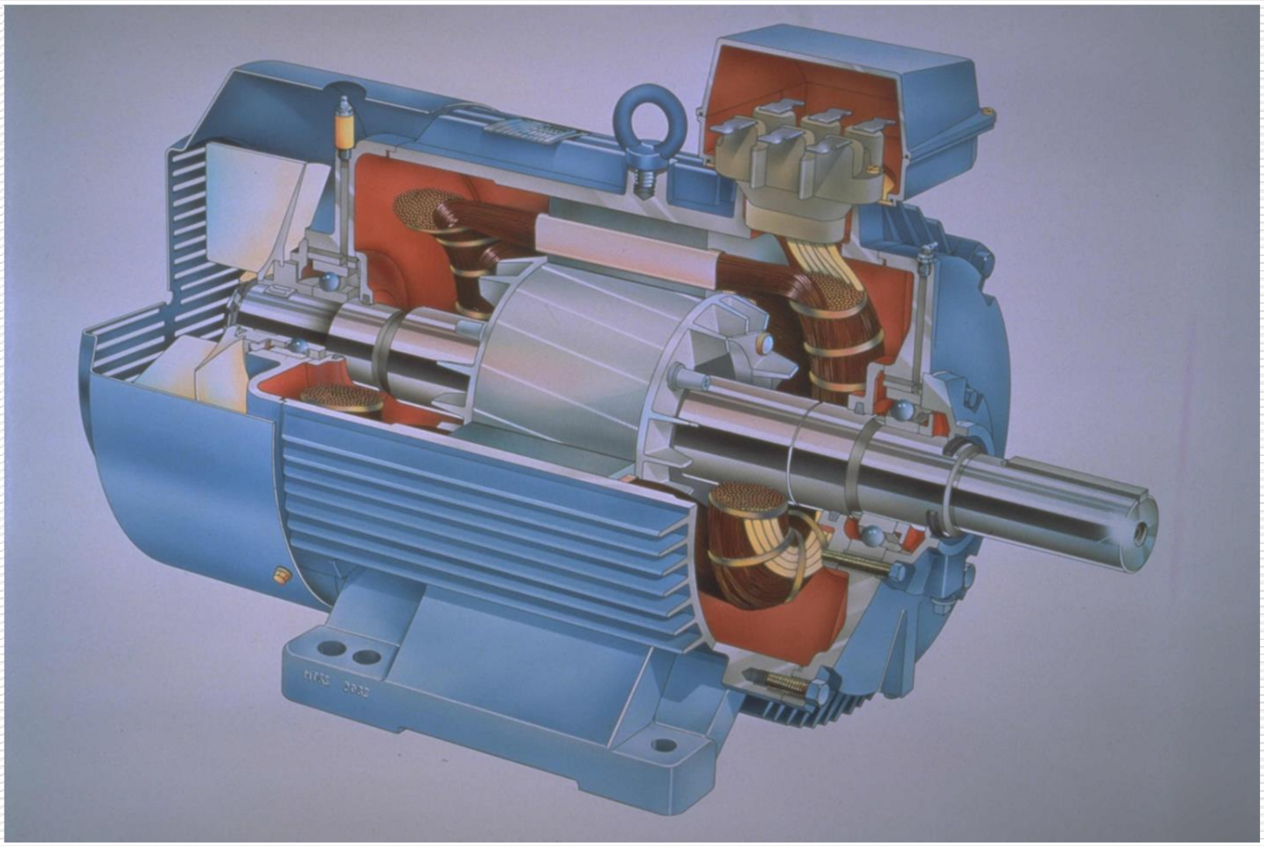
Induction Motors



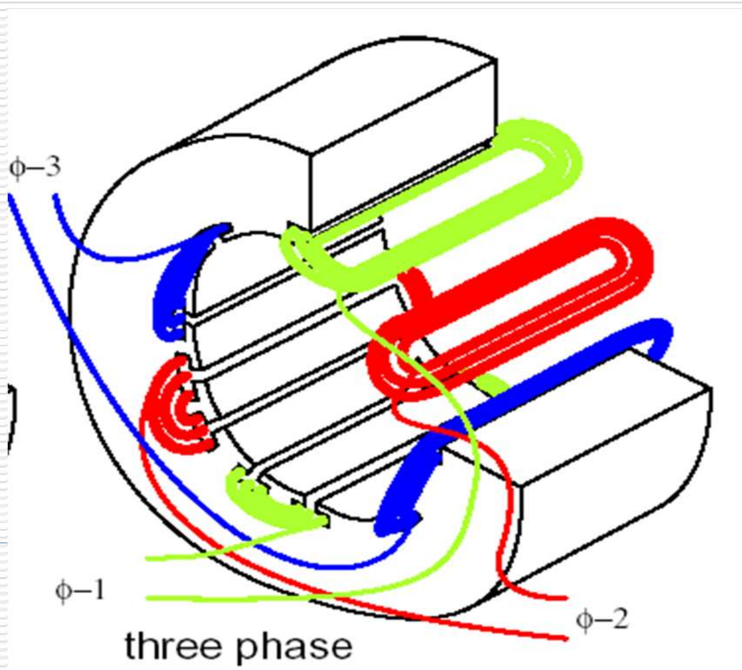
Introduction

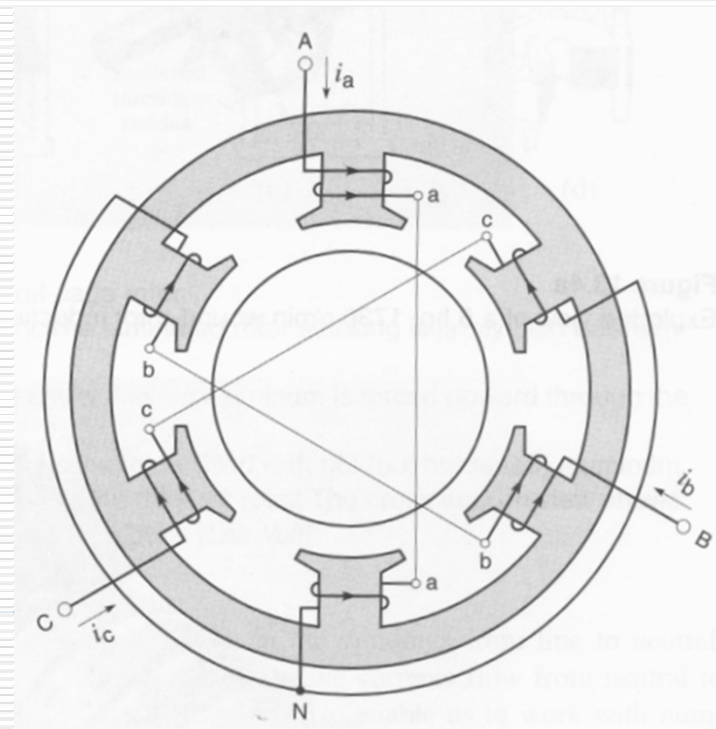
- Three-phase induction motors are the most common and frequently encountered machines in industry

  - simple design, rugged, low-price, easy maintenance
  - wide range of power ratings: fractional horsepower to 10 MW
  - run essentially as constant speed from no-load to full load
  - Its speed depends on the frequency of the power source
  - not easy to have variable speed control
  - Speed is determined by the supply frequency
  - To vary its speed need a variable frequency supply

Construction

- An induction motor has two main parts
  - a stationary stator
    - consisting of a steel frame that supports a hollow, cylindrical core
    - core, constructed from stacked laminations (why?), having a number of evenly spaced slots, providing the space for the stator winding





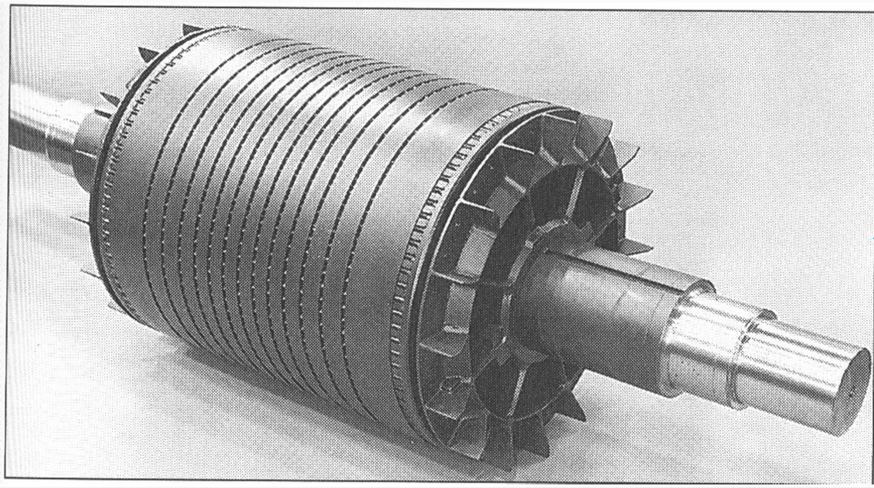
Construction

- a revolving rotor
  - composed of punched laminations, stacked to create a series of rotor slots, providing space for the rotor winding
  - one of two types of rotor windings
  - conventional 3-phase windings made of insulated wire (wound-rotor) » similar to the winding on the stator
  - aluminum bus bars shorted together at the ends by two aluminum rings, forming a squirrel-cage shaped circuit (squirrel-cage)

➢ Two basic design types depending on the rotor design

- squirrel-cage: conducting bars laid into slots and shorted at both ends by shorting rings.
- wound-rotor: complete set of three-phase windings exactly as the stator. Usually Y-connected, the ends of the three rotor wires are connected to 3 slip rings on the rotor shaft. In this way, the rotor circuit is accessible.

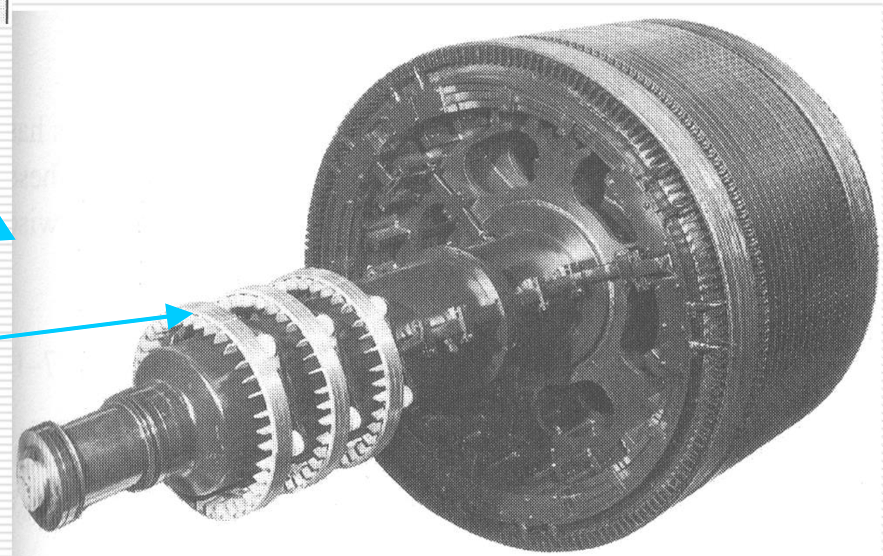
Construction



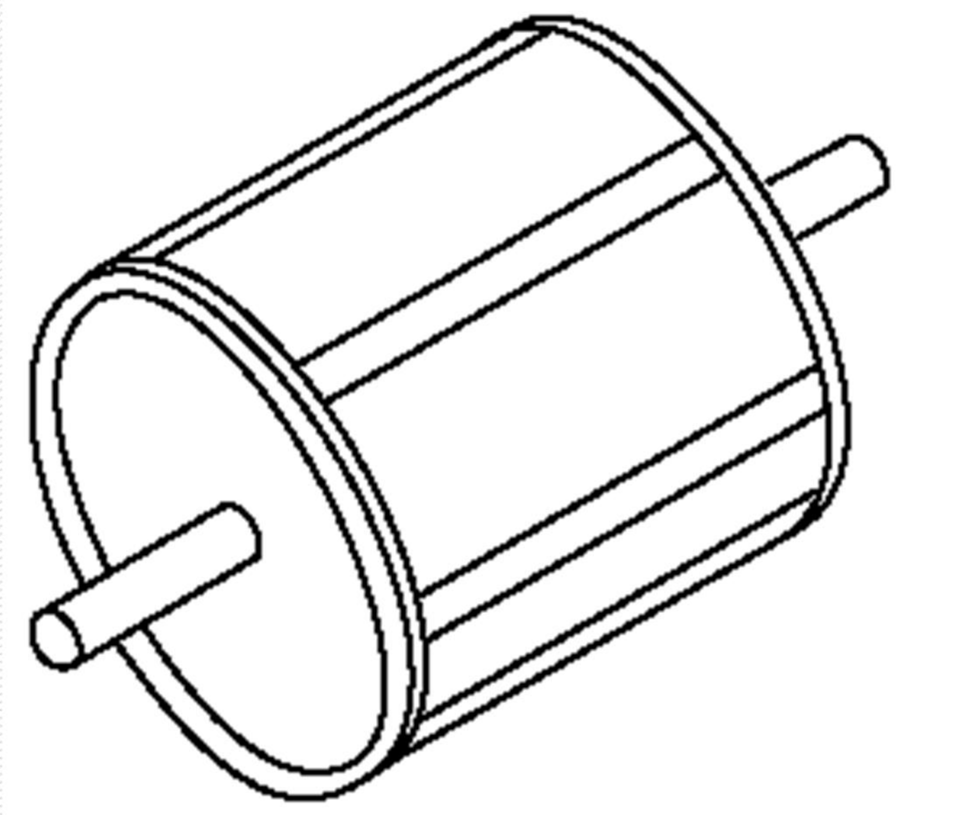
Squirrel cage rotor

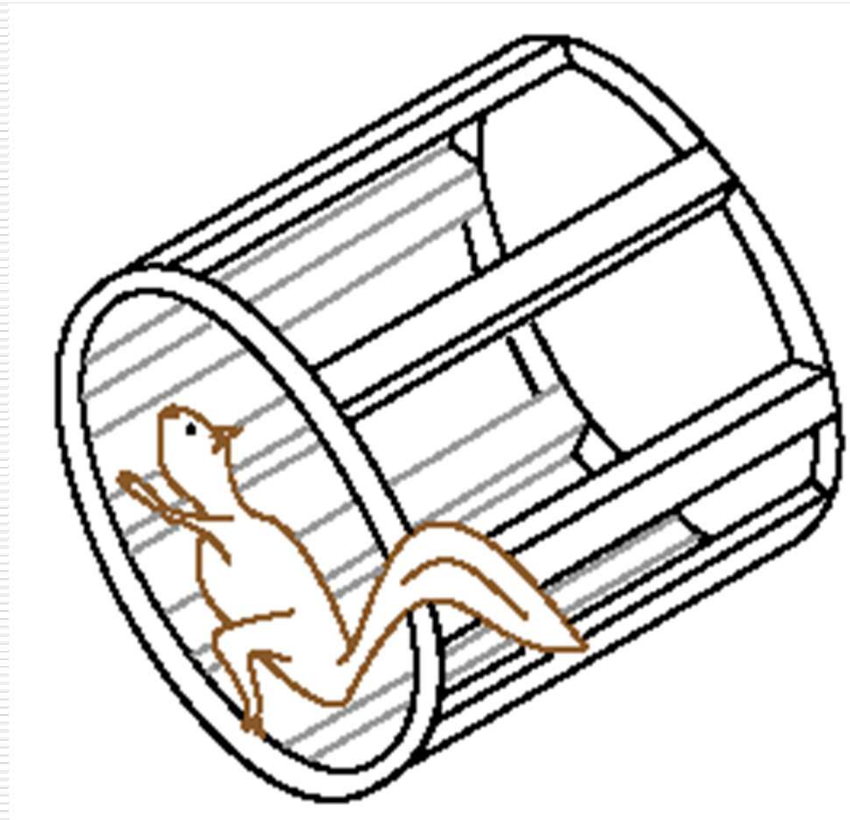
Wound rotor

Notice the slip rings



Squirrel cage rotor

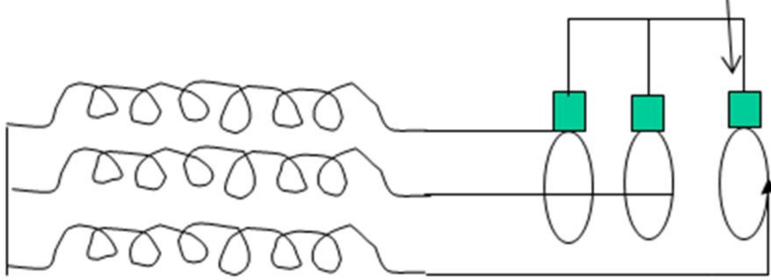




Slip Ring Rotor

- The rotor contains windings similar to stator.

- The connections from rotor are brought out using slip rings that are rotating with the rotor and carbon brushes that are static.



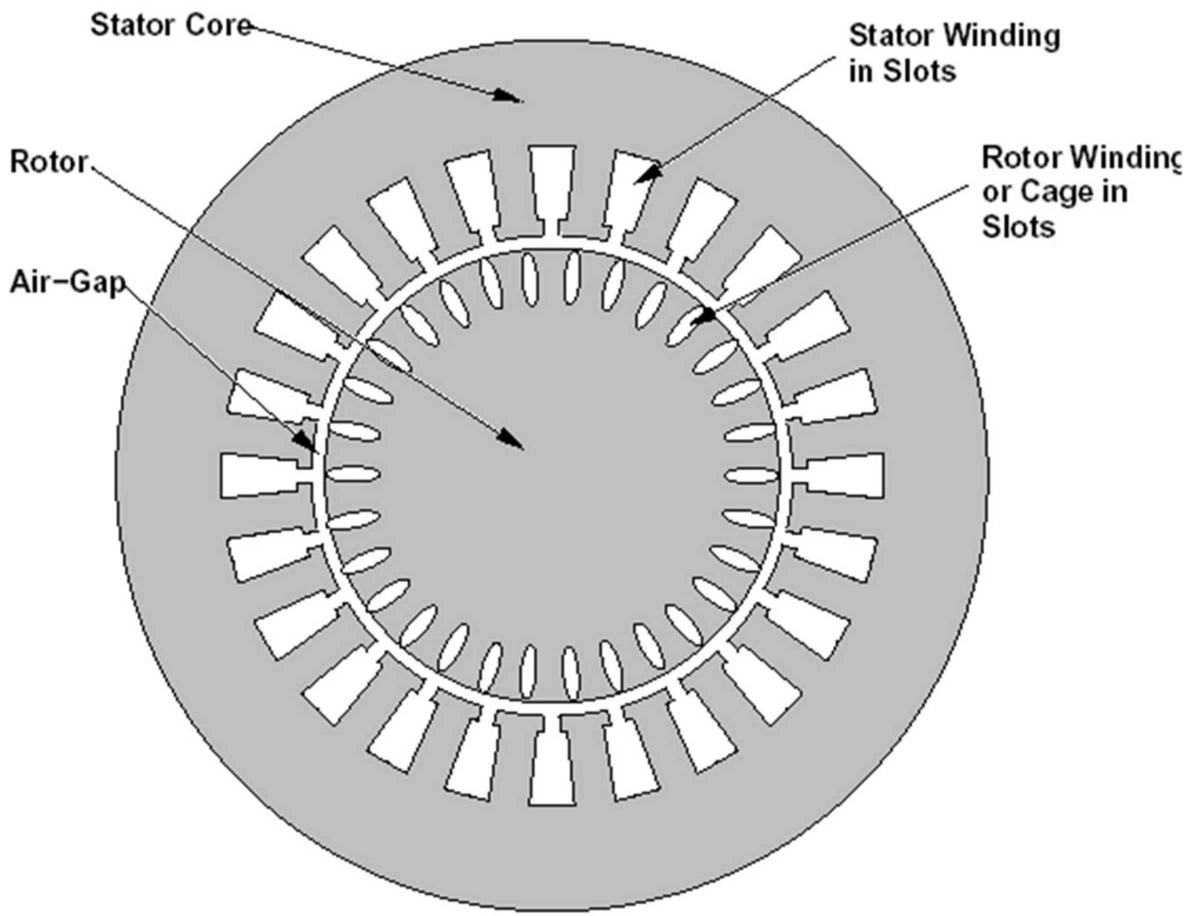
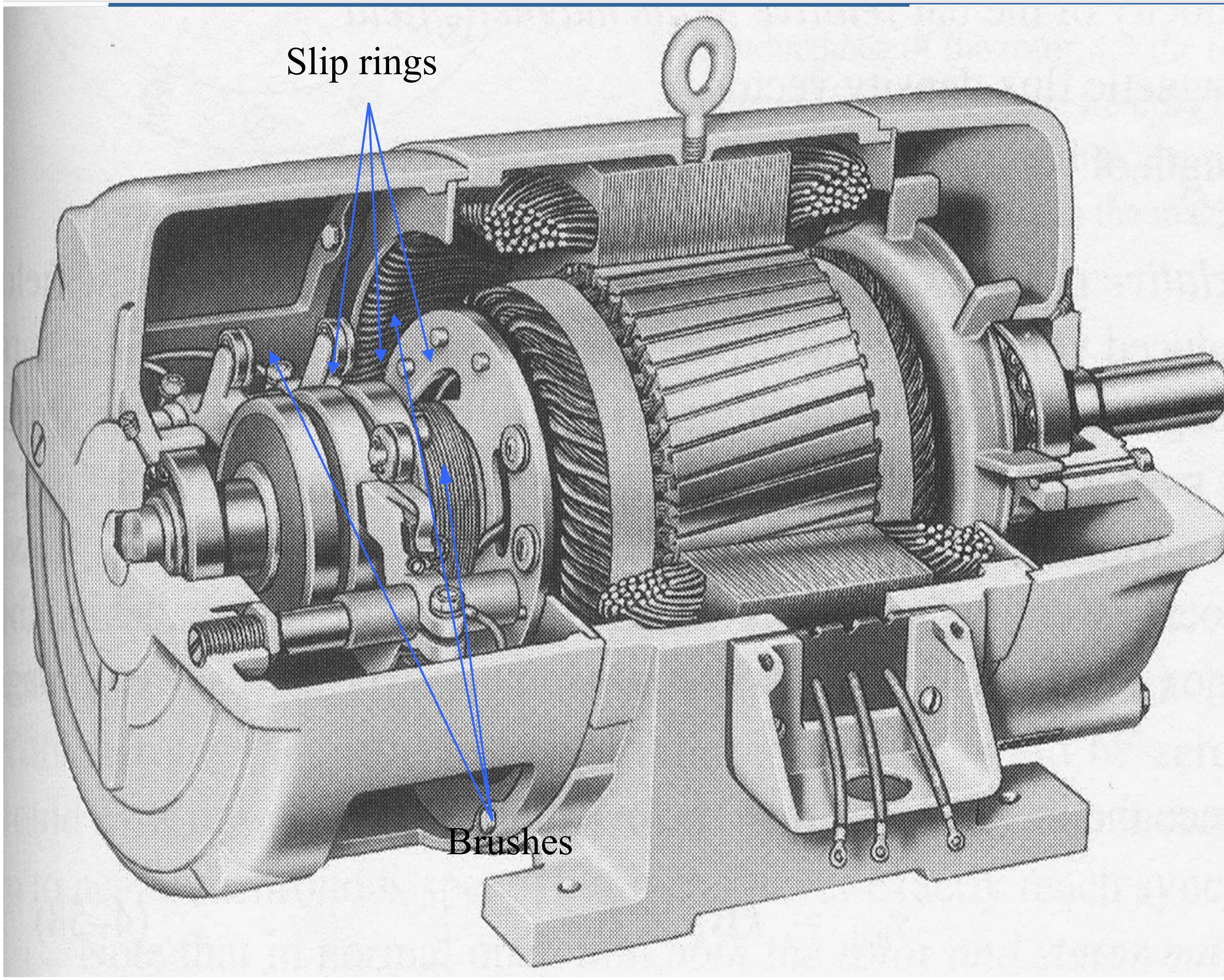


Figure 1: Axial View of an Induction Machine

Construction



Cutaway in a typical wound-rotor IM. Notice the brushes and the slip rings

Rotating Magnetic Field

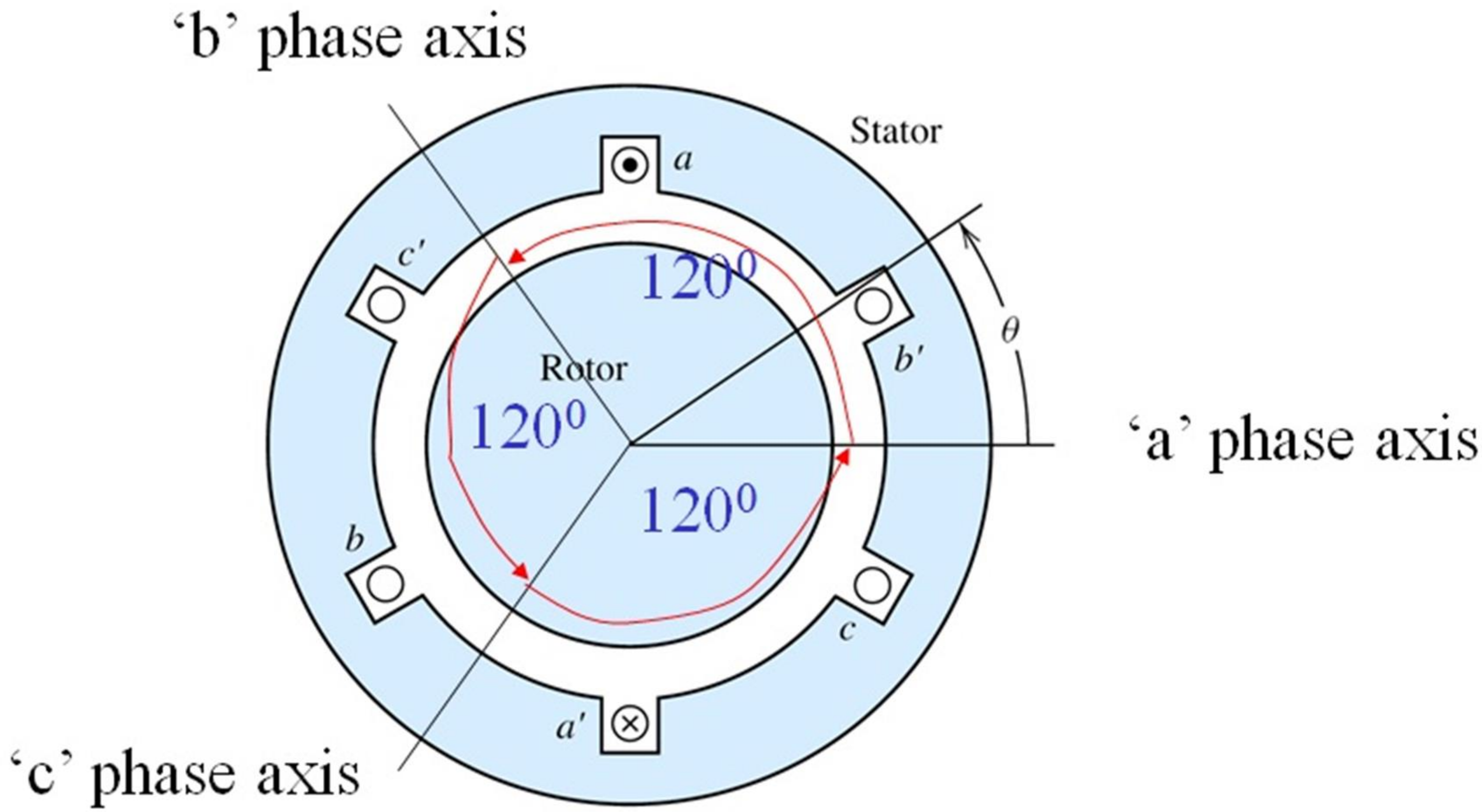
- Balanced three phase windings, i.e. mechanically displaced 120 degrees from each other, fed by balanced three phase source
- A rotating magnetic field with constant magnitude is produced, rotating with a speed

$$n_{sync} = \frac{120 f_e}{P} \quad rpm$$

Where f_e is the supply frequency and

P is the no. of poles and n_{sync} is called the synchronous speed in *rpm* (revolutions per minute)

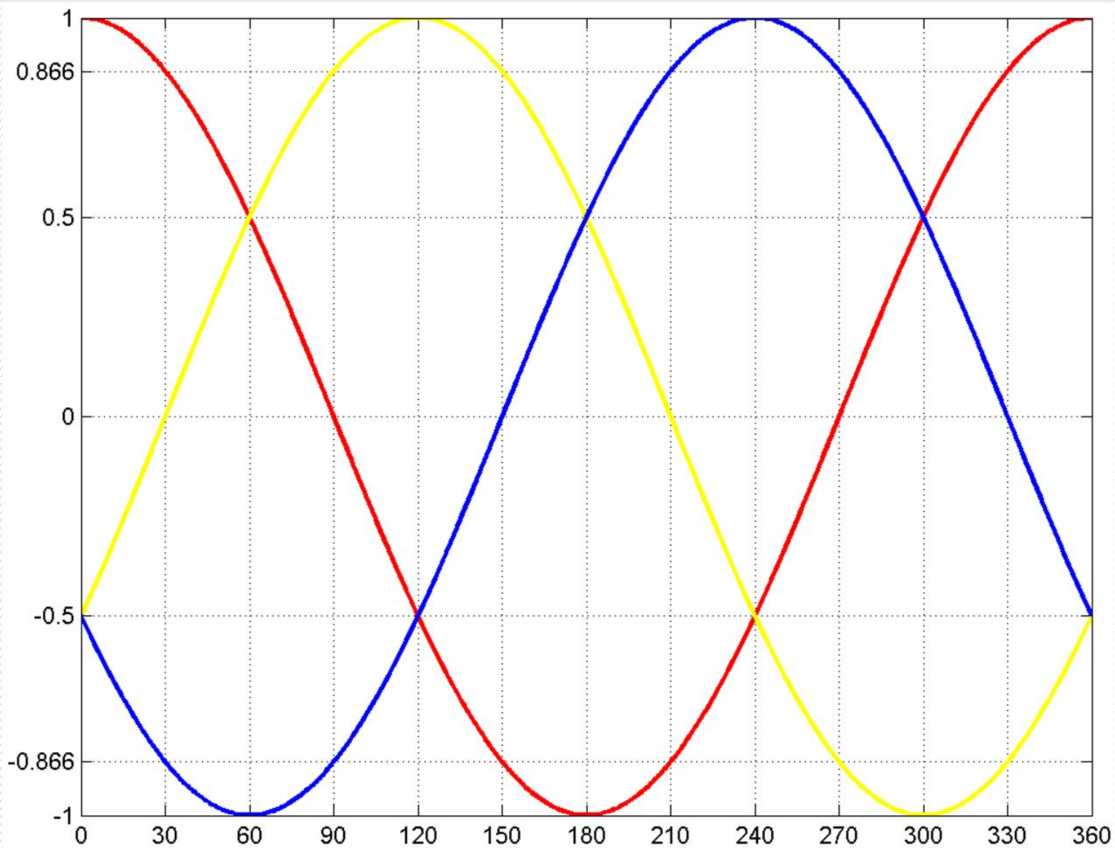
AC Machine Stator

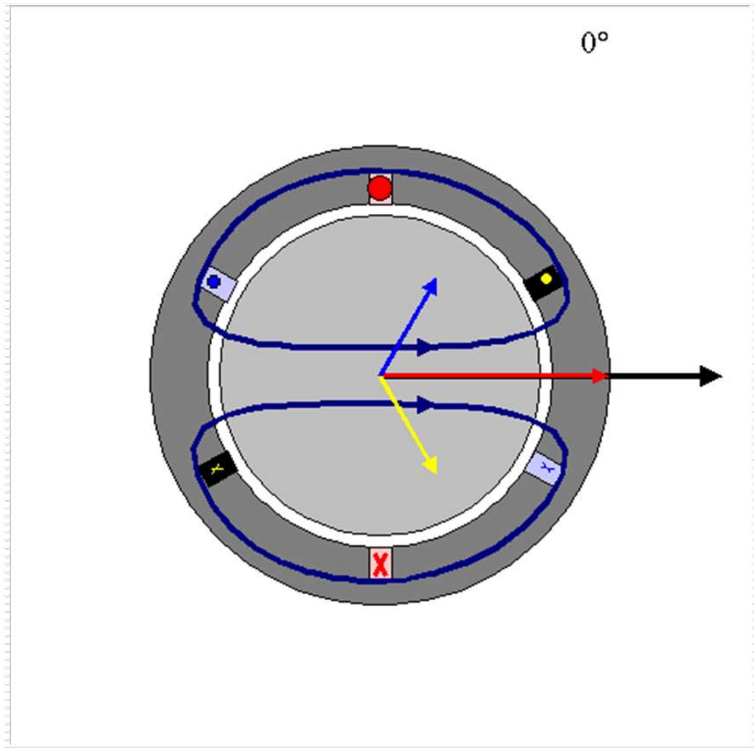


Synchronous speed

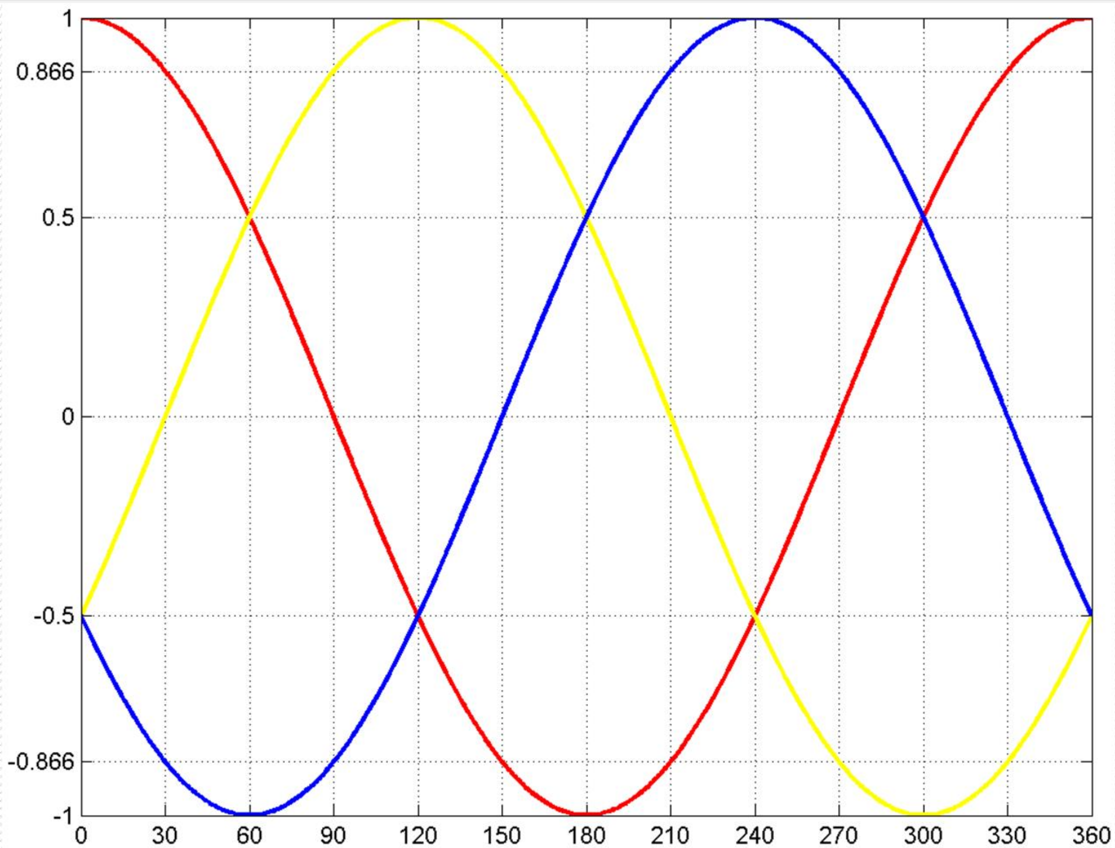
P	50 Hz	60 Hz
2	3000	3600
4	1500	1800
6	1000	1200
8	750	900
10	600	720
12	500	600

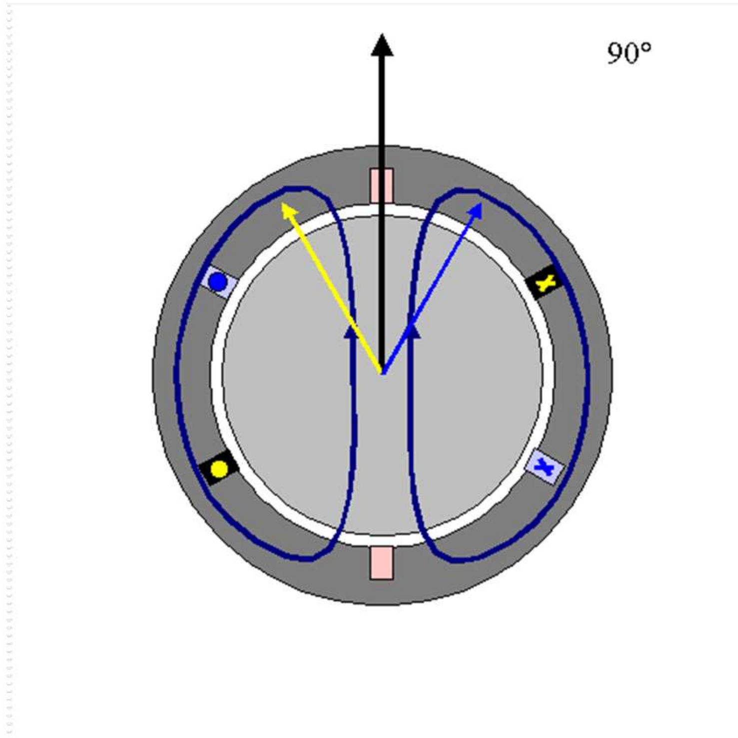
Rotating Magnetic Field





Rotating Magnetic Field





Principle of operation

- This rotating magnetic field cuts the rotor windings and produces an induced voltage in the rotor windings
- Due to the fact that the rotor windings are short circuited, for both squirrel cage and wound-rotor, and induced current flows in the rotor windings
- The rotor current produces another magnetic field
- A torque is produced as a result of the interaction of those two magnetic fields

$$\tau_{ind} = kB_R \times B_s$$

Where τ_{ind} is the induced torque and B_R and B_S are the magnetic flux densities of the rotor and the stator respectively

Induction motor speed

- At what speed will the IM run?
  - Can the IM run at the synchronous speed, why?
  - If rotor runs at the synchronous speed, which is the same speed of the rotating magnetic field, then the rotor will appear stationary to the rotating magnetic field and the rotating magnetic field will not cut the rotor. So, no induced current will flow in the rotor and no rotor magnetic flux will be produced so no torque is generated and the rotor speed will fall below the synchronous speed
  - When the speed falls, the rotating magnetic field will cut the rotor windings and a torque is produced

Induction motor speed

- So, the IM will always run at a speed lower than the synchronous speed
- The difference between the motor speed and the synchronous speed is called the *Slip*

$$n_{slip} = n_{sync} - n_m$$

Where n_{slip} = slip speed

n_{sync} = speed of the magnetic field

n_m = mechanical shaft speed of the motor

The Slip

$$s = \frac{n_{sync} - n_m}{n_{sync}}$$

Where s is the *slip*

Notice that : if the rotor runs at synchronous speed

$$s = 0$$

if the rotor is stationary

$$s = 1$$

Slip may be expressed as a percentage by multiplying the above eq. by 100, notice that the slip is a ratio and doesn't have units

Induction Motors and Transformers

- Both IM and transformer works on the principle of induced voltage
  - Transformer: voltage applied to the primary windings produce an induced voltage in the secondary windings
  - Induction motor: voltage applied to the stator windings produce an induced voltage in the rotor windings
  - The difference is that, in the case of the induction motor, the secondary windings can move
  - Due to the rotation of the rotor (the secondary winding of the IM), the induced voltage in it does not have the same frequency of the stator (the primary) voltage

Frequency

- The frequency of the voltage induced in the rotor is given by

$$f_r = \frac{P \times n}{120}$$

Where f_r = the rotor frequency (Hz)

P = number of stator poles

n = slip speed (rpm)

$$f_r = \frac{P \times (n_s - n_m)}{120}$$

$$= \frac{P \times sn_s}{120} = sf_e$$

Frequency

- What would be the frequency of the rotor's induced voltage at any speed n_m?

$$f_r = s\, f_e$$

- When the rotor is blocked (s=1) , the frequency of the induced voltage is equal to the supply frequency
- On the other hand, if the rotor runs at synchronous speed (s = 0), the frequency will be zero

Effect of slip on Rotor Parameters :-

(1) Rotor frequency :-

we know, $N_s = \frac{120f}{P}$;

$$\therefore f = \frac{N_s \cdot P}{120} \text{ Hz.} \quad \text{——— (1)}$$

When rotor rotates at a speed 'N' then the rotor conductors cut the rotating magnetic field at the relative speed i.e., $N_s - N$. The frequency of rotor induced emf or current 'f_r' can be expressed as

$$f_r = \frac{(N_s - N) P}{120} \text{ Hz.} \quad \text{——— (2)}$$

Dividing equation (2) by (1), we get

$$\frac{f_r}{f} = \frac{\frac{(N_s - N) \times P}{120}}{\frac{N_s \cdot P}{120}} = \frac{N_s - N}{N_s} = s$$

$$\Rightarrow \boxed{f_r = s \cdot f}$$

So, rotor frequency in running condition is slip times the supply frequency.

→ At start, $N = 0 \Rightarrow s = \frac{N_s - 0}{N_s} = 1$

$$\Rightarrow \boxed{f_r = f}$$

② Effect of slip on rotor Induced Emf :—

When the rotor is stationary or stand still i.e., Slip $= \frac{N_s - N}{N_s} = \frac{N_s - N}{N_s}$

~~[illegible] under stationary [illegible]~~

Since $N = 0$ when rotor is stationary

$\Rightarrow S = \frac{N_s - 0}{N_s}$

$\Rightarrow \boxed{S = 1}$

at $S = 1$, the emf Induced in the rotor is maximum.

→ As rotor gains speed, the relative speed decreases. This reduces the magnitude of Induced Emf in the rotor, proportionately.

At stand still : rotor Induced Emf/phase at standstill,

$$E_2 \propto N_s \quad \text{(Since } N = 0 \text{ at start)} \longrightarrow ③$$

Under Running Conditions : rotor Induced Emf/phase under running condition,

$$E_{2r} \propto (N_s - N) \longrightarrow ④$$

dividing equation ④ by ③, we get

$$\frac{E_{2r}}{E_2} = \frac{N_s - N}{N_s} = S$$

$$\Rightarrow \boxed{E_{2r} = S \cdot E_2}$$

∴ Rotor Induced emf under running Condition will be slip times the rotor Induced emf at stand still.

Effect of slip on Rotor Reactance, Resistance and Impedence

Rotor Reactance :

Let L_2 = Inductance of rotor per phase

At standstill : $f_r = f$, hence rotor Reactance/phase at standstill,

$$X_2 = 2\pi f_r L_2$$

$$\boxed{X_2 = 2\pi f L_2}$$

Under Running Condition : i.e., at slip 's', $f_r = sf$ hence

Rotor reactance/phase under running condition,

$$X_{2r} = 2\pi f_r L_2$$

$$= 2\pi (sf) \cdot L_2$$

$$= s (2\pi f L_2)$$

$$\boxed{X_{2r} = s X_2}$$

∴ Rotor Reactance under Running Condition will be slip times the rotor reactance at stand still.

Rotor Resistance : Let R_2 = Stand still rotor resistance/phase.

Rotor resistance is Independent of frequency and hence rotor resistance remains same as 'R_2' Ω/phase at standstill and in running Condition

Rotor Impedence :

Rotor Impedence/phase at Standstill is $Z_2 = R_2 + jX_2 = \sqrt{R_2^2 + X_2^2}$

Rotor Impedence/phase at running Condition is $Z_{2r} = R_2 + j(sX_2) = \sqrt{R_2^2 + (sX_2)^2}$

Effect of Slip on Rotor Current and power factor :—

At stand still : Let I_2 = Standstill rotor current/phase.

$\cos\phi_2$ = Power factor of rotor at stand still.

then equivalent circuit of rotor at stand still is shown in fig ①

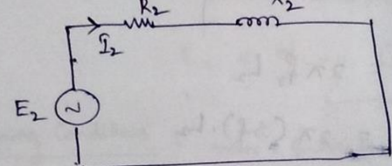


fig ①

$Z = R_2 + jX_2$

$|Z| = \sqrt{R_2^2 + X_2^2}$

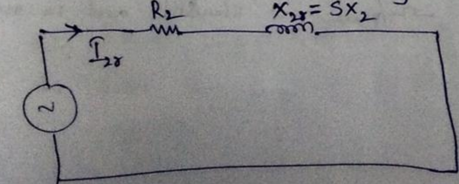
$$\therefore I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \quad \text{and}$$

$$\cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

Where E_2 = stand still rotor Induced Emf/phase.

At Running Condition :

Equivalent circuit under running Condition is shown in fig ②



$Z = R_2 + j(sX_2)$

$|Z| = \sqrt{R_2^2 + (sX_2)^2}$

∴ Rotor Current / phase under running Condition.

$$I_{2r} = \frac{E_{2r}}{Z_{2r}} = \frac{S \cdot E_2}{\sqrt{R_2^2 + (sX_2)^2}} \qquad (\because E_{2r} = sE_2)$$

$$\cos\phi_{2r} = \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Torque Equation of an Induction Motor :—

The Torque, T of an Induction motor is proportional to the product of stator flux per pole (ϕ), rotor current (I_2) and the p.f of rotor. The Torque of an Induction motor is due to the Interaction of rotor and stator fields

Thus, $T \propto \phi\, I_{2r} \cos\phi_{2r}$

Since rotor Induced emf / phase, E_{2r} is proportional to stator flux, ϕ

i.e., $E_{2r} \propto \phi$

$$\therefore T \propto E_{2r}\, I_{2r} \cos\phi_2$$

$$\Rightarrow T \propto E_2 \cdot \left(\frac{S \cdot E_2}{\sqrt{R_2^2 + (sX_2)^2}}\right) \cdot \left(\frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}\right)$$

$$\Rightarrow T \propto \frac{S E_2^2 \cdot R_2}{R_2^2 + (sX_2)^2}$$

$$\Rightarrow \boxed{T = K \cdot \frac{S E_2^2 \cdot R_2}{R_2^2 + (S X_2)^2}} \longrightarrow (1)$$

where K = Constant of proportionality $= \frac{3 \times 60}{2\pi N_s}$.

Starting torque :-

The torque produced by motor at start is called starting torque T_{st}.

at start, $N = 0$ and hence slip, $S = 1$ $\left(\because S = \frac{N_s - N}{N_s}\right)$

$\therefore$ Starting torque is obtained by substituting $S = 1$ in equation (1)

$$\Rightarrow \boxed{T_{st} = K \cdot \frac{E_2^2 \cdot R_2}{R_2^2 + X_2^2}}$$

From this equation, it can be observed that, by changing R_2, starting torque can be controlled, which is possible in case of slip ring Induction motor.

Condition for Maximum Torque :-

We know $T = K \cdot \frac{S E_2^2 R_2}{R_2^2 + (S X_2)^2}$ —— (2)

The condition for maximum torque can be obtained by differentiating torque w.r.to slip and equating it to zero.

$$\frac{dT}{dS} = 0.$$

$$\frac{d}{ds}\left[K.\frac{SE_2^2 R_2}{R_2^2+(sx_2)^2}\right]=0$$

$$K.\frac{d}{ds}\left[\frac{sE_2^2 R_2}{R_2^2+(sx_2)^2}\right]=0 \quad \text{—— (3)}$$

equation (3) is in the form of $\frac{d}{ds}(u/v) \Rightarrow \frac{v.du-u.dv}{v^2}$

$$\Rightarrow K.\left[\frac{[R_2^2+(sx_2)^2](E_2^2R_2)-(sE_2^2R_2)(0+(2sx_2)x_2)}{(R_2^2+(sx_2)^2)^2}\right]=0.$$

$$\Rightarrow [R_2^2+(sx_2)^2](E_2^2R_2)-(sE_2^2R_2)(2sx_2^2)=0.$$

$$[R_2^2+(sx_2)^2]E_2^2R_2 = s.E_2^2R_2(2sx_2^2)$$

$$R_2^2+(sx_2)^2 = 2s^2x_2^2$$

$$R_2^2 = 2s^2x_2^2 - s^2x_2^2$$

$$R_2^2 = (sx_2)^2$$

Condition for maximum torque $\rightarrow$ $\therefore$ $\boxed{R_2 = sx_2}$ $\longrightarrow$ (4)

$$\therefore \boxed{S=\frac{R_2}{x_2}}$$

this slip at which the torque is maximum is denoted as S_m.

$$\therefore \boxed{S_m=\frac{R_2}{x_2}}$$

The condition for maximum torque at starting can be obtained by substituting $S = 1$ in equation (4), we get

$$\Rightarrow \boxed{R_2 = X_2}$$

∴ Starting torque is maximum, when rotor resistance is equal to rotor reactance at stand still.

Expression for the Maximum Torque :-

We know,

$$T = K . \frac{S . E_2^2 . R_2}{R_2^2 + (SX_2)^2}$$

The maximum torque can be obtained by substituting $R_2 = SX_2$ in torque equation (∵ $R_2 = SX_2$ is the condition for max torque)

$$\therefore T_{max} = \frac{K . SE_2^2 (SX_2)}{(SX_2)^2 + (SX_2)^2}$$

$$T_{max} = \frac{K . S E_2^2 (SX_2)}{2 (SX_2)^2} = \frac{K . S E_2^2}{2 S X_2}$$

$$\therefore \boxed{T_{max} = \frac{K . E_2^2}{2 X_2}}$$

→ From the expression of T_{max}, it can be observed that

(i) Maximum Torque is independent of rotor resistance (R_2)

(ii) T_{max} is inversely proportional to stand still rotor reactance (X_2)

(iii) T_{max} is directly proportional to square of the stand still rotor induced emf. (E_2)

Torque-slip & Torque-Speed characteristics :-

The curve obtained by plotting torque against slip is called Torque - slip characteristics of Induction motor.

We know, torque $T = K \cdot \frac{S E_2^2 \cdot R_2}{R_2^2 + (S X_2)^2}$

→ if supply voltage is const then E_2 is also const.

→ We can evaluate value of torque at different values of slip in the range from 0 to 1.

(i) When slip $S = 0$, $N = N_S$ and hence torque, $T = 0$. Motor cannot run at synchronous speed.

(ii) When slip 'S' is very low i.e., the speed is very near to synchronous speed, then the term $(S X_2)^2$ is very small and can be neglected in comparision with R_2^2.

$$\therefore T \propto \frac{S}{R_2} \quad \text{(or)}$$

$$\boxed{T \propto S} \quad \text{if } R_2 \text{ is constant}$$

Hence for low values of slip, the torque-slip curve is a straight line.

(iii) When slip increases i.e., the speed decreases (with increase load), the torque increases and becomes maximum when

$$S = S_m = \frac{R_2}{X_2}.$$

i.e., $T = T_{max}$ when $S = S_m$.

Maximum torque is called as pull out or break down torque.

(iv) When slip is further increases beyond $S = S_m$, then the term R_2^2 is very small as compared to $(SX_2)^2$ and may be neglected.

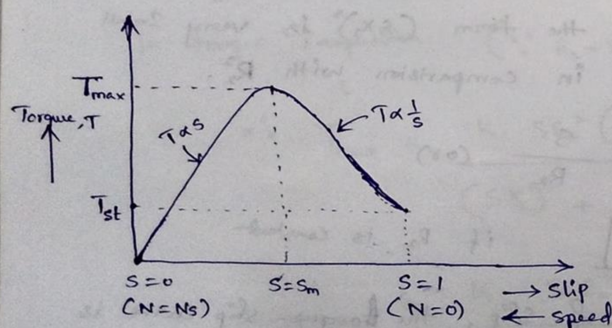
$$\therefore \quad T \propto \frac{S \cdot R_2}{(S X_2)^2} \quad \text{(or)}$$

$$\boxed{T \propto \frac{1}{S}} \quad \text{if } R_2 \text{ \& } X_2 \text{ are constants.}$$

Hence for higher values of slip, the torque slip curve is a rectangular hyperbola.

(v) If slip, $S = 1$, then motor is stationary hence corresponding torque is nothing but starting torque.

i.e., $T = T_{st}$ when $S = 1$.



Torque slip & torque speed characteristics.