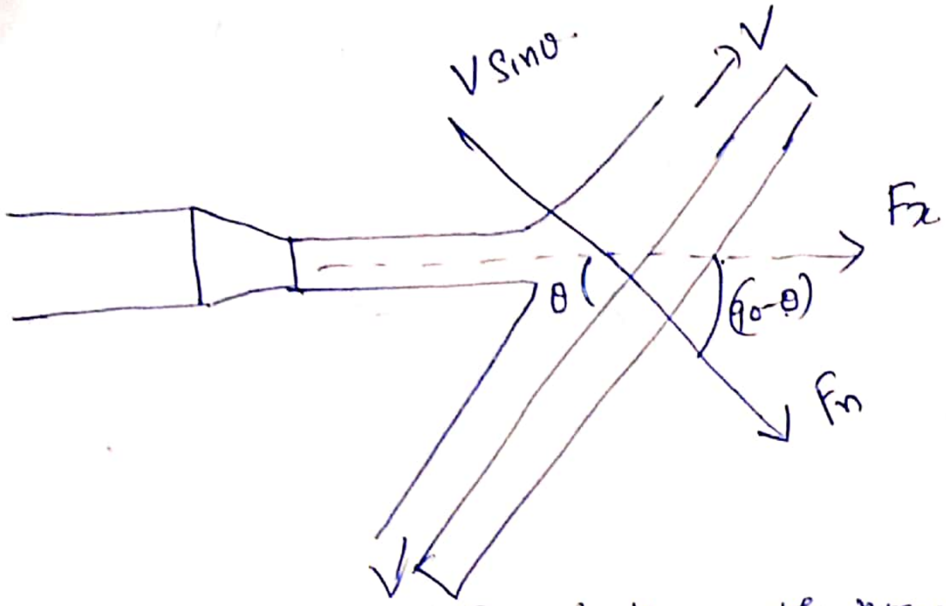
= Force × Distance/sec in the direction of force

$= F_x \times u = \rho a V(V-u) \times u$.

$\therefore K.E/sec = \frac{1}{2} mV^2 = \frac{1}{2}(\rho a V) \times V^2 = \frac{1}{2} \rho a V^3$.

$$\therefore \eta = \frac{\rho a V(V-u) \times u}{\frac{1}{2} \rho a V^3}$$

$$= \frac{2(V-u) \times u}{\cancel{\rho a} V^2}$$



Fn = mass [IV of jet before striking in the normal direction —
FV " aft " "

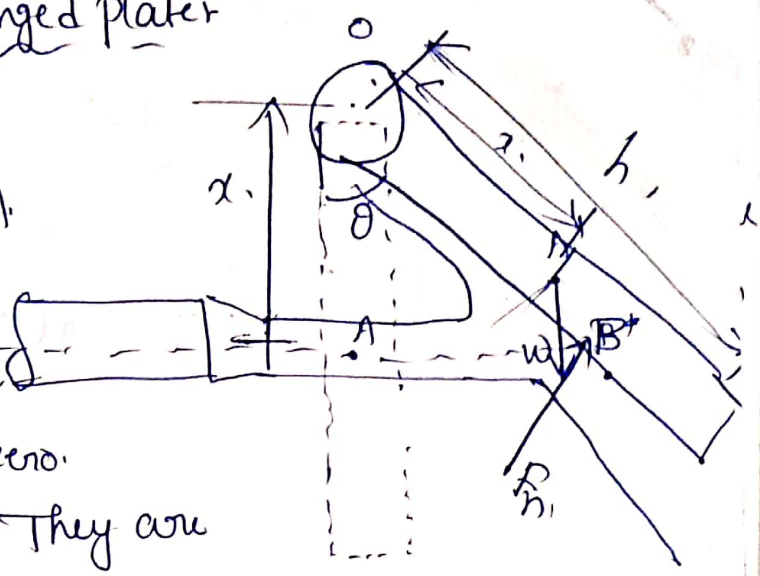
$F_n = \rho a v(v\sin\theta - 0) = \rho a v^2 \sin\theta$.

This force can be resolved into two components -
In the direction of the jet   ⊥er to the direction of the jet

$f_x = F_n \cos(90-\theta) = F_n \sin\theta$

$f_y = F_n \sin(90-\theta) = F_n \cos\theta$

Force exerted by a jet on a hinged plate:



$OA = OA' = x$.

weight of plate is acting at A.

when the plate is in equilibrium after jet strikes the plate, the moment of all the forces about hinge is zero.

Two forces are acting on plate. They are

(1) $F_n = \rho a v^2 \sin\theta'$

θ' = Angle between jet and plate $= 90° - \theta)$

(2) weight of the plate w.

Moment of force F_n about hinge $= F_n \times OB$

$= \rho a v^2 \sin(90 - \theta) \times OB$

$= \rho a v^2 \cos\theta \frac{OA}{\cos\theta} = \rho a v^2 \times OA$

$= \rho a v^2 x x$.

Moment of weight w about hinge $= w \times OA' \sin\theta = w \times x \times \sin\theta$.

$\therefore$ for equilibrium of the plate, $\rho a v^2 x x = w \times x \times \sin\theta$

$\sin\theta = \frac{\rho a v^2}{w}$.

The angle of swing of the plate about hinge can be calculated from above equation.

Problem: $d = 2.5 cm$; $V = 10 m/s$ $w = 98.1 N$

Sol: $\theta = 29.96°$.