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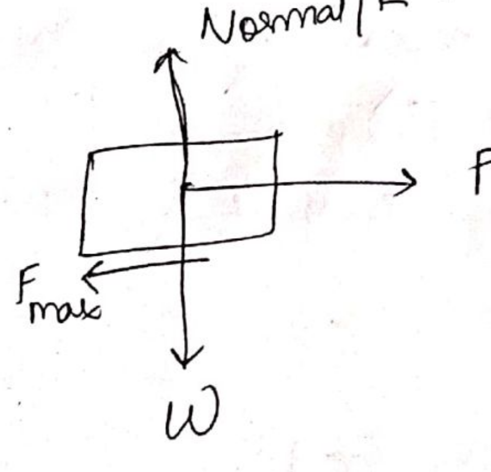
FRICTION

Friction:- When a body moves or tends to move over another body, a force opposing the motion develops at the contact surface which is tangential to the contact surface. and this is called as frictional force or friction.

Limiting frictional force:- The maximum frictional force that is developed at the surface when the block is at the verge of motion (Impending Motion).

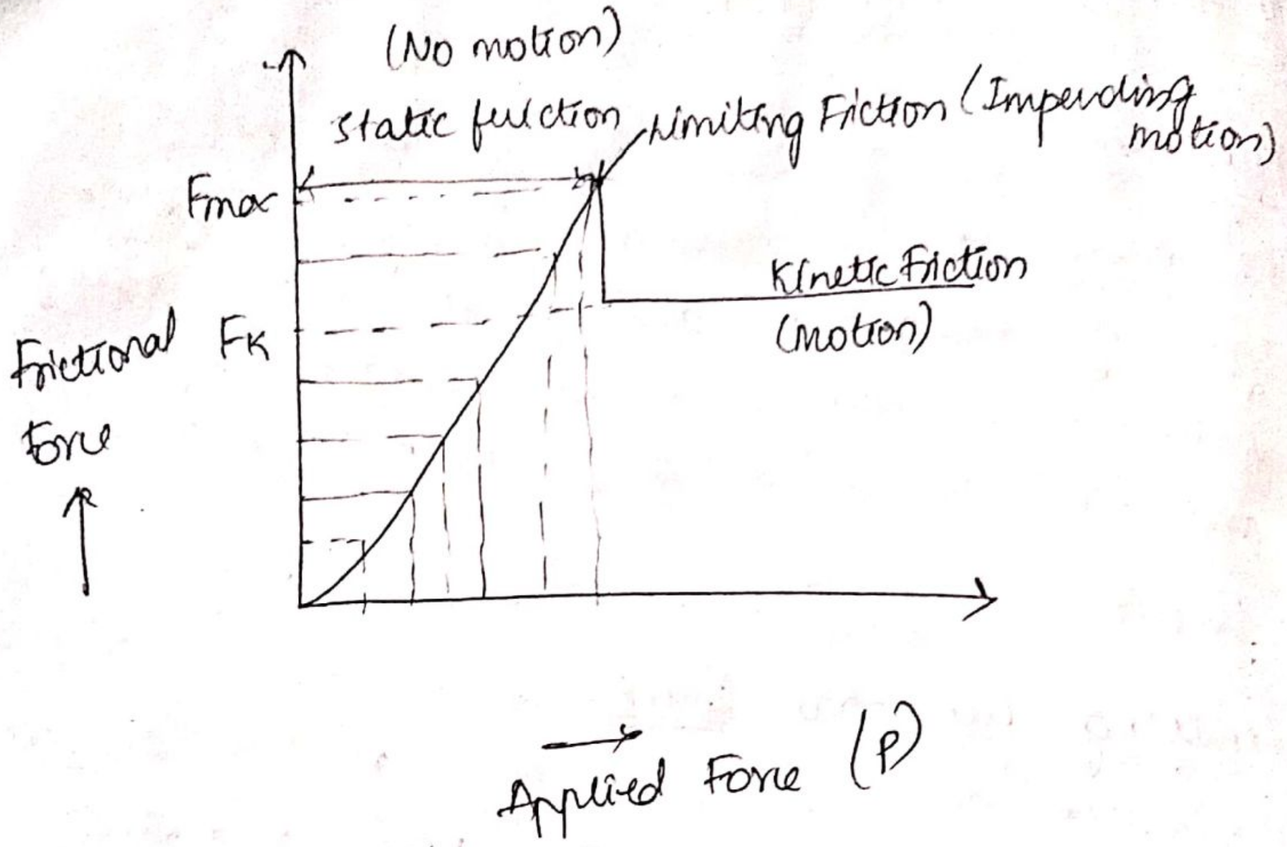


$F_{max} \propto N$

$F_{max} = \mu_s N$ [∵ μ_s = coeff of static friction]

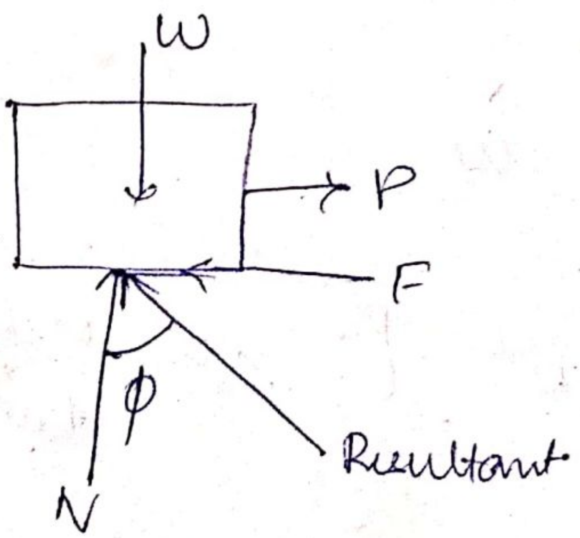
$\mu_s = \frac{F_{max}}{N}$; $\mu_k = \frac{F_{max}}{N}$.

$F_K < F_{max}$.



Angle of Friction:-

It is the angle made by the Resultant of Limiting Frictional Force and the Normal Reaction with the Normal Reaction.



$F_{max} = R \sin\phi$

$\mu_s N = R \sin\phi \quad \text{--- (1)}$

$N = R\cos\phi$ — ②

① ÷ ②

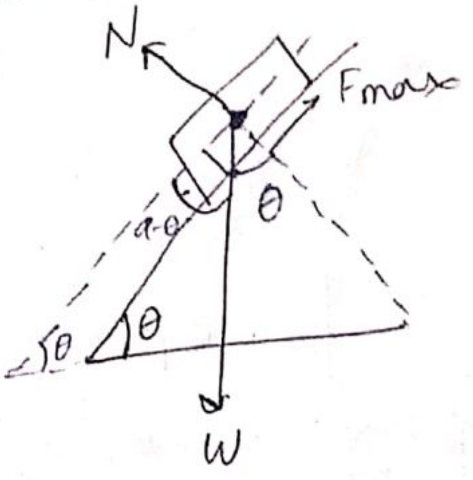
$$\frac{\mu_s N}{N} = \frac{R\sin\phi}{R\cos\phi}$$

$$\tan\phi = \mu_s$$

$$\phi = \tan^{-1}(\mu_s)$$

Angle of Repose

It is the minimum angle of inclination of a plane with the horizontal which the body kept will just slide down on it without of application of external force.



$\Sigma F_x = 0$.

$F_{max} - W\sin\theta = 0$

$F_{max} = W\sin\theta$

$\mu_s N = W\sin\theta$ — ①

$\Sigma F_y = 0 \Rightarrow N - W\cos\theta = 0$

$N = W\cos\theta$ — ②

① ÷ ②.

$\mu_s = \tan\theta$

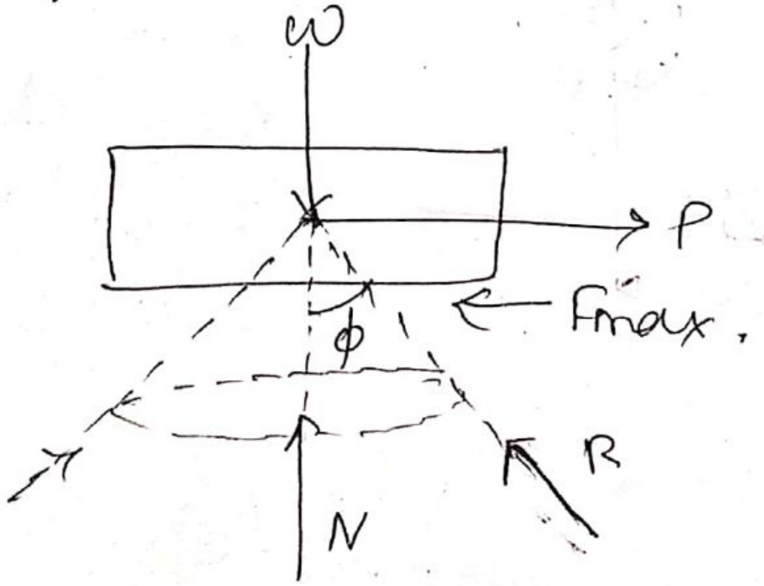
$\theta = \tan^{-1}(\mu_s)$

- Angle of friction = Angle of repose

Cone of Friction :-

When the applied force (P) is just ~~to~~ sufficient to produce the impending motion of given body, Angle of friction (ϕ) is obtained which is the angle made by Resultant of Maximum Frictional Force and Normal Reaction with Normal Reaction.

- If the direction of applied force is changed through 360°, the resultant (R) generates a right circular cone with semi vertex angle equal to ϕ. This is called cone of friction.



Laws of Friction:-

① The frictional force is always tangential to the contact surface and acts in a direction opposite to that in which the body tends to move.

② The magnitude of frictional force is self-adjusting to the applied force till the limiting friction is reached and at the maximum frictional [illegible] force the body will have impending motion.

③ Limiting frictional force is directly proportional to Normal Reaction.

$$F_{max} \propto N$$

$$F_{max} = \mu_s N$$

④ For a body in motion F_K developed is less than that of Maximum frictional force and the relation $\boxed{F_K = \mu_K N}$ [illegible] holds good.

⑤ Frictional Force depends on Roughness of the surface and the Material in contact

⑥ Frictional Force is independent of area of surface and speed of the Body.

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(7)

$\mu_s > \mu_k$

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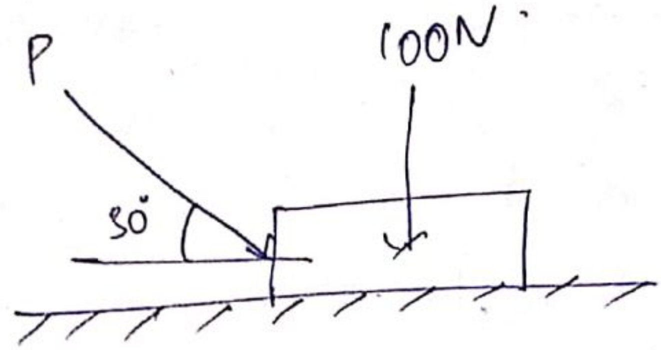
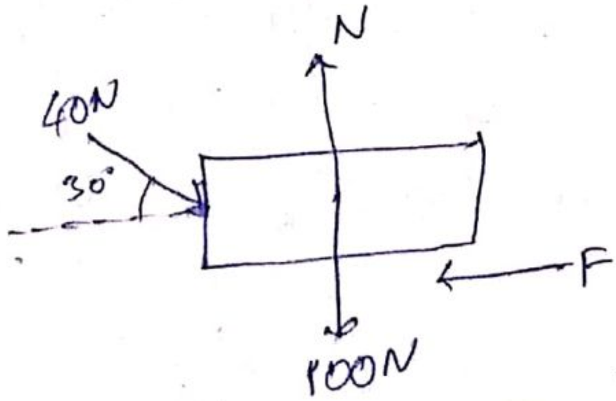
(1) Determine the frictional force developed on the block as shown in below figure when (i) P = 40N (ii) P = 80 N take $\mu_s = 0.3$ and $\mu_k = 0.25$

(iii) Also find the value of P when the block is about to move.

Sol:



(i)



$\Sigma F_x = 0$

$40 \cos 30° - F = 0$

$F = 34.64 \ N$

actual

$\Sigma F_y = 0$

$N - 100 \sin 30° - 100 = 0$

$N = 120\ N$

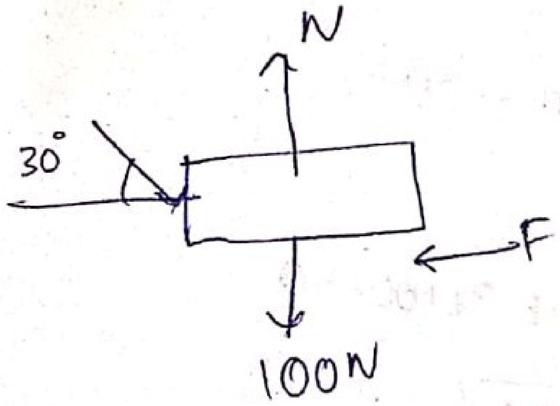
$F_{max} = \mu_s N$

$= 0.3\ (120)$

$F_{max} = 36 N$

$F_{max} > F_{actual}$ [Body is at Rest].

(iii)



$\Sigma F_x = 0$

$F = 80 \cos 30°$

$F_{actual} = 69.28\ N$

$\Sigma F_y = 0$

$N - 80 \sin 30° - 100 = 0$

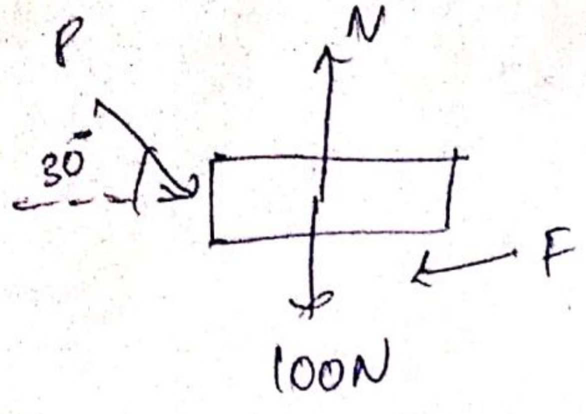
$N = 140\ N$

$F_{max} = \mu_s N$

$= 0.3 \times 140 = 42 N$

$F_{max} < F_{actual}$ (Body is Moving)

(iii)



$P = F_{max}$

$\Sigma F_x = 0$

$P\cos 30 - F = 0$

$P\cos 30 - \mu_s N = 0$ — (1)

$\Sigma F_y = 0$

$N - P\sin 30 - 100 = 0$

$N = 0.5P + 100$ — (2)

eq (1) & (2)

$P(0.86) - 0.3(0.5P + 100) = 0$

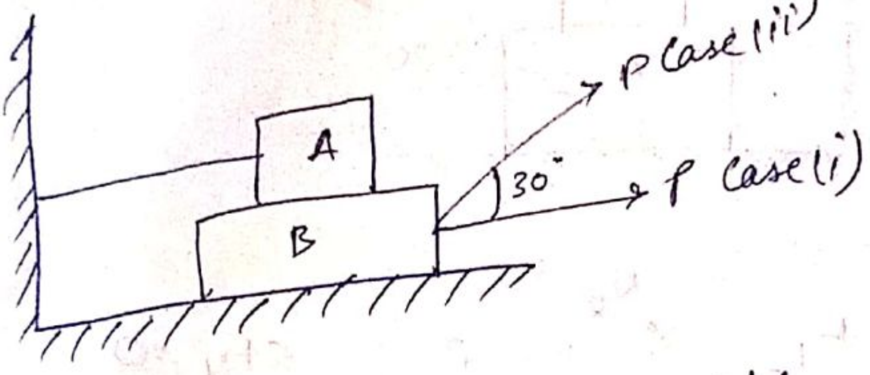
$$\boxed{P = 42.25N}$$

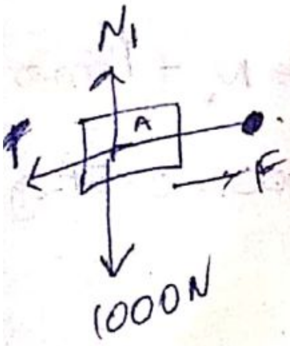
(2) Block A weighing 1000N rests over block B which weighs 2000 N. Block A is tied to wall with a horizontal string. If coeff of friction b/w block A and B is 0.25 and b/w floor and block B is 1/3. What should be the value of P to move the block B. If

(i) P is horizontal

(ii) P acts at 30° upwards to the Horizontal

Sol





$\Sigma F_y = 0 \Rightarrow N = 1000\,N$

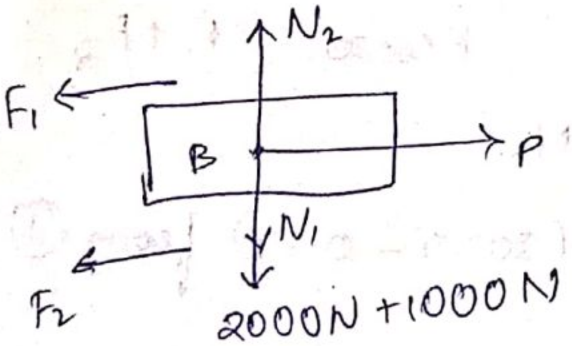
$\Sigma F_x = 0 \Rightarrow F = T$

$\mu N = T = 0.25 \times 1000$

$T = 250\,N$

$T = F = 250\,N$

Case (i)



$\Sigma F_x = 0$

$P - F_1 - F_2 = 0$

$P = F_1 + F_2$

$= \mu_{AB} N_1 + \mu_{BFloor} N_2$

$= (0.25)\,1000 + (0.33)(3000)$

$\boxed{P = 1250\,N}$

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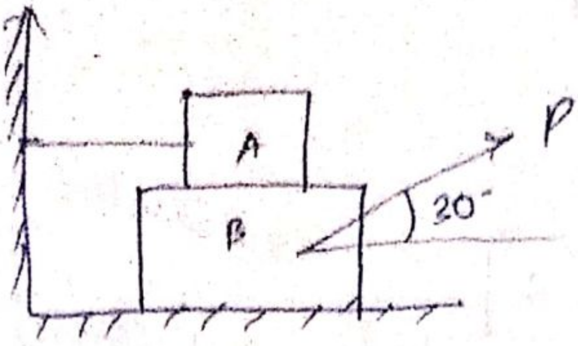
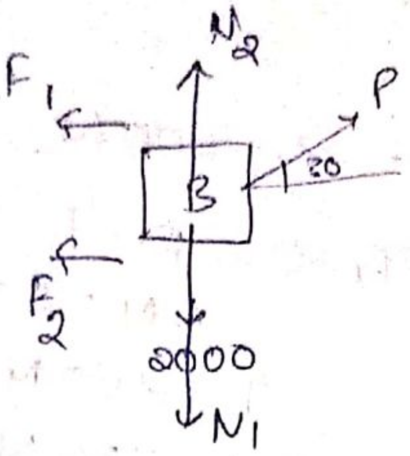
case (ii) :-





$\Sigma F_y = 0$

$2000 + N_1 = N_2 + P\sin 30$

$N_2 = 3000 - 0.5P$ — (1)

$\Sigma F_x = 0$

$P\cos 30 = F_1 + F_2$

$0.866P = \mu_1 N_1 + \mu_2 N_2$

$= 250 + \frac{1}{3}(3000 - 0.5P)$ from (1)

$0.866P + 1.026P = 1250$

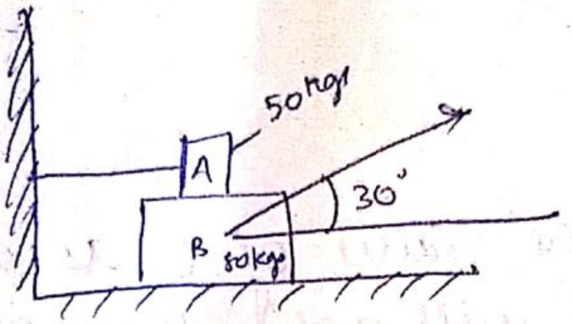
$P = 1218.32N$

(3) Two blocks A and B weighing 50 kgs and 80 kgs respectively are in Equillibrium in the position as shown in below figure. Calculate the Force P required to move the lower block B and tension in the cable take $\mu = 0.3$ for all contact surfaces.

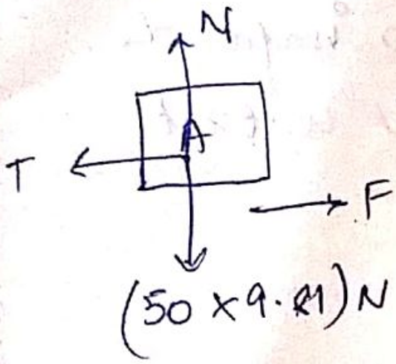
(ii) Also calculate 'P' when it is acting in horizontal position towards right side

H.W.

Sol



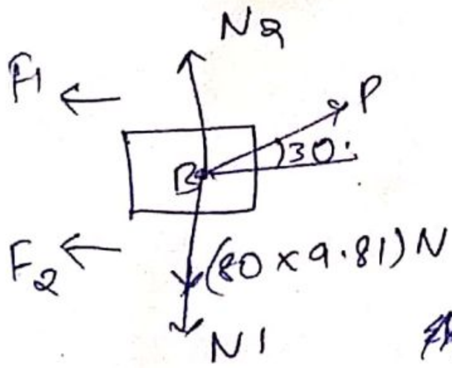
$\mu = 0.3$.



$N_1 = 50 \times 9.81$
$= 490.5 N$

$T = F$
$= \mu N = 0.3 (490.5)$

$T = 147.15 N$



$\Sigma F_y = 0$.

$P \sin 30° + N - 784.8 = 0$.

$0.5P + N_2 - 490.5 - 784.8 = 0$

$N_2 = 1275.3 - 0.5P$ — (1).

$\Sigma F_x = 0$

$P \cos 30° - F_1 - F_2 = 0$

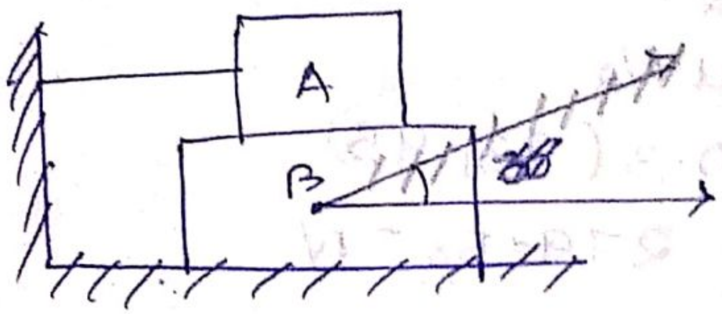
$0.866P - \mu_1 N_1 - \mu_2 N_2 = 0$.

$0.866P = (0.3 \times 490.5) + (0.3)(1275.3 - 0.5P)$

P = 521.4N

H/W

③ (i)



F.B.D :-

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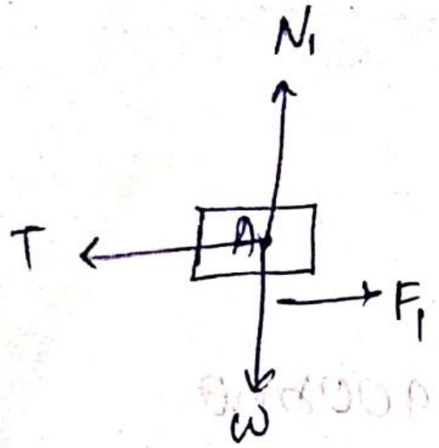
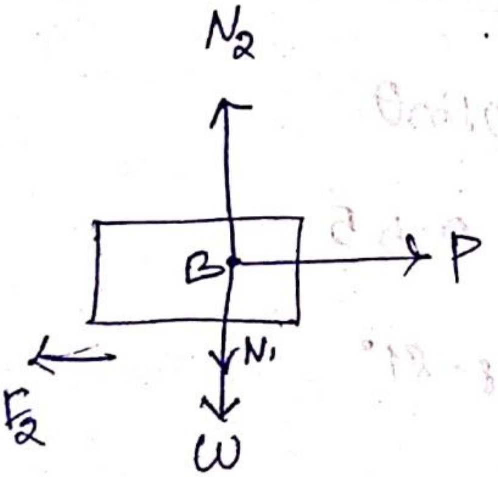
FBD



$N_1 = 50 \times 9.81$
$= 490.5N$

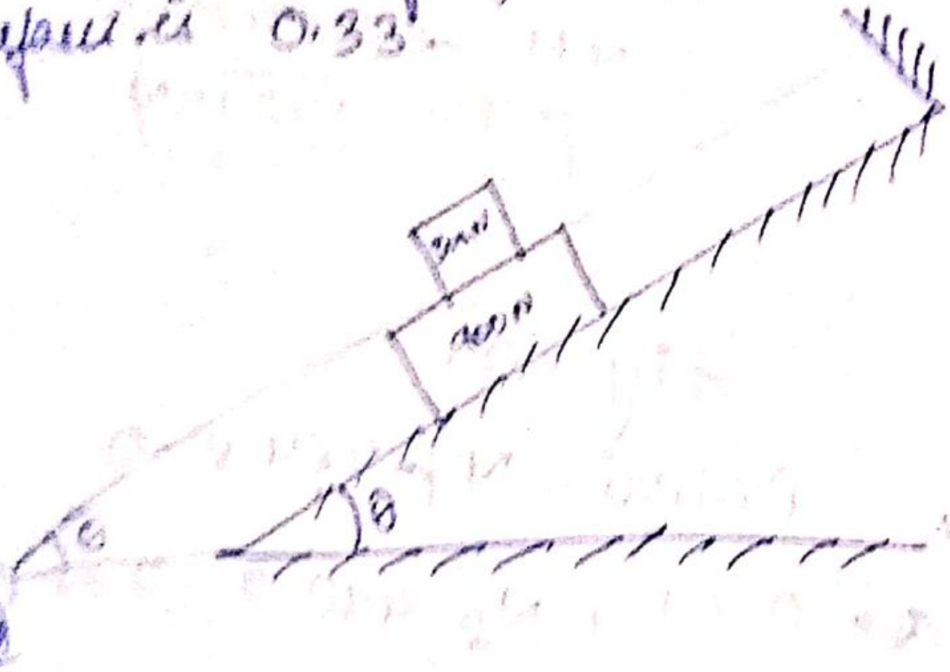
$T = F$
$T = \mu R.$
$= 490.5 \times (0.3)$
$= 147.15N$



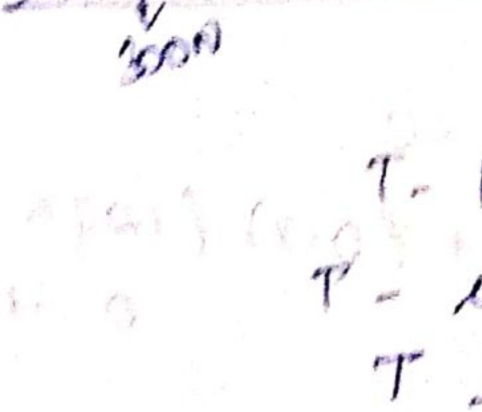
$N_2 = N_1 + W$
$= (490.5)(0.3) + 80(9.81)$
$= 147.15 + 784.8$
$= 931.95N$

$P = F_2$
$= \mu N_2$
$= 0.3(931.95)$
$= 279.585 N$

(4) What should be the value of θ as shown in below figure that will make the motion of 400N block down the plane to impend. The coefficient of friction for all the contact surfaces is 0.33.



Sol



$\Sigma F_y = 0$

$N_1 = 300\cos\theta$

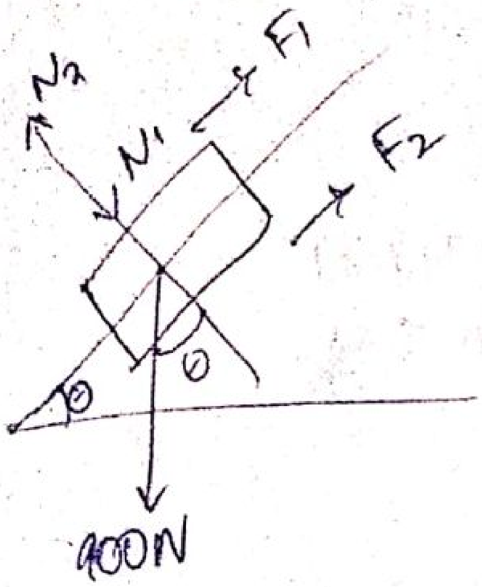
$N_1 = 300\cos\theta$ — (1)

$\Sigma F_x = 0$

$T - F - 300\sin\theta = 0$

$T - \mu_1 N_1 = 300\sin\theta$

$T - 0.33(300\cos\theta) = 300\sin\theta$ — (2)



$\Sigma F_y = 0$

$N_2 - N_1 - 900\cos\theta = 0$

From ①

$N_2 = 300\cos\theta + 900\cos\theta$

$$N_2 = 1200\cos\theta$$

$\Sigma F_x = 0$

$F_1 + F_2 - 900\sin\theta = 0$

$\mu_1 N_1 + \mu_2 N_2 = 900\sin\theta$

$(0.33)(300\cos\theta) + 0.33(1200\cos\theta) = 900\sin\theta$

$495\cos\theta = 900\sin\theta$

$\tan\theta = 0.55$

$\theta = 28.81^\circ$

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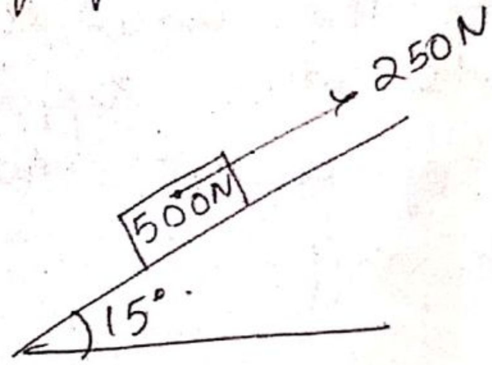
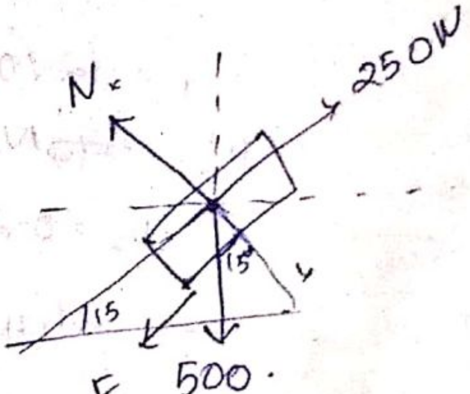
H/W

① A force of 250N pulls a body of weight 500N up an inclined plane, the force being applied parallel to the plane. If the inclination of the plane to the horizontal is 15°, find the coefficient of friction.

Sol



F.B.D:-



$\Sigma F_y = 0$

$N - 500\cos 15 = 0$

$\Sigma F_x = 0$

$250 - F - 500\sin 15 = 0$

$F = 120.59\ N$

$N = 482.96\ N$

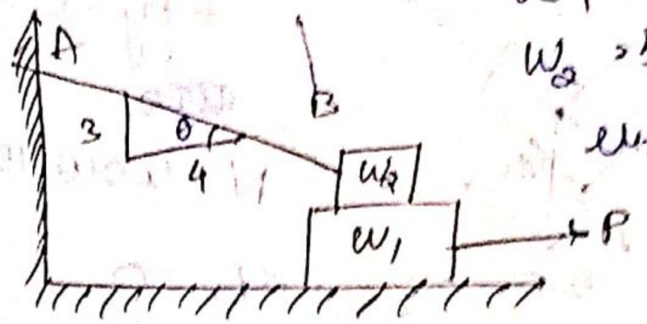
$N \times \mu = F$

$120.59 = \mu(482.96)$

$\boxed{\mu = 0.2496}$

Q.) A block of weight $W_1 = 1290N$ rests on a horizontal surface and supports another block of weight $W_2 = 570N$ on top of it. Block of W_2 is attached to a vertical wall by an inclined string AB. Find the force 'P' applied to the lower block that will be necessary to cause slipping to impend. Coefficient of friction b/w block (1) and (2) = 0.25 & coeff. b/w (1) & Horizontal surface is 0.4.

Sol



$W_1 = 1290N$

$W_2 = 570N$

$\mu_2 = 0.25$

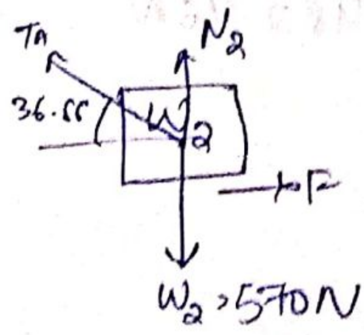
$\mu_{1,4} = 0.4$

Sol

$\tan\theta = 3/4$

$\theta = 36.86°$

F.B.D



$\Sigma F_y = 0$

$N_2 + T_A \sin 36.86° - 570 = 0$

$N_2 = 570 - 0.59\, T_A$

$\Sigma F_x = 0$

$F - T_A \cos 36.86° = 0$

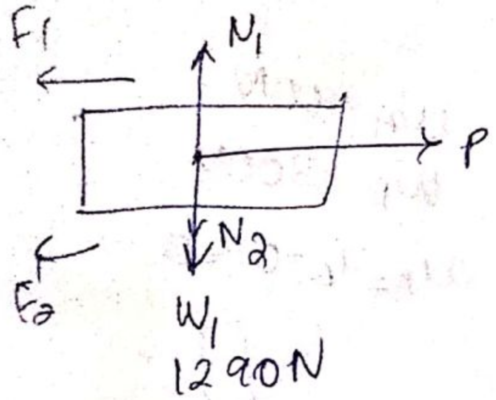
$F = T_A(0.80)$

$\mu N_2 = T_A (0.80)$

$0.8 T_A = 0.25 \times (570 - 0.59 T_A)$

$0.8 T_A + 0.1475 T_A = 142.5$

$T_A = 150.39 N$



$\Sigma F_x = 0$

$P - F_1 - F_2 = 0$

$P = \mu_{12} N_1 + \mu_{2A} N_2$

$= 0.25 N_1 + 0.4 (570 - 0.59 T_A)$

$= 0.25 N_1 + 228 - 0.236 T$ — ③

$\Sigma F_y = 0$

$R_1 - R_2 - 1290 = 0$

$R_1 = 1290 + 570 - 0.59 T$ — ④

$P = 0.25 (1290 + 570 - 0.595) + 228 - 0.23 T$

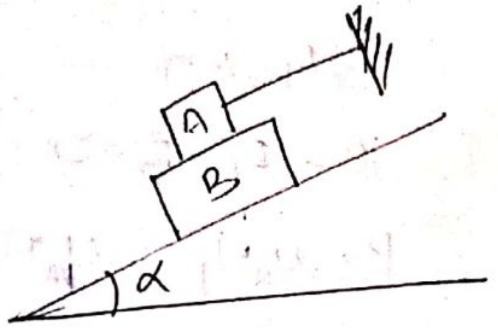
$P = 693 - 0.377 T$

$T = 150.39 N$

$P = 636.30 N$

③ Two blocks A and B of weight 100N and 300N respectively are resting on rough inclined plane. Find the value of angle (α) when the block B is about to slide. Take coeff of friction b/w two blocks as well as block B and Inclined plane is 0.25.

Sol

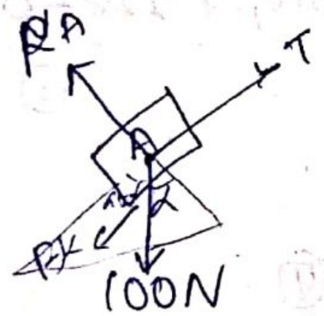


$W_A = 100N$

$W_B = 300N$

$\mu_A = \mu_B = 0.25$

F.B.D



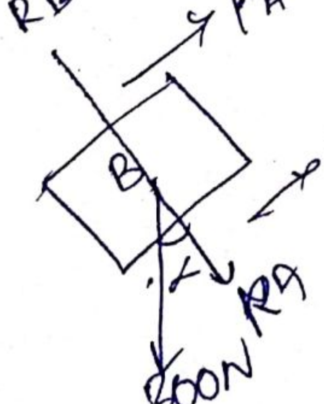
$\Sigma F_x = 0$

$T - F - 100 \sin\alpha = 0$

$\Sigma F_y = 0$

$R_A - 100\cos\alpha = 0$

$R_A = 100\cos\alpha$



$\Sigma F_x = 0$

$F_A + F_B - 300\sin\alpha = 0$

$\mu_A R_A + \mu_B R_B = 300\sin\alpha$

$(100\cos\alpha) 0.25 + 0.25 R_B = 300 \sin\alpha.$

$\Sigma F_y = 0$

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$R_B - R_A = 300 \cos\alpha$

$R_B = 100 \cos\alpha + 300 \cos\alpha$

$R_B = 400 \cos\alpha$

$25 \cos\alpha + 0.25(400 \cos\alpha) = 300 \sin\alpha$

$125 \cos\alpha = 300 \sin\alpha$

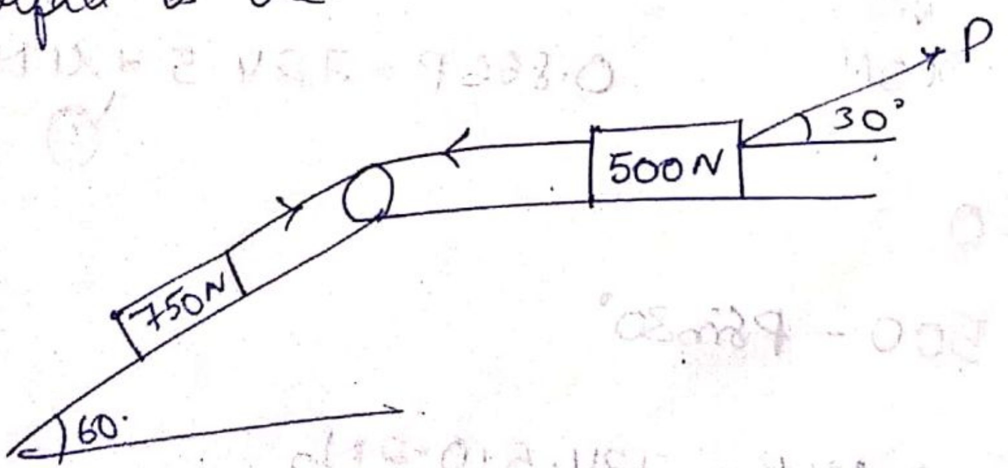
$\tan\alpha = 0.41$

$\alpha = 22.29°$

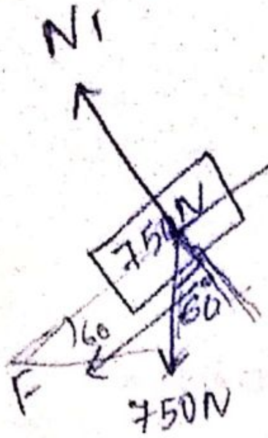
(4) ☆☆☆

What is the value of P in the system to cause the motion of 500 N block to the right side assume the pulley is smooth and the coefficient of friction b/w other contact surface is 0.2.

Sol:



F.B.D



$\Sigma F_x = 0$

$T_1 - 750 \sin 60° - F = 0$

$T_1 = F + 649.51$

$= \mu N_1 + 649.51$

$\Sigma F_y = 0$

$N_1 = 750 \cos 60°$

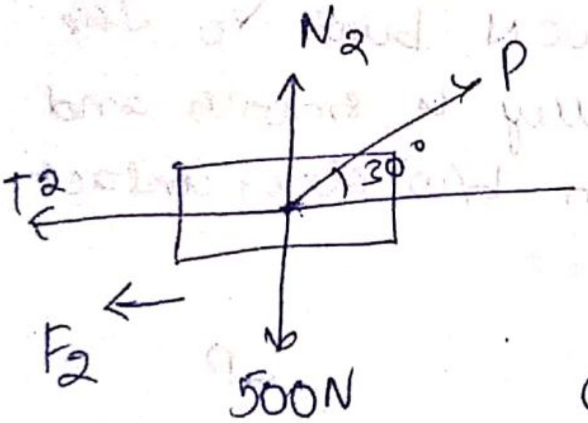
$= 375 N$

$= (0.2)(375) + 649.5)$

$= 75 + 649.5$

$T_1 = 724.5 N$

$T_1 = T_2$



$\Sigma F_x = 0$

$P \cos 30 - F_2 - T_2 = 0$

$P(0.866) - \mu N_2 - 724.5 = 0$

$0.866P = 724.5 + \mu N_2$ — (1)

$\Sigma F_y = 0$

$N_2 = 500 - P \sin 30°$

(1) => $0.866P = 724.5 + 0.2N_2$

$= 724.5 + 0.2(500 - P(0.5))$

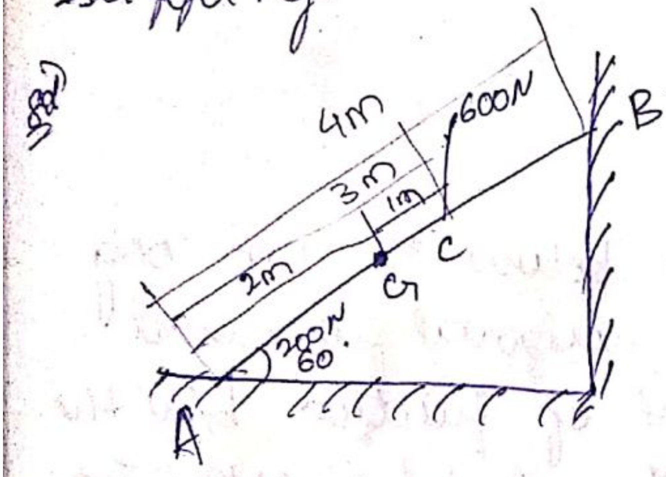
$0.966P = 824.5$

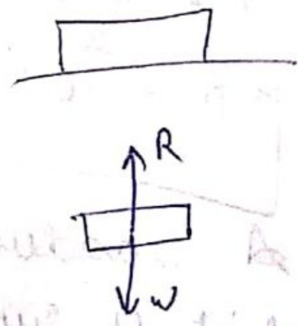
$P = 853.51 N$

26/8/19

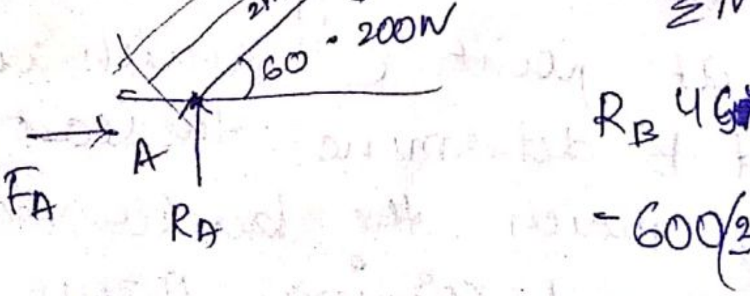
① A ladder of 4m weighing 200N is placed against a vertical wall. The coefficient of friction b/w wall & ladder is 0.2 & b/w floor & ladder is 0.3. In addition to self-weight the ladder has to support a man weighing 600N at a distance of 3m from A. Calculate the minimum horizontal force to be applied at pt A to prevent slipping.

Sol)





F.B.D



$\Sigma M = 0$ at (A)

$R_B \, 4 \sin 60^\circ + F_B \, 4 \cos 60^\circ - 600(3 \cos 60^\circ) - 200(2 \cos 60^\circ) = 0$

$\mu_B = 0.2$
$\mu_A = 0.3$

$R_B(3.46) + 2(\mu_B) R_B = 1100$

$R_B = 284.97 N$

$\Sigma F_x = 0$

$F_A + P - R_B = 0$

$\mu_A R_A + P = R_B$ ——— ②

$P = 284.97 - 0.3 R_A$

$\Sigma F_y = 0$

$R_A + F_B - 200 - 600 = 0$

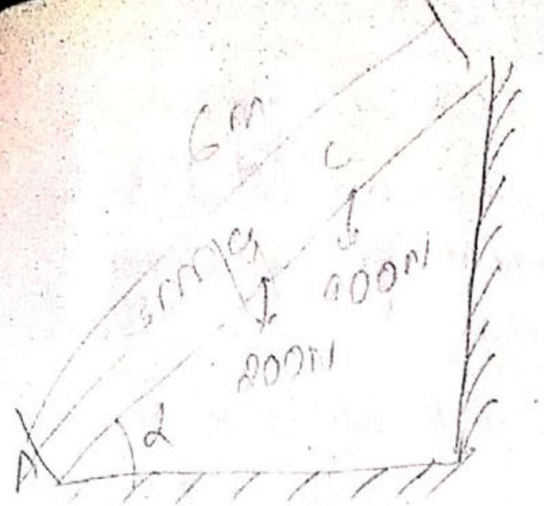
$R_A = -\mu_B R_B + 800$

$R_A = 743 N$

$P = 284.97 - 22.9$

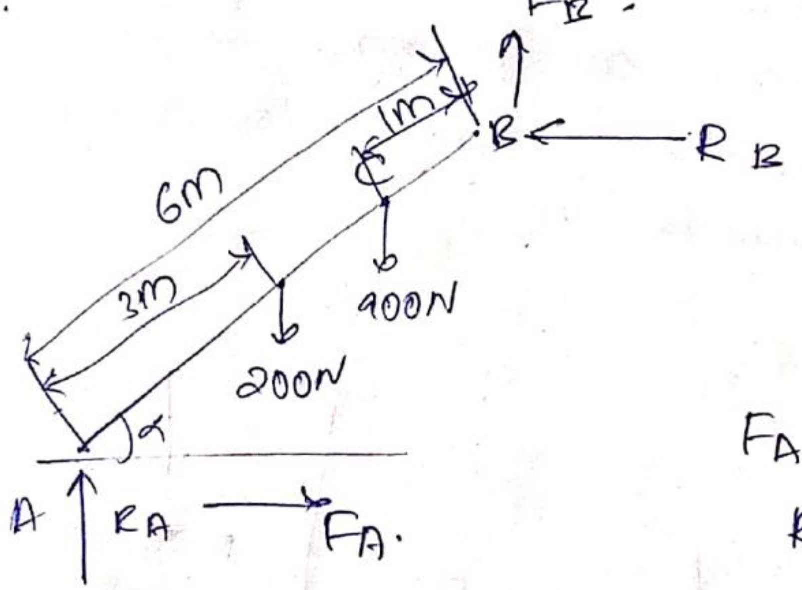
$P = 62.07 N$

② A ladder as shown below is 6m long and it is supported by a horizontal floor and vertical wall. The coefficient of friction b/w the floor and ladder is 0.25 and b/w wall & ladder is 0.4. The self weight of a ladder is 200N. And may be considered as concentrated at G. The ladder also supports a vertical load of 900N at point C which is at 1m from pt B determine the least value of α at which the ladder may be placed without slipping. Determine the Reactions developed at that stage.



$\mu_A = 0.25$

$\mu_B = 0.4$



$\Sigma F_x = 0$

$F_A - R_B = 0$

$R_B = \mu_A R_A$

$R_B = 0.25 R_A$

$R_B = 0.25(1000)$

$= 250N$

$\Sigma F_y = 0$

$F_B + R_A - 200 - 900 = 0$

$\mu_B R_B + R_A = 1100$

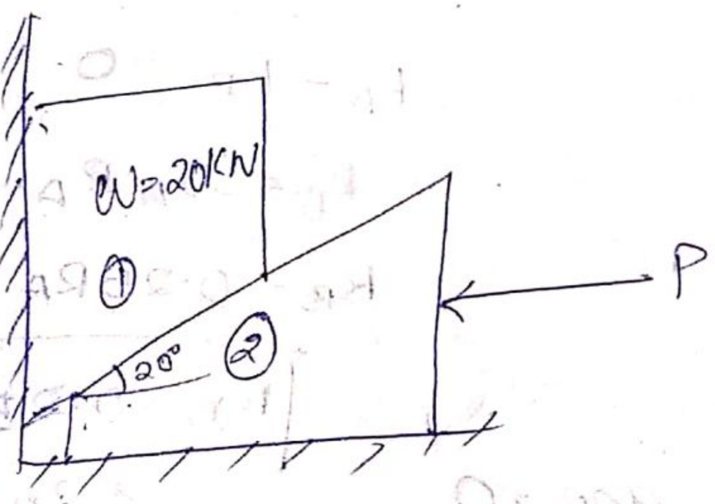
$(0.4)(0.25)R_A + R_A = 1100$

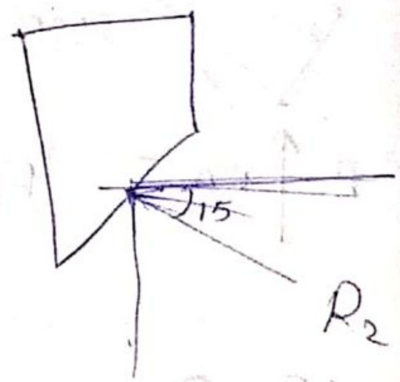
$1.1 R_A = 1100 \quad \therefore \quad R_A = 1000N$

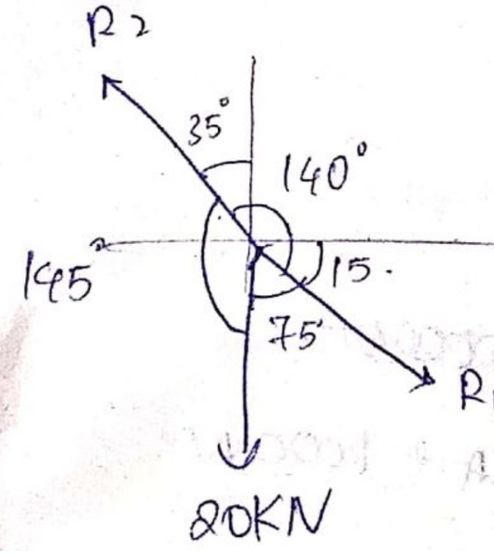
- wedge friction: 3/9/18

wedges are small pieces of materials with two of their opp surfaces are not parallel.
- They are used to slightly lift heavy blocks. The weight of wedge is very small as compared to the weight lifted.
Hence in all problems the self-weight of the wedge is neglected.

① Determine the minimum force required to move the wedge as shown in below figure. The angle of friction for all contact-surfaces is 15°.







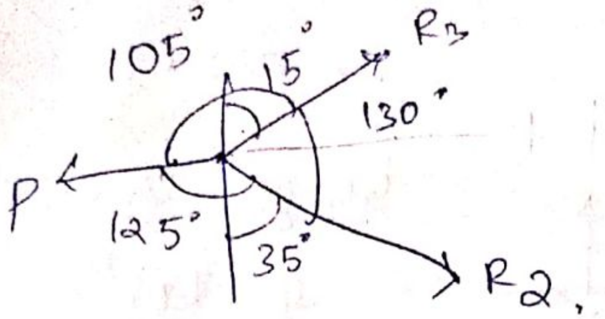
$$\frac{R_2}{\sin 75} = \frac{R_1}{\sin 145} = \frac{20}{\sin 140},$$

$R_2 = \sin 75 \times 31.11$

$= 30.054 KN$

→ $R_1 = \sin 145 \times 31.11$

$= 17.84 KN$



$$\frac{R_3}{\sin 125} = \frac{R_2}{\sin 105} = \frac{P}{\sin 130}$$

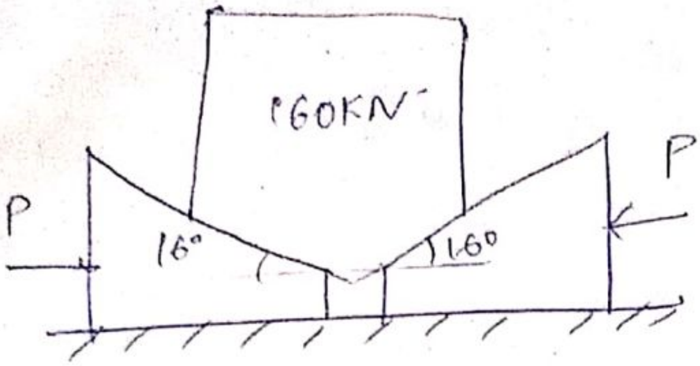
$$R_3 = \frac{30.054}{\sin 105} \times \sin 125$$

$= 25.48 KN$

$$P = \frac{30.054}{\sin 105} \times \sin 130°$$

$P = 23.83 KN$

② The block 'C' weighing 160KN is to be raised by means of driving wedge A and B. As shown in below figure find the value of 'P' for impending motion of the block upwards, If the coefficient of friction is 0.25 for all the contact surfaces self weight of wedge may be neglected.

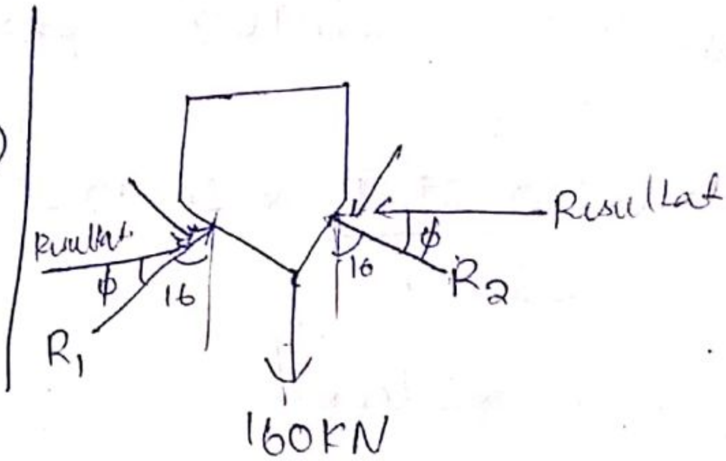


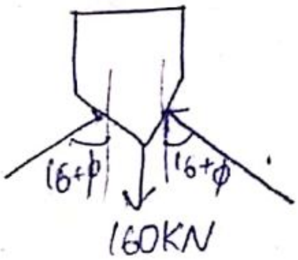
$\mu = 0.25$

$\tan\phi = 0.25$

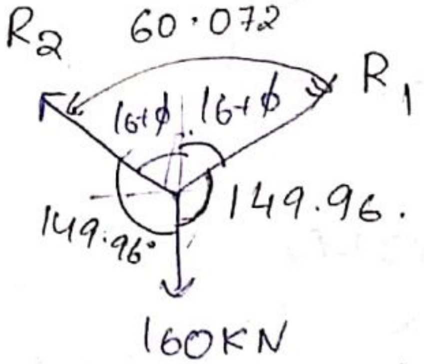
$\phi = \tan^{-1}(0.25)$

$= 14.03^\circ$





⇒

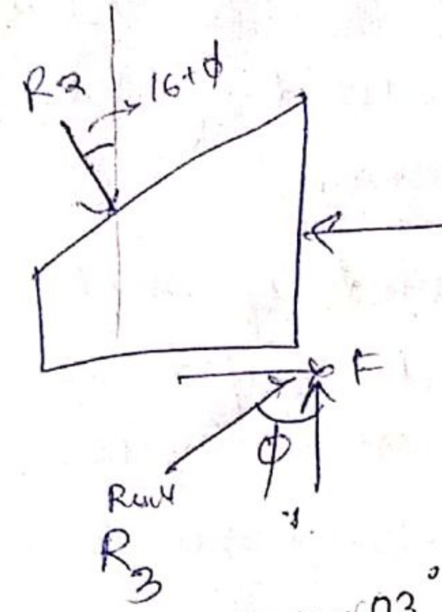


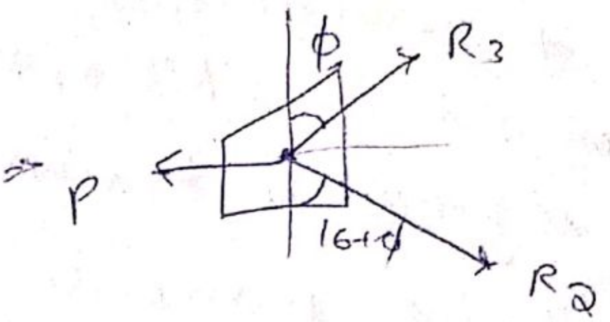
$$\frac{R_1}{\sin(149.96)} = \frac{R_2}{\sin(149.96)} = \frac{160}{\sin(60.072)}$$

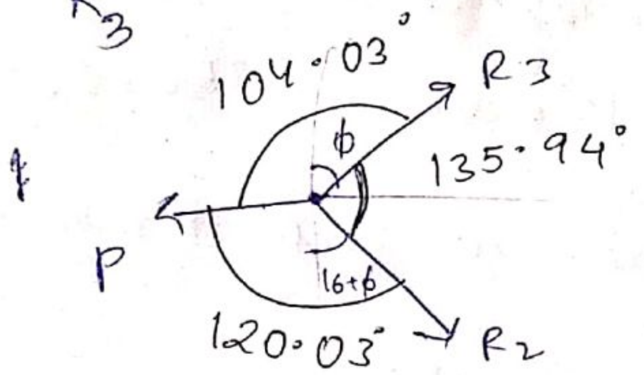
$R_1 = 184.618 \times \sin(149.96)$

$= 92.420\ KN$

$$R_2 = 184.618 \times \sin(149.96)$$
$$= 92.420 \text{ KN}$$







$$\frac{R_3}{\sin(120.03^\circ)} = \frac{R_2}{\sin(104.03)} = \frac{P}{\sin(135.94)}$$

$$R_3 = \frac{92.420}{\sin(104.03)} \times \sin(120.03)$$

$$= 82.474 \text{ KN}$$

$$P = \frac{92.420}{\sin(104.03)} \times \sin(135.94)$$

$$= 66.24 \text{ KN}$$

UNIT-3

ANALYSIS OF PERFECT FRAMES (TRUSSES)

- A structure made up of several Bars or members. Ribetted or welded joints together is known as frames.
- If the frame is composed of such members which are just sufficient to keep the frame in equillibrium, when the frame is supporting an external load. Then the frame is known as TRUSS.

1/9/19

Types of Trusses:-

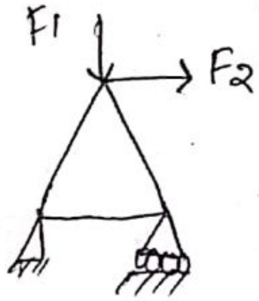
Trusses are divided into 2 types:-

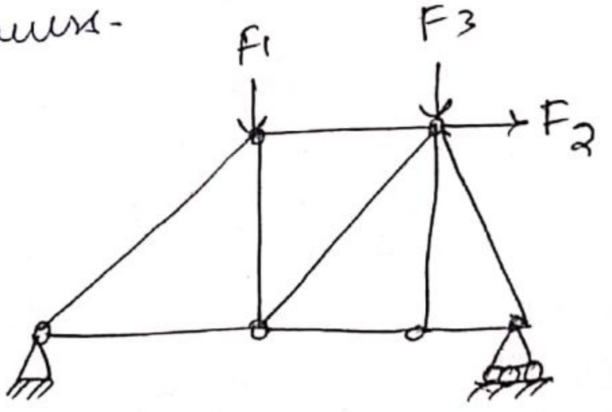
1. Perfect Truss
2. Imperfect truss

Trusses
- perfect
- Imperfect
  - Deficient
  - Redundant

Perfect Truss:-

A pin jointed truss which has got just sufficient no: of members to resist the load without undergoing any deformation in shape is called perfect truss.





$$m + 3 = 2j$$

Here
m = no: of members
j = no: of joints

Case 1:-

$m + 3 = 2j$

$3 + 3 = 2 \times 3$

$6 = 6$

Case 2:-

$m + 3 = 2j$

$9 + 3 = 2 \times 6$

$12 = 12$

Imperfect Truss:- ① Deficient Truss
② Redudant Truss

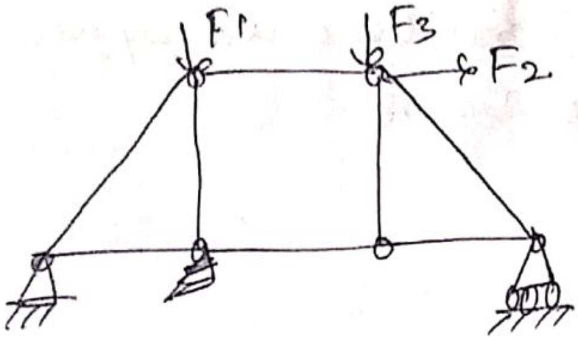
1) Deficient Truss:- A truss which doesnot satisfy the relation [$m+3=2j$] then it is called Imperfect Truss.

→ A truss which satisfy the below relation

$m+3<2j$ is called Deficient truss

→ It is unstable and may collapse under external force

→ In the below figure $m=8$; $j=6$.



$m+3<2j$

$8+3<2\times6$

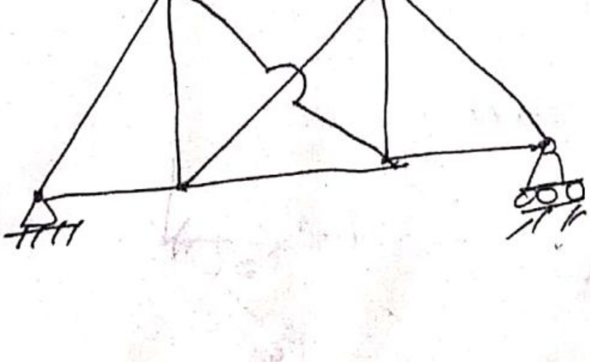
$11<12$

Reduntant Truss:-

A Truss which satisfies the below relation.

$m+3>2j$ is called Reduntant truss.

- It is over rigid truss



Assumptions for a Perfect Truss :-

① All the members of truss are straight and connected to each other at their ends by frictionless pins.

② The loading (external force) on truss is applied at the joints only.

③ All the members are assumed to be weight-less.

④ Static equilibrium conditions are applicable for Analysis of perfect Truss.

$$\Sigma F_X = 0$$
$$\Sigma F_Y = 0$$
$$\Sigma M = 0.$$