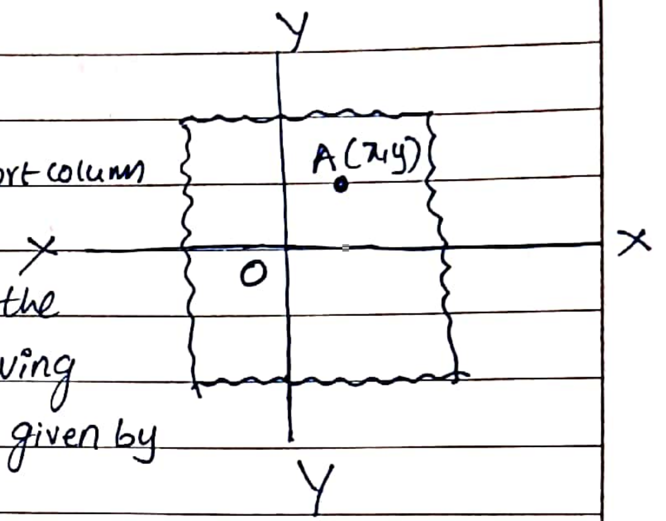
COLUMN ANALOGY METHOD.

- The column analogy method was developed by Prof. Hardy Cross. The method is useful in determining the fixed end moments, as well as the stiffness and carry-over factors, for a beam element with constant or variable moment of Inertia. Also, it is useful in the complete Analysis of symmetrical (or) unsymmetrical rigid frames, either with two fixed supports or with one closed cell.

SHORT COLUMNS :-



- Stress at any point in a short column loaded eccentrically, varies linearly with the position of the point. At any point "A" having co-ordinates (x,y) the stress is given by

$$\cancel{\sigma = ax + by}$$

$$\sigma \; \cancel{f} = a + bx + cy \qquad a, b \,\&\, c \text{ are constants}$$

To calculate constants a, b & c.

- consider small area "δA" at point "A".

force on elementary area will be.

$$\delta P = \sigma \, \delta A.$$

Total forces on the section.

$$P = \int \sigma \cdot dA.$$

$$P = \int (a + bx + cy)\, dA$$

$$P = a\int dA + b\int x\, dA + c\int y\, dA.$$

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In case the reference axes passes throught C.G.
[point passes through "O" C.G]

$$\int x\,dA = 0 \;;\; \int y\,dA = 0.$$

$$\therefore P = a\int dA = a\cdot A$$

$$\boxed{a = \frac{P}{A}}$$

$\therefore A \rightarrow$ total area of the section.

→ The moment of elementary force "δP" about x-axis is

$$\delta M_x = y\cdot\delta P.$$

$\therefore$ Total moment.

$$M_x = \int y\cdot dP = \int y\cdot\sigma\,dA.$$

$$= \int y\,(a+bx+cy)\cdot dA.$$

$$M_x = a\int y\,dA + b\int xy\,dA + c\int y^2\cdot dA.$$

$$\boxed{M_x = 0 + b\,I_{xy} + c\cdot I_x} \quad \text{— (1)}$$

I_{xy} = Product M.O.I ; I_x = M.O.I about x-axis

→ Moment of elementary force δP about y-axis.

$$\delta M_y = x\cdot\delta P = x\,\sigma\,\delta A.$$

Total moment $M_y = \int x\,(a+bx+cy)\cdot dA$

$$= a\int x\cdot dA + b\int x^2\,dA + c\int xy$$

$$\boxed{M_y = 0 + b\,I_y + c\cdot I_{xy}} \quad \text{— (2)}$$

Solving Equations ① & ② for b & c.

$$b = \frac{M_y I_x - M_x \cdot I_{xy}}{I_y \cdot I_x - I_{xy}^2}$$

$$c = \frac{M_x I_y - M_y I_{xy}}{I_y \cdot I_x - I_{xy}^2}$$

$$\sigma = \frac{P}{A} + \frac{M_y I_x - M_x I_{xy}}{I_y I_x - I_{xy}^2} \times x + \frac{M_x I_y - M_y I_{xy}}{I_y I_x - I_{xy}^2} \times y.$$

Where $P = P_1 + P_2 + P_3 + \cdots$ = Sum of vertical forces acting on the section.

$M_x = P_1 y_1 + P_2 y_2 + P_3 y_3 + \cdots$ = Sum of the moments of vertical forces about x-axis

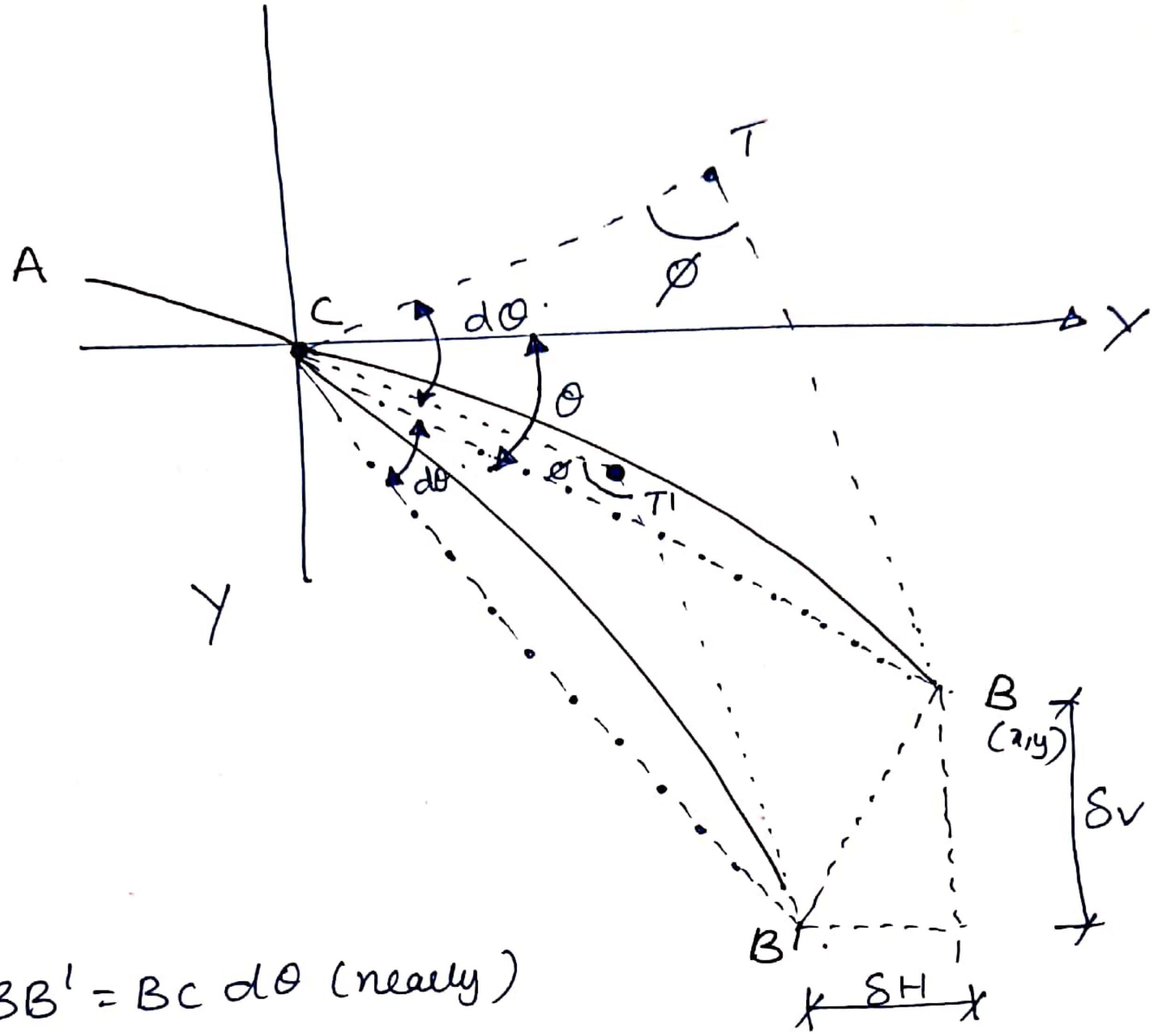
$M_y = P_1 x_1 + P_2 x_2 + P_3 x_3 + \cdots$ = Sumen of the moments of vertical forces about y-axis

and $(x_1, y_1), (x_2, y_2), (x_3, y_3) \cdots$ are the coordinates of $P_1, P_2, P_3 \cdots$ If the references axes are principal axes.

$$\boxed{I_{xy} = 0}$$

$$\boxed{\sigma = \frac{P}{A} + \frac{M_y}{I_y} \times x + \frac{M_x}{I_x} \times y}$$

BENDING OF CURVED BARS



$BB' = BC\, d\theta$ (nearly)

$\delta v = BB' \cos\theta$

$= BC \cos\theta\, d\theta$

$$\boxed{\delta v = x\, d\theta}$$

$\delta H = BB' \sin\theta$

$\delta H = BC \sin\theta\, d\theta$

$$\boxed{\delta H = y \cdot d\theta}$$

- Considering whole bar as rigid excepting small length "δs" at "C".

- The change in slope "$\delta\theta$" at "C" will be $\frac{M\delta s}{EI}$, where "M" is the B.M at C.
- The change in slope at 'B' will also be $\frac{M\delta s}{EI}$.

$$\delta\phi = \frac{M\delta s}{EI}$$

$$\delta v = \frac{Mx \cdot \delta s}{EI}$$

$$\delta H = \frac{My \cdot \delta s}{EI}$$

THEORY OF COLUMN ANALOGY:

→ Indeterminate structures of a single span (or) single bay (or) closed rings can be analysed by this method. The bending moment at any point in such structures will consist of bending moment M_s, considering the structure as a statically determinate one, i.e., by removing the redundancy of the structure, and M_i bending moment due to redundancy.

B.M at any section $= M_s + M_i$.

Let, θ be the relative rotation of the ends of the structure.

H & V are relative horizontal & vertical displacements at the ends.

→ In case relative θ, H & V are zero.

$$\Rightarrow \theta = 0 = \int \frac{M\,ds}{EI}$$

$$= \int \frac{(M_s + M_i)\,ds}{EI}.$$

$$\therefore \int \frac{M_i\,ds}{EI} = \int - \frac{M_s\,.ds}{EI}.$$

$$\Rightarrow H = 0 = \int \frac{My\,ds}{EI} = \int \frac{(M_s + M_i)\cdot y\cdot ds}{EI}.$$

$$\therefore \int \frac{M_i \cdot y\,ds}{EI} = \int - \frac{M_s\,ds}{EI} \times y$$

$$\Rightarrow V = 0 \quad \int \frac{Mx\,ds}{EI} = \int \frac{M_s + M_i}{EI}\,x\,ds.$$

$$\therefore \int \frac{M_i \cdot x\,ds}{EI} = \int - \frac{M_s\,ds}{EI} \times x.$$

Consider a short column of width at any section equal to $\frac{1}{EI}$ and the load intensity as $-M_s$.

$$P = \text{Total load} = \int -\frac{M_s \cdot ds}{EI}.$$

$$\therefore \int \frac{M_i ds}{EI} = P \quad \therefore dP = \frac{M_i ds}{EI}.$$

$$\int \frac{M_i y\, ds}{EI} = \int y\, dP$$

$$\int \frac{M_i x\, ds}{EI} = \int x\, dP.$$

→ comparing the expressions with the expressions for stress in columns with eccentric load.

$$\int \sigma\, dA = P$$

$$\int \sigma y\, dA = M_x = \int y\, dP.$$

$$\int \sigma \cdot x\, dA = M_y = \int x \cdot dP.$$

→ In the analogous column, $\frac{ds}{EI}$ will be the elementary area, "σ" will be the stress at any point due to the load of intensity "$-M_s$" or total load $\int -\frac{M_s \cdot ds}{EI}$

$$M_i = \frac{P}{A} + \frac{M_y I_x - M_x I_{xy}}{I_y I_x + I^2_{xy}} \times x + \frac{M_x I_y - M_y I_{xy}}{I_y I_x - I^2_{xy}} \times y$$

$$M_i = \frac{P}{A} + \frac{\overbrace{M_y - M_x \frac{I_{xy}}{I_x}}^{M_y'}}{\underbrace{I_y\left[1 - \frac{I^2_{xy}}{I_x I_y}\right]}_{I'_y}} \times x + \frac{\overbrace{M_x - M_y \frac{I_{xy}}{I_y}}^{M_x'}}{\underbrace{I_x\left[1 - \frac{I^2_{xy}}{I_x \cdot I_y}\right]}_{I'_y}} \times y$$

$$\boxed{M_P = \frac{P}{A} + \frac{M_{y'}}{I_{y'}} + \frac{M_{x'}}{I_{x'}} \times y}$$

Final $B.M = M_s + M_i$

$M_s \rightarrow +ve \rightarrow$ Tension in the inside fibres.
$M_s \rightarrow -ve \rightarrow$ Tension in the outer fibres
$P \rightarrow$ will be tensile for +ve, M_s is negative.
$P \rightarrow$ will be compress for −ve, M_s is positive.
$M_i \rightarrow +ve \rightarrow \sigma$ is compressive
$M_i \rightarrow -ve \rightarrow \sigma$ is tensile.

- For Hinge $\rightarrow$ Any Rotation is possible $\rightarrow \frac{1}{EI}$ is infinity
- Fixed end. $\rightarrow$ Infinite rigidity $\rightarrow \frac{1}{EI}$ is zero.

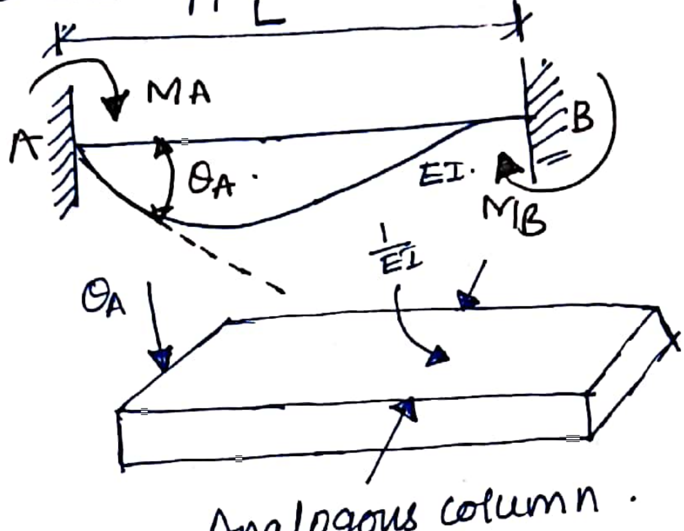
STIFFNESS AND CARRY-OVER FACTORS BY COLUMN ANALOGY METHOD:

Stiffness:-

→ It is defined as the value of moment to be applied at an end to cause slope of one radian.

Carry-over factor:-

→ It is defined as the ratio of the moment produced at far end to the applied moment.



Beam:-

→ Consider a prismatic beam of flexural rigidity EI. Let θ_A be the slope at ~~other~~ one end and slope at other end be zero.

Analogous column

→ Width of Analogous column will be $\frac{1}{EI}$.

→ Loading on the column will be θ_A at "A".

$$\sigma = \frac{P}{A} + \frac{P \times e \times y}{I_y}$$

$$\sigma_A = \frac{\theta_A}{L \times \frac{1}{EI}} + \frac{\theta_A \times \frac{L}{2} \times \frac{L}{2}}{\frac{L^3}{12} \times \frac{1}{EI}}$$

$$\sigma_A = \frac{EI\theta_A}{L} + \frac{3EI\theta_A}{L} = \frac{4E\theta_A}{L}$$

$$M_A = 0 + \frac{4EI\theta_A}{L} = \frac{4EI\theta_A}{L}$$

When $\theta_A = 1$, $M_A = \frac{4EI}{L}$.

$\therefore$ Stiffness is $\frac{4EI}{L}$.

$$\sigma_B = +\frac{P}{A} - \frac{P \times e \times y}{I_y}$$

$$= \frac{EI\theta_A}{L} - \frac{3EI\theta_A}{L}$$

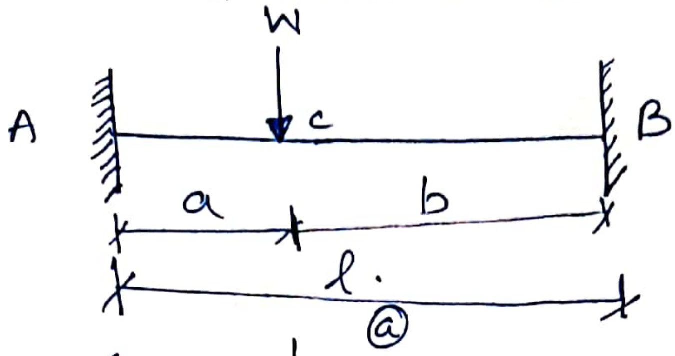
$$\sigma_B = -\frac{2EI\theta_A}{L}.$$

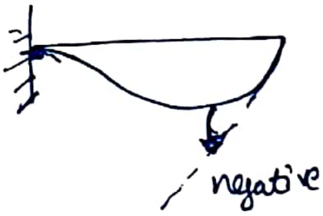
$$M_B = 0 + \sigma_B = 0 + -\frac{2EI\theta_A}{L} = -\frac{2EI\theta_A}{L}.$$

$$\boxed{M_B = -\frac{1}{2} M_A}$$

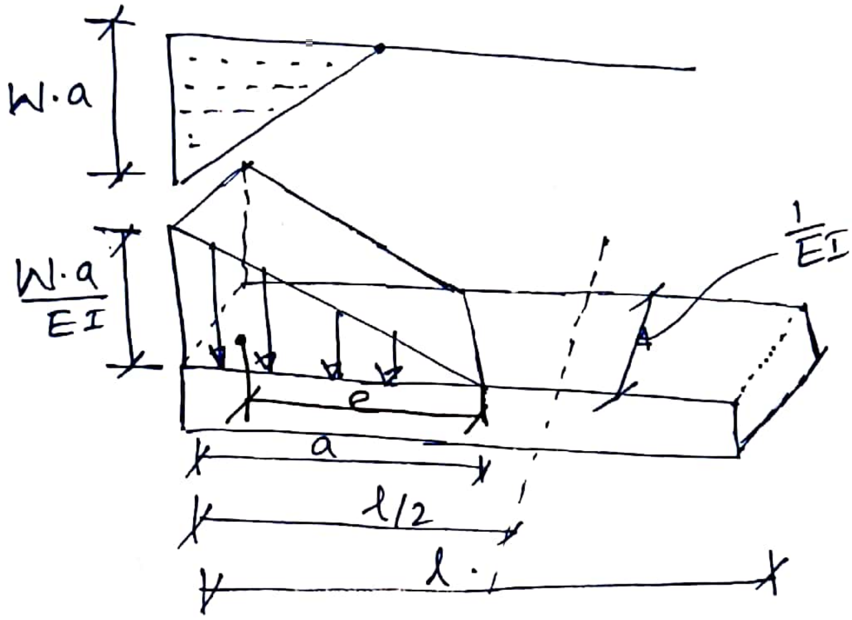
→ Carry over factor is $-\frac{1}{2}$.

① Find the fixed end moments for the beam loaded.





b) Basic determinate structure



$$P = \frac{Wa}{EI} \times \frac{1}{2} \times a = \frac{Wa^2}{2EI}.$$

$$e = \frac{l}{2} - \frac{a}{3} = \frac{3l-2a}{6}.$$

$$A = l \times \frac{1}{EI} = \frac{l}{EI}.$$

$$I_y = \frac{l^3}{12} \times \frac{1}{EI} = \frac{l^3}{12EI}.$$

The loading intensity on column will be $-M_s$ & hence compressive.

$$\sigma_A = +\frac{Wa^2}{2EI \times \frac{l}{EI}} + \frac{\frac{Wa^2}{2EI} \times \frac{(3l-2a)}{6} \times (+l/2)}{\frac{l^3}{12EI}}$$

$$= +\frac{Wa^2}{2l} + \frac{Wa^2(3l-2a)}{2l^2}.$$

$$= \frac{Wa^2}{2l^2}(l+3l-2a) = +\frac{Wa^2}{l^2}(2l-a)$$

$$M_A = M_s + M_i$$

$$= -W \cdot a + \frac{Wa^2}{l^2}(2l-a)$$

$$= -\frac{Wa}{l^2}\left[l^2 - 2al + a^2\right]$$

$$= -\frac{Wa}{l^2}[l-a]^2$$

$$\boxed{M_A = -\frac{Wab^2}{l^2}}$$

$$\sigma_B = +\frac{Wa^2}{2l} - \frac{Wa^2}{2l^2}(3l-2a]$$

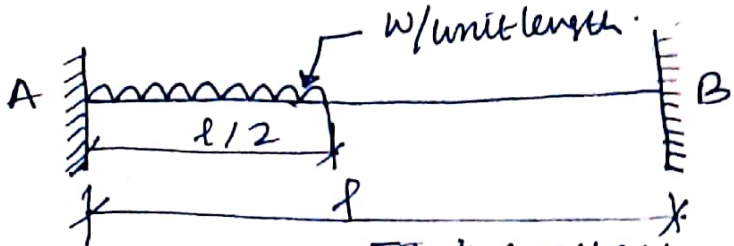
$$= \frac{Wa^2}{2l^2}\left[l-3l+2a\right] = -\frac{Wa^2b}{l^2}.$$

$$\boxed{M_B = M_s + M_i}$$

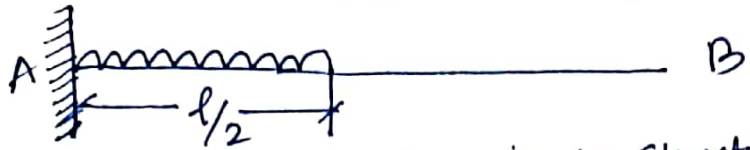
$$M_B = 0 - \frac{Wa^2b}{l^2}$$

$$\boxed{M_B = -\frac{Wa^2b}{l^2}}$$

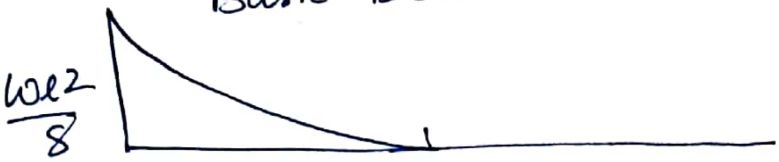
(2)



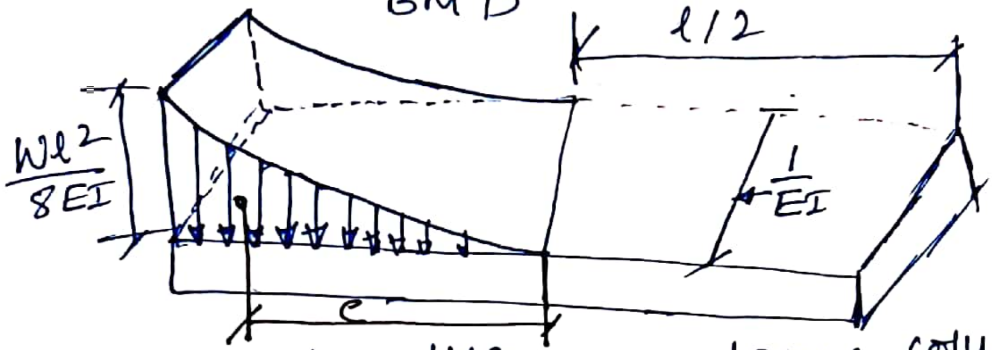
EI is constant



Basic Determinate Structure



BMD



Loading on analogous column.

→ The loading intensity on column will be $-M_s$ and hence compressive

$$I_y = \frac{l^3}{12EI} \; ; \; A = \frac{l}{EI} \; ; \; e = \frac{3}{4} \times \frac{l}{2} = \frac{3}{8} l .$$

$$P = \frac{wl^2}{8EI} \times \frac{1}{3} \times \frac{l}{2} = \frac{wl^3}{48EI}$$

$$\sigma_A = \frac{P}{A} + \frac{P \cdot e \cdot y}{I_y}$$

$$= \frac{\frac{wl^3}{48EI}}{\frac{l}{EI}} + \frac{wl^3}{48EI} \times \frac{\frac{3l}{8} \times \frac{l}{2}}{\frac{l^3}{12EI}} \qquad [x = +l/2]$$

$$= +\frac{wl^2}{48} + \frac{3}{64}wl^2 = +\frac{13}{192}wl^2$$

$$M_A = M_s + M_i = -\frac{wl^2}{8} + \frac{13}{192}wl^2$$

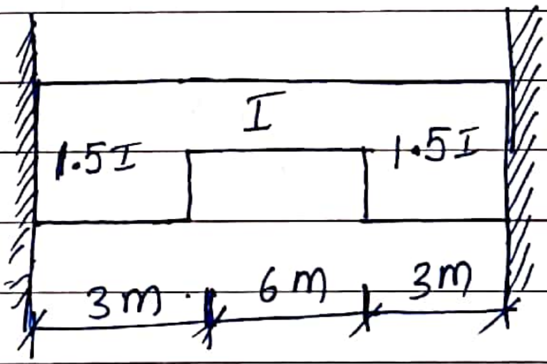
$$= \frac{-11wl^2}{192}.$$

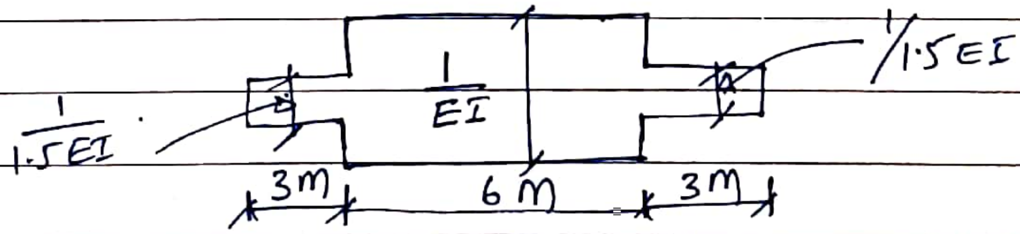
$$\sigma_B = \frac{P}{A} - \frac{P \times e \times l/2}{I_y} = +\frac{wl^2}{48} - \frac{3}{64}wl^2 \quad [x = -l/2]$$

$$= \frac{-5wl^2}{192}$$

$$M_B = 0 - \frac{5wl^2}{192} = -\frac{5wl^2}{192}.$$

3) Find the Stiffness & carryover factors for the fixed beam.





Analogous column.

θA

Loading of analogous colum.

$$A = 6 \times \frac{1}{1.5EI} + \frac{6}{EI} = \frac{10}{EI}.$$

$$I_y = \frac{1}{12} \times \frac{1}{1.5EI} \times 12^3 + \frac{1}{12}\left[\frac{1}{EI} - \frac{1}{1.5EI}\right] \times 6^3$$

$$= \frac{96}{EI} + \frac{6}{EI} = \frac{102}{EI}.$$

$P = \theta_A$, Eccentricity $e = 6m$ ($L/2$)

$$\sigma_A = \frac{\theta_A}{\frac{10}{EI}} + \frac{\theta_A \times 6 \times 6}{\frac{102}{EI}}.$$

$$= EI\theta_A\left[\frac{1}{10} + \frac{6}{17}\right] = \frac{77EI\theta_A}{10 \times 17}$$

$$\therefore M_A = \frac{77EI\theta_A}{170}.$$

When $\theta_A = 1$, $M_A = \frac{77}{170}EI$.

stiffness is $\frac{77EI}{170}$.

$$\sigma_B = \frac{\theta_A}{10} - \frac{\theta_A \times 6 \times 6}{\frac{102}{EI}}.$$

$$= -\frac{43}{170}EI\theta_A.$$

$$M_B = -\frac{43}{170}EI\theta_A.$$

$$\text{Carry over factor} = \frac{M_B}{M_A} = \frac{-\frac{43}{170}EI\theta_A}{\frac{77EI\theta_A}{170}} = -\frac{43}{77}.$$

If sign convention of moment distribution is adopted COF is $\boxed{+\frac{43}{77}}$.

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COLUMNS AND STRUTS

Column:-

→ It is a long vertical slender bar or vertical member, subjected to an axial compressive load and fixed rigidly at both ends.

Strut

→ A strut is a slender bar or member in any position other than vertical, subjected to a compressive load and fixed rigidly or hinged or pin jointed at one or both the ends. Eg:- Truss.

Failures :- i) By pure compression
ii) By Buckling.
iii) By pure compression & buckling based up on slenderness ratio.

Slenderness ratio (K)

→ It is the ratio of unsupported length of the column to the minimum radius of gyration of the cross-sectional ends of the column. It has no unit whatsoever.

Buckling factor:-

→ It is the ratio between the "equivalent length" of the column to the minimum radius of gyration.

Buckling load -

→ The maximum limiting load at which the column tends to have lateral displacement or tends to buckle is called buckling or crippling load. The buckling takes place about the axis having minimum radius of gyration, or least moment of inertia.

Safe load :- It is the load at which a column is actually subjected ~~to~~ and is well below buckling load. $\boxed{\text{Safe load} = \frac{\text{Buckling load}}{\text{FOS}}}$

Classification of columns :-

→ Depending up on the slenderness ratio or length to diameter ratio, columns can be classified in to three types.

SHORT COLUMN

→ If $\boxed{\frac{L}{D} < 8}$ & $\boxed{K < 32}$ are called short columns or stocky struts.

→ When short columns are subjected to compressive loads, their buckling is generally negligible, and it is always subjected to direct compressive stress only.

→ These columns fails suddenly.

→ High load carrying capacity.

→ FOS is adopted to avoid failure

→ It fails by crippling.

→ All are short columns since R.C.C. columns in general.

Design for short column

$$\boxed{P = \sigma A}$$

P = Load carrying capacity

σ = Allowable strength or permissible strength of column.

A = cross sectional area of column.

For Composite Short column

$$P_u = P_{cc} + P_{sc} \quad [LSM]$$

$$\boxed{P_u = (0.4 f_{ck})(A_c) + (0.67 f_y) \cdot A_{sc}}$$

$$\boxed{P = \sigma_{cc} \cdot A_c + \sigma_{sc} A_s}$$

MEDIUM SIZE COLUMNS

- $\frac{L}{D} = 8$ to 30 ; $K = 32$ to 120.
- In these columns both buckling as well as direct stresses are of significant values. Hence, both the values must be considered.

LONG COLUMNS :-

- $\frac{L}{D} > 30$; $K \geq 120$.
- These are usually subjected to buckling stress only.
- Direct compressive stress is very less, and hence it is neglected.
- Gradual failure
- Less Load carrying capacity.
- Fails by buckling.
- Most of the steel columns are long.

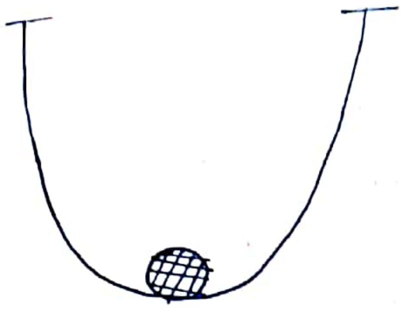
i) STRENGTH OF COLUMNS :-

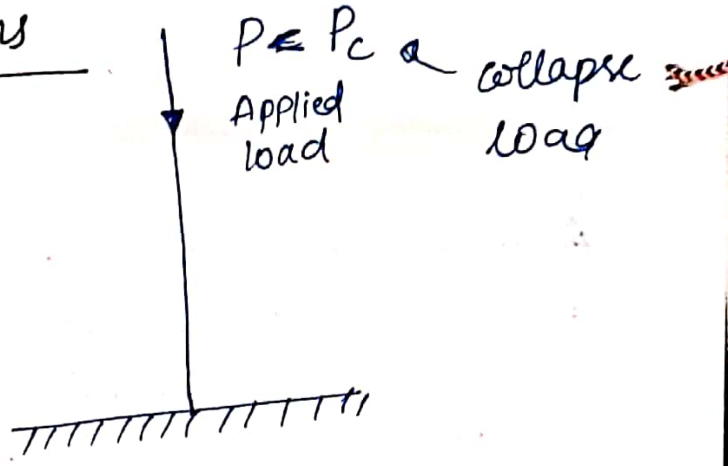
- In case of long column, Primary design criteria is "STABILITY" and then strength.
- The Strength of the column depends up on the Slenderness ratio and also end conditions.
- If Slenderness ratio is increased the compressive strength of a column decreases as the tendency to buckle increases.

ii) STABILITY OF COLUMNS

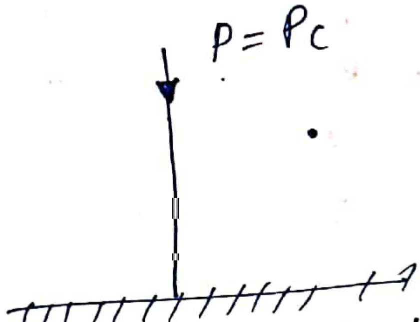
- To Decide the stability of a long column, equilibrium conditions will be used.

1. Stable Equilibrium conditions



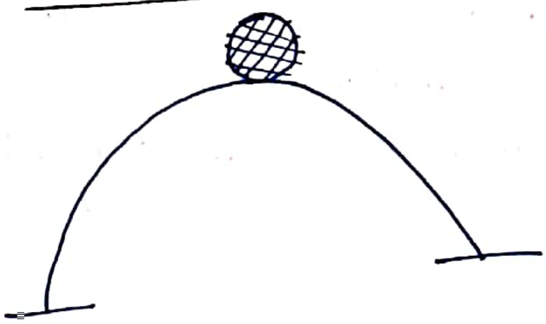


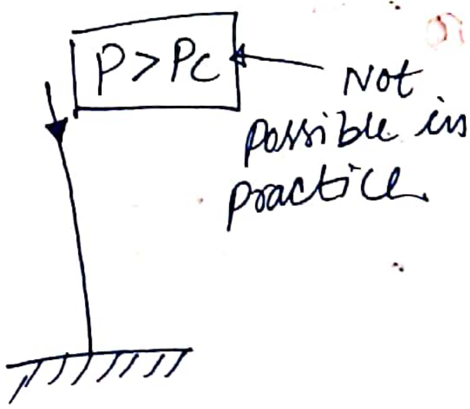
2. Neutral equilibrium condition :-



→ Neutral equilibrium condition is the condition of column just before failure

3. unstable equilibrium condition :-





EQUIVALENT LENGTH (l_e)

→ The distance between adjacent points of inflexion (zero BM) is called equivalent length (or) effective length or Simple column length.

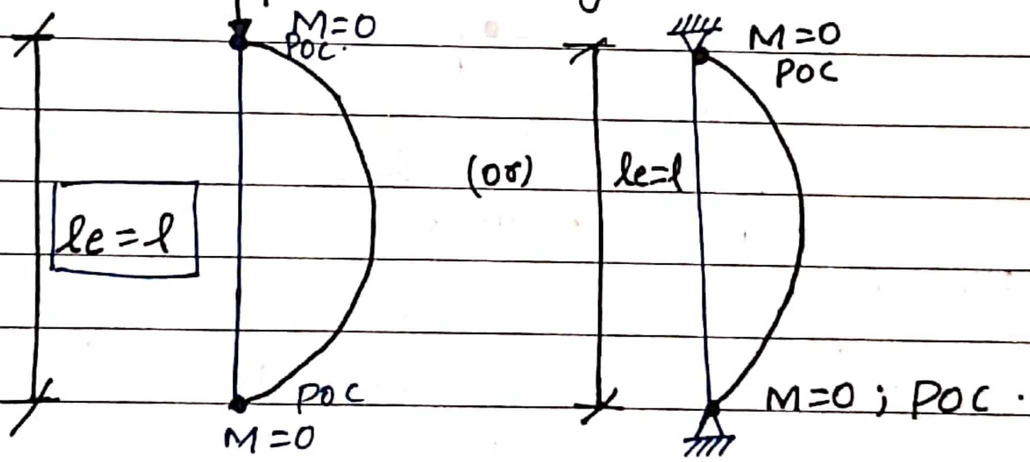
Point of inflexion

→ A point of inflexion is found at every column end that is free to rotate and at every point where there is a change of the axis.

- Effective length is independent of load on column, depends on support (or) end conditions.

End Conditions:-

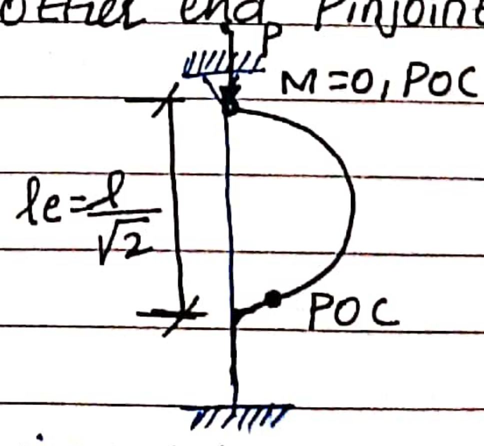
i> Both ends pin jointed or hinged or Rounded or free.



$$\boxed{\text{Equivalent length} = \text{Actual length}}$$

ii> One end fixed and other end Pinjointed.

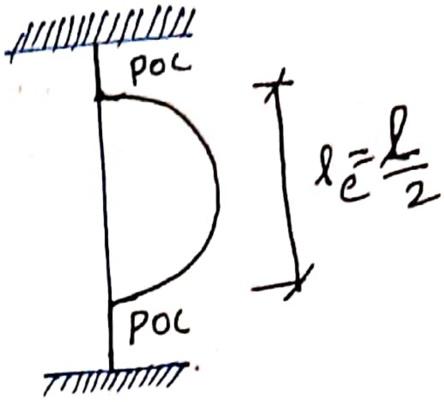
$$\boxed{l_e = \frac{l}{\sqrt{2}}}$$



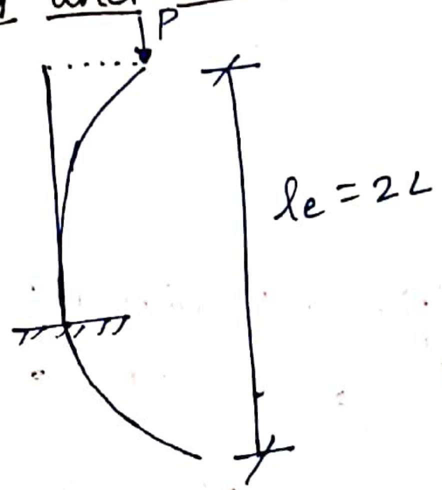
→ Between the top of the column and the inflexion point.

iii> Both ends fixed :-

$$\boxed{l_e = \frac{l}{2}}$$



iv) One ends fixed and other end free :-



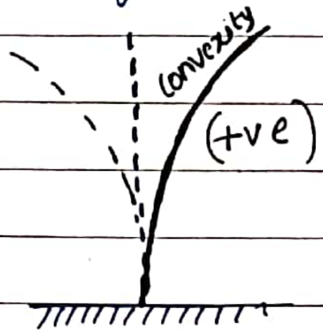
Eulers Theory for Long columns :-

Assumptions :-

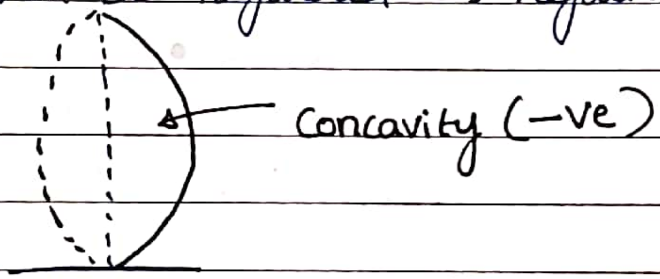
1. The column is initially straight and of uniform lateral dimension.
2. The compressive load is exactly axial and it passes through the centroid of the column section.
3. The material of the column is perfectly Homogeneous & isotropic.
4. Pin Joints are frictionless (No moment constraint) & fixed ends are perfectly rigid (No rotation).
5. The weight of the column itself is neglected.
6. The column fails by buckling alone, This is true when compressive stress does not exceed the yield strength (σ_y)
7. Limit of proportionality is not exceeded.
8. The cross-section of the column is uniform throughout its length.
9. Length of column is greater than C/S of the column
10. The direct stress is very small as compared to the bending stress (Material is compressed only within the elastic range of strains)

SIGN CONVENTIONS FOR BENDING MOMENTS.

1. → A Bending moment which bends the column so as to present convexity towards the initial centre line of the member will be regarded as Positive.



2. A bending moment which bends the column as to present concavity towards the initial centre line of the member will be regarded as negative.



Column buckling -

(i) → Column buckling is not related to the strength of the material.

→ A column buckling analysis consists of determining the maximum load a column can support before it collapses. But, for long columns, the column has nothing to do with material yield. It is instead governed by the column's stiffness, both material and geometric.

Limitation for the use of Eulers formula.

1. It is applicable to an ideal Strut only and in Practice, there is always crookedness (Bends, curves or angles) in the column and the load applied may not be exactly coaxial.
2. Direct stress < Bending stress (Long columns)
3. Bending stress < Direct stress (short columns) (Takes more buckling load)

Applicability of Euler theory.

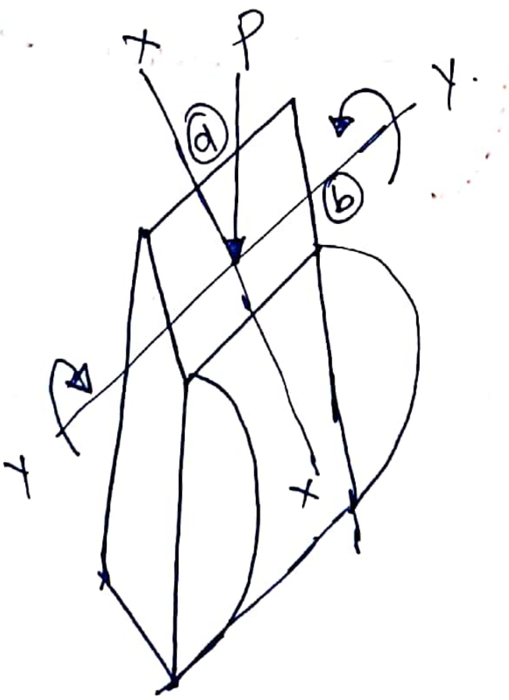
→ Applicable for Slender (long) columns.

If Max. Allowable Compressive Stress in the Strut is "σ_c".

P = Breaking or crushing load / strength

$P = \sigma_c \cdot A$.

$P > \sigma_c \cdot A$ → Strut will break in crushing.



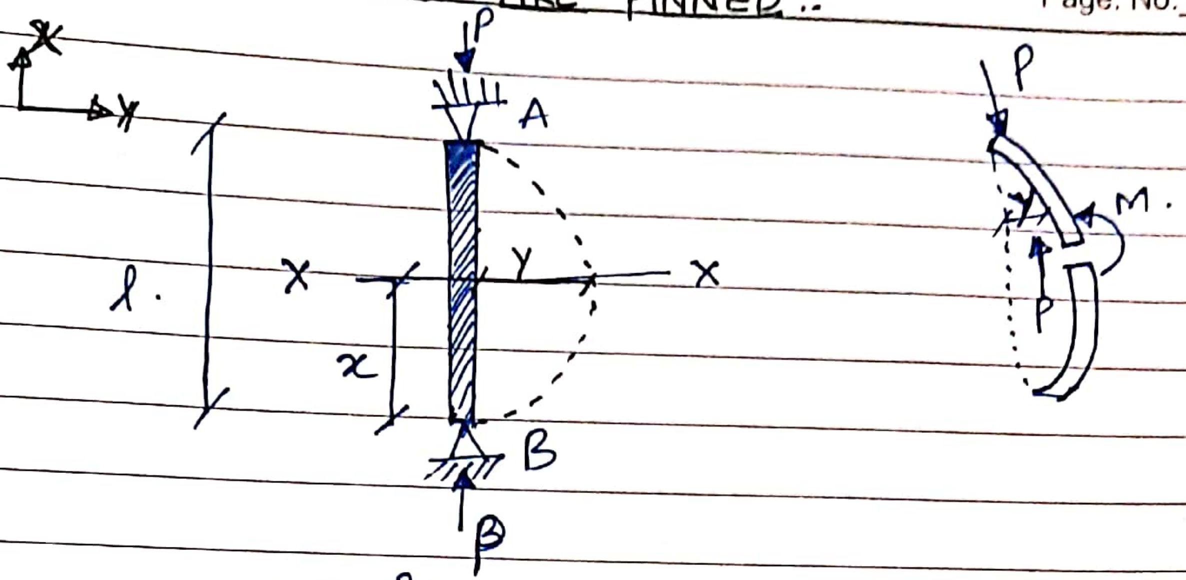
llel to axis / ⊥lar to axis

$$I_{min} = I_{yy} = \frac{db^3}{12}.$$

$$I_{max} = I_{xx} = \frac{bd^3}{12}$$

For solid circular $I_{min} = I_{max} = \frac{\pi}{64} d^2$.

Case i :- BOTH ENDS ARE PINNED..



$$EI\frac{d^2y}{dx^2} = -M_{xx}$$

$$EI\frac{d^2y}{dx^2} = -P \times y$$

$$EI\frac{d^2y}{dx^2} + P.y = 0$$

$$y\left[EI\,D^2 + P\right] = 0$$

$$\frac{d^2y}{dx^2} = \frac{-P.y}{EI}.$$

$$y\left[D^2 + \frac{P}{EI}\right] = 0$$

$$\frac{P}{EI} + D^2 = 0$$

$$D^2 = \frac{-P}{EI}.$$

$$D = \pm i\sqrt{P/EI}$$

$$D = \alpha \pm i\beta.$$

Complimentary function

$$C.F = e^{\alpha x}\left[a\cos\beta x + b\sin\beta x\right]$$ - from Malthematics

$$C.F = a\cos\sqrt{\frac{P}{EI}}\cdot x + b\sin\sqrt{\frac{P}{EI}}\cdot x.$$

Particular integral $PI = 0$.

$$y = CF + PI.$$

$$y = a\cos\sqrt{\frac{P}{EI}}\cdot x + b\sin\sqrt{\frac{P}{EI}}\cdot x \rightarrow ①$$

Put, $x = 0$; $y = 0$

$$0 = a\cos 0^\circ \;;\; \boxed{a = 0.}$$

Put, $x = l$; $y = 0$

$$0 = 0 + b\sin\sqrt{\frac{P}{EI}}\cdot l$$

$$\boxed{\begin{array}{l} b \neq 0 \\ \sin\sqrt{\frac{P}{EI}}\cdot l = 0 \end{array}}$$

$$\cancel{\sin}\sqrt{\frac{P}{EI}}\cdot \times l = \cancel{\sin}\pi.$$

$$\sqrt{\frac{P}{EI}} \times l = \pi.$$

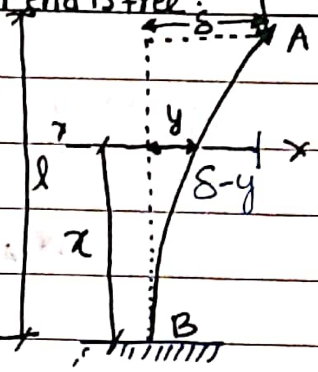
Squaring on b-sides

$$\frac{P}{EI} \times l^2 = \pi^2$$

$$\boxed{P_{cr} = \frac{\pi^2 EI}{l^2}}$$

→ In above calculations, If $a = 0$; & $b = 0$; it means y becomes zero from Eqn ①, which represents no buckling of column, which is impossible.

Case II When one end is fixed & other end is free.



$$EI\frac{d^2y}{dx^2} = +P(\delta - y)$$

$$EI\frac{d^2y}{dx^2} + Py = P\delta.$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}\cdot y = \frac{P\delta}{EI}.$$

The solution to the above differential equation is

$$y = C_1 \cos\left[x\sqrt{P/EI}\right] + C_2 \sin\left[x\sqrt{P/EI}\right] + \delta.$$

$$\left[\frac{P\delta/EI}{P/EI} = \delta\right]$$

where C_1 & C_2 are constants of integration

At B, deflection is zero.

At, $x=0$; $y=0$.

$0 = C_1 + \delta$; (or) $\boxed{C_1 = -\delta.}$

The slope at any section is given by.

$$\frac{dy}{dx} = -C_1\cdot\sqrt{P/EI}\,\sin\left(x\cdot\sqrt{P/EI}\right) + C_2\cdot\sqrt{P/EI}\cdot\cos\left[x\cdot\sqrt{P/EI}\right]$$

At B, the slope is zero.

$x = 0$; $\frac{dy}{dx} = 0$ ∴

$0 = C_2\sqrt{P/EI}$ (or) $C_2 = 0$.

At A, the deflection is δ

$x = l$; $y = \delta$.

$\cancel{\delta} = -\delta\cos\left[l\sqrt{P/EI}\right] + \cancel{\delta}$ (or) $\cos\left[l\sqrt{P/EI}\right] = 0$

$\boxed{\delta \neq 0}$

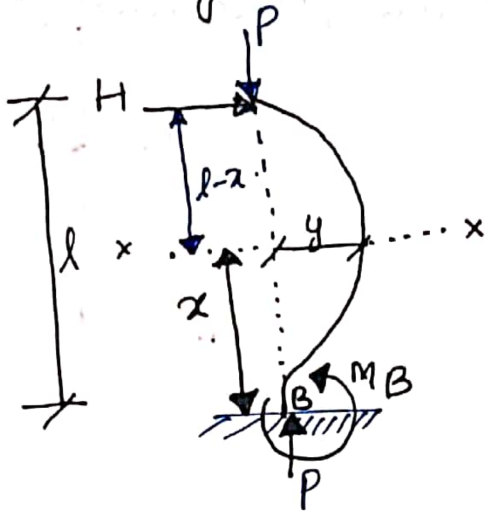
$\cos l\sqrt{P/EI} = \cos \pi/2.$

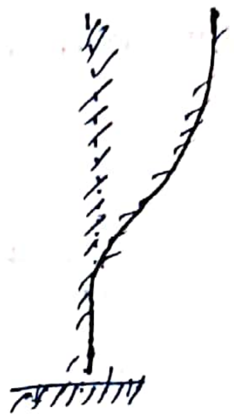
$l\sqrt{P/EI} = \pi/2$

Squaring on B. sides

$$\boxed{P = \frac{\pi^2 EI}{4l^2}}$$

Case III When one end of the column is fixed and the other Pinned or Hinged.





$$EI\frac{d^2y}{dx^2} = -\underset{\text{Concavity}}{Py} + \underset{\text{convexity}}{H(l-x)}$$

The solution to above differential equation is

Deflection

$$y = C_1 \cdot \cos\left[x \cdot \sqrt{\frac{P}{EI}}\right] + C_2 \cdot \sin\left[x\sqrt{\frac{P}{EI}}\right] + \frac{H}{P}(l-x) \quad -(1)$$

C_1 & C_2 are constants of integration.

Slope

$$\frac{dy}{dx} = -C_1 \cdot \sqrt{\frac{P}{EI}} \sin\left[x \cdot \sqrt{P/EI}\right] + C_2 \cdot \sqrt{P/EI} \cos\left[x \cdot \sqrt{\frac{P}{EI}}\right] - \frac{H}{P}$$

At B, the deflection is zero.

At $x=0$; $y=0$.

$$0 = C_1 + \frac{H}{P} \times l \qquad \boxed{C_1 = -\frac{H}{P} \times l}$$

At B, the slope is zero:-

At $x=0$; $\frac{dy}{dx} = 0$.

$$0 = C_2 \cdot \sqrt{P/EI} - H/P \qquad \boxed{C_2 = \frac{H}{P} \times \sqrt{\frac{EI}{P}}}$$

At A, the deflection is zero.

At $x = l$; $y = 0$.

$$0 = -\frac{\cancel{H}}{\cancel{P}}\, l \cos\left[l \cdot \sqrt{P/EI}\right] + \frac{\cancel{H}}{\cancel{P}} \cdot \sqrt{\frac{EI}{P}} \sin\left[l \cdot \sqrt{\frac{P}{EI}}\right]$$

Simplifying, we get.

$$\tan . l . \sqrt{P/EI} = l . \sqrt{P/EI} .$$

$$\tan . l . \sqrt{P/EI} = \tan(\sqrt{2} . \pi)$$

$$= 4.44 \simeq 4.5 \text{ radians}.$$

$$l \cdot \sqrt{P/EI} = 4.5$$

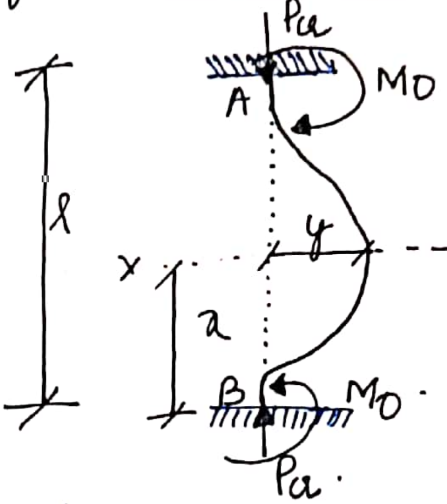
squaring on both sides

$$l^2 \times P/EI = 4.5^2$$

$$P = \frac{20.25\, EI}{l^2} .$$ $\left[20.25 \simeq 2\pi^2\right]$

$$\boxed{P = \frac{2\pi^2 EI}{l^2}}$$

Case iv :- When both ends of the column are fixed.



$$EI\frac{d^2y}{dx^2} = M_0 - P\cdot y.$$

$$\therefore EI\frac{d^2y}{dx^2} + P\cdot y = M_0$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}\cdot y = \frac{M_0}{EI}.$$

The solution to the above differential equation.

Deflection

$$y = C_1\cdot\cos\left[x\cdot\sqrt{P/EI}\right] + C_2\cdot\sin\left[x\sqrt{P/EI}\right] + \frac{M_0}{P}.$$

Where, C_1 & C_2 are constants of integration.

Slope.-

$$\frac{dy}{dx} = -C_1\cdot\sqrt{P/EI}\cdot\sin\left[x\cdot\sqrt{P/EI}\right] + C_2\cdot\sqrt{P/EI}\cdot\cos\left[x\cdot\sqrt{P/EI}\right].$$

At B, the deflection is zero.

At $x=0;\ y=0.$

$$0 = C_1 + \frac{M_0}{P} \quad (\text{or}) \quad \boxed{C_1 = -\frac{M_0}{P}}.$$

At B, the slope is zero.

$$\text{At } x=0;\ \frac{dy}{dx}=0 \therefore 0 = C_2\cdot\sqrt{P/EI} \text{ or, } \boxed{C_2=0}$$

At A, the deflection is zero.

At $x=l;\ y=0.$

$$0 = -\frac{M_0}{P}\cos\left[l\cdot\sqrt{P/EI}\right] + \frac{M_0}{P}. \quad (\text{or}) \quad \frac{M_0}{P}\left[1-\cos\left[l\cdot\sqrt{P/EI}\right]\right] = 0.$$

$$\cos\cdot l\sqrt{P/EI} = 1. \Rightarrow \cos\cdot l\sqrt{P/EI} = \cos 2\pi.$$

$$l\cdot\sqrt{P/EI} = 2\pi$$

Squaring on b sides

$$\boxed{P = \frac{4\pi^2 EI}{l^2}}$$

RANKINE'S HYPOTHESIS FOR STRUTS / COLUMNS.

- Acceptable theoretical expressions are available for both very short and very long struts. The Rankine hypothesis is designed to link these two known results to obtain an expression applicable to all dimensions.

Now, for a very short strut, collapse will result from direct crushing, and crippling load is

$$\boxed{P_c = \sigma_c \cdot A}$$

For a long strut Euler formula applies.

$$\therefore \left[K = \sqrt{I/A}\right]$$

$$P_{Euler} = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EAK^2}{l_e^2} = \pi^2 EA\left[\frac{K}{l_e}\right]^2$$

The Rankine hypothesis is.

$$\frac{1}{P} = \left[\frac{1}{P_{Rankine}}\right] = \frac{1}{P_c} + \frac{1}{P_{Euler}}.$$

Where, $P_{Rankine}$ is the actual crippling load for a strut.

- If the strut is very short, (l_e) P_{Euler} becomes very large and $\frac{1}{P_{Euler}} = 0$, so that $\boxed{P = P_c}$
- If the strut is very long strut, $\frac{1}{P_{Euler}}$ becomes very large so that, $\boxed{P = P_{Euler}}$.
- Hence, it may be assumed that if the Rankine hypothesis is true for both very long and very short struts, it will also be true for struts of other dimensions.

... Euler formula :

$$\Rightarrow \frac{1}{P} = \frac{1}{\sigma_c \cdot A} + \frac{1}{\pi^2 EA \cdot \left[\frac{k}{l_e}\right]^2} \quad (\text{or}) \quad \frac{A}{P} = \frac{1}{\sigma_c} + \frac{1}{\pi^2 E \left[\frac{k}{l_e}\right]^2}$$

$$\therefore \frac{P}{A} = \frac{1}{\frac{1}{\sigma_c} + \frac{l_e^2}{\pi^2 E k^2}}.$$

$$\frac{P}{A} = \frac{\sigma_c}{1 + \frac{\sigma_c}{\pi^2 E} \cdot \left[\frac{l_e}{k}\right]^2}.$$

$$\boxed{P_{Rankine} = \frac{\sigma_c \cdot A}{1 + a\left[\frac{l_e}{k}\right]^2}.}$$

→ Hence, this is the Rankine formula for the mean breaking stress of a strut/column, where $a = \frac{\sigma_c}{\pi^2 E}$.

→ "σ_c" & "a" are constants.

→ $1 + a\left[\frac{l_e}{k}\right]^2$ introduced to take account the buckling effect.

Johnsons parabolic formula.

$$\frac{P}{A} = \sigma_c \left[1 - a\left[\frac{l_e}{k}\right]^2\right] \text{ approxiamaletly}$$

$$\frac{P}{A} = \sigma_c - b\left[\frac{l}{k}\right]^2.$$

'b' is approximately $\left[\frac{\sigma_c^2}{4\pi^2 E}\right]$ & Johnson accepted the value $\frac{\sigma_c^2}{64E}$ for pinned ends.

Columns Subjected to eccentric loading

a) Rankines Method :-

Let, P = An eccentric load to which a short column is subjected.

e = Eccentricity from the geometric axis.

A = Area of cross section of the member.

σm = Max. Compressive stress.

σ = Safe stress of the column,

le = effective length of the column.

Now,
$$\sigma max = \frac{P}{A} + \frac{P \cdot e}{I} \cdot y.$$

$$= \frac{P}{A} + \frac{P \cdot e}{AK^2} \cdot y$$

$$= \frac{P}{A}\left[1 + \frac{ey}{K^2}\right]$$

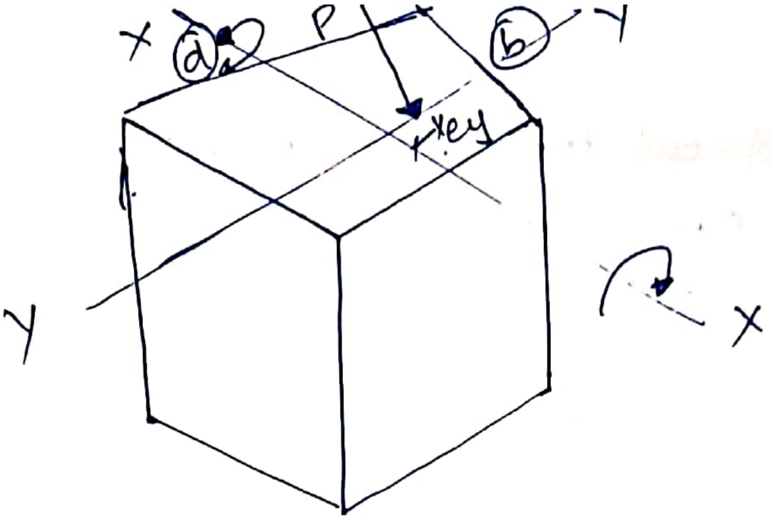
$$\boxed{P = \frac{\sigma max \cdot A}{1 + \frac{ey}{K^2}}}$$

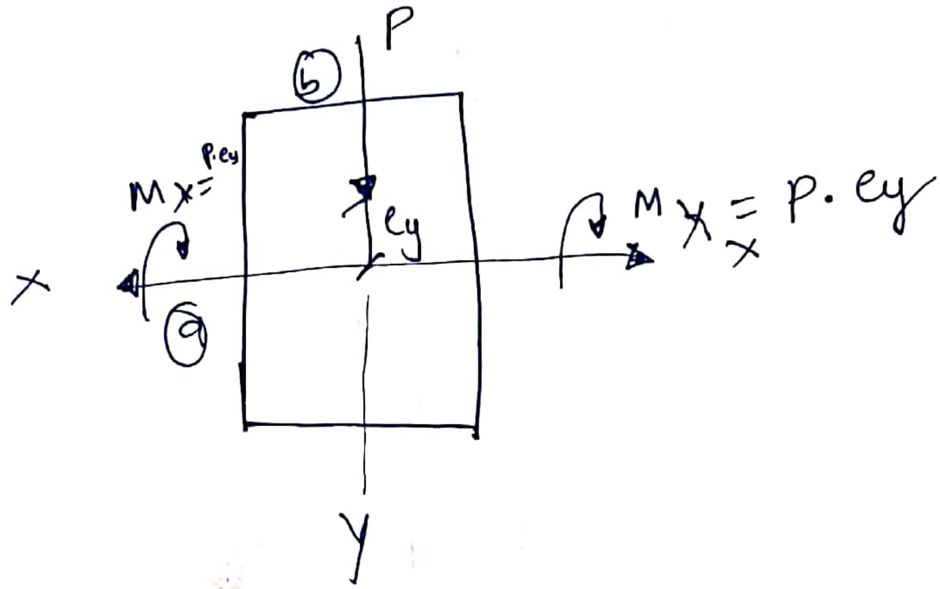
and, Safe load for the column at the eccentricity 'e'

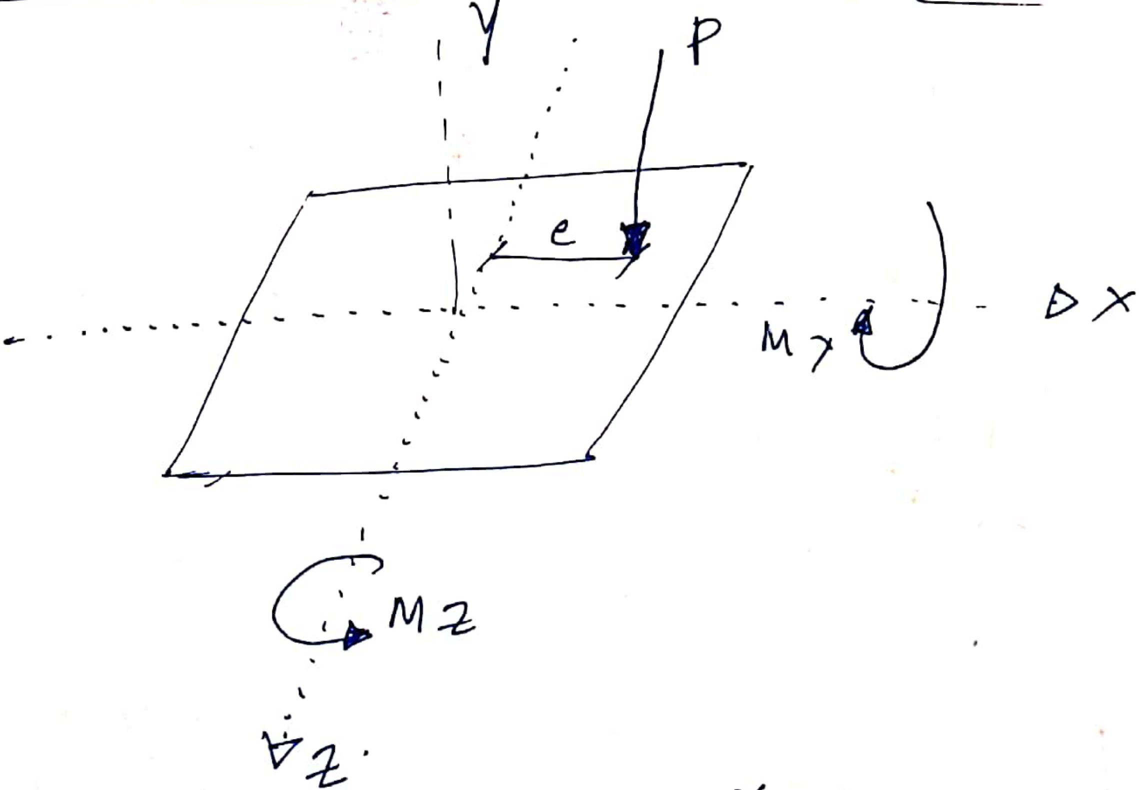
$$\boxed{P = \frac{\sigma_c \cdot A}{1 + \frac{ey}{K^2}}}$$

$$\boxed{P = \frac{\sigma_c \cdot A}{\left[1 + \frac{e \cdot y}{K^2}\right]\left[1 + a \cdot \frac{le^2}{K^2}\right]}}$$

Effect of buckling is also included

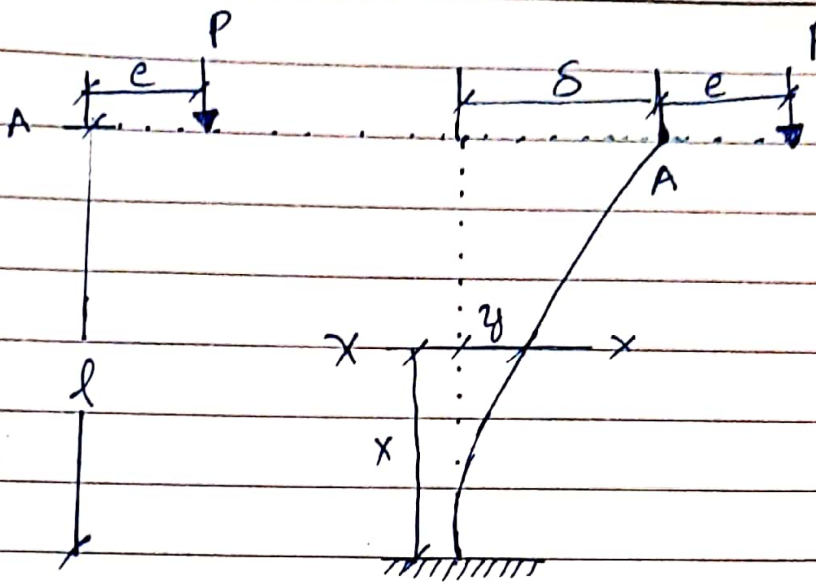






$$\sigma_{max} = \frac{\frac{P}{A}}{\sigma_{all\ conc}} + \frac{\frac{M_x Z_{max}}{I_{xx}}}{\sigma_{all\ bend}} + \frac{\frac{M_z x_{max}}{I_z}}{\sigma_{al}(bend)} \leq 1$$ for eccentric loading

Eulers Method:-



Let, y = Deflection at any section "x" distant "x" from the fixed end B, and

δ = Deflection at "A".

The bending moment at the section is given by.

$$EI\frac{d^2y}{dx^2} = P(\delta + e - y)$$

$$\therefore \frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{P(\delta+e)}{EI}.$$

The Solⁿ to the above D.E is given by.

$$y = C_1 \cdot \cos x \cdot \sqrt{P/EI} + C_2 \sin x\sqrt{P/EI} + (\delta + e)$$

The Slope at any section

$$\frac{dy}{dx} = -C_1 \cdot \sqrt{P/EI}\,\sin x \cdot \sqrt{P/EI} + C_2\sqrt{P/EI}\cos x\sqrt{P/EI}$$

At B, $x=0$ & $y=0$ & $\frac{dy}{dx}=0$.

Constants

$\boxed{C_2 = 0 \;\&\; C_1 = -(\delta+e)}$ $0 = C_1 + (\delta+e)$ and $0 = C_2\sqrt{P/EI}$

At A, $x = l$; $y = \delta$

$\therefore \delta = -(\delta+e)\cdot \cos l\sqrt{P/EI} + (\delta+e)$.

$\therefore \delta = \delta + e\left[1 - \cos l\sqrt{P/EI}\right]$

$\therefore (\delta+e)\cos\sqrt{P/EI} = e$.

$$\boxed{\delta + e = e \sec l\sqrt{P/EI}}$$

$\therefore$ The max. Bending moment for the column occurs at B and is equal to $P(\delta+e)$

$\therefore$ Max B.M $= M = P\cdot e\sec l\sqrt{P/EI}$.

→ Hence the max. compressive stress for the column section at "B".

$$\boxed{\sigma_{max} = \underset{\text{direct}}{\sigma_d} + \underset{\text{Bendig}}{\sigma_b} = \frac{P}{A} + \frac{P\cdot e\sec l\sqrt{P/EI}}{Z}}$$

<u>If Both ends are hinged</u>:-

(or) for one fixed & other free.

$$\boxed{\sigma_{max} = \sigma_d + \sigma_b = \frac{P}{A} + \frac{P\cdot e\sec \frac{le}{2}\sqrt{P/EI}}{Z}}$$

→ For short columns moment = P.e, but for long columns moment is increased by $P\cdot e\cdot \sec\frac{le}{2}\sqrt{P/EI}$ times.

6

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PROF. PERRY'S FORMULA :-

→ In cases, where we have to determine the safe load that can be applied on a column at a given eccentricity Prof. perry's formula is quite useful.

Let, σ_d = stress due to direct load ($= P/A$)

σ_{max} = Maximum permissible stress.

l_e = Effective length of the column, and

σ_b = Max. Compressive stress due to B.M.

$$= \frac{M}{Z} = \frac{M \cdot y_c}{A \cdot k^2}$$

$$= \frac{P \cdot e \cdot \sec \frac{l_e}{2} \cdot \sqrt{P/EI}}{Ak^2} \cdot y_c$$

$$= \frac{P \cdot e \cdot y_c}{Ak^2} \sec \frac{\pi}{2} \cdot \sqrt{\frac{P}{P_{Euler}}}$$

$$\left[P_{Euler} = \frac{\pi^2 EI}{l_e^2} \right]$$

$$\sigma_{max} = \frac{P}{A} + \frac{P \cdot e \cdot y_c}{Ak^2} \cdot \sec \cdot \frac{\pi}{2} \times \sqrt{\frac{P}{P_{Euler}}}$$

$$\boxed{\sigma_{max} = \sigma_d \left[1 + \frac{e y_c}{k^2} \cdot \sec \cdot \frac{\pi}{2} \cdot \sqrt{\frac{P}{P_{Euler}}} \right]}$$

According to prof. Perry's.

$$\sec \frac{\pi}{2}\sqrt{\frac{P}{P_{Euler}}} = \frac{1.2 P_{Euler}}{P_{Euler} - P} \text{ (Approx)}$$

Let, $\sigma_{Euler} = \frac{P_{Euler}}{A}$.

$$\sec \cdot \frac{\pi}{2} \cdot \sqrt{P/P_{Euler}} = \frac{1.2 P_{Euler}}{P_{Euler} - P} = \frac{1.2 \sigma_{Euler}}{\sigma_{Euler} - \sigma_d}.$$

$$\sigma_{max} = \sigma_d \left[1 + \frac{e \cdot y_c}{K^2} \cdot \frac{1.2 P_{Euler}}{P_{Euler} - P} \right]$$

$$\sigma_{max} = \sigma_d \left[1 + \frac{e y_c}{K^2} \cdot \frac{1.2 \sigma_{Euler}}{\sigma_{Euler} - \sigma_d} \right]$$

$$\frac{\sigma_{max}}{\sigma_d} - 1 = \frac{e y_c}{K^2} \cdot \frac{1.2 \sigma_{Euler}}{\sigma_{Euler} - \sigma_d}$$

$$\left[\frac{\sigma_{max}}{\sigma_d} - 1 \right] \left[\frac{\sigma_{Euler} - \sigma_d}{\sigma_{Euler}} \right] = \frac{1.2 \cdot e y_c}{K^2}$$

$$\boxed{\left[\frac{\sigma_{max}}{\sigma_d} - 1 \right] \left[1 - \frac{\sigma_d}{\sigma_{Euler}} \right] = \frac{1.2\, e y_c}{K^2}}$$

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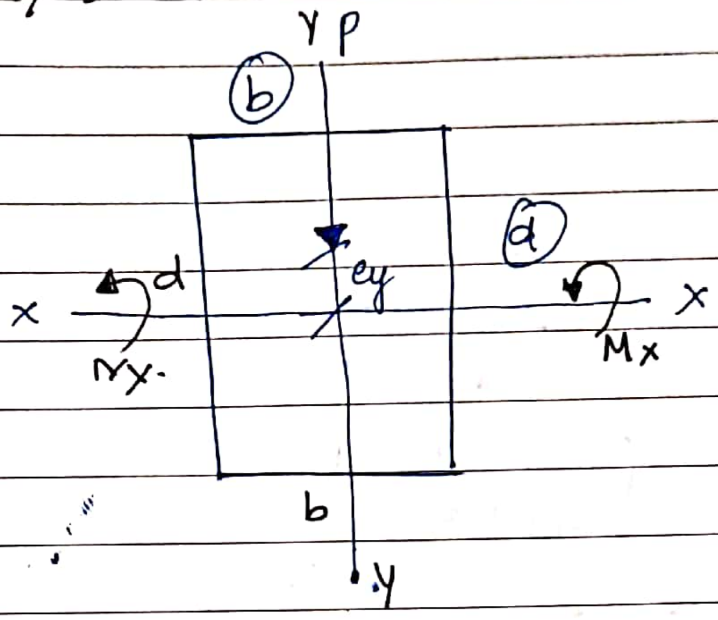
Minimum eccentricity for No tension ($\sigma_{min} = 0$)

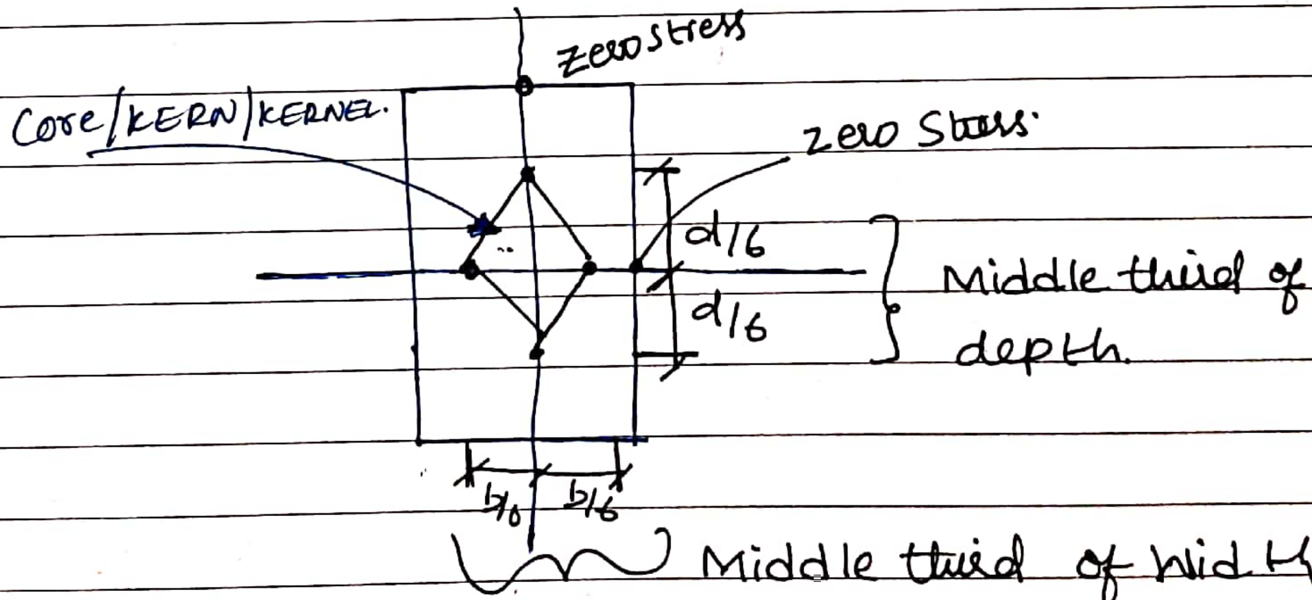
Rectangle/square.



$$\sigma_{min} = 0 = \frac{P}{A} - \frac{M_x}{I_{xx}} \cdot y_{max}$$

$$0 = \frac{P}{b \cdot d} - \frac{P \cdot e_y}{\frac{bd^3}{12}} \cdot d/2$$

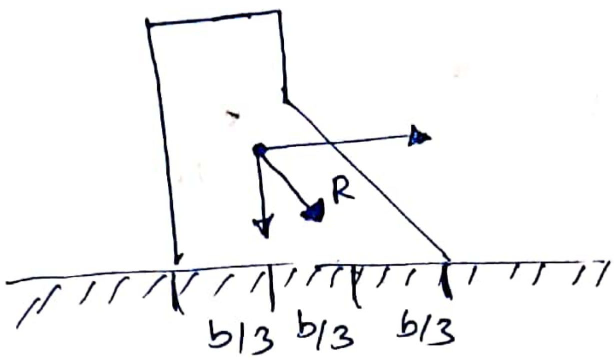
$$\boxed{e_y = \frac{d}{6}}$$



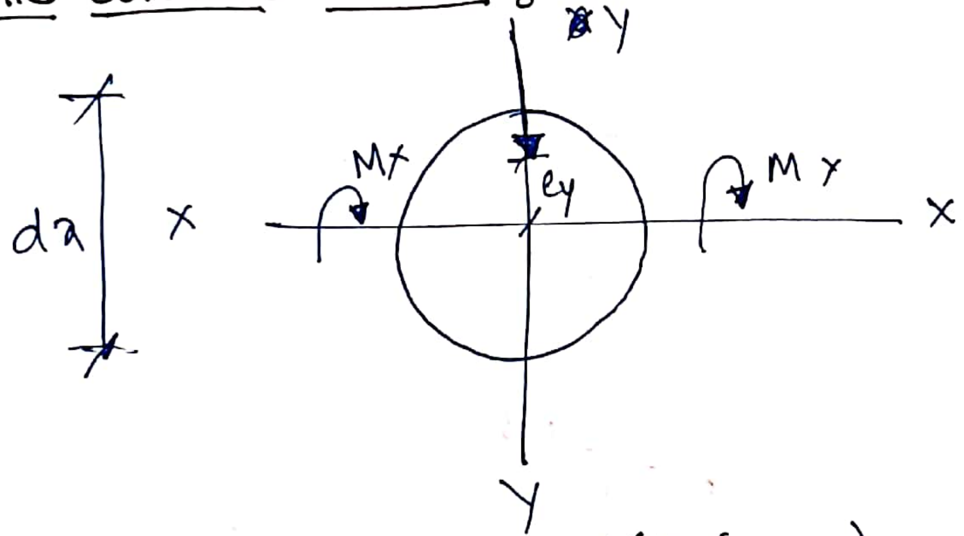
1. Inside core → Entire cross section in compression.
2. core opposite fibre → Zero stress.
3. Outside the core → opposite extreme fibre is under tension

Middle third rule [$d/3$]

- Middle third rule applicable to rectangle (or) square section as long as the load applied is with in middle third of a side No tension occurs.
- Application in Dams, we must ensure the resultant should fall in the middle third, to get no tension at the base



② Solid circular section :-

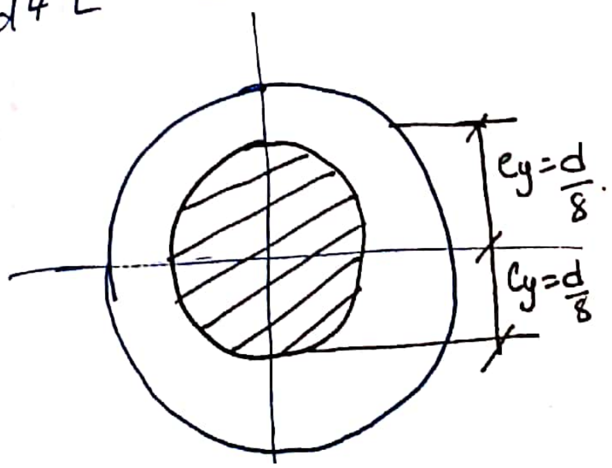


$$\sigma_{min} = 0 = \frac{P}{A} - \frac{M_x}{I_x}(y_{max})$$

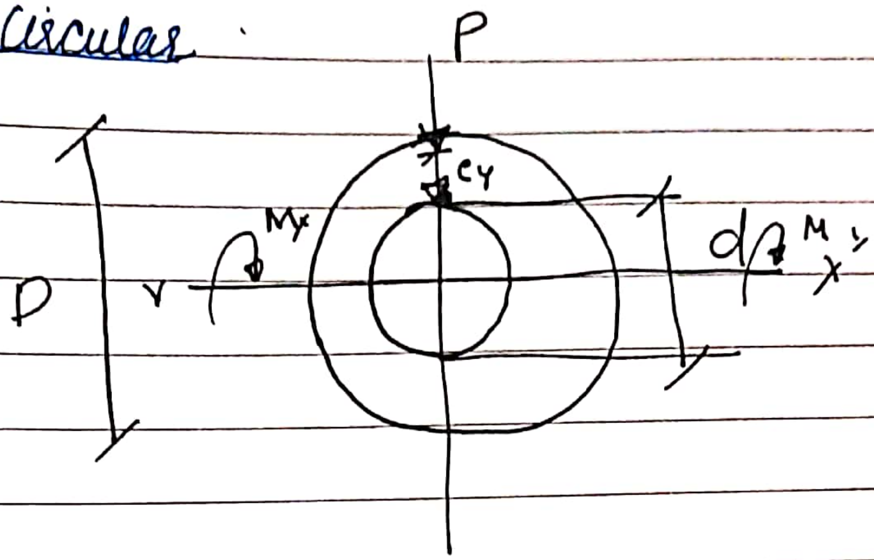
$$0 = \frac{P}{\frac{\pi}{4}d^2} (-) \frac{P.e_y}{\frac{\pi}{64}d^4}\left[d/2\right]$$

$$\boxed{e_y = d/8}$$

$$\boxed{\text{Middle fourth rule} = \frac{d}{4}}$$



3. Hollow circular.



$$\sigma_{min} = 0 = \frac{P}{A} - \frac{M_x}{I_x}(y_{max})$$

$$0 = \frac{P}{\frac{\pi}{4}(D^4 - d^4)} - \frac{P.e_y}{\frac{\pi}{64}(D^4 - d^4)} \cdot \left[\frac{D}{2}\right]$$

$$\boxed{e_y = \frac{D^2 + d^2}{8D}}$$