19) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 = -4$ at $(-1, 2, 1)$

20) If $\bar{F} = (5xy - 6x^2)\bar{i} + (2y - 4x)\bar{j}$, evaluate $\int_c \bar{F} \cdot d\bar{R}$ along the curve C in the xy plane given $y = x^3$, from the point $(1, 1)$ to $(2, 8)$

21) prove that $\nabla \cdot (\nabla \times \bar{A}) = 0$, where $\vec{A}$ is a vector point function

22) Apply Green's theorem to evaluate $\oint_c e^{-x} \sin y \, dx + e^{-x} \cos y \, dy$, where c is the rectangle whose vertices are $(0, 0)$, $(\pi, 0)$, $(\pi, \pi/2)$, $(0, \pi/2)$

23) verify Green's theorem for $\oint_c (xy^2 + 2xy)dx + x^2 dy$ where C is the boundary of the region enclosing $y^2 = 4x$, $x = 3$

24) using Green's theorem evaluate $\int_c (y - \sin x)dx + \cos x \, dy$ where c is the triangle enclosed by the lines $y = 0$, $x = 2\pi$, $\pi y = 2x$.

25) verify Stokes theorem for $\bar{F} = (y - z + 2)\bar{i} + (yz + 4)\bar{j} - xz\bar{k}$ where s is the surface of cube $x = 0$, $y = 0$, $z = 0$ and $z = 2$ above xy plane

26) verify Stokes theorem for $\vec{F} = (x^2 - y^2)i + 2xy\,j$ around the rectangular bounded by $x = 0$, $x = a$, $y = 0$, $y = b$.

27) Verify Gauss divergence theorem for $\bar{V} = 2xy\bar{i} + 6yz\bar{j} + 3zx\bar{k}$ and D is the region bounded by the coordinate planes and $x + y + z = 2$.

28) If $F = (2x - z)\bar{i} + x^2 y\,j - xz^2 k$ and S is the surface bounded by $x = 0, y = 0, z = 0$, $x = 1, y = 1, z = 1$, then evaluate $\iint_S \bar{F} \cdot \bar{n} \, ds$ by using Gauss divergence theorem

29) verify Gauss divergence theorem for $\bar{F} = (x^3 - yz)\bar{i} - 2x^2 y\,j + z\,k$ taken over entire surface of cube $0 \le x \le a$, $0 \le y \le a$, $0 \le z \le a$.

UNIT-5 Vector Calculus

1) Find the unit normal to the surface $x^2 - y^2 + z = 9$ at $(1, -1, 2)$

2) If $F = (3x^2 + 6y)i - 14yz\,j + 20xz^2 k$ then evaluate $\int_c F \cdot dr$ where c is the straightline joining $(0,0,0)$ to $(1,1,1)$

3) Find the directional derivative of $f(x,y,z) = xy + yz + zx$ in the direction vector $3i + 3j + 6k$ at $P(3, 1, 2)$

4) If $\bar{a}$ is a constant vector and $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ then find curl $(\bar{a} \times \bar{r})$.

5) Find the angle between the surfaces $x^2+y^2+z^2=22$, $z = x^2+3y$ at $(3,-2,3)$.

6) If $\vec{a}$ is a constant vector $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$. then find curl$(\bar{a} \times \bar{r})$

7) Evaluate $\int_c \bar{v} \cdot dr$ where $\bar{v} = x\bar{i} + y\bar{j} + z\bar{k}$ and c is the line segment ~~a~~ from $A(1, 2, 2)$ to $B(3, 6, 6)$.

8) Find the angle between the surfaces $x^2+y^2+z^2=9$, $z+3 = x^2+y^2$ at $(-2,1,2)$

9) Find a scalar function $f(x,y,z)$ such that $\bar{v} = \nabla f = 12x\bar{i} - 15y^2\bar{j} + k$

10) If $\vec{a} = a_1\bar{i} + a_2\bar{j} + a_3k$. is a constant vector, $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$. then evaluate div$(\vec{a} \times \vec{r})$

11) Find the equation of the tangent to the surface $x^2+y^2-z=0$ at $(2,-1,5)$

12) Evaluate $\nabla^2\left(\frac{x}{r^3}\right)$, where $r^2 = x^2+y^2+z^2$

13) Find the directional derivative of $f(x,y,z) = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the direction of the vector $2\bar{i} - \bar{j} - 2\bar{k}$

14) If $\vec{a}$ is a constant vector and $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, P.T $\nabla \cdot (\bar{a} \times \bar{r}) = 0$

15) State stokes theorem.

16) Find the gradient of $f(x,y,z) = \log(x^2+y^2+z^2)$ at $(1,1,1)$

17). Show that $\vec{v} = 12x\bar{i} - 15y^2\bar{j} + \bar{k}$ is irrotational

18) find ∇f at $(1, 2, -1)$ if $f(x,y,z) = \log_e(x+y+z)$.

30) Evaluate $\oint_c \bar{F} d\bar{r}$, where $\bar{F} = (4xy - 3x^2 z)\bar{i} + 2x^2 j - 2x^3 z K$ and c is $y = x^2$ from $x=0$ to $x=1$

31) If $\bar{F} = x^2 y \bar{i} + xy^2 z \bar{j} - yz^2 K$, find grad div F

find the constants a, b and c such that $\bar{V} = (3x + ay + z)\bar{i} + (2x - y + bz)\bar{j} + (x + cy + z)\bar{k}$ is irrotational.

32) find the directional derivative of $f = x^2 y^2 + 2z^2$ at the point P(1,2,3) in the direction of the line PQ, where Q (5,0,4).

33) prove that $\nabla \cdot (\nabla \times \vec{F}) = 0$, where $\bar{F}(x, y, z)$ is vector field

34) Find the work done by the force $\bar{F} = (2y+3)\bar{i} + xzj + (yz - x)\bar{k}$ when it moves a particle from (0,0,0) to (2,1,1) along $x = 2t^2, y = t, z = t^3$

35) Evaluate $\oint_c x\,dy - y\,dx$, where c is the triangle with vertices at (0,0), (2,0) and (0,1) using Green's theorem.

36) Find the constants a, b, c such that $\bar{F} = (2x + 3y + az)i + (bx + 2y + 3z)\bar{j} + (2x + cy + 3z)\bar{k}$ is irrotational and find the scalar function f such that $\bar{F} = \nabla f$.

37) compute the gradient of the scalar function $f = e^{xy}(x + y + z)$ at (2, 1, 1)

38) If f is a diff'be scalar field, then show that $\nabla \times (\nabla f) = 0$

39) Show that the vector function $\vec{V} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational and find its scalar potential.

40) use Green's theorem to evaluate the line integral $\oint_c (xy + x^2)dx + (x^2 + y^2)dy$, where c is the closed curve of the region bounded by $y = x$ and $y = x^2$.

1) Find the radius of curvature at (0,0) on the curve $y^2 = 4x$.

UNIT-2

2) State C.M.T. $f(x) = \log x$, $g(x) = \frac{1}{x}$ in $(1, e)$

3) Find the radius of curvature $x^3 + y^3 = 3axy$ at $(\frac{3a}{2}, \frac{3a}{2})$

4) Verify Rolles theorem for $f(x) = \frac{\sin x}{e^x}$ on $[0, \pi]$

5) Find the envelope of the family of straightlines $x\cos\alpha + y\sin\alpha = \alpha$ where α is parameter.

6) S.P Cauchy's mean value theorem

7) Find the evolute of the curve $x = a\cos^3 t$, $y = a\sin^3 t$.

8) Find the circle of the curvature of the curve $xy = 9$ at $(1,9)$

9) Find 'c' value of Rolles theorem for $f(x) = x(x+3)e^{-x/2}$ in $(-3, 0)$

10) Find the envelope of the the family of $y = mx - 2am - am^3$, m pa

11) Find the centre of curvature of the curve $y = x^3 - 6x^2 + 3x + 1$ at $(1, -1$

12) State Cauchy M.V.T. find 'c'. $f(x) = \frac{1}{x^2}$, $g(x) = \frac{1}{x}$ on $[3, 6]$

13) Find the envelope of the family of curve $y = 3px - p^3$ 'p' is para

14) Discuss the applicability of L.M.V $f(x) = x^2 - 9x + 18$ on $[3.6]$

15) Find the circle of the curvature of the curve $y = x^3 - 6x^2 + 3x + 1$ at the point $(1, -1)$.

16) Find the asymptotes of the curve $x^2(x^2+y^2) = a^2(y^2-x^2)$.

17) S. P. C.M.T

18) Find the asymptotes of the curve $x^2y^2 - x^2y - xy^2 + xy + y + 1 = 0$ parallel to x-axis.

19) Find the asymptotes of the curve $x^2 = \frac{y+2}{y-2}$

20) Find the radius of curvature of the curve $y = x^2 - 3x + 5$ at $(1, 3)$

21) Find the envelope of the family of straightline $y = mx + \frac{a}{m}$ where 'm' is parameter, a constant

22) Verify L.M.T. for $f(x) = x(x-1)(x-2)$, $x \in (0, 1/2)$

23) Find the evolute of the parabola $x^2 = 4ay$

24) Find the radius of curvature of $y = x^2 - 3x + 5$ at $(1, 3)$

25) Discuss the applicability of Rolles theorem $f(x) = |x|$ in $(-2, 2)$

26) Find the curvature of the curve $y^2 = x^3$ at $(1, 1)$

27) S.P. L.M.T.

28) Find the envelope of the family of $x\tan p + y\sec p = 5$, p is

29) S. P. C.M.T

30) Find a point on the curve $f(x) = x^2 - 2x$ in $(0, 2)$ at which tangent is parallel to x-axis

31) Obtain the envelope of the family of curve $y = cx + \frac{3}{2c}$. c -

32) Find all asymptotes to the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$

33) Find radius of curvature of the curve $r = a(1 + \cos\theta)$ at any θ.

34) Expand $f(x) = e^x \cdot \sin x$ in power of x upto term x^2

35) State C.M.T. and verify $f(x) = e^{-x}$, and $g(x) = e^x$ in $[a, b]$

36) find all asymptote to the curve $y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 = 0$.

37). S.P. R.T

36) find the Taylor series of $f(x) = \sin x$ about $x = \pi/4$

37) find the envelope of $y = (x - p)^2$, p is parameter

38). S.P. L.M.T

39) find the evolute of $y^2 = 4ax$.

UNIT-3

(1) Evaluate $\lim_{(x,y)\to(0,0)} \frac{x-y}{x+y}$, if it exists

(2) If $f(x,y) = \begin{cases} \frac{xy(5x^2-4y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ Then S.T $f_{xy}(0,0) = f_{yx}(0,0)$

(3) S.T $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2+y^2}$ does not exist

(4) If $f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ then evaluate $f_x(0,0)$ $f_y(0,0)$.

(5) Discuss the continuity of $f(x,y) = \begin{cases} \frac{(x-y)^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (0,0) \end{cases}$ at $(0,0)$

6) Determine $\lim_{(x,y)\to(1,2)} \frac{xy}{x^2+y^2}$

7) If $f(x,y) = \begin{cases} \frac{y(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ at $(0,0)$ if they exists

8) If $z = y + f(u)$, $u = \frac{x}{y}$. S.T $u\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$

9) If $u = x^2+y^2$, $v = 2xy$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

10) Examinine for maxima and minima vale of $f(x,y) = x^2 - 3xy + y^2 + 2x$

11) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$, at $(0,0)$ give $f(x,y) = \begin{cases} \frac{xy}{x^2+2y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

12) Determine $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2}$ if it exist

13) If $u = f(2x-3y, 3y-4z, 4z-2x)$, P.T $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$

14) If $x = r\cos\theta$, $y = \sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

15) If $x = u(1+v)$, $y = v(1+u)$, then evaluate $\frac{\partial(x,y)}{\partial(u,v)}$

16) If $u = 2xy$, $v = x^2-y^2$, $x = r\cos\theta$, $y = r\sin\theta$, evaluate $\frac{\partial(u,v)}{\partial(r,\theta)}$

17) If $x = r\cos\theta$, $y = r\sin\theta$, $z = z$ find $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}$

18) If $f(x,y) = \begin{cases} \frac{x^2+y^2}{x-y}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ at $(0,0)$

19) Discuss the maxima and minima of the function $f(x,y) = x^3 + y^3 - 12x - 3y + 20$

20) Find the maximum value of $x+y+z$, subject to the condition $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$.

21) Examine for maximum and minimum values of the fun $f(x,y) = x^4 + 2x^{2y} - x^2 + 3y^2$

22) Find the Taylor Series ex of $f(x,y) = \frac{1}{1-x-y}$ around $(0,0)$.

23) Expand $f(x,y) = e^x \cdot \sin y$ in powers of x and y upto third degree.

24) Expand $f(x,y) = \tan^{-1}(x \cdot y)$ as a Taylor series about $(1,1)$ upto terms of second degree.

25) Find the maxima and minima of $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$

26) If $z = f(x,y)$, $x = u\cos\alpha - v\sin\alpha$, $y = u\cos\alpha + v\cos\alpha$

P.T $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}$

27) Expand $f(x,y) = \sin xy$ in powers of x and y, upto 3^{rd} degree t_2

28) If $z = f(x,y)$ and $x = e^u + e^{-v}$, $y = e^{-u} - e^v$. P.T

$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.

29) S.T $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$ does not exists.

30) ~~Expand~~ S.T $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ is not diff'ble at $(0,0)$.

31) P.T $\lim_{(x,y)\to(0,0)} x^2+y^2 = 0$

32) If $z = \log(u^2+v)$, $u = e^{x^2+y^2}$, $v = x^2+y$ find $\frac{\partial z}{\partial x}$ at $(1,0)$

33) Find the Local max and min of $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

34) P.T $f(x,y) = \sqrt{x^2+y^2}$ is not diff'ble at $(0,0)$.

(1) Evaluate $\int_0^{\pi/4} \int_{\sin x}^{\cos x} dy\, dx$

(2) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} 6\, dz\, dy\, dx.$

14) a) Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)}\, dx\, dy$ by changing to polar co-ordinates

b) Find the volume of the unit sphere $x^2+y^2+z^2=1$.

17 a) Evaluate $\int_0^2 \int_0^2 2y^2 \sin xy\, dx\, dy$ changing order of integration.

$\int_{y=0}^{1} \int_{x=y}^{y^{1/3}} e^{x^2}\, dx\, dy$ chng ord & int

UNIT - 4

1) $\int_0^{f} \int_0^1 x \cos xy\, dx\, dy$

2) $\int_0^{\pi/6} \int_0^1 \int_{-2}^{3} y \sin z\, dx\, dy\, dz.$

3) $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2+y^2)\, dy\, dx$ Changing to polar Co-ordinate

4) $\int_0^{1} \int_0^{y+4} \frac{2y+1}{x-1}\, dx\, dy$ change of order of integral