Transportation Problem

is a special kind of LPP in goods are transported from a set of sources to set of destinations subject to to supply and demand of the sources and destination.

Let m, be the sources

n be the destination

a_i be supply at the source i

b_j be demand at the destination

C_{ij} be the cost of transportation/unit from source i to destination j.

X_{ij} be the number of units to be transported from the source i to the destination j.

Mathematical model for Transportation problem.

$$\text{Min } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Destination

$$\begin{array}{c|ccccc|cc}
 & 1 & 2 & \cdots & j & \cdots\ n & \text{Supply} & \\
\hline
1 & c_{12} & c_{13} & \cdots & & c_{1n} & a_1 & \\
\text{Source } 2 & c_{21} & c_{22} & \cdots & & c_{2n} & a_2 & a_i \\
3 & c_{31} & & & & & & \\
\vdots & \vdots & & & & & \vdots & \\
m & c_{m1} & & & & & a_n & \\
\hline
 & b_1 & b_2 & \cdots & b_j & \cdots\ b_n & &
\end{array}$$

Demand

Subject

$$\sum_{j=1}^{n} X_{ij} \le a_i \qquad i = 1,2,3 \cdots m$$

$$\sum_{i=1}^{m} X_{ij} \ge b_j \qquad j = 1,2,3, \cdots n$$

Type of transportation problem

(a) **Balanced transportation:** If the supplies of all the sources is equal to the sum of the demands of all the destination

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Unbalanced: $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$.

Destination

Sources	1	2	3	Supply
1	30	50	15	300
2	35	70	20	200
3	20	45	60	500
Demand	300	200	400	900/1000

$$\sum_{i=1}^{3} a_i \neq \sum_{j=1}^{3} b_j$$

Destination

Sources	1	2	3	4	Supply
1	30	50	16	0	300
2	35	70	20	0	200
3	20	45	60	0	5000
demand	300	200	400	100	1000 / 1000

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Methods to solve the Transportation Problem

→ North west corner

→ Least cost cell method

→ Vogels approximation method / penalty method

NWC:

Destination

Sources	1	2	3	4	Supply
1	3 $\boxed{250}$	1	7	4	~~300~~ 50
2	2	6	5	9	400
3	8	3	3	2	500
Demand	~~250~~ 0	350	400	200	

1 $\boxed{50}$	7	4	~~50~~ 0
6	5	9	400
3	3	2	500
~~350~~ 300	400	200	

6 $\boxed{300}$	5	9	~~400~~ 100
3	3	2	500
~~300~~ 0	400	200	

5 $\boxed{100}$	9	~~100~~ 0
3	2	500
~~400~~ 300	200	

3 $\boxed{300}$	2	~~500~~ 200
300	200	

2 $\boxed{200}$	200

3 [250]	1 [80]	7	4
2	6 [300]	5 [100]	9
8	3	3 [300]	2 [200]

Total cost = $3 \times 250 + 6 \times 300 + 5 \times 100 + 3 \times 300 + 2 \times 200 = 4400$ Rs.

Least cost cell Method:

Destination

Source					Supply
	4 [20]	6	8	8	~~40~~ 20
	6	8	6	7	60
	5	7	6	8	50
Demand	~~20~~ 0	30	50	50	150 / 150

6 [20]	8	8	~~20~~ 0
8	6	7	60
7	6	8	50
~~30~~ 10	50	50	

If there is a tie choose the box whose demand is more

8	50 6	7	60 10
7	6	8	60
10	50	50	

Vogel Approximation:

	1	2	3	4	Supply	Row diff (penalty)			
1	3	300 1	7	4	300 0	2	3	—	—
2	250 2	6	5	150 9	400 150	3	1	1	—
3	8	3	3	200 2	500 300	1	1	1	1
demand	250 0	350 50	400	200 50	1200 / 1200				
colom diff	1	2	2	2					
	—	2	2	2					
		3	2	7					

Source

6	5	150	1	—
50 3	250 3	300 250	0	—
50	400 250			
3	2			
	2			

5 150 | 150 0

150

0

400
250
150

(6) Intial basic feasible solution using VAM

3	[300] 1	7	4	300
[250] 2	6	[150] 5	9	400
8	[50] 3	[250] 3	[200] 2	500
250	350	400	200	

Total cost:

$1\times300+2\times250+5\times150+3\times50+3\times250+2\times200 =$ Rs. 2850.

U-V Method: (MODI Method)

	1	2	3	4	
1	[250] 3	[50] 1	7	4	300
2	2	[300] 6	[100] 5	9	400
3	8	3	[300] 3	[200] 2	500
	250	350	400	200	1200 / 1200

Application of NWC the initial basic feasible solution obtained.

$3\times250+1\times50+6\times300+5\times100+3\times300+2\times200 = 4400$

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(7) Application of U-V method to optimize the sol.

	$v_1 = 3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$
$u_1 = 0$	250 / 3 (−)	50 / 1 (+)	7	4
$u_2 = 5$	2 (+) New basic cell	300 / 6 (−)	100 / 5	9
$u_3 = 3$	8	3	300 / 3	200 / 2

The no. of basic cell in the set of IBFS is equal to $m+n-1 = 3+4-1 = 6$

finding of U and V: $u_i + v_j = c_{ij}$

c_{ij} = allocated cells

$m+n-1 = 6$

$3+4-1 = 6$

$0 + v_1 = 6$

$v_1 = 6$

$u_1 + v_1 = 3$

$u_1 + v_2 = 1$

$u_1 + v_3 = 7$

(2) penalties for non basic cells

$$\boxed{P_{ij} = u_i + v_j - c_{ij}}$$

$u_1 + v_1 = 3$

$0 + \boxed{v_1 = 3}$

$u_1 + v_2 = 1$

$v_2 = 1$

$u_1 + v_3 = 7$

$C_{13} = 0 + 0 - 7 = -7$

$C_{14} = 0 - 1 - 4 = -5$

(Maximum +ve value)

$C_{21} = 5 + 3 - 2 = 6$

$C_{24} = 5 - 1 - 9 = -5$

$C_{31} = 3 + 3 - 8 = -2$

$C_{32} = 3 + 1 - 3 = 1$

$u_2 + v_1 = 6$

$u_i + v_j = c_{ij}$

$u_1 + v_1 = 6$

(Q)

$v_1 = -3 \quad v_2 = 1 \quad v_3 = 0 \quad v_4 = -1$

	$v_1 = -3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$
$u_1 = 0$	3	1 [300]	7	4
$u_2 = 5$	2 [250]	6 [50] (−)	5 [100] (+)	9
$u_3 = 3$	8	3 (+)	3 [300] (−)	2 [200]

m+n-1

$u_i + v_j = c_{ij}$

$$\boxed{P_{ij} = u_i + v_j - c_{ij}}$$

$C_{11} = 0 - 3 - 3 = -6$

$C_{13} = 0 + 0 - 7 = -7$

$C_{24} = 5 - 1 - 9 = -5$ $\qquad C_{14} = 0 - 1 - 4 = -5$

$C_{31} = 3 - 3 - 8 = -8$

$C_{32} = 3 + 1 - 3 = +1$

The solution in the table is not optimal because the highest of penality of nonbasic +ve

$v_1 = -2 \quad v_2 = 1 \quad v_3 = 1 \quad v_4 = 0$

	$v_1 = -2$	$v_2 = 1$	$v_3 = 1$	$v_4 = 0$
$u_1 = 0$	3	1 [300]	7	4
$u_2 = 4$	2 [250]	6	5 [150]	9
$u_3 = 2$	8	3 [50]	3 [250]	2 [200]

$C_{11} = 0 - 2 - 3 = -5$ $\qquad C_{24} = 4 + 0 - 9 = -5$

$C_{13} = 0 + 1 - 7 = -6$ $\qquad C_{31} = 2 - 2 - 8 = -8$

$C_{14} = 0 + 0 - 4 = -4$

$C_{22} = 4 + 1 - 6 = -1$

(9)

Hence optimality is Reached. all the Values are negative.

Total cost =

= $1\times300+2\times250+5\times150+3\times50+3\times250+2\times200$ = Rs. 2850

	1	2	3	4	5	
1	~~10~~ 20	2	3	18	9	~~25~~ 5
2	~~5~~	10	15	2	4	30
3	~~15~~	5	14	7	15	20
4	~~20~~	15	13	∞	8	30
	~~20~~ 0	20	30	10	25	105

~~2~~ 5	~~3~~	~~15~~	~~9~~	~~5~~
10	15	2	4	30
5	14	7	15	20
15	13	∞	8	30
~~20~~ 15	30	10	25	

~~10~~ (15)	15	2	4	~~25~~
~~5~~	14	7	15	20
~~15~~	13	∞	8	30
~~15~~	30	10	25	

~~15~~ (15)	~~2~~	~~4~~	~~15~~ 0
14	7	15	20
13	∞	8	30
~~30~~ 15	10	25	

~~14~~ (15)	7	15	~~20~~ 5
~~13~~	∞	8	30
~~15~~	10	25	

~~7~~ (5)	~~15~~	~~5~~ 0
∞	8	30
~~10~~ 5	25	

∞ (5)	8 (25)	~~30~~ ~~25~~ 0
~~5~~	~~25~~ 0	

Stepping Stone Method :

	1	2	3	supply.
1	2	7	4	5
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand	7	9	18	34

1	2 (5)	7	4	~~5~~ 0
2	3 (2)	3 (6)	1	~~8~~ ~~6~~ 0
3	5	4 (3)	7 (4)	~~7~~ 4
4	1	6	2 (14)	14
	~~7~~ ~~2~~	~~9~~ ~~3~~ 0	~~18~~ 14	34

2 (5)	7	4
3 (2)	3 (6)	1
5	− 4 (3)	+ 7 (4)
1	+ 6	− 2 (14)

6 − 4 + 7 − 2 ⇒ 7

Total cost $= 2\times5 + 3\times2 + 3\times6 + 4\times3 + 7\times4 + 2\times14$

$= 10 + 6 + 18 + 12 + 28 + 28$

$= 46 + 56$

$= 102/-$

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2 (5)	7	4
3 (2)	3 (6)	1
5	4 (3)	7 (4)
1	6	2 (14)

$\Rightarrow +7 - \cancel{3} + \cancel{3} - 2 = 5$

$= +4 - 2 + \cancel{3} - \cancel{3} + 4 - 7$

$\Rightarrow 8 - 9 \Rightarrow -1$

$+1 - 3 + 4 - 7 = -5$

$+5 - \cancel{3} + \cancel{3} - 4 = 1$

$+1 - \cancel{3} + \cancel{3} - 4 + 7 - 2 \Rightarrow 2$

	1	2	3
1	2 [5]	7	4
2	3 [2]	- 3 [6]	1 +
3	5	+ 4 [3]	7 - [4]
4	1	6	2 [14]

2 [5]	7	4
3 [2]	3 [2]	1 [4]
5	4 [7]	7
1	6	2 [14]

$7 - 2 + 3 - 3 \rightarrow 5$

56
10
12
78

Total cost $= 2\times5 + 3\times2 + 3\times2 + 4\times7 + 2\times14$
$= 10 + 6 + 6 + 28 + 28$
$= 78$

Since Reallocation in any other unoccupied cell Cannot decrease transportation cost.

Assignment problem is a special kind of transportation problem in which each source should fulfill the demand of any of the destination. In other words any operator should be able to perform any job regardless of his skills although the cost will be more if the job doesn't match with the worker skill.

General format of AP

operators

$$
\begin{array}{cc|ccccccc|}
 & & 1 & 2 & \cdots & j & \cdots & m \\
\hline
 & 1 & t_{11} & t_{12} & \cdots & t_{1j} & \cdots & t_{1m} \\
 & 2 & t_{21} & t_{22} & \cdots & t_{2j} & \cdots & t_{2m} \\
 & \vdots & \vdots & \vdots & & & & t_{im} \\
\text{job} & i & t_{i1} & t_{i2} & \cdots & t_{ij} & \cdots & \vdots \\
 & \vdots & \vdots & \vdots & & \vdots & & \vdots \\
 & m & t_{m1} & t_{m2} & & t_{mj} & & t_{mm} \\
\hline
\end{array}
$$

m be the number of jobs, and operators

t_{ij} be the assigned processing time of the job i if it is assigned to operator j.

Main objective is assign the job to the operator such that total processing time is minimized.

objective function

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{m} t_{ij} x_{ij}$$

t_{ij} - processing time.

$x_{ij} = 1$ if the row i assigned to the column j

$= 0$, otherwise

Subject $\sum_{j=1}^{m} x_{ij} = 1$, $i = 1,2,3,\dots m$.

$\sum_{j=1}^{m} x_{ij} = 1$, $j = 1,2,\dots m$.

Eg/1 :

	1	2	3	4	5
1	10	12	15	12	8
2	7	16	14	14	11
3	13	14	7	9	9
4	12	10	11	13	10
5	8	13	15	11	15

Develop a zero-one programming model.

Sol let $x_{ij} = 1$ if the job i is assigned to the operator j.

$x_{ij} = 0$ otherwise

Minimize $z = 10x_{11} + 12x_{12} + 15x_{13} + 12x_{14} + 8x_{15}$

subject to

$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 1$

$x_{ij} = 0$ or 1 , $i = 1,2,3,4,5$ $j = 1,2,3,4,5$

Hungarian Method:-

operator

job	1	2	3	4	5	Row min
1	10	12	15	12	8	8
2	7	16	14	14	11	7
3	13	14	7	9	9	7
4	12	10	11	13	10	10
5	8	13	15	11	15	8

Phase : I

The row Reduction are carried out. operator

~~10~~ ~~12~~ 15 ~~12~~ ~~8~~ 2

~~7~~

job	1	2	3	4	5
1	2	4	7	4	0
2	0	9	7	7	4
3	6	7	0	2	2
4	2	0	1	3	0
5	0	5	7	3	7
	0	0	0	2	0

The column Reduction is carried out.

	1	2	3	4	5
1	2	4	7	2	0
2	0	9	7	5	4
3	6	7	0	0	2
4	2	0	1	1	0
5	0	5	7	1	7

Matrix after column Reduction.

In the next phase the optimum solution is obtained in an interactive manner.

Iteration 1 : Row scanning column.

	1	2	3	4	5
1	2	4	7	2	$\boxed{0}$
2	$\boxed{0}$	9	7	5	4
3	6	7	$\boxed{0}$	0	2
4	2	$\boxed{0}$	1	1	0
5	$\boxed{0}$	5	7	1	7

	1	2	3	4	5
1	2	4	7	2	$\boxed{0}$
2	$\boxed{0}$	9	7	5	4
3	6	7	$\boxed{0}$	0	2
4	2	$\boxed{0}$	1	1	0
5	$\boxed{0}$	5	7	1	7

The nor of squares marked $\neq$ nor of rows.

$4 \neq 5$

The optimality not reached.

Iteration 2:

	1	2	3	4	5
1	2	4	6	2	$\boxed{0}$
2	$\boxed{0}$	9	6	5	4
3	7	8	$\boxed{0}$	$\boxed{6}$	3
4	2	$\boxed{0}$	0	0	0
5	0	5	6	$\boxed{0}$	7

$5 = 5$

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Optimal solution

Job	Operator	Time
1	5	8
2	1	7
3	3	7
4	2	10
5	4	11

43 hrs

Total processing time = 43

21
14
21
35
98
73

21
14
18
3

Consider the problem of assigning four sales persons to four different sales Region such that the total sales is maximized. find the optimal allocation of sales/person. to different region

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 10 & 22 & 12 & 14 \\ 2 & 16 & 18 & 22 & 10 \\ 3 & 24 & 20 & 12 & 18 \\ 4 & 16 & 14 & 24 & 20 \end{array}$$

Sales

$\begin{array}{r} 24 \\ 16 \\ \hline 8 \end{array}$ $\begin{array}{r} 24 \\ 18 \\ \hline 6 \end{array}$

Minimizing the total cost of assignment
The maximum ✗.
convert the max into usual minimization

The maximum value from the matrix. is 24 and substract 24 for each value from the matrix.

$$\begin{array}{c|cccc|c} & 1 & 2 & 3 & 4 & \text{min Row value} \\ \hline 1 & 14 & 2 & 12 & 10 & 2 \\ 2 & 8 & 6 & 2 & 14 & 2 \\ 3 & 0 & 4 & 12 & 6 & 0 \\ 4 & 8 & 10 & 0 & 4 & 0 \end{array}$$

Row Reduction:

$$\begin{bmatrix} 12 & 0 & 10 & 8 \\ 6 & 4 & 0 & 12 \\ 0 & 4 & 12 & 6 \\ 8 & 10 & 0 & 4 \end{bmatrix}$$

Column Reduction :

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 12 & \boxed{0} & 10 & 4 \\ 2 & 6 & 4 & \boxed{0} & 8 \\ 3 & \boxed{0} & 4 & 12 & 2 \\ 4 & 8 & 10 & 9 & \boxed{0} \end{array}$$

The nor of cells marked with square is 4 which is equal to nor of rows of the matrix. hence the solution is feasible and optimal

the sale assigned

1 - 2 ⇒ 22

2 - 3 ⇒ 22

3 - 1 ⇒ 24

4 - 4 ⇒ 20

Max. profit = Rs. 88 lakhs.

Problems with restrictions :

Solve for optimal Assignment (cost min.)

In the modification of a plant layout of a factory four new machines $M_1, M_2, \ldots M_4$ location are to be installed in a machine shop. There are five vacant locations A, B, C D & E available. Because of limited space machine M_2 cannot be placed at C and M_3 cannot be placed at A. The cost of placing the the machines to these location is shown in matrix.

find the optimal assignment schedule.

Machines	Locations (Cost) A	B	C	D	E
M_1	9	11	15	10	11
M_2	12	9	—	10	9
M_3	—	11	14	11	7
M_4	14	8	12	7	8

problem

Step 1:

	A	B	C	D	E
M_1	9	11	15	10	11
M_2	12	9	∞	10	9
M_3	∞	11	14	11	7
M_4	14	8	12	7	8

Step 2: The no of machines $\neq$ no of ws!

adde a dummy row.

	A	B	C	D	E
M_1	9	11	15	10	11
M_2	12	9	∞	10	9
M_3	∞	11	14	11	7
M_4	14	8	12	7	8
M_5	0	0	0	0	0

Step 3: Row reduction.

	A	B	C	D	E
M_1	0	2	6	1	2
M_2	3	0	∞	1	0
M_3	∞	4	7	4	0
M_4	7	1	5	0	1
M_5	0	0	0	0	0

Column Reduction

$$\begin{array}{c|ccccc|}
 & A & B & C & D & E \\
M_1 & 0 & 2 & 6 & 1 & 2 \\
M_2 & 3 & 0 & \infty & 1 & 0 \\
M_3 & \infty & 4 & 7 & 4 & 0 \\
M_4 & 7 & 1 & 5 & 0 & 1 \\
M_5 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{c|ccccc|}
 & A & B & C & D & E \\
M_1 & \boxed{0} & 2 & 6 & 1 & 2 \\
M_2 & 3 & \boxed{0} & \infty & 1 & 0 \\
M_3 & \infty & 4 & 7 & 4 & \boxed{0} \\
M_4 & 7 & 1 & 5 & \boxed{0} & 1 \\
M_5 & 0 & 0 & \boxed{0} & 0 & 0
\end{array}$$

$M_1 \rightarrow A = 9$

$M_2 \rightarrow B = 9$

$M_3 \rightarrow E = 7$

$M_4 \rightarrow D = 7$

$M_5 \rightarrow C = 0$

Total cost = 32