Engineering Mechanics.

- It is a branch of Physics which deals with study of effect of force system acting on a particle or a Rigid Body. which may be at rest or in motion. The purpose of mechanics is to explain and predict physical phenomena and thus to lay the foundation for Engineering Applications.

Particle

- It is a matter having considerable mass but negligible Dimensions. A body whose shape and size is not considered in analysis of problem and all the forces acting on a given body is assumed to act a single point is considered to be a particle.

Rigid Body (No Deformation Takes place)

- It is a matter having considerable mass as well as Dimensions.
- The body which is capable to with-stand its shape le size and does not deform under action of forces is termed as Rigid Body.

Eg. Beams and Columns of Building structures to Deform under the action

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of loads, But the actual Deformation that has taken place in structures are negligible &
Therefore the body is assumed to be Rigid Body.

NOTE :- It is a hypothetical concept, that no body is perfectly rigid in the Universe so assumption of a rigid body is made.

Classification of Mechanics:-

- Mechanics of Deformable Bodies
- Mechanics of Rigid Bodies
  - Static
  - Dynamics
    - Kinematics
    - Kinetics
- Mechanics of Fluids.

Static

- It is the study of the effect of force system acting on a particle or rigid bodies which is at rest.

Dynamic:

- It is the study of the effect of force system acting on a particle or a rigid body which is in motion.

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- It can also be ~~stated~~ stated as the study of Geometry of Motion with or without reference to the cause of motion.

Geometry of Motion:-

The study of Geometry of Motion means relationship between Displacement, Velocity, Acceleration and time.

- with reference to cause of motion means mass and the force causing motions are considered.
- The sub branches of Dynamics are Kinematics and Kinetics.

Kinematics **

- It is the study of geometry of motion without reference to cause of motion. (i.e mass and force causing motion are not considered).

Kinetics **

- It is the study of geometry of motion with reference to cause of motion. (i.e mass and force causing motion are considered).

Laws of Mechanics:-

Newton's first law of motion:-

Every body continues in its state of rest or uniform motion in a straight line unless an external unbalanced force acts on it.

- Newtons first Law contains the principle of equilibrium of forces, which is the main topic of Concerned in static

Newtons second law of Motion:-

- The rate of change of momentum of a Body is directly proportional to force acting on it and takes place in the direction of Applied force

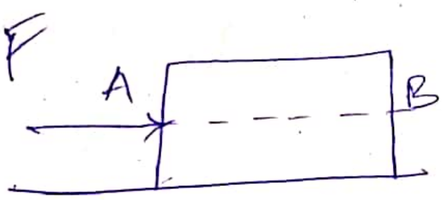
$$\boxed{F = \frac{m(v-u)}{t}}$$

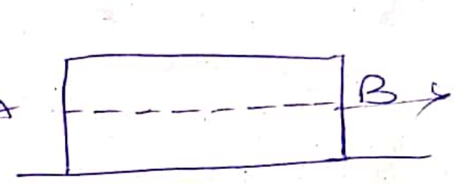
Newtons third Law of Motion:-

- To every action there is an equal and opposite Reaction.
- The forces of action & Reaction between two intersecting Bodies are equal in magnitude opposite in Direction and collinear.

Principle of Transmissibility of Force

- It states that the condition of Equillibrium or uniform motion of Rigid Body will Remain unchanged if the point of application of force acting on a Rigid Body is Transmitted to act at any other point along its line of action





Concept of Force :-

Force :- An external Agency which changes or tends to change the state of rest or of uniform motion of a body upon which it acts is known as Force

1N of force

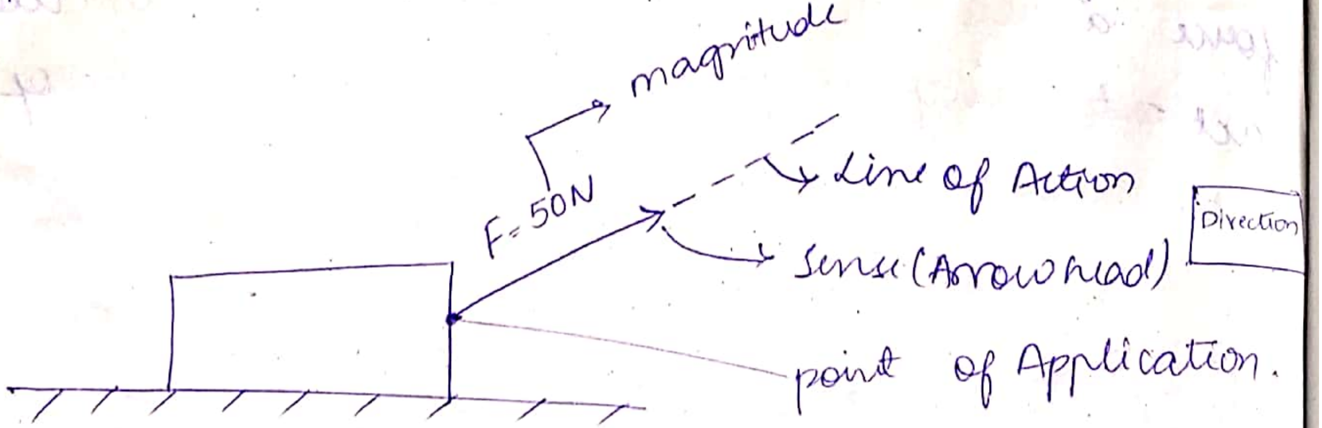
- It is the force required to produce an acceleration of $1m/s^2$ in a body of mass 1kg

$$F = m.a$$
$$= 1kg \cdot 1m/s^2$$

$$\boxed{1N = 1\ kg\ m/s^2}$$

Characteristics of Force:-

(i) magnitude
(ii) Direction (Line of Action and Sense).
(iii) point of Application.



Resultant Force

It is a single force which can replace the given force system or which produces the same effect as the original system of force.

Frictional Force:-

It is a force which opposes the relative motion of two bodies and acts tangential at the contact of surface due to roughness of surface and material in contact

Force System:-

When a number of forces act simultaneously on a body then they are said to form a force system.

Classification of force system.

(1) Coplanar Force system.

If the line of action of all the forces acting on a body lies on the same plane then it is called coplanar force system.

(2) Non-coplanar Force system:

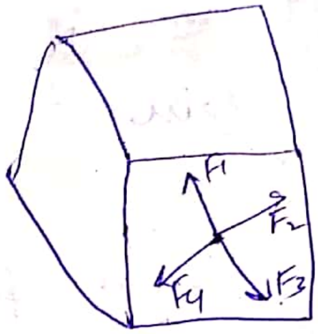
If the line of action of all the forces in the system do not lie on the same plane it is called Non-coplanar force system.

FORCE SYSTEM

- Coplanar F.S
  - Concurrent F.S
  - Parallel F.S
    - Like
    - Unlike
  - Non-concurrent & Non-parallel (general F.S)
- Non-coplanar F.S
  - Concurrent F.S
  - parallel F.S
  - Non-concurrent & Non-parallel F.S.

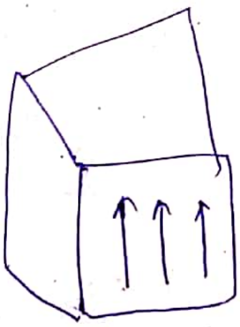
Concurrent F.S (coplanar).

If the line of action of all forces in the system passes through single point and lies in the same plane then it is called concurrent force systems

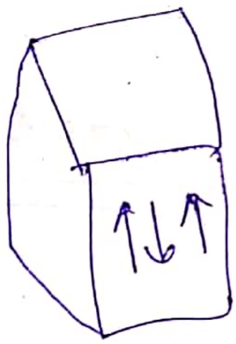


Coplanar parallel Force system:-

If the line of action of all the forces in the system are parallel to each other & lies in the same plane



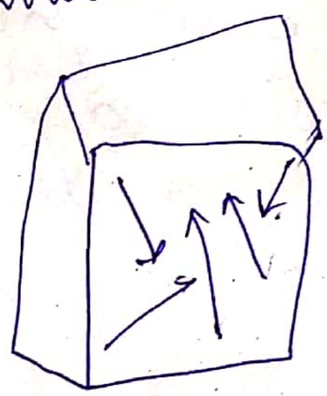
Cop- Parallel- like F-S



Unlike

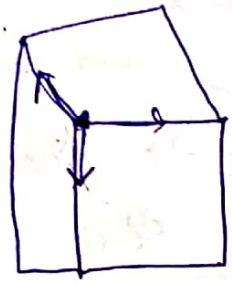
General Force System

If the line of action of all the forces in the systems are neither parallel nor concurrent then it is known as General Force system.



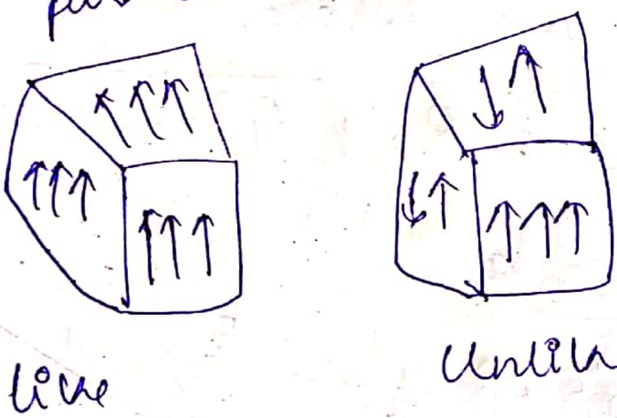
Non coplanar concurrent F·S:-

If the line of action of all forces in the system passes through single point and lies in different planes.

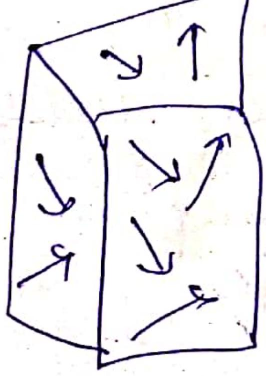


Non-coplanar Parallel F·S:-

If the line of action of all forces in the system are parallel and lies in different planes.



Non-coplanar General F·S:-

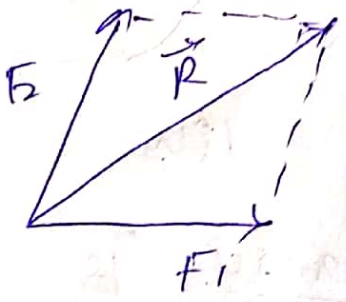


Law of Parallelogram of Forces:-

The resultant of any two Non-collinear concurrent forces may be found by the law which states that. "If two forces acting simultaneously on a body are represented in magnitude & direction by the two adjacent sides of a parallelogram then their resultant is represented in Magnitude and Direction by the Diagonal of parallelogram which passes through the point of Intersection of two sides representing the forces.

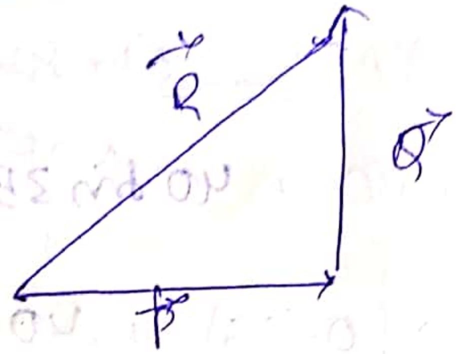
mathematically ; $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$$\tan\theta = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$$



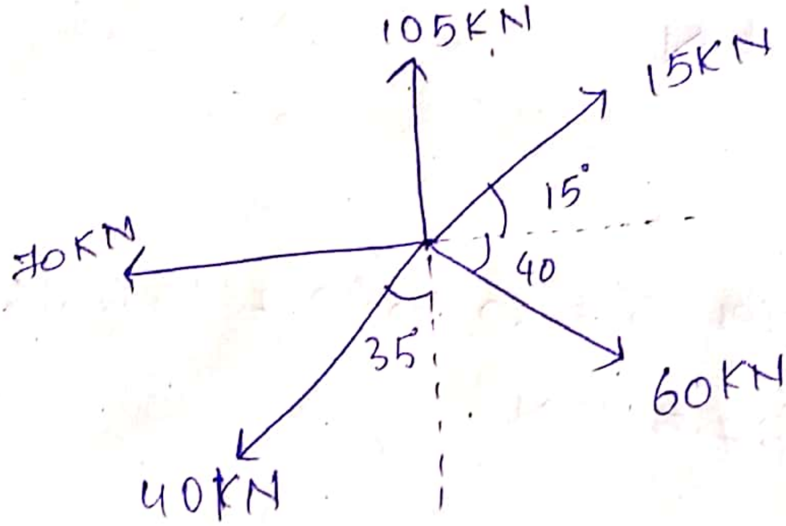
Triangle law of Forces:-

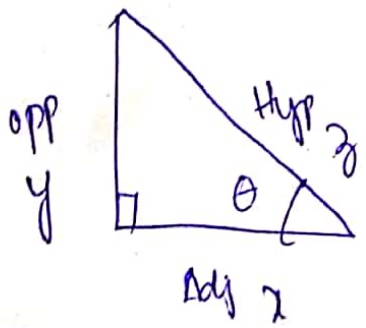
If two forces are represented by their force vectors placed Tip to Tail, their resultant is vector directed from the tail of first vector to the tip of second Vector.



(1) Determine the Resultant of the concurrent forces as shown in Fig.

Sol





$\cos\theta = \frac{x}{z}$

$x = z \cdot \cos\theta$ (Horizontal)

$\sin\theta = \frac{y}{z}$

$y = z \cdot \sin\theta$ (Vertical)

$\sin\theta = \frac{Opp}{Hyp}$

$\cos\theta = \frac{Adj}{Hyp}$

$\tan\theta = \frac{Opp}{Adj}$

$\Sigma F_x = 0$.

$70 - 15\cos 15^\circ - 60\cos 40 + 40\sin 35$

$70 - 15(0.96) - 60(0.76) + 40(0.57)$

$= 70 - 14.4 - 45.6 + 22.8$

$= 32.58$ KN

$\Sigma F_y = 0$.

$105 - 40\cos 35^\circ - 60\sin 40 + 15\sin 15$.

$\Sigma F_y = 37.5$ KN

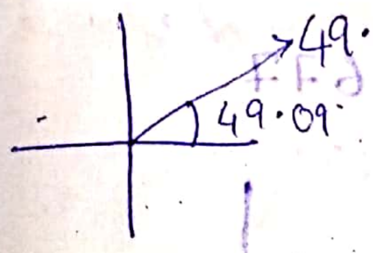
$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{1055.6001 + 1406.25}$

$= \sqrt{8501}$ $= 49.61$ KN

$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$; $\theta = \tan^{-1}\left(\frac{37.5}{32.49}\right)$

$= 49.09^\circ$



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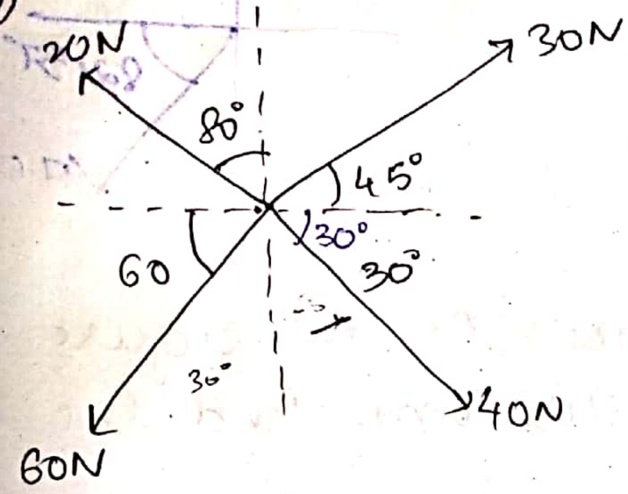


Find the magnitude and direction of the resultant force.

Sol: At Equilibrium, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\Sigma F_x = 30\cos 45^\circ + 40\cos 30^\circ - 60\cos 60^\circ - 20\sin 80^\circ$

$= 6.158\ N$

$\Sigma F_y = 30\sin 45^\circ + 20\cos 80^\circ - 60\sin 60 - 40\sin 30$

$= -47.275 N$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

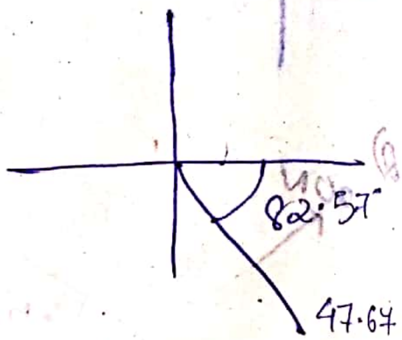
$= \sqrt{(6.158)^2 + (-47.275)^2}$

$= \sqrt{37.920 + 2234.925}$

$= \sqrt{2272.845} = 47.67 N$

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{-47.245}{6.158} = -7.677$$

$$\theta = \tan^{-1}(-7.677)$$
$$= -82.57^\circ$$



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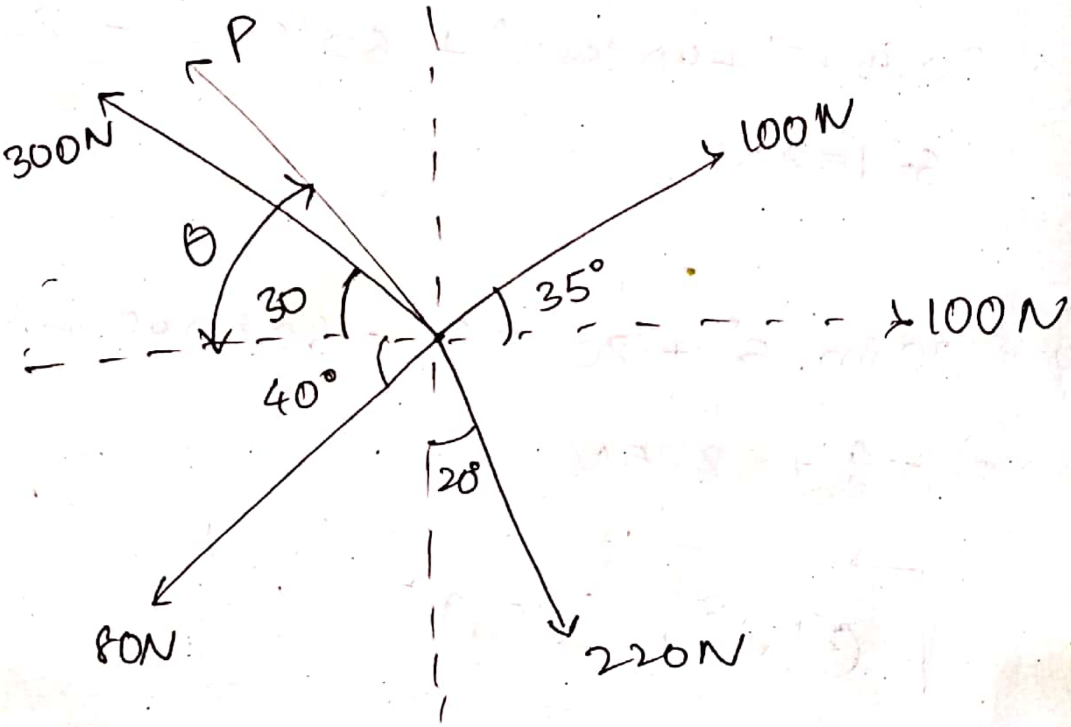
(1) If the system of forces in the figure shown below is in Equilibrium. Find the values of P and θ.



0-7944

$$\Sigma F_x = 0$$

$$100 + 100\cos 35 + 220\sin 20 - 80\cos 40 - 300\cos 30'$$
$$- P\cos\theta = 0$$

$P\cos\theta = -63.93N$ —— (1)

$\Sigma F_y = 0$

$100\sin 35 + 300\sin 30 + P\sin\theta - 220\cos 20 - 80\sin 40 = 0$

$P\sin\theta = 50.79\,N$ —— (2)

DIV $\frac{(2)}{(1)} \Rightarrow \frac{\cancel{P}\sin\theta}{\cancel{P}\cos\theta} = \frac{50.79}{-63.93}$

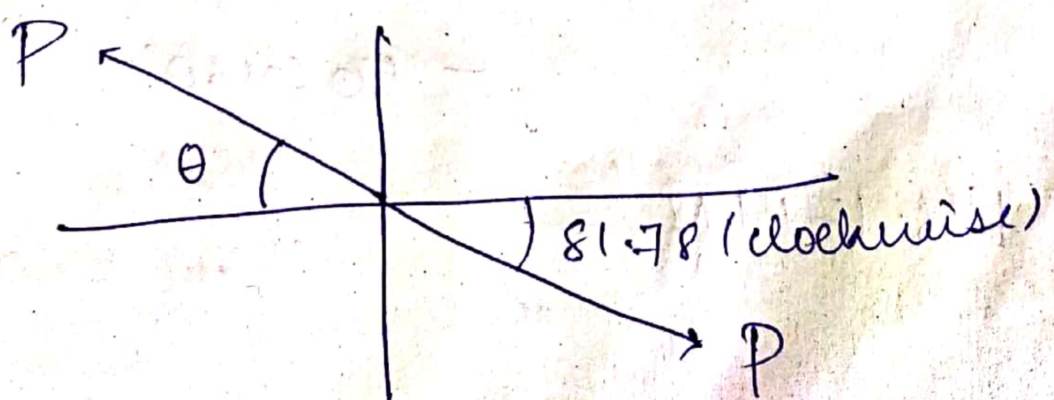
$\tan\theta = (-0.79)$

$\theta = \tan^{-1}(-0.79)$

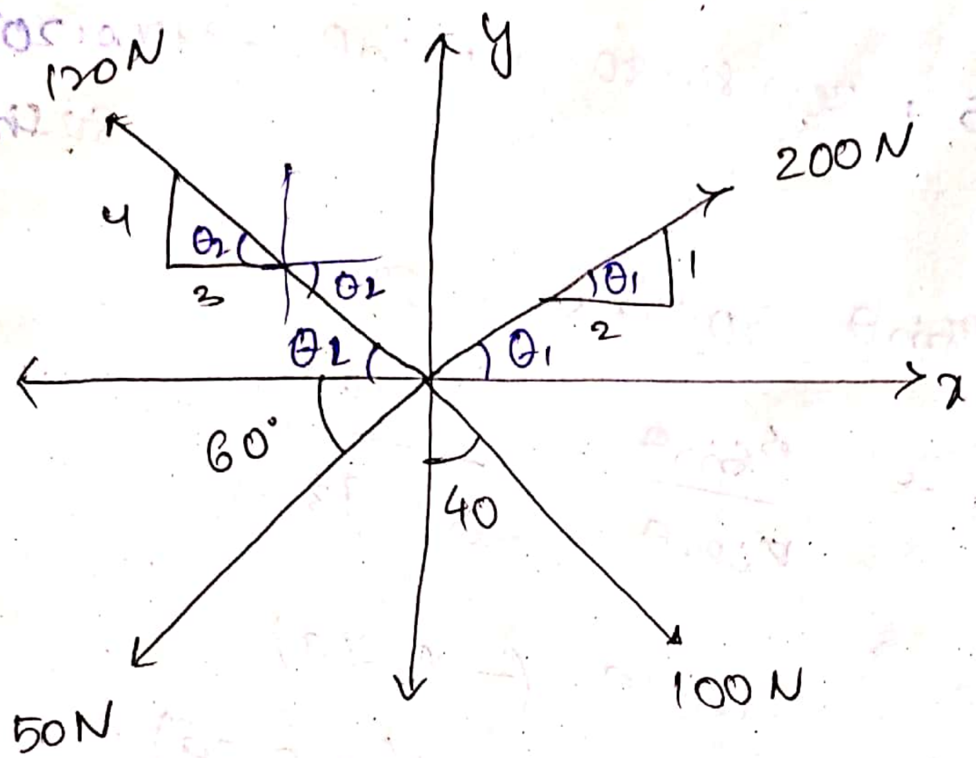
$\theta = -38.46°$ —— (3)

put (3) in (2) for P =

$P = \frac{+50.79}{\sin(-38.46)} = \frac{50.79}{-0.621} = -81.78N$



② Determine the resultant and direction of force system as shown below.



$\tan\theta_1 = \frac{1}{2}$; $\theta_1 = \tan^{-1}(0.5)$

$\boxed{\theta_1 = 26.56°}$

$\tan\theta_2 = \frac{4}{3}$; $\theta_2 = \tan^{-1}(1.333)$

$\boxed{\theta_2 = 53.13°}$

$\Sigma F_x = 200\cos 26.56 + 100\sin 40° - 120\cos 53.13 - 50\cos 60°$

$= 146.1719\ N$

$\Sigma F_y = 200 \sin 26.56 + 120 \sin 53.13$

$- 50 \sin 60 - 100 \cos 40$

$= 65.52 N$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

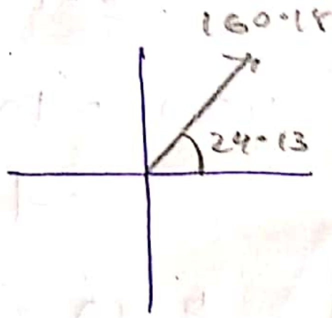
$= \sqrt{4292.84 + 21365.66}$

$= \sqrt{\cancel{25658.1449}}$

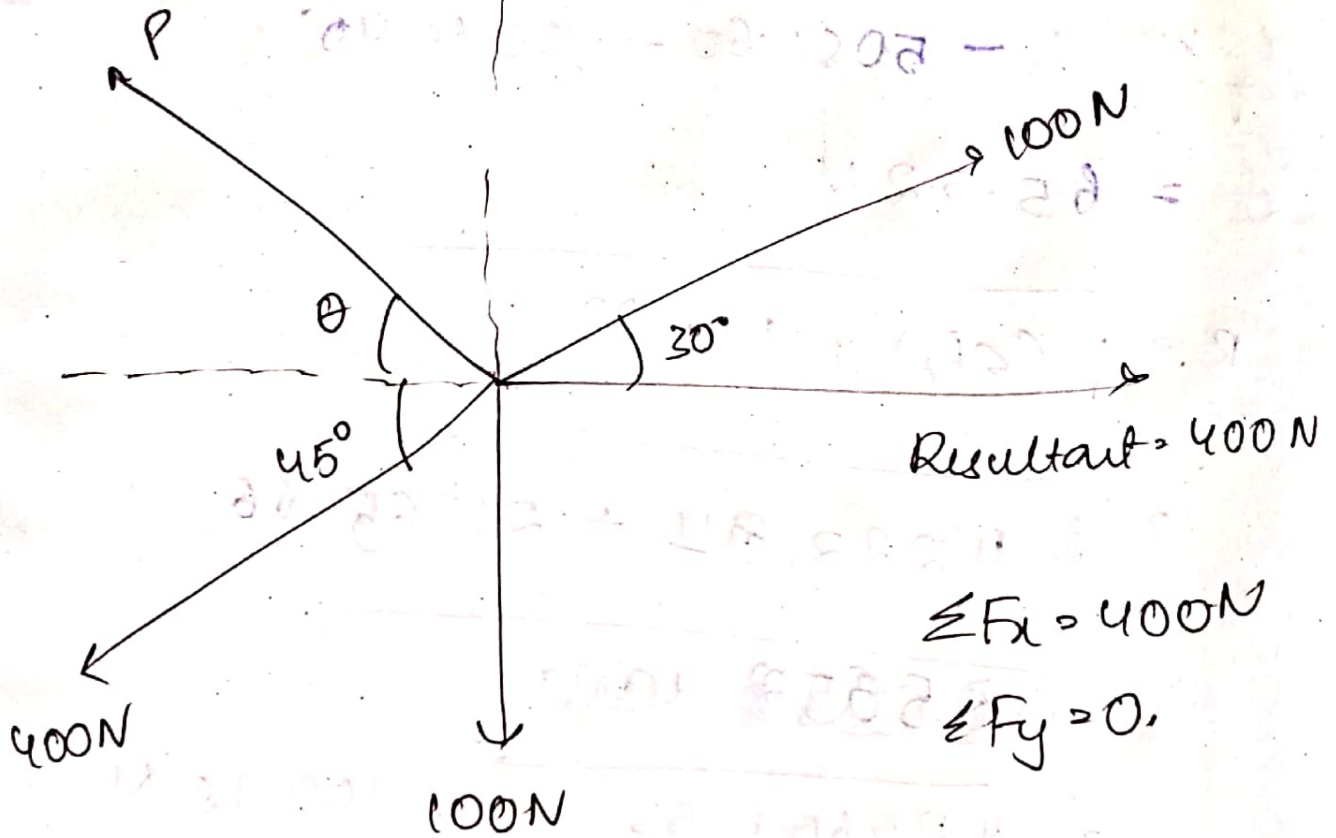
$= \sqrt{25658.53} = 160.18 N$

$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}(0.448)$

$= 24.13°$



③ The Four coplanar forces are acting at a point as shown in below. One of the force is Unknown and Its Magnitude is shown by P. The Resultant is having a magnitude of 400 N. and it is acting along X-axis. Determine the unknown force P and its inclination with X-axis.



$\Sigma F_x = 400N$

$\Sigma F_y = 0.$

Sol

$\Sigma F_x = 400N$

$400 = 100\cos 30 - P\cos\theta - 400\cos 45$

$= -196.24 - P\cos\theta$

$P\cos\theta = -400 - 196.24$

$P\cos\theta = -596.24$ ——— ①

$\Sigma F_y = 0.$

$0 = P\sin\theta + 100\sin 30 - 400\sin 45° - 100$

$P\sin\theta = 332.8427$ ——— ②

Divide ② with ①

$$\frac{P \sin\theta}{P \cos\theta} = \frac{332.8421}{-596.24}$$

$$\tan\theta = -0.558.$$

$$\theta = \tan^{-1}(-0.558)$$

$$\theta = -29.161^\circ$$

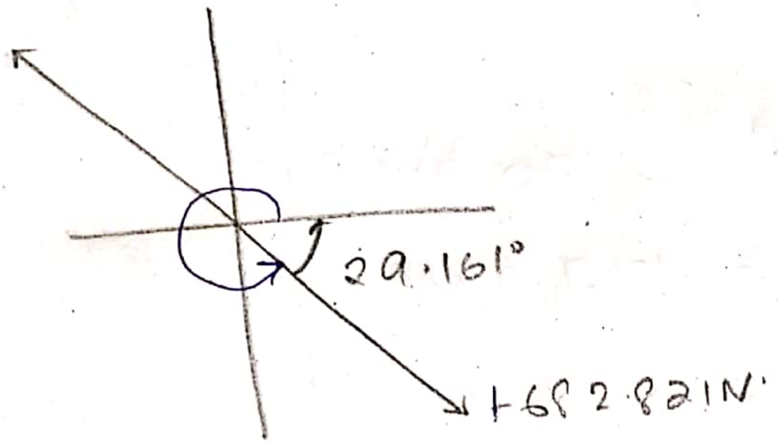
$360 - 29.16$

$=$

from ①

$$P = \frac{-596.24}{\cos(-29.161)} = \frac{-596.24}{0.8732}.$$

$$P = -682.821\ N$$



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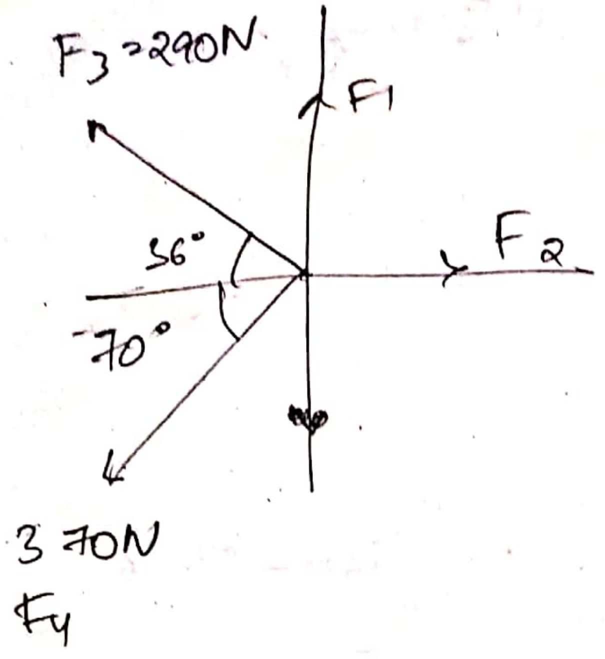
① Determine the magnitude and direction of force F_1 and F_2 as shown in below given figure when the resultant of force system is found to be 800N acting along the x-axis



Sol

$$\Sigma F_x = 800.$$

$$F_2 - 290 \cos 36° - 370 \cos 70 = 800$$

$$F_2 = 1161.1 N$$

$$\Sigma F_y = 0.$$

$$F_1 + 290 \sin 36° - 370 \sin 70 = 0$$

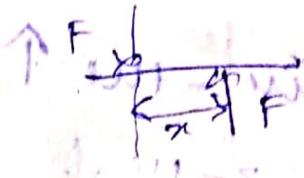
$$F_1 = 177.22 N.$$

Couple Forces:-

Two Non-collinear forces of equal magnitude and in opposite direction. form a couple

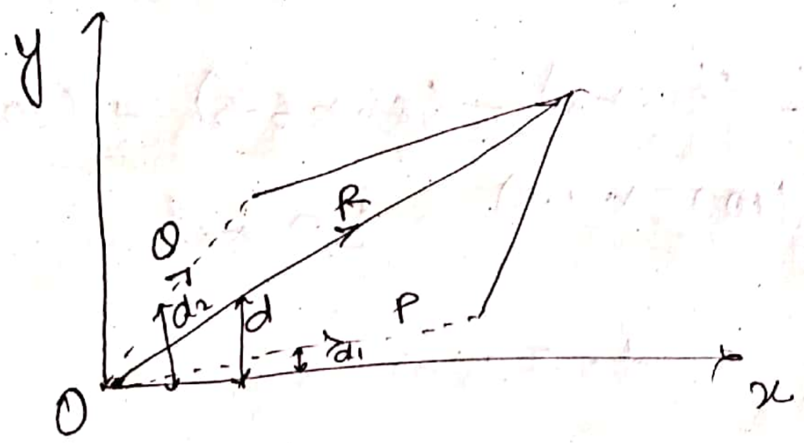
moment of couple:

The magnitude of rotation ~~are~~ known as moment of couple. It is the product of common magnitude of the two forces (F) and of the perpendicular distance (x) between the lines of action. $\boxed{M = F \times x}$



Varignon's Theorem:-

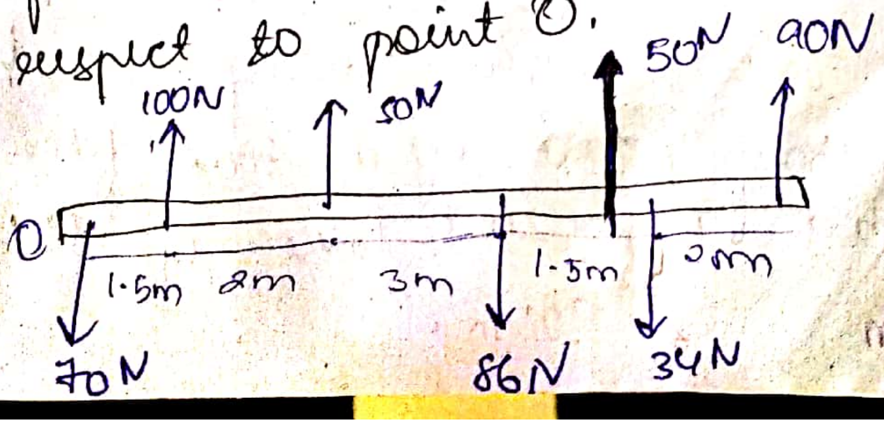
- It states that moment of Resultant of all the forces in a plane about any point is equal to the algebraic sum of moments of all the forces about the same point.



$$\Sigma M = \text{Resultant} \times \text{distance}$$

$$(R \times d) = (P \times d_1) + (Q \times d_2)$$

① Find the Resultant of the following force system as shown in figure below. And also find the position of the Resultant with respect to point O.



Anticlockwise = +ve ↑

clockwise = −ve ↓

Sol: $\Sigma F_y = 0$

$$\Sigma F_y = 100 + 50 + 90 - 70 - 86 - 34$$

$$= 50\,N$$

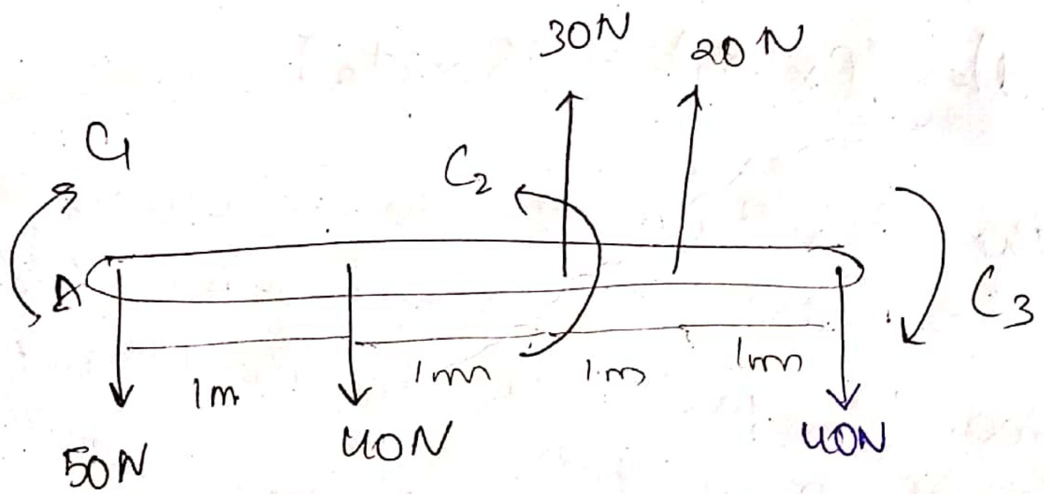
$$\Sigma M = R \times d$$

$$(90 \times 10) - (34 \times 8) - (86 \times 6.5) + (50 \times 3.5) + (100 \times 1.5) = 50 \times d$$

$$d = 7.88\,m$$

$$394 = 50 \times d$$

2)



Replace the force system acting on a bar as shown in above figure by a single force and also calculate its position. Take $C_1 = 85\,Nm$, $C_2 = 65\,Nm$; $C_3 = 90\,Nm$.

$\Sigma F_y = 0$

$-50 - 40 + 30 + 20 - 40 = 0$

$\Sigma F_y = -80N$

$\Sigma M = R \times d$

$(-40 \times 4) + (-40 \times 1) + (20 \times 3) + (30 \times 2)$

$-85 + 65 - 90 = -80 \times d$

-90
50
-40

$-160 - 40 + 60 + 60 - 85 + 65 - 90 = -80 \times d$

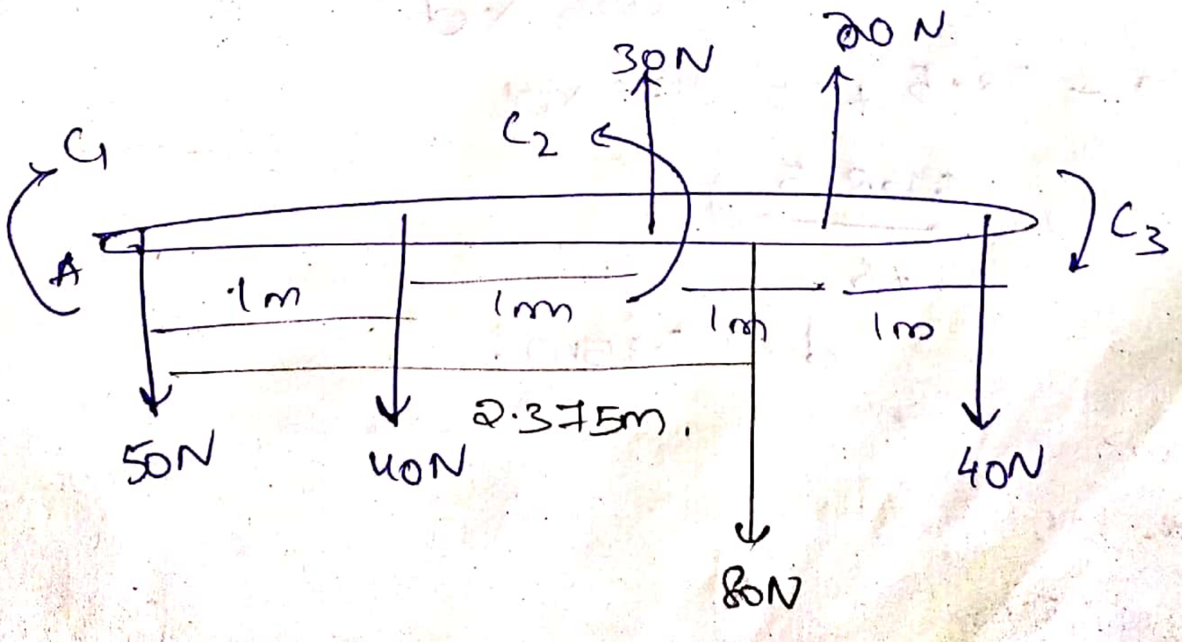
$-200 + 120 + 65 - 175 = -80 \times d$

$-80 - 175 + 65 = -80 \times d$

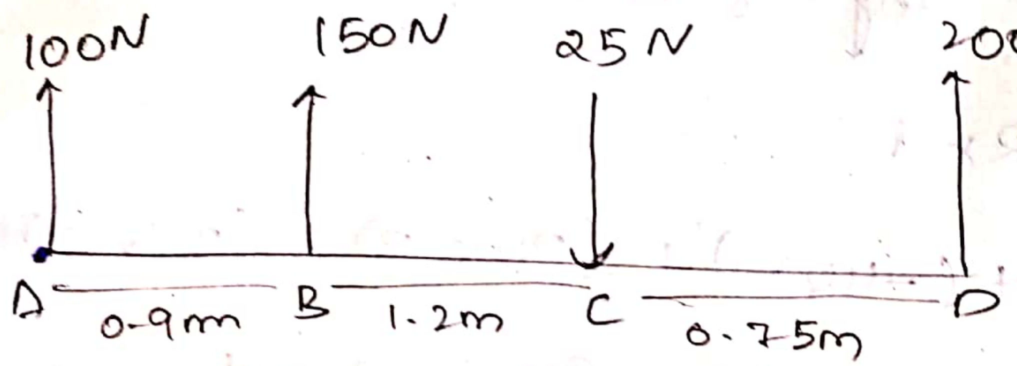
$-255 + 65 = -80 \times d$

$\frac{-190}{-80} = d$

$d = 2.375\,m$



5) Four parallel forces of magnitude 100N, 150N, 25N and 200N are as shown. Determine the magnitude of the resultant and also the distance of the resultant from point A.



Sol

$\Sigma F_y = 0$ +R.

$\Sigma F_y = 100 + 150 - 25 + 200$

$= 450 - 25$

$= 425$

$\Sigma M = R \times d$

0.9
1.2
2.1

$(200 \times 2.85) - (25 \times 2.1) + (150 \times 0.9)$

0.75
2.85

$= 425 \times d.$

$570 - 52.5 + 135 = 425 \times d$

$\frac{652.5}{425} = d.$

$d = 1.535m.$

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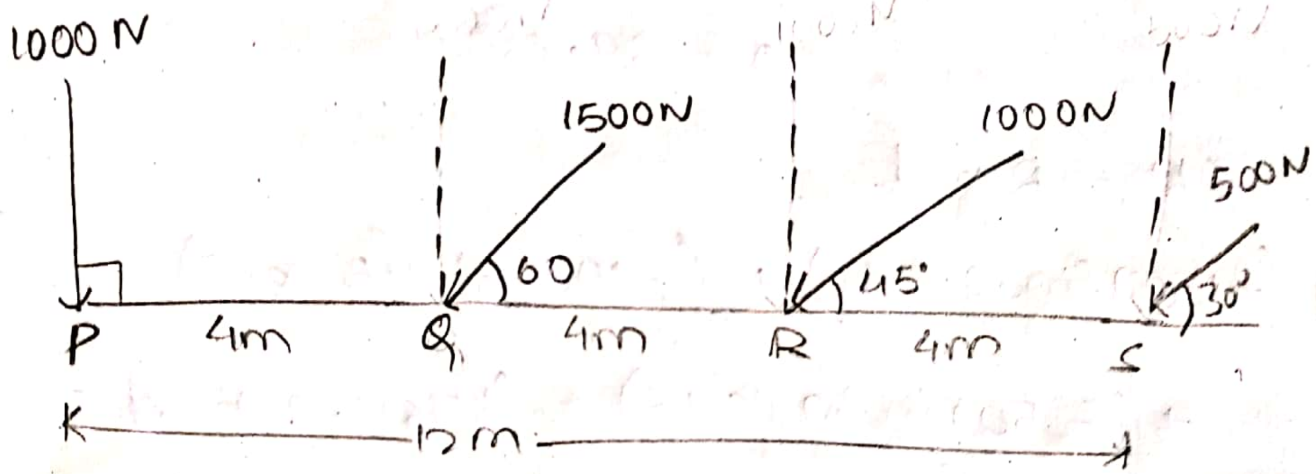
16/7/19

1. A Horizontal line PQRS is 12m long where PQ = QR = RS = 4m forces of 1000N, 1500N, 1000N, 500N act at P, Q, R and S respectively with Downward direction. The lines of action of these forces make angles of 90°, 60°, 45°, 30° respectively with PS. Find the magnitude, direction and position of the Resultant force.



PQ = RS = QR = 4m

$\Sigma F_y = 0$

$-1000 - 1500 \sin 60 - 1000 \sin 45 - 500 \sin 30° = 0$

$\Sigma F_y = -3256.14 N$

$\Sigma F_x = 0$

$0 - 1500 \cos 60 - 1000 \cos 45 - 500 \cos 30° = 0$

$\Sigma F_x = -1890 N$

$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$

$= \sqrt{(-1890.1)^2 + (-3256.14)^2}$

$= \sqrt{14174925.7}$

$R = 3764.9\,N$

$\theta = Tan^{-1}\left(\frac{\Sigma Fy}{\Sigma Fx}\right) = Tan^{-1}\left(\frac{-3256.4}{-1890.1}\right)$

$\theta = 59.86°$

$\Sigma M = R \times d$

$(-1500 \sin 60 \times 4) + (-1000 \sin 45 \times 8)$

$+ (-500 \sin 30 \times 12) = -(3764.9 \times \sin 59.86$

$-8761.8378 = d \times (-3255.8896)$

$d = 2.6910 m$

2. A force of 15N applied perpendicular to the edge of a door 0.8m wide. Find the moment of force about the hinge if the force is applied at an angle of 60° to the edge of the same door

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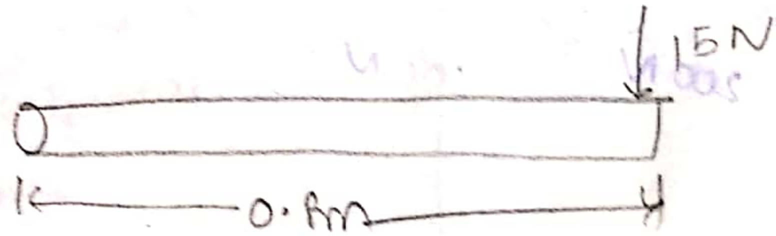
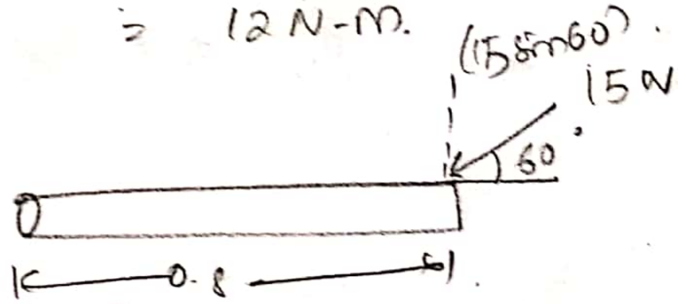


Hinge = Force × ~~for~~ distance

$= 15 \times 0.8$

$= 12\ N\text{-}m.$

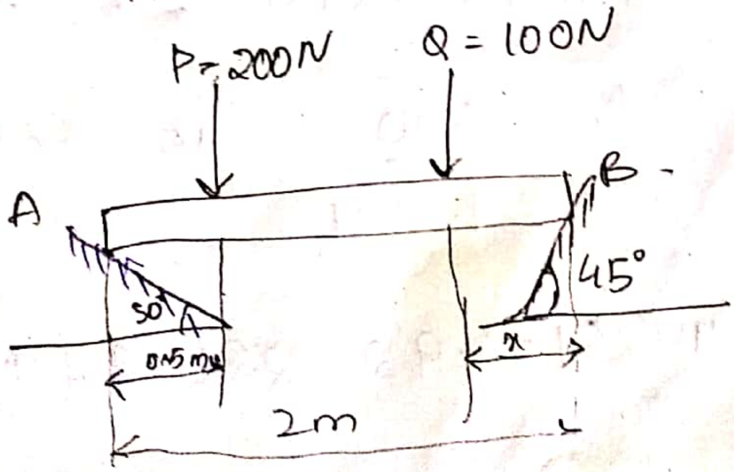


$M_H = 15 \sin 60 \times 0.8$

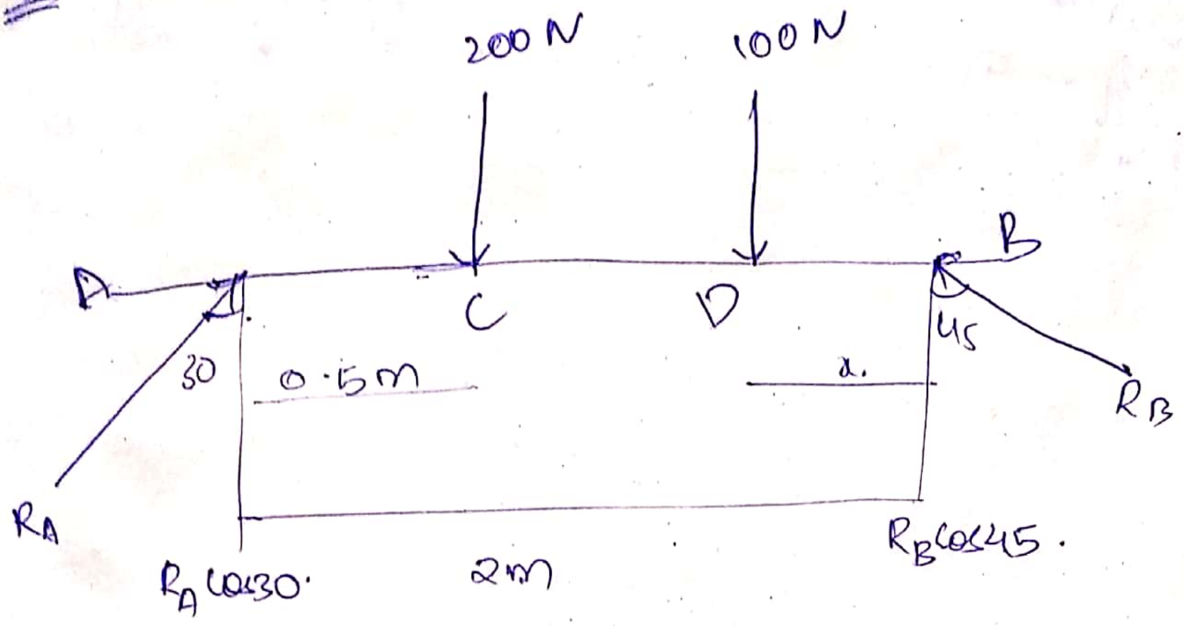
$= 10.3\ N\text{-}m$

18/8/19

① A bar 2m long and of negligible weight rests in horizontal position on two smooth inclined plane as shown in fig. Determine the distance 'x' at which the load Q = 100N should be placed from point B to keep the path horizontal.



SSP



$\Sigma F_y = 0$.

$-200 - 100 + R_A \cos 30 + R_B \cos 45 = 0$.

$R_A\, 0.866 + 0.70 R_B = 300$ —— (1).

$\Sigma F_x = 0$.

$R_B \cancel{\cos} \sin 45 - R_A \sin 30 = 0$

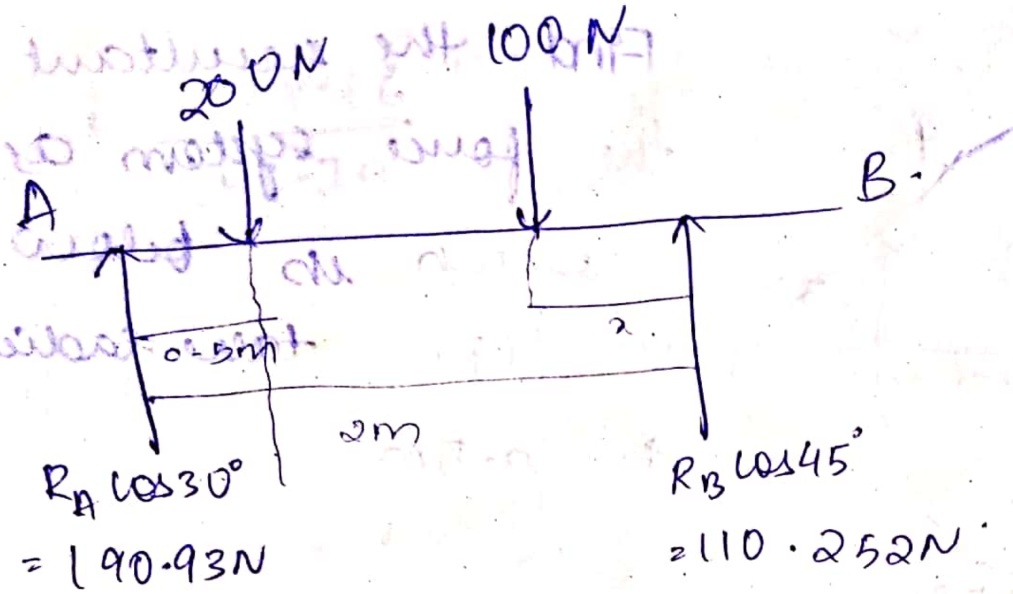
$R_A = R_B (1.414)$. —— (2)

(2) in (1)

$\cancel{R_A}$ $(1.414)(0.866) . R_B + 0.70 R_B = 300$.

$R_B = \dfrac{300}{1.924} = 155.92 N$

$R_A = 1.414 \times 155.92 = 220.47$ N



$\Sigma M = 0$

Taking moments about A.

$$110.252 \times 2 - (100 \times (2-x)) - 200 \times 0.5 = 0.$$

$$220.504 - 200 + 100x - 100 = 0$$

$$100x = 100 + 200 - 220.504$$

$$= 300 - 220.504$$

$$x = \frac{79.49}{100} = 0.7949m$$

$$\boxed{x = 0.7949m}$$

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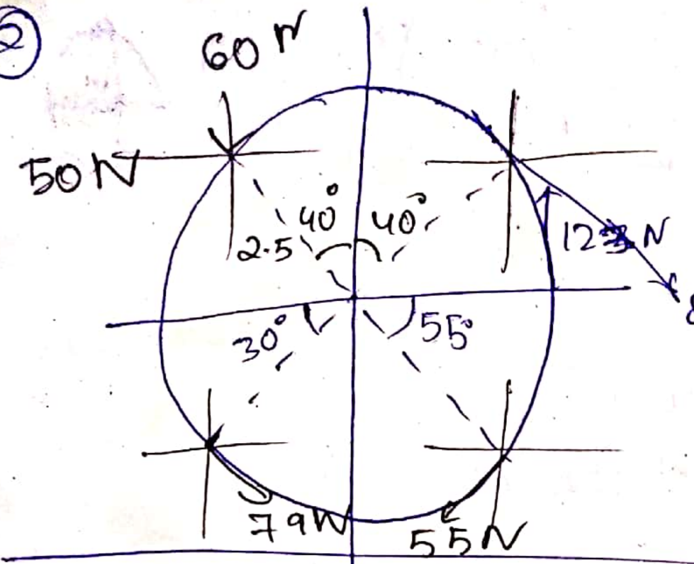
(2) Find out the resultant of the force system as shown in below figure. take radius as 2.5m



$\Sigma F_y = 0$; $123 - 60 - 84\cos 50° - 55\sin 35 - 79\sin 60$

$= -90.95\,N$

$\Sigma F_x = 0$; $50 + 84\sin 50 - 55\cos 35 + 79\cos 60$

$= 108.79$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(108.79)^2 + (-90.95)^2} = \sqrt{20107.166}$

$R = 141.799\,N$

$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{-90.95}{108.79}$; $\theta = -39.89°$

$\Sigma M = 0$ (Taking moments about pt 'O')

$(-84)(2.5) + (-55)(2.5) + 79(2.5) + 123(2.5)$
$+ (-50)(2.5\cos 40) + 60(2.5\sin 40) = 0$

$\Sigma M = 158.16\ N\text{-}m$

$\Sigma M = R \times d$

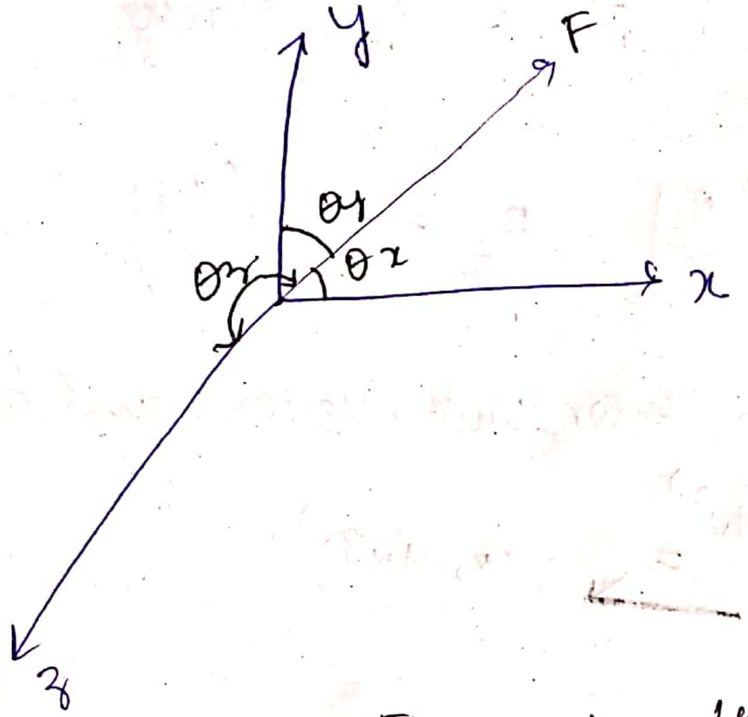
$158.16 = 141.79 \times d$

$d = \frac{158.16}{141.79} = 1.11 m$

$\boxed{d = 1.11 m}$

NON-COPLANAR FORCE SYSTEM :- or

3-D Force system or Spacial F.S



Magnitude of Forces along x, y, z :-

$F_x = F\cos\theta_x$

$F_y = F\cos\theta_y$

$F_z = F\cos\theta_z$.

Expression of Force in the form of Vector:-

$\vec{F} = F_x \bar{i} + F_y \bar{j} + F_z \bar{k}$

Magnitude of Force vector $= \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2}$

for Direction Cosines : squaring the above Eqn

$F^2 = (F_x)^2 + (F_y)^2 + (F_z)^2 = (F\cos\theta_x)^2 + (F\cos\theta_y)^2 + (F\cos\theta_z)^2$

$F^2 = F^2\cos^2\theta_x + F^2\cos^2\theta_y + F^2\cos^2\theta_z$

$F^2 = F^2(\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z)$

$$\cancel{F^2} = \cancel{F^2}(\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z)$$

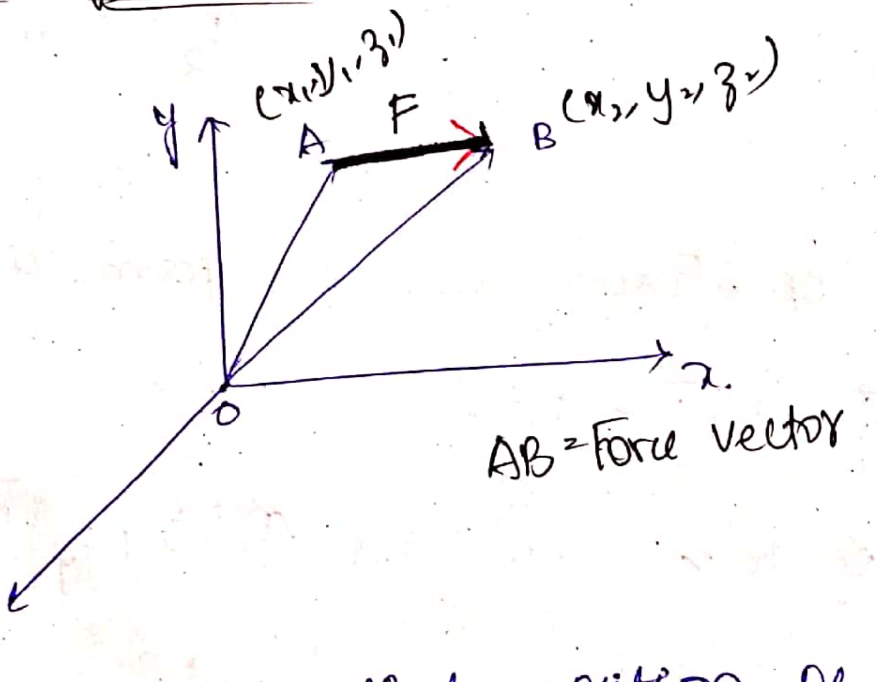
$$\boxed{1 = \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z.}$$

$$F_x = F\cos\theta_x.$$

$$\boxed{\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right).}$$ Similarly

$$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right) \; ; \; \theta_z = \cos^{-1}\left(\frac{F_z}{F}\right)$$

Calculation of Position Vector, Unit Vector and Force Vector.



AB = Force vector

The vector which indicates position of the force is called position vector.

$$\overline{OA} + \overline{AB} = \overline{OB}$$

$$\overline{OB} - \overline{OA} = \overline{AB}$$

$\overline{AB} = (x_2 i + y_2 j + z_2 k) - (x_1 i + y_1 j + z_1 k)$

$\overline{AB} = (x_2 - x_1) i + (y_2 - y_1) j + (z_2 - z_1) k$

Magnitude of Position Vector

$$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$\boxed{\text{Unit vector } (\lambda_{AB}) = \frac{\text{position vector}}{\text{magnitude of position vector}}}$$

Unit Vector :- $\frac{\overline{AB}}{|\overline{AB}|} = \lambda_{AB} = \frac{\text{position vector}}{\text{Magnitude of Position Vector}}$

$$= \frac{(x_2-x_1) i + (y_2-y_1) j + (z_2-z_1) k}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

$\boxed{\textbf{Force Vector } (\vec{F}) = F_{AB} \cdot \lambda_{\overline{AB}}}$

$\boxed{\text{Force Multiplier} = \frac{F_{AB}}{|\overline{AB}|}}$

$$= F_{AB} \left[\frac{(x_2-x_1) i + (y_2-y_1) j + (z_2-z_1) k}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}} \right].$$

$$= x \cdot \left[(x_2-x_1) i + (y_2-y_1) j + (z_2-z_1) k \right]$$

$\boxed{\text{Force vector} = Ai + Bj + Ck}$

$A = F_x$; $B = F_y$; $C = F_z$.

① A force acts at the origin of a coordinate system in a direction defined by the angles. $\theta_x = 70.9°$ and $\theta_y = 144.9°$. Knowing that 'z' component of the force is $-52N$. Determine

① the Angle θ_z ② The other components and the magnitude of force

Sol) $\theta_x = 70.9°$, $\theta_y = 144.9°$

$F_z = -52N$

$F = ?$ $\theta_z = ?$; $F_x = ?$, $F_y = ?$

we know,

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos^2\theta_z = 1 - (\cos^2\theta_y + \cos^2\theta_x)$$

$$\cos\theta_z = \sqrt{0.223} \quad ; \quad \theta_z = 188.03$$

$\cos\theta_3 = 0.4722$

$$F_z = F\cos\theta_z$$

$$F = \frac{F_z}{\cos\theta_z} = \frac{-52}{-0.47} = 110.63\,N$$

$$\boxed{F = 110.63\,N}$$

$$F_x = F\cos\theta_x = 36.20\,N$$

$$F_y = F\cos\theta_y = -90.51\,N$$

22/7/19

① A force 1000N forms angle of 60°, 45° and 120° with x, y, z axis Respectively. Write the equation of the force in the form of vector form.

Sol

$\theta_x = 60°, \quad \theta_y = 45°, \quad \theta_z = 120°$

$\vec{F} = F\cos\theta_x i + F\cos\theta_y j + F\cos\theta_z k$

$= F_x i + F_y j + F_z k$

$= 1000\cos 60° \vec{i} + 1000\cos 45 \vec{j} + 1000\cos 120° \vec{k}$

$\vec{F} = 500\vec{i} + 707.106\vec{j} - 500\vec{k}$

② A force of Magnitude 650N passes through a point P (0, 3, 0) to Q (5, 0, 4). Write the Equation of force in the form of vector form.

Sol

$F = 650N$

$F = F \cdot \lambda_{PQ} = 650 \dfrac{5i - 3j + 4k}{\sqrt{25+9+16}}$

$= \dfrac{\cancel{650}^{130}}{\cancel{5}\sqrt{2}} (5i - 3j + 4k)$

$= 91.93 (5i - 3j + 4k)$

$F = 459.61\vec{i} + (-275.7)\vec{j} + 367.75\vec{K}$

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$F_x = 459.61 N$

$F_y = -275.7 N$

$F_z = 367.75 N$

(3) A Force acts at origin in a Direction defined by angles $\theta_y = 65°$; $\theta_z = 40°$. knowing that x component of force is $-750 N$. Find

(i) the other components

(ii) Magnitude of Force

(iii) The value of θ_x

Sol

$\theta_y = 65°$; $\theta_z = 40°$

$F_x = -750N$

0.17

0.586

0.27

$\cos^2\theta_x = 1 - \cos^2\theta_y - \cos^2\theta_z$

$= 1 - 0.17 - 0.58$

$= 0.27$

$\cos\theta_x = \sqrt{0.234} = -0.483$

$\theta_x = \cos^{-1}(-0.483) = 118.8°$

$\boxed{\theta_x = 118.8°}$

$F_x = F\cos\theta_x$; $F = \dfrac{F_x}{\cos\theta_x} = \dfrac{-750}{-0.483} = 1552.7N$

$F_y = F\cos\theta_y = 1552.7\cos 65$

$= 656.19N$

$F_z = F\cos\theta_z = 1552.7\cos 40^\circ$

$= 1189.42N$

(4) A Force acts at the origin of co-ordinate system in a direction defined by the angles $\theta_x = 69.3^\circ$ and $\theta_z = 57.9^\circ$ and knowing that y component of force is $-170N$. Determine the angle θ_y, the other components & the magnitude of the force

Sol $\theta_x = 69.3^\circ$, $\theta_z = 57.9^\circ$.

$F_y = -170N$

$\cos^2\theta_y = 1 - \cos^2\theta_x - \cos^2\theta_z$

$= 1 - (0.353)^2 - (0.531)^2$

$= 1 - 0.12 - 0.28$

$= 0.6.$

0.35
0.53

$\cos\theta_y = -0.77$

$\theta_y = \cos^{-1}(-0.77)$

$= 140.35^\circ$

$$F = \frac{F_y}{\cos\theta_y} = \frac{-170}{-0.77} = 220.77 N$$

$$F_y = F\cos\theta_y$$

$$= 220.77 \cos 140.35°$$

$$= 220.77(-0.77)$$

$$= -169.99 N$$

$$F_z = F\cos\theta_z$$

$$= 220.77 \cos 57.9$$

$$= 117.31 N$$

θ_y $F_x = F\cos\theta_x$

$$= 220.77 \cos(69.3)$$

$$= 78.03 N$$

⑤ A Force 'F' of magnitude 210N acts at the origin of a co-ordinate system knowing that Force along x-direction $F_x = 80N$, $\theta_z = 151.2°$, and F_y is less than zero. Determine

① the Components F_y, F_z

② The angles θ_x, θ_y.

Sol $F = 210N$; $F_x = 80N$; $\theta_z = 151.2°$

$\boxed{F_y < 0}$.

$\cos\theta_x = \frac{F_x}{F} = \frac{80}{210} = 0.38$

$\theta_x = \cos^{-1}(0.38)$

$= 67.6°$

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\cos^2\theta_y = 1 - 0.145 - (-0.87)^2$

$= 1 - 0.144 - 0.7676$

$\cos\theta_y = \sqrt{0.089} = 0.28$

$\cos\theta_y = -0.28$

$\theta_y = \cos^{-1}(-0.28)$

$= 106.285°$

$F_y = F\cos\theta_y$

$= 210\cos(106.285)$

$= 210(-0.28)$

$= -58.8N$

$F_z = F\cos\theta_z$

$= 210\cos(151.2)$

$= 210(-0.87)$

$= -184.02N$

$F_y = -58.8N$

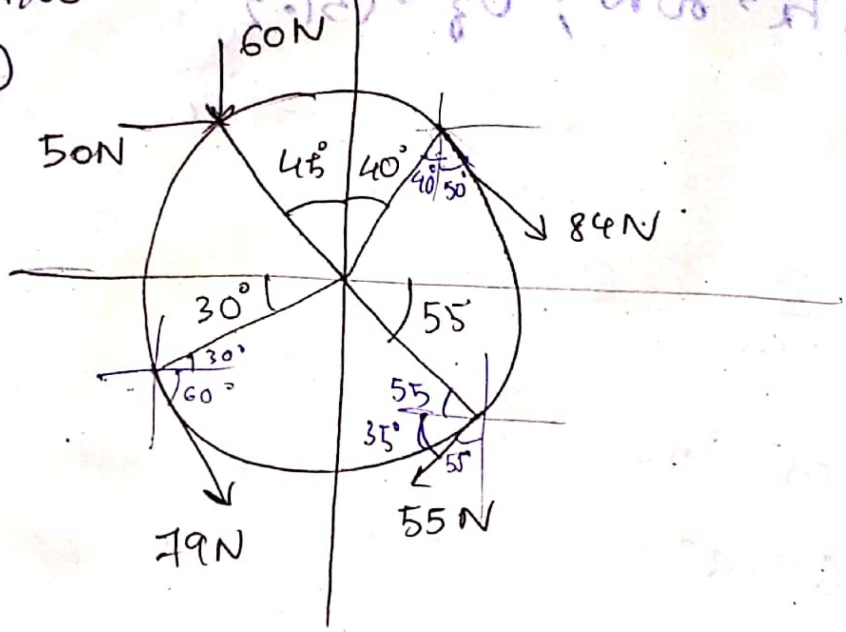
$\theta_x = 67.6$

$\theta_y = 106.26°$

$F_z = -184.02N$

H/W

①



$$\Sigma F_y = -84\cos 50° - 55\sin 35 - 79\sin 60 - 60$$

$$= -213.95 \text{ N}$$

$$\Sigma F_x = +50 + 84\sin 50 - 55\cos 35 + 79\cos 60°$$

$$= 108.79° \text{ N}$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{45774.60 + 11835.2641}$$

$$= \sqrt{57609.8641}$$

$$= 240.020 \text{ N}$$

$$\tan\theta = \frac{-213.95}{108.79} = -1.9666$$

$$\theta = \tan^{-1}(1.966)$$

$$\theta = 63.03°$$

$\Sigma M = 0.$

$-84(2.5) + (-55)(2.5) + 79(2.5)$
$+ (-50)(2.5\cos 40) + 60(2.5\sin 40) = 0.$

$\Rightarrow -210.0 - 137.5 + 197.5 - 95.75$
$+ 96.418 = 0.$

$\Sigma m = -149.332$

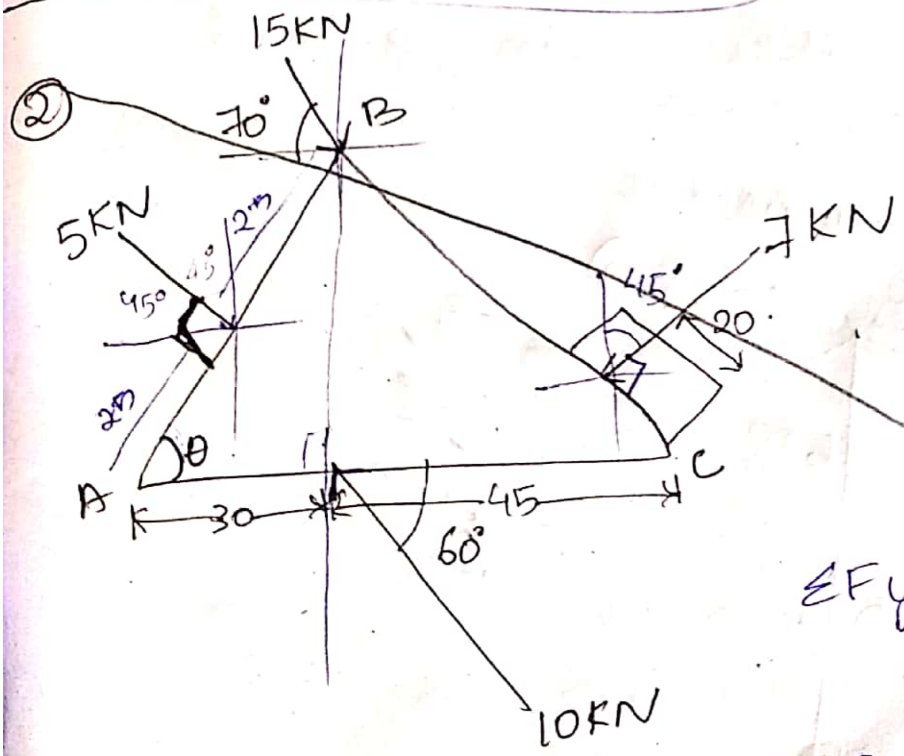
$\Sigma m = R \times d$

$-149.332 = 240.020 \times d$

$\boxed{d = 0.622\,m}$

---

(2)



$\cos\theta = \frac{30}{50}.$

$\theta = \cos^{-1}(0.6)$

$=$

$\Sigma F_y = -15\sin 70° - 7\cos 45°$
$+ 10\sin 60 - 5\sin 45°$

$= -13.920\,N.$

$\Sigma F_x = 15\cos 70° - 7\sin 45° - 10\cos 60 + 5\sin 45$

$= -1.283\,N$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

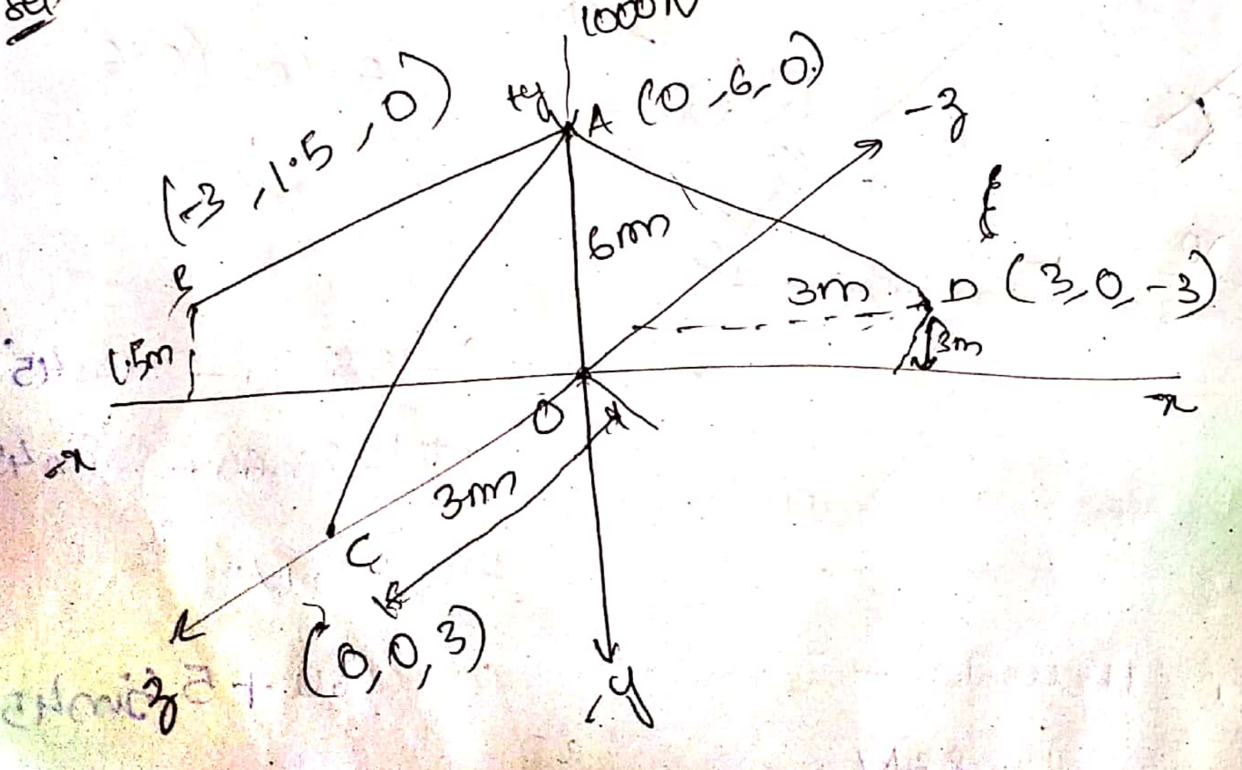
$$= \sqrt{(-13.920)^2 + (-1.283)^2}$$

$$= \sqrt{193.766} = 13.919$$

23/7/19

1) A vertical load of 1000N is supported by 3 Bars as shown in fig. find the forces in each bar.

Sol)



$$\vec{F}_{DA} = F_{DA}\,\lambda_{DA}$$

$$= F_{DA}\,\frac{(-3i + 6j + 3k)}{\sqrt{9 + 36 + 9}}$$

$$= 0.13\,F_{DA}\,(-3i + 6j + 3k).$$

$$= F_{DA}(-0.40i + 0.81j + 0.408k)$$

$$\vec{F}_{CA} = F_{CA}\cdot\lambda_{CA}$$

$$= F_{CA}\cdot\frac{(6j - 3k)}{\sqrt{36 + 9}}$$

$$= 0.14\,F_{CA}\,(6j - 3k) = F_{CA}(0.89j - 0.44k)$$

$$\vec{F}_{BA} = F_{BA}\cdot\lambda_{BA}$$

$$= F_{BA}\cdot\frac{(3i + 4.5j + 0k)}{\sqrt{9 + 20.25}}$$

$$= 0.18\,F_{BA}\,(3i + 4.5j + 0k)$$

$$= F_{BA}(0.54i + 0.81j)$$

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$F + F_{BA} + F_{CA} + F_{DA} = 0$

$-1000j + [0.55i + 0.83j + 0k] F_{BA} +$
$[0i + 0.89j + (-0.44)k] F_{CA} +$
$[-0.40i + 0.8j + 0.408k] F_{DA} = 0.$

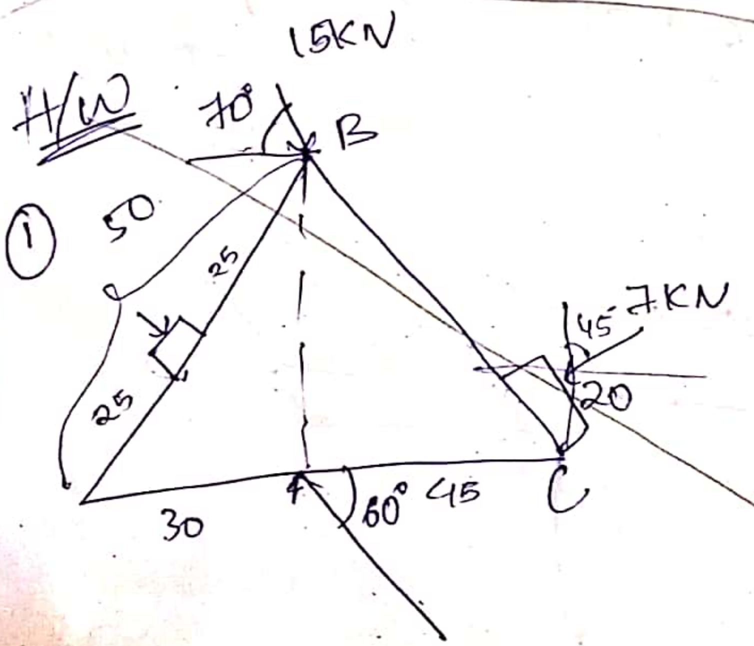
$0.55i F_{BA} + 0i F_{CA} - 0.40i F_{DA} = 0$

$0.83j F_{BA} + 0.89j F_{CA} + 0.8 F_{DA} j = 1000$

$0k F_{BA} - 0.44k F_{CA} + 0.408k F_{DA} = 0$

H/W

(1)



$\Sigma F_y = -15\sin 70° - 7\cos 45° + 10\sin 60 - 5\sin 45°$

$= -13.920N.$

$\Sigma F_x = 15\cos 70° - 7\sin 45° - 10\cos 60 + 5\sin 45$

$= -1.283N$

$R = \sqrt{(\Sigma(F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(-13.920)^2 + (-1.28)^2}$

$= \sqrt{193.766}$

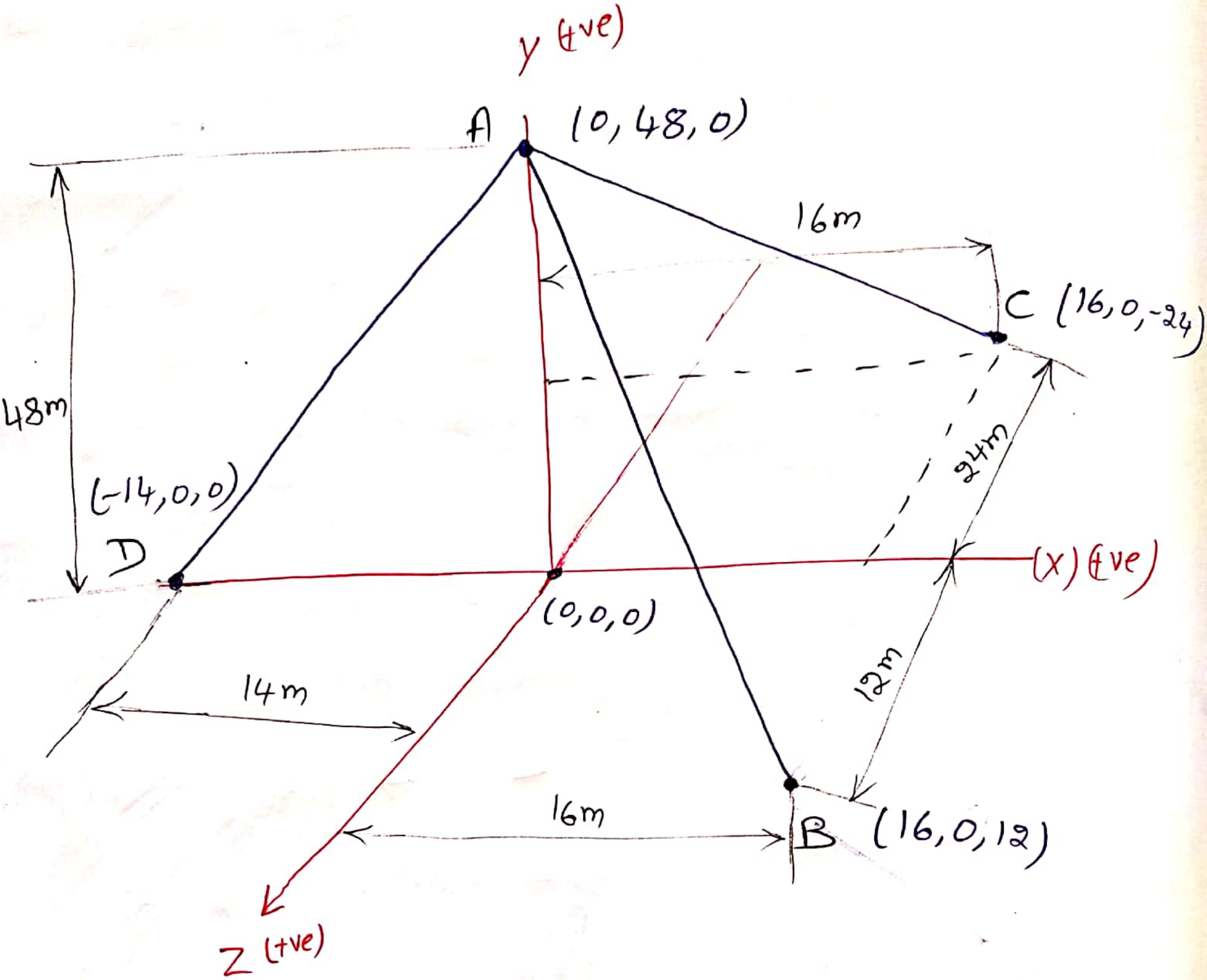
$= 13.919N$; $\tan\theta = \left(\frac{-1.28}{-13.92}\right)$

$\theta = 5.266°$

$R = 13.919N$

$\theta = 5.266°$

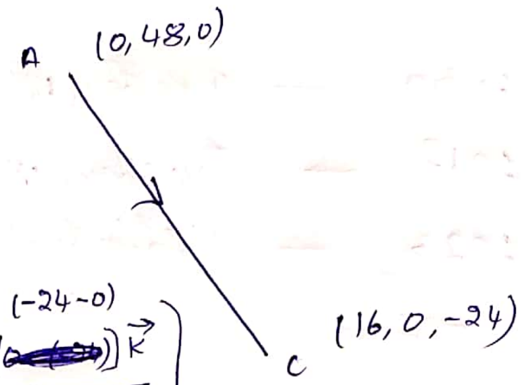
$M \Rightarrow (7\cos 45°)$



Q) Knowing that the tension AC is $T_{AC} = 20\,kN$ Determine the required values of Tension T_{AB} and T_{AD} so that the resultant of the three forces applied at A is vertical and calculate resultant.

Sol Given Data

$T_{AC} = 20KN$



We know

$$\vec{T_{AC}} = T_{AC}\,\lambda_{AC}$$

$$= T_{AC}\left[\frac{(16-0)\vec{i} + (0-48)\vec{j} + (-24-0)\vec{k}}{\sqrt{(16)^2 + (-48)^2 + (-24)^2}}\right]$$

$$= 20\left[\frac{16\vec{i} - 48\vec{j} + (-24)\vec{k}}{\sqrt{(256)+(2304)+(576)}}\right]$$

$$= \frac{20}{56}\left[16\vec{i} - 48\vec{j} - 24\vec{k}\right]$$

$$= 0.35\,(16\vec{i} - 48\vec{j} - 24\vec{k}]$$

$$\boxed{\vec{T_{AC}} = 5.6\vec{i} - 16.8\vec{j} - 8.4\vec{k}} \quad \text{— (1)}$$

ii) Force vector in AB

$$\vec{F_{AB}} = T_{AB}\cdot\lambda_{AB}$$

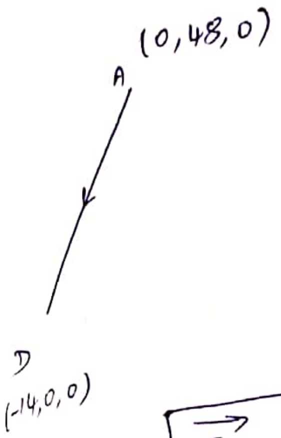
$$= T_{AB}\cdot\left[\frac{(16-0)\vec{i} + (0-48)\vec{j} + (12-0)\vec{k}}{\sqrt{(16)^2 + (-48)^2 + (12)^2}}\right]$$

$$= T_{AB}\left[\frac{16\vec{i} - 48\vec{j} + 12\vec{k}}{\sqrt{2704}}\right]$$

$$= \frac{T_{AB}}{52}\left[16\vec{i} - 48\vec{j} + 12\vec{k}\right]$$

$$= \left[0.30\vec{i} - 0.92\vec{j} + 0.23\vec{k}\right]T_{AB}$$

$$\boxed{\vec{F_{AB}} = 0.30\vec{i}\,T_{AB} - 0.92\vec{j}\,T_{AB} + 0.23\vec{k}\,T_{AB}} \quad \text{— (2)}$$



$\vec{T}_{AD} = T_{AD} \cdot \lambda_{AD}$.

$= T_{AD} \left[\dfrac{(-14-0)\vec{i} + (0-48)\vec{j} + (0-0)\vec{k}}{\sqrt{(-14)^2 + (-48)^2 + (0)^2}} \right]$

$= T_{AD} \left[\dfrac{-14\vec{i} - 48\vec{j} + 0\vec{k}}{\sqrt{2500}} \right.$

$$\boxed{\vec{T}_{AD} = -0.28\vec{i}\,T_{AD} - 0.96\vec{j}\,T_{AD} + 0\vec{k}\,T_{AD}} \quad - (3)$$

we know the equilibrium Conditions

$\Sigma F_x = 0; \quad \Sigma F_y = 0; \quad \Sigma F_z = 0$

In this question he mention $\Sigma F_y = R$

$\underline{\Sigma F_x = 0}$ (Adding i terms)

$5.6 + 0.30\,T_{AB} - 0.28\,T_{AD} = 0$

$0.30\,T_{AB} - 0.28\,T_{AD} = -5.6$ — (4)

$\Sigma F_z = 0$

$-8.4 + 0.23\,T_{AB} + 0\,T_{AD} = 0$

$0.23\,T_{AB} = 8.4$

$$\boxed{T_{AB} = \frac{8.4}{0.23} = 36.52\,KN}$$

Substitute T_{AB} value in eqn (4)

$0.30\,(36.52) - 0.28\,T_{AD} = -5.6$

$10.956 - 0.28\,T_{AD} = -5.6$

$10.956 + 5.6 = 0.28\,T_{AD}$

$$\boxed{T_{AD} = \frac{16.556}{0.28} = 59.12\,KN}$$

$\Sigma y = R$

$-16.8 - 0.92\,T_{AB} - 0.96\,T_{AD} = R$

$-16.8 - 0.92\,(36.52) - 0.96\,(59.12) = R$

$R = -16.8 - 33.59 - 56.75$

$\boxed{~~R = ... KN~~}$ $\boxed{R = -107.14\,KN}$

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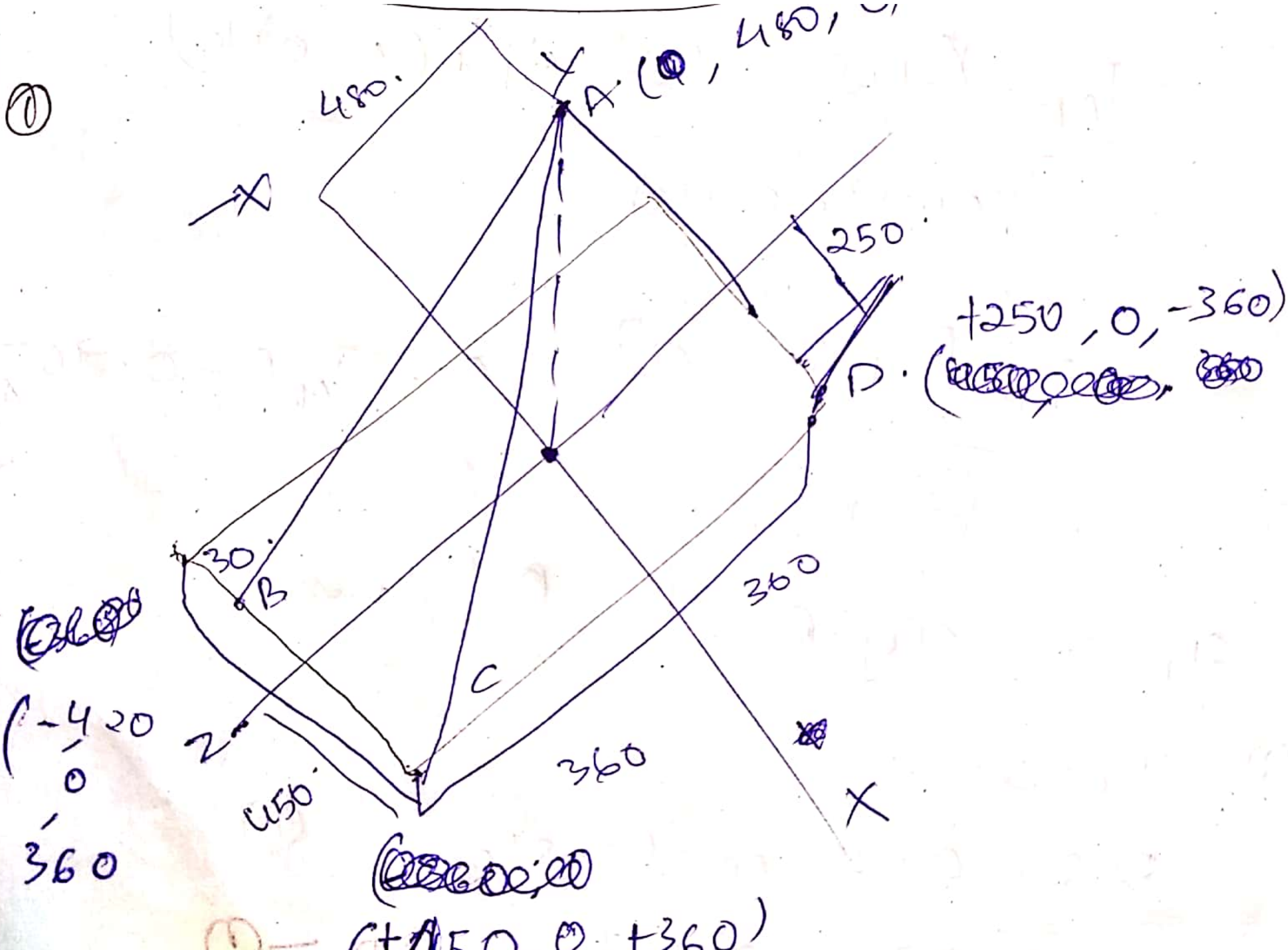
①



① — (+450, 0, +360)

A Rectangular plate is supported by 3 Cables as shown in above figure. Knowing that Tension in cable AD is 520 N. Find the components of force at point D.

$T_{AD} = 520\,N$

$\vec{F}_{AD} = F_{AD} \cdot \lambda_{AD}$

$$= 520 \left[\frac{-250i + 480j + 360k}{\sqrt{62500 + 230400 + 129600}} \right]$$

$$= 520 \left[\frac{-250i + 480j + 360k}{\sqrt{422500}} \right]$$

$$= \frac{520}{650} \left[-250i + 480j + 360k \right]$$

$$= 0.8 \left(-250i + 480j + 360k \right)$$

$$= -200i + 384j + 288k.$$

$F_x = -200\,N$

$F_y = 384\,N$

$F_z = 288\,N$

$T_{AC} = ?$

$\vec{F}_{AC} = F_{AC} \cdot \lambda_{AC}$

A (0, 480, 0) C(450, 0, 360)

$$\vec{F}_{AC} = T_{AC}\left[\frac{(450)i + (480j) + 360k}{\sqrt{(450)^2 + (480)^2 + (360)^2}}\right]$$

$$= T_{AC}\left[\frac{450i \mp 480j + 360k}{\sqrt{202500 + 230400 + 129600}}\right]$$

$$= T_{AC}\left[\frac{450}{750}i \mp \frac{480}{750}j + \frac{360}{750}k\right]$$

$$= T_{AC}\left[0.6i \mp 0.64j + 0.48k\right]$$

$$\vec{F}_{AB} = T_{AB}\,\lambda_{AB}$$

$A(0, 480, 0)$ $\quad B(-420, 0, 360)$.

$$= T_{AB}\left[\frac{-420i + (-480)j + 360k}{\sqrt{176400 + 230400 + 129600}}\right]$$

$$= T_{AB}\left[\frac{-420}{732.39}i - \frac{480}{732.39}j + \frac{360}{732.39}k\right]$$

$= T_{AB}\left[-0.57i - 0.655j + 0.49k\right]$

on comparing force along x, y & z to get value of T_{AB} and T_{AC} and R

- As the forces are acting towards point A (vertically)

i.e $\Sigma F_y = R$.

$\Sigma F_x = 0$; $\Sigma F_z = 0$

$200 + 0.6T_{AC} - 0.57T_{AB} = 0$.

$0.6T_{AC} - 0.57T_{AB} = -200$. — (1)

$-288 + 0.48T_{AC} + 0.49T_{AB} = 0$

$0.48T_{AC} + 0.49T_{AB} = 288$ — (2)

Solving (1) & (2) we get

$T_{AC} = 116.56\ N$

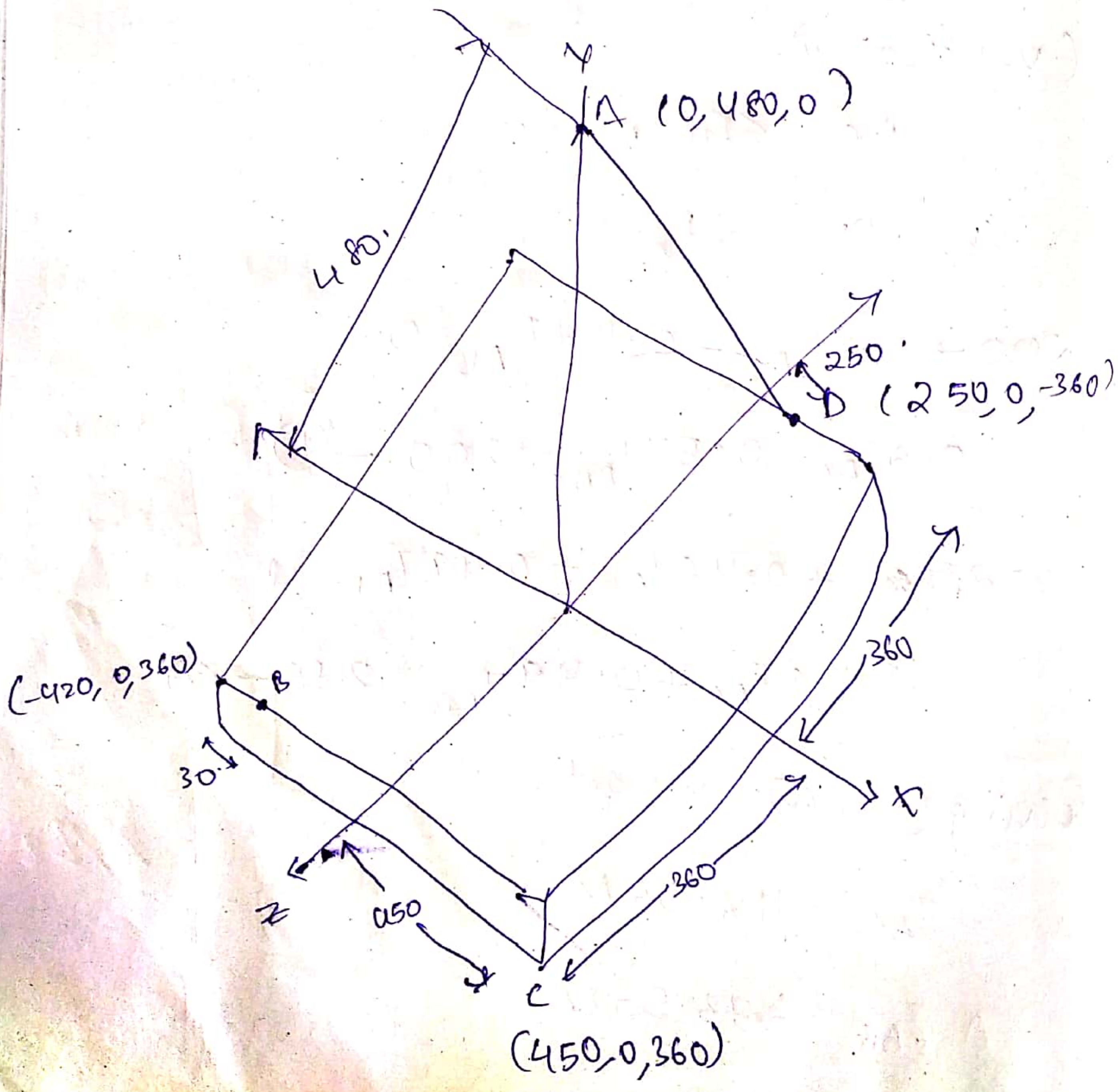
$T_{AB} = 473.57N$

$\Sigma F_y = R.$

$-384 + (-0.64)\, T_{AC} + (-0.655)\, T_{AB} = R.$

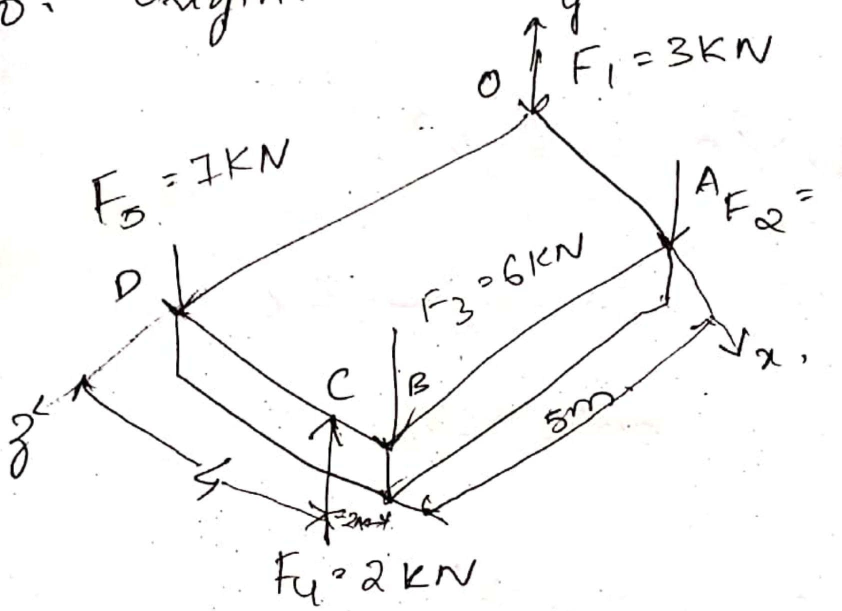
$-384 + (-0.64)(116.56) + (-0.655)(473.57) = R$

$R = -768.78\,N$



NON COPLANAR PARALLEL FORCE SYSTEM

① 5 vertical forces are acting on a horizontal plane as shown in below figure. Find the resultant of the forces and point of application with respect to Origin.



Sol

Force vector $(\vec{F}) = F\lambda$

$\vec{F_1} = -3j$; $\vec{F_2} = -5j$; $\vec{F_3} = -6j$; $\vec{F_4} = 2j$

$\vec{F_5} = -7j$

$R = \Sigma F = -19j$

$\Sigma M = R \times d$

In Non-coplanar $A \cdot B = B \cdot A$

$A \times B \neq B \times A$

$M_1 = (0i + 0j + 0k)(0i - 3j + 0k)$

$= 0$ N-m

$m_2 = (6i + 0j + 0k)(0i - 5j + 0k)$

$$\begin{vmatrix} i & j & k \\ 6 & 0 & 0 \\ 0 & -5 & 0 \end{vmatrix} \Rightarrow -30k$$

$m_3 = (6i + 0j + 5k)(0 - 6j + 0k).$

$$\begin{vmatrix} i & j & k \\ 6 & 0 & 5 \\ 0 & -6 & 0 \end{vmatrix} \Rightarrow i(0 + 30) - 36k$$

$$= 30i - 36k.$$

$m_4 = (4i + 0j + 5k)(0i + 2j + 0k).$

$$\begin{vmatrix} i & j & k \\ 4 & 0 & 5 \\ 0 & 2 & 0 \end{vmatrix} = -10i + 8k.$$

$m_5 = (0i + 0j + 5k)(0i - 7j + 0k)$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & 5 \\ 0 & -7 & 0 \end{vmatrix} = 35i$$

$m_5 = 35i$

$\Sigma M = R \times d$

$0 + (-30k) + 30i - 36k - 10i + 8k + 35i$

$\Sigma M = 55i - 58k$

$\boxed{\Sigma M = R \times d}$

$M_x = R \times d$

$$\begin{vmatrix} i & j & k \\ x & 0 & 3 \\ 0 & -19 & 0 \end{vmatrix}$$

$i(0\times0 - (-19\,3)) - j(x\times0 - 3\times0) + k[x\times(-19) - (0\times0)]$

$M_x = 193\vec{i} + (-19x)\vec{k}$

Apply varignon theorem

$\Sigma M = M_x$

$55\vec{i} - 58\vec{k} = 193\vec{i} - 19x\vec{k}$

$55\vec{i} = 193\vec{i}$

$3 = \frac{55}{19}$

$\boxed{3 = 2.89\,m}$

$-58\vec{k} = -19x\vec{k}$

$x = \frac{58}{19}$

$\boxed{x = 3.05\,m}$

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