UNIT - I

18/7/2017

LINEAR WAVE SHAPING

Any Electrical Network comprises of both active and passive elements.

Active elements are energy sources like Voltage sources and current sources and this energise the network.

Resistor consumes power, 'L' and 'C' store energy. The energy consumed by Resistor descipates in the form of heat and energy stored in magnetic field of 'L' and electrostatic field of 'C'.

Resistor, C and L are the linear circuit elements since the current through them is proportional to voltage and resulting current ie, There is linear relationship b/w applied voltage and resulting current.

A N/w comprising of these linear elements is termed as linear Network.

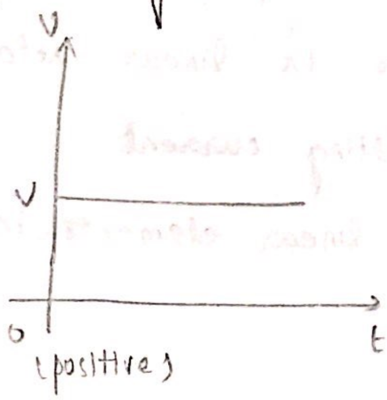
Linear wave Shaping:-

When a sinusoidal voltage is applied to linear circuit, the output voltage is also sinusoidal in nature. There is no difference what so ever as far as the waveform is concerned the amplitude of i/p signal may change and there be phase displacement b/w input signal and output voltage but there is no distortion of wave shape in this respect. The sinusoidal signal is unique.

when a non sinusoidal is transmitted through a linear network the wave form of output voltage bears no resemblances to the wave of the input signal. This process by which the wave form of a non sinusoidal signal is altered by transmitting. the signal through a linear network is termed as Linear wave shaping

STUDY OF STEP, PULSE, RAMP, EXPONENTIAL AND SQUARE WAVE VOLTAGES:-

(1) Step voltage



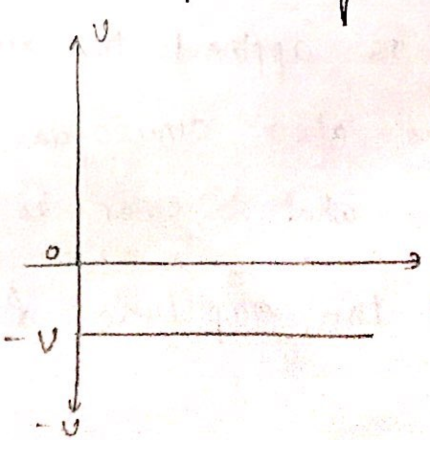
magnitude = V

V=0 for t<0 and

V=V for t>0

(for all values of t>0 the voltage maintains a constant level just like a direct voltage of constant magnitude)

Negative step voltage



magnitude = -V

V=0 for t<0 and

V=-V for t>0

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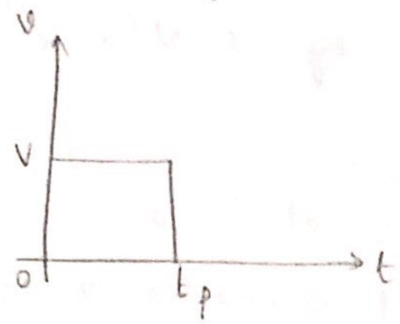
(2) Pulse voltage



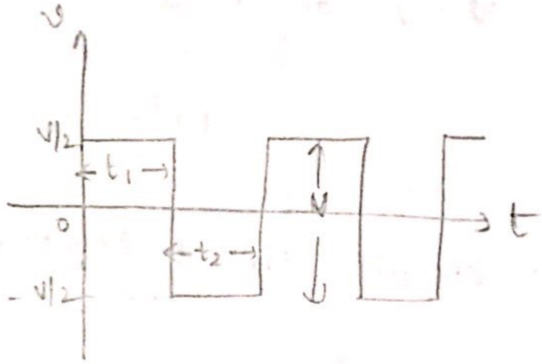
magnitude of the pulse = V

we have $v = V$ for $0 < t < t_p$

$v = 0$ for $t < 0$ & $t > t_p$

where t_p is pulse width

(3) Square wave :-



Here $t_1 = t_2 = T/2$

$T \rightarrow$ period

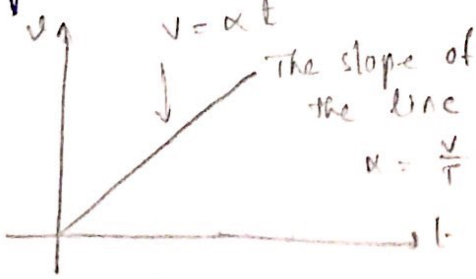
for a symmetrical sq. wave the upper +ve dc level & lower -ve dc level are equal $t_1 = t_2$. If t_1, t_2 are unequal it represents a repetitive pulse.

Eg:-

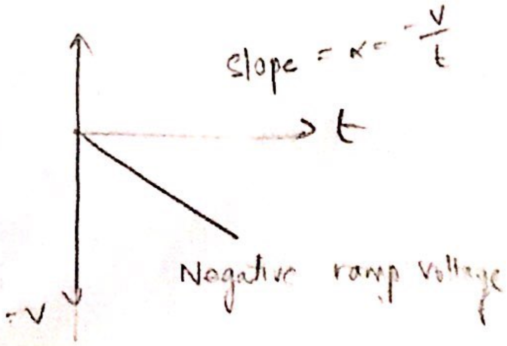
$t_1 < t_2$

(4) Ramp voltage:-

A Ramp voltage increases or decreases at a constant rate w.r.t time. If it increases it is called positive ramp and if it decreases it is called Negative Ramp.



positive ramp voltage



Negative ramp voltage

(5) Exponential voltage:

An exponential voltage given by $v = V(1 - e^{-t/\tau})$ increases with time

$v = 0$ at $t = 0$

As t increases v increases exponentially

$v = V$ at $t = \infty$

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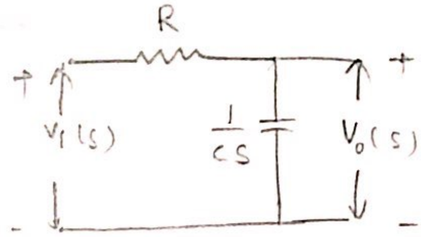
Response of Low Pass RC circuit for a sinusoidal



From KVL

$$V_o(s) = \frac{\frac{1}{CS}}{R + \frac{1}{CS}} \cdot V_i(s)$$

$$= \frac{1}{1 + R.CS} \cdot V_i(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + RC.j\omega}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + j\omega RC}$$

$$= \frac{1}{1 + j2\pi f RC}$$

$$= \frac{1}{1 + j.\frac{f}{\left(\frac{1}{2\pi RC}\right)}}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{1 + j \cdot \frac{f}{f_H}}$$ is the Transfer function.

where f_H is the upper cuttoff frequency

$$f_H = \frac{1}{2\pi RC}$$

f_H is constant for given R & c values.

$\Rightarrow$ If $f < f_H$ then frequencies are passed through circuit

$\Rightarrow$ If $f > f_H$ then frequencies are not passed through the circuit

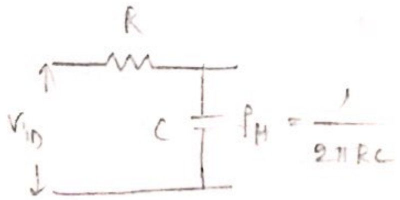
Polar form

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{1 + j \cdot \frac{f}{f_H}}$$

$\left(\because \; a + jb = M \angle \phi \right.$

$M = \sqrt{a^2 + b^2}$

$\phi = \tan^{-1}\left(\frac{b}{a}\right)$



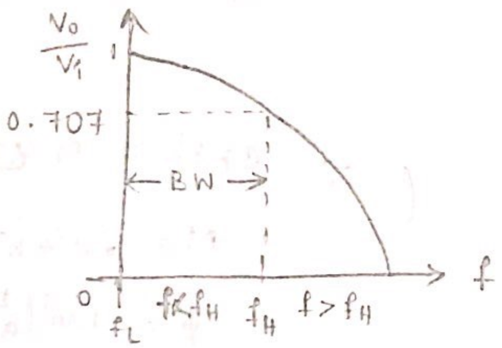
where f = frequency of i/p signal varies from 0 to ∞.

Polar form :-

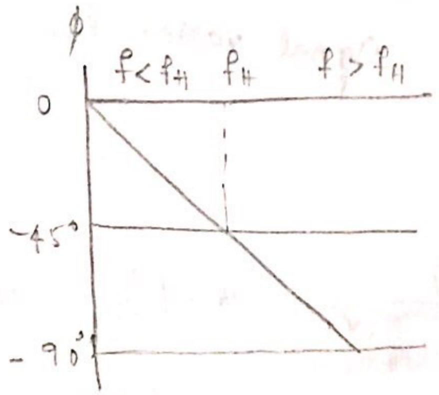
	$\frac{V_0}{V_i} = \frac{1}{\sqrt{1 + \left(\frac{f}{f_H}\right)^2}}$ (Magnitude)	$\phi = \angle \frac{V_0}{V_i} = -\tan^{-1}\left(\frac{f}{f_H}\right)$ (Phase)
at $f = 0$	$\frac{f}{f_0} < 1 \cdot = 1$ ie, $V_0 = V_i$	0° $(\because \tan^{-1}(0))$
at f_1 but $f < f_H$	$\simeq 1$ but $V_0 < V_i$	$0 < \lvert\phi\rvert < -45^\circ$

at $f = f_H$	$\frac{1}{\sqrt{2}} = 0.707$ $V_o = 70.7\ \%$ of V_i	$-45°$
at $f \uparrow$ but $f > f_H$	$\frac{V_o}{V_i}$ reduces	$-45° < \phi < 90°$
at $f = \infty$	$V_o = 0$	$\phi = 90°$

Magnitude Response for LPRC circuit :-



Phase Response -



Bandwidth : It is the Range of frequencies over which the gain will be greater than or equal to 70.7 % of the maximum value.

$$(B.W)_{LPRC} = f_H - f_L \quad \text{where } f_L = 0.$$
$$= f_H - 0$$
$$= f_H \text{ (Upper cutoff frequency)}$$

* LPRC circuit acts as Low Pass filter with upper cutoff frequency as f_H. and the Bandwidth of LPRC is f_H ie., $\frac{1}{2\pi RC}$. The response of Linear circuit is always sinusoidal when sinusoidal input is applied.

Capacitor (C) :-

Main function of the capacitor is to store energy temporarily in the form of electric field. (or) voltage.

Let W = Energy stored by the capacitor

$$= \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{Q.V}{2}$$

where Q is the charge stored by the capacitor.

The ability to store energy by capacitor is defined as capacitance of the capacitor. The units are farads.

Time constant of RC circuit :-

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Consider a series circuit comprising of Resistance R and capacitance 'c'. Let a step voltage 'v' be applied to the circuit. Let V_0 be the voltage across the capacitor. It can be shown that the voltage V_0 increases exponentially after a fairly long time at $t=\infty$. It attains the value 'V'. At $t=0$ the instant of application of the input voltage the capacitor acts as short circuited and a charging current i flows. The capacitor charges and a voltage develops across the capacitor progressively increases. and the transient current is given as $i(t) = \frac{V}{R} e^{-t/RC}$.

The instantaneous voltage across the capacitor is given as $V_0 = \frac{1}{c}\int_0^t i(t)\,dt$.

putting $i(t) = \frac{V}{R} e^{-t/RC}$

$$V_0 = \frac{1}{c}\int_0^t \frac{V}{R} e^{-t/RC}\,dt.$$

$$= \frac{1}{c}\cdot\frac{V}{R}\cdot(-RC)\left[e^{-t/RC}\right]_0^t$$

$$= -V\left[e^{-t/RC} - 1\right]$$

$$V_0 = V\left[1 - e^{-t/RC}\right]$$

if $t = RC$ then $V_o = 0.632\,V$.

It means that after time $t = RC$ the voltage across the capacitor becomes equal to 63.2 % of the final study value. The product RC is termed as time constant of the RC circuit. denoted as τ.

Hence $\tau = RC$.

Definition of Time Constant :-

Time constant of the RC circuit is the time required for the output voltage (or voltage across the capacitor to attain 63.2 % of the final steady value. The τ is expressed in sec.

The voltage across capacitor in any instant of time

$$\boxed{V_c(t) = V_f - (V_f - V_i).e^{-(t-t_i)/RC}}$$

where V_f is the voltage across capacitor at $t = \infty$.

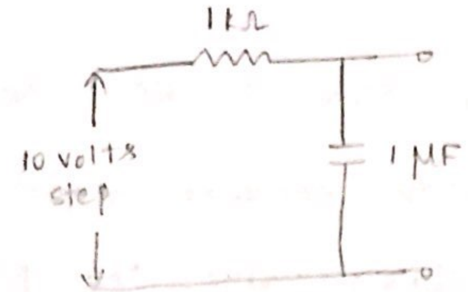
→ t_i is the time at which V_s is applied. (initial time).

→ V_i is the voltage across capacitor at $t = t_i$

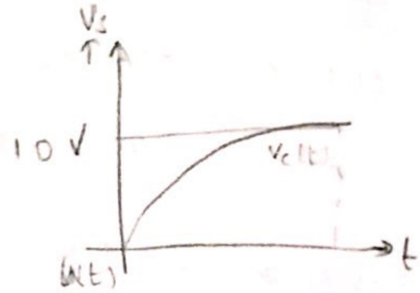
→ RC is the time constant (τ) of the given circuit.

PROBLEM :-

1.



Find $V_c(t)$ at $t = 1\tau, 2\tau, 5\tau$



Sol:- $V_f = 10\ V.$

$t_i = 0$

$V_i = 0$

$\tau = RC = 1k\Omega \times 1\ MF$

$= 1\ ms.$

At $t = 1\tau = 1\ ms$

$V_c(t) = V_f - (V_f - V_i)\, e^{-(t-t_i)/RC}$

$= 10 - (10-0)\, e^{-(10^{-3}-0)/10^{-3}}$

$= 10 - 10 \cdot e^{-1}$

$= 6.32.\ V$

At $t = 2\tau = 2 \times 1\ ms = 2ms$

$V_c(t) = V_f - (V_f - V_i)\, e^{-(t-t_i)/RC}$

$= 10 - (10-0)\, e^{-(2 \times 10^{-3})/10^{-3}}$

$= 10 - 10\, e^{-2}$

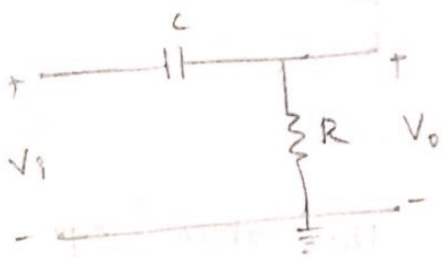
$= 8.64$

At $t = 5\tau$

$$V_c(t) = 10 - 10e^{-5}$$
$$= 9.93$$

Hence capacitor requires minimum of 5 time constants of the time to reach final value

High Pass RC circuit :-



If the input signal V_i is not sinusoidal it can be visualized ~~of~~ as comprising of several sine waves of frequencies which are multiples of the frequency of the signal and there will be made D.C component also. Since the reactance of the capacitor decreases as the frequency increases

$$X_c = \frac{1}{2\pi f C}$$

The capacitor offers very little impedance to harmonics of higher orders. Hence the high frequency components of the input voltage through the network with very little attenuation passes through it. For this reason, the circuit is termed as High Pass RC circuit. Applying KVL to the circuit.

$$V_C + V_R = V_i$$

But $V_C = \frac{1}{C}\int i\,dt$ and $V_R = V_o$ the o/p voltage

$$\therefore \frac{1}{C}\int i\,dt + V_o = V_i$$

current $i = \frac{V_R}{R} = \frac{V_o}{R}$

$$\therefore \frac{1}{C}\int \frac{V_o}{R}\,dt + V_o = V_i$$

Differentiating w.r.t 't' the above eqn

$$\frac{1}{C}\left(\frac{V_o}{R}\right) + \frac{dV_o}{dt} = \frac{dV_i}{dt}$$

(or) $\left(\frac{1}{CR}\right) V_o + \frac{dV_o}{dt} = \frac{dV_i}{dt}$

Ignoring Transient Response

$\left(\frac{1}{RC}\right) V_o = \frac{dV_i}{dt}$ (or)

$$V_o = RC\frac{dV_i}{dt}$$

i.e., $V_o \propto \frac{d}{dt} V_i$

This proves that HPF acts as Differentiator

Condition for perfect Differentiator :

When a sine input is applied

$$\phi = \tan^{-1}\left(\frac{f_L}{f}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

if $\omega RC = 0.1$

$$\phi = \tan^{-1}\left(\frac{1}{0.1}\right)$$

$$= \tan^{-1}(10)$$

$$= 84.29$$

if $\omega RC = 0.01$

$$\phi = \tan^{-1}\left(\frac{1}{0.01}\right)$$

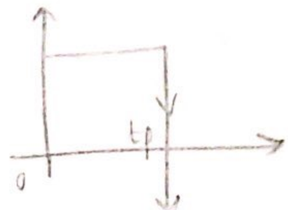
$$= 89.42 \simeq 90^{\circ}$$

$\therefore \omega RC = 0.01$

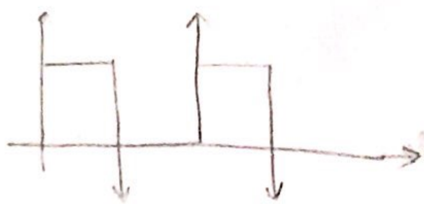
$$RC \le \frac{0.01}{\omega}$$

$$\le \frac{0.01\,T}{2\pi}$$

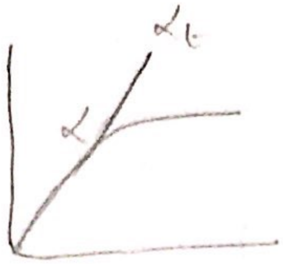
step i/p → Impulse o/p



pulse i/p → Two impulses in Sequence



Square wave i/p → Series of +ve & -ve impulses



ramp i/p → step o/p

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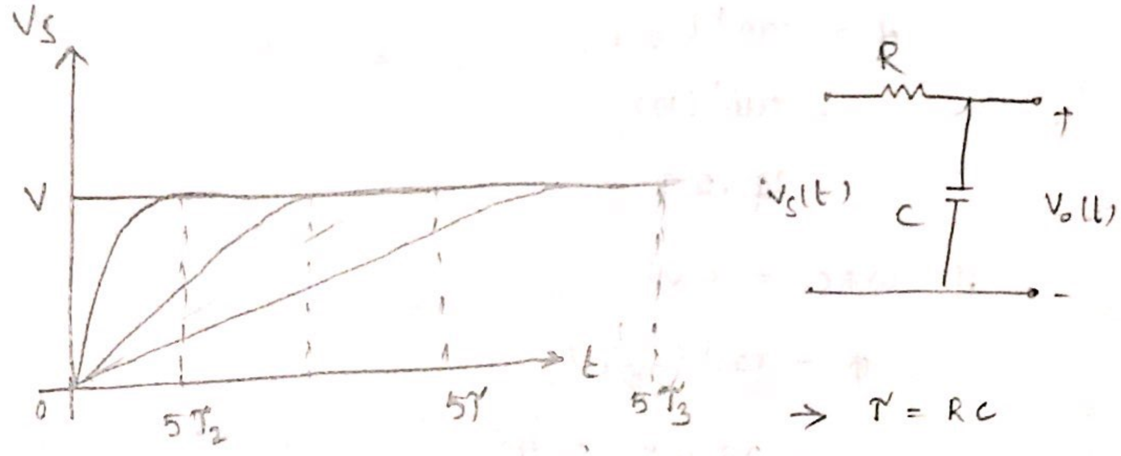
RESPONSE OF LPRC circuit for step input :-



$(\tau = RC)$

→ $\tau = RC$

→ $\tau_2 = R_2 C$

→ if $\tau_2 < \tau$

→ $\tau_3 = R C_2$

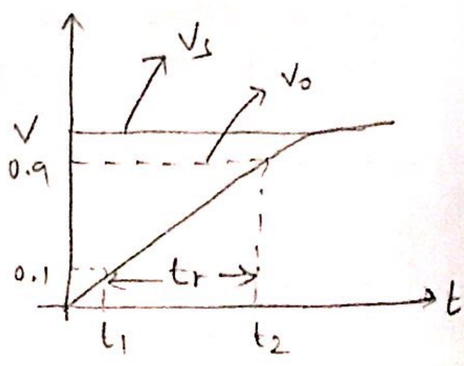
$\tau_3 > \tau$

Voltage across capacitor at any instant of time

$$V_c(t) = V_f - (V_f - V_i)\, e^{-(t-t_i)/RC}.$$

Calculation of Rise time :-

Rise time is defined as the time required for the output to reach from 10% to 90% of its maximum value



$$\boxed{t_r = t_2 - t_1}$$

Let t_1 = time required to reach 10% of its max from initial value. $(0.1V)$

t_2 = time required to reach 90% of its max from initial value. $(0.9V)$

t_r = rise time $= t_2 - t_1$

calculation of t_1 :-

$V_c(t) = 0.1V$, $V_f = V$, $V_i = 0V$, $t_i = 0$, $t = t_1$

$$\therefore \quad V_c(t) = V_f - (V_f - V_i)\, e^{-(t-t_i)/RC}$$

$$0.1V = V - (V-0)\, e^{-(t_1-0)/RC}$$

$$0.1\cancel{V} = \cancel{V}\left[1 - e^{-t_1/RC}\right]$$

$$e^{-t_1/RC} = 1 - 0.1$$

$$e^{-t_1/RC} = 0.9$$

$$-t_1/RC = \ln(0.9)$$

$$-t_1/RC = -0.1$$

$$t_1 = 0.1\, RC$$

Calculation of 't_2'.

$V_c(t) = 0.9V$, $V_f = V$, $V_i = 0$, $t_i = 0$, $t = t_2$

$$V_c(t) = V_f - (V_f - V_i)\, e^{-(t-t_i)/RC}$$

$$0.9V = V - (V-0)\, e^{-(t_2-0)/RC}$$

$$0.9\cancel{V} = \cancel{V}\left[1 - e^{-t_2/RC}\right]$$

$$e^{-t_2/RC} = 1 - 0.9$$

$$e^{-t_2/RC} = 0.1$$

$$-\frac{t_2}{RC} = \ln(0.1)$$

$$\cancel{-}\frac{t_2}{RC} = \cancel{-}2.3$$

$$t_2 = 2.3\ RC$$

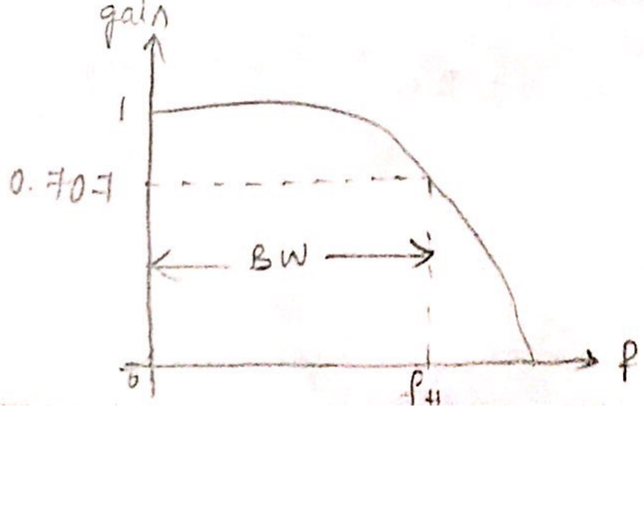
$\therefore$ Rise time $t_r = t_2 - t_1$

$$= 2.3\,RC - 0.1\,RC$$

$$t_r = 2.2\ RC$$

$\Rightarrow$ 't_r' depends on RC not on Applied voltage.

Relation between Rise time and Bandwidth:



frequency response of LPRC circuit

We know that

$$t_r = 2.2\ RC \longrightarrow ①$$

Bandwidth of LPRC $= f_H - 0 = f_H$

where $f_H = \dfrac{1}{2\pi RC} = BW.$

$$\therefore\ RC = \frac{1}{2\pi(BW)} \longrightarrow ②$$

Substitute eqn ② in eqn ①

$$t_r = 2.2 \times \frac{1}{2\pi(B.W)}$$

$$= \frac{0.35}{B.W}$$

$$\therefore\ t_r = \frac{0.35}{B.W} = \frac{0.35}{f_H}$$

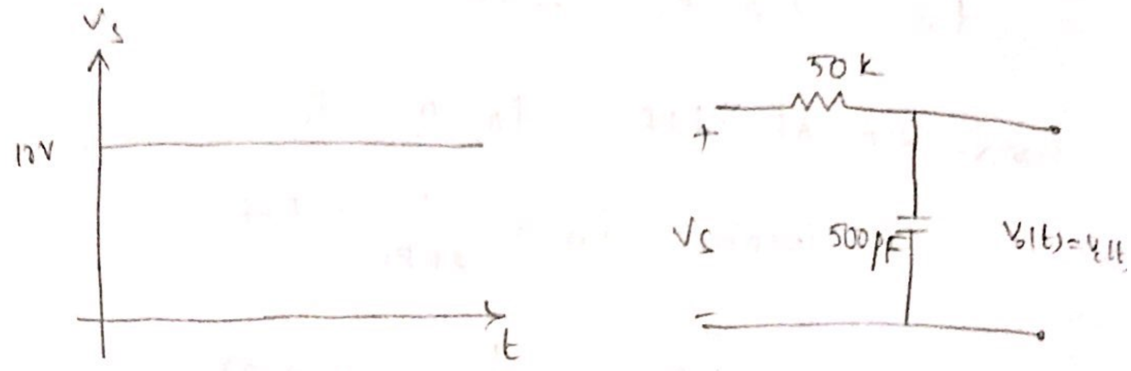
$$\boxed{\therefore\ t_r . B.W = 0.35.}$$

i.e., $\boxed{t_r \propto \dfrac{1}{B.W}}$

$$\Rightarrow\ t_r \propto \frac{1}{f_H}$$

Rise time is inversely proportional to cutt off frequency.

1. Calculate



(i) Rise time

(ii) Time required for the output to reach 63.2 % of the final value.

(iii) Voltage across the capacitor at $t = 100\ \mu s$.

(iv) Time required to reach 7.8 Volts by the output.

(v) Time required to reach 4.8 Volts by voltage across the resistor.

Sol:- (i) $T_r = 2.2\ RC$

$= 2.2 \times 50 \times 10^3 \times 500 \times 10^{-12}$

$T_r = 55\ \mu s.$

(ii) We know that

Time constant τ is $= 63.2$ % of final value

$= RC$

$= 50k \times 500pF$

$= 25\ \mu s.$

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(iii) $t = 100\ \mu s$

$V_f = 10V$, $V_i = 0$, $t_i = 0$, V_c at $100\mu s$ $RC = 25\ \mu s$

$$V_c(t) = 10 - [10 - 0]\, e^{-(100\mu s - 0)/25\mu s}$$

$$= 10 - [10 \times e^{-\frac{100}{25}}$$

$$= 10 - [10 \cdot e^{-4}]$$

$$= 9.81\ V.$$

(IV) $V_c(t) = 7.8\ V$, $V_f = 10V$, $V_i = 0V$, $t_i = 0$, $t = ?$

$$V_c(t) = V_f - (V_f - V_i)\, e^{-(t_f - t_i)/RC}$$

$$7.8\ V = 10 - (10 - 0)\, e^{-(t-0)/25\mu s}$$

$$7.8 = 10 - 10\, e^{-\frac{t}{25\mu s}}$$

$$10 \cdot e^{-\frac{t}{25\mu s}} = 2.2$$

$$e^{-\frac{t}{25 \times 10^{-6}}} = 0.22$$

$$-\frac{t}{25 \times 10^{-6}} = \ln(0.22)$$

$$= -1.51$$

$$t = 1.51 \times 25 \times 10^{-6}$$

$$= 3.78 \times 10^{-5}$$

$$t = 37.8\ \mu s$$

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(v) $\tau = RC = 25\,\mu s.$

$$V_s = V_R + V_C$$

$$V_C = V_S - V_R$$

$$= 10 - 4.8$$

$$\therefore \; V_c = 5.2.$$

$\therefore \; V_c(t) = 5.2, \quad V_i = 10, \; V_0 = 0, \; t_i = 0, \; t = ?$

$$5.2 = 10 - (10 - 0)\, e^{-t/25\,\mu s}$$

$$5.2 = 10 - 10\, e^{-\frac{t}{25 \times 10^{-6}}}$$

$$10\, e^{-\frac{t}{25 \times 10^{-6}}} = 4.8$$

$$e^{-\frac{t}{25 \times 10^{-6}}} = 0.48$$

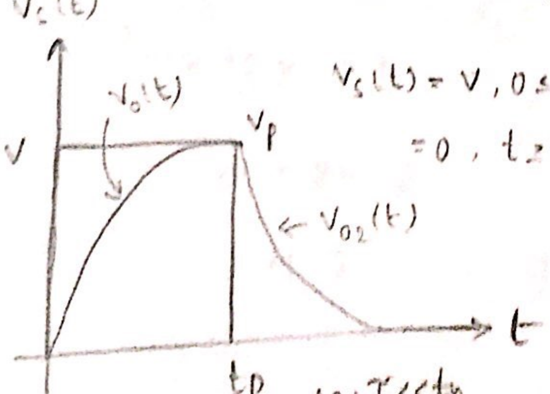
$$\cancel{-}\frac{t}{25 \times 10^{-6}} = \cancel{-}0.733$$

$$t = 1.834 \times 10^{-5}$$

$$\boxed{t = 18.34\,\mu s.}$$

25/07/2017

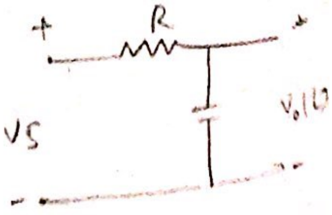
RESPONSE OF LPRC CIRCUIT FOR PULSE INPUT:-



(a) $\tau \ll t_p$

$V_s(t) = V, \; 0 \le t \le t_p$

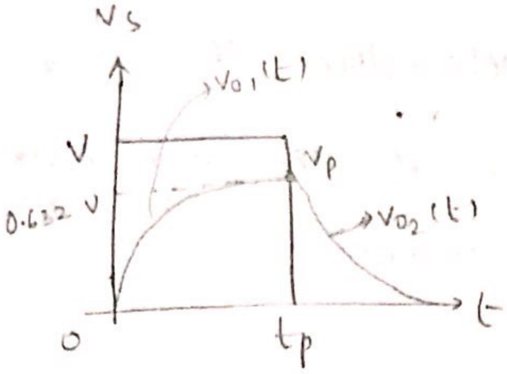
$= 0, \; t > t_p$



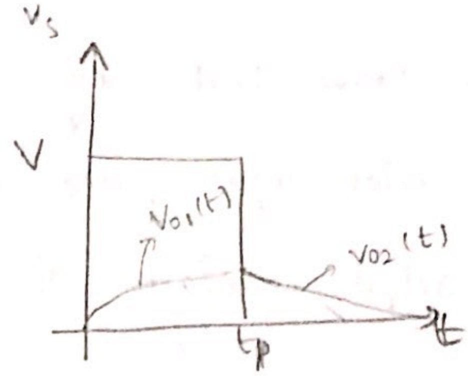
(i) During $0 < t < t_p$

a) $\tau \ll t_p$

(ii) During $t > t_p$



(b) $\tau = t_p$



(c) $\tau >> t_p$

$V_{01}(t)$ = output waveform during $0 < t < t_p$

$V_{02}(t)$ = output waveform during $t \geq t_p$

V_p = Capacitor voltage at $t = t_p$

Voltage across capacitor at any instant of time is

$$V_c(t) = V_f - (V_f - V_i)\, e^{-(t-t_i)/RC}.$$

$0 \leq t \leq t_p$:-

$V_f = V$, $t_i = 0$, $V_i = 0$

$$V_{01}(t) = V - (V-0)\, e^{-t/RC}$$

$$= V(1 - e^{-t/RC}) \longrightarrow (1)$$

At $t = t_p \Rightarrow V_{01}(t = t_p) = V_p = V(1 - e^{-t/RC}) \longrightarrow (2)$

$t \geq t_p$:-

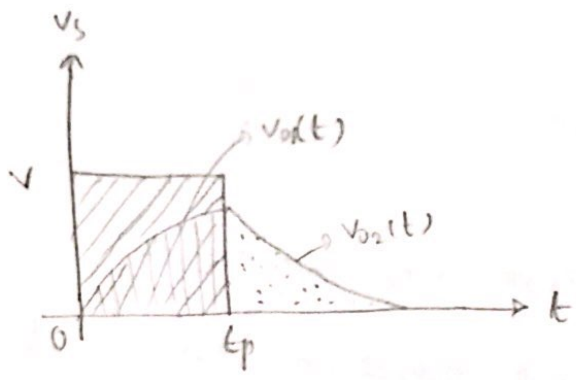
$V_f = 0$, $t_i = t_p$, $V_i = V_p$

$$V_{02}(t) = 0 - (0 - V_p)\, e^{-(t-t_p)/RC}$$

$$= V_p \cdot e^{-(t-t_p)/RC} \longrightarrow (3)$$

Q1:- Prove that by direct integration, the area under input pulse is same as the area under output waveform of LPRC circuit.

Sol:-



Total Area of output waveform = Area under $V_{o1}(t)$ + Area under $V_{o2}(t)$

where $V_{o1}(t) = V(1 - e^{-t/RC})$

$$V_{o2}(t) = V_p \cdot e^{-(t-t_p)/RC}$$

Area under $V_{o1}(t)$:-

$$A_1 = \int_0^{t_p} V_{o1}(t)\, dt$$

$$= \int_0^{t_p} V(1 - e^{-t/RC})\, dt$$

$$= V\left[t - \frac{e^{-t/RC}}{-1/RC}\right]_0^{t_p}$$

$$= V\left[t + RC \cdot e^{-t/RC}\right]_0^{t_p}$$

$$= V\left[t_p + RC \cdot e^{-t_p/RC} - 0 - RC\right]$$

$$= V t_p - RCV(1 - e^{-t_p/RC})$$

$\therefore\ A_1 = V \cdot t_p - RC \cdot V_p. \longrightarrow (1)$

Area under $V_{02}(t)$:-

$$A_2 = \int_{t_p}^{\infty} V_{02}(t)\, dt$$

$$= \int_{t_p}^{\infty} V_p \cdot e^{-(t-t_p)/RC}\, dt$$

$$= \left[V_p \cdot \frac{e^{-(t-t_p)/RC}}{-1/RC} \right]_{t_p}^{\infty}$$

$$= \left[-V_p \cdot RC \cdot e^{-(t-t_p)/RC} \right]_{t_p}^{\infty}$$

$$= -V_p \cdot RC \left[e^{-\infty} - e^{0} \right]$$

$$= -V_p \cdot RC\,(0-1)$$

$$= V_p \cdot RC. \longrightarrow (2)$$

$\therefore$ Total Area of output waveform $= A_1 + A_2$

$$= V \cdot t_p - \cancel{V_p RC} + \cancel{V_p RC}$$

$$= V \cdot t_p.$$

= Area under input pulse

1. Draw the output Response of LPRC circuit for a pulse input with 1 μs duration when

(a) $f_H = 10$ MHz.

(b) $f_H = 1$ MHz

(c) $f_H = 0.1$ MHz.

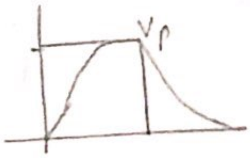
Sol:- (a) $f_H = 10$ MHz

$$f_H = \frac{1}{2\pi RC}$$

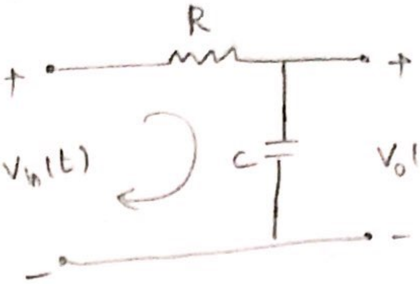
$$RC = \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 10 \times 10^6} = 1.59 \times 10^{-8} = 15.9 \times 10^{-9}.$$

Given that $t_p = 1$ μs.

$\therefore$ $RC << t_p$.



LPRC CIRCUIT AS INTEGRATER :-



To prove

$$V_o(t) \propto \int V_{in}(t)\, dt$$

$$V_o(t) = \frac{1}{C} \int i(t)\, dt$$

where $i(t) = \frac{V_R(t)}{R}$

if $\boxed{RC >> T}$ ('T' is the time duration

ie, is less than 5τ where input signal is present)

Here $V_C(t)$ is very small.

The input signal duration is very low

if $V_C(t) \cong$ very small.

$$V_R(t) \cong V_{in}(t)$$

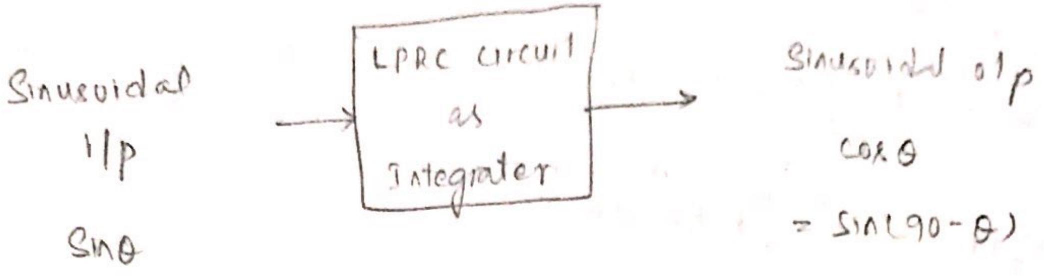
$$i(t) = \frac{V_{in}(t)}{R}$$

For $RC >> T$, $V_o(t) = \frac{1}{C} \int \frac{V_{in}(t)}{R}\, dt$

$$V_o(t) = \frac{1}{RC} \int_0^t V_{in}(t)\, dt$$

$$\boxed{\therefore V_o(t) \propto \int_0^t V_{in}(t)\, dt}$$

Condition for Perfect Integrater :-



Perfect Integrater gives 90° phase shift.

$$\phi = -\tan^{-1}\left(\frac{f}{f_H}\right) = -\tan^{-1}(2\pi f RC)$$

$$\phi = -\tan^{-1}(\omega RC) = -90^\circ$$

$$|\phi| = \tan^{-1}(\omega RC) = 90^\circ$$

If $\omega RC = 14$, $\phi = \tan^{-1}(14) = 84^\circ$

If $\omega RC = 20$, $\phi = \tan^{-1}(20) = 87^\circ$

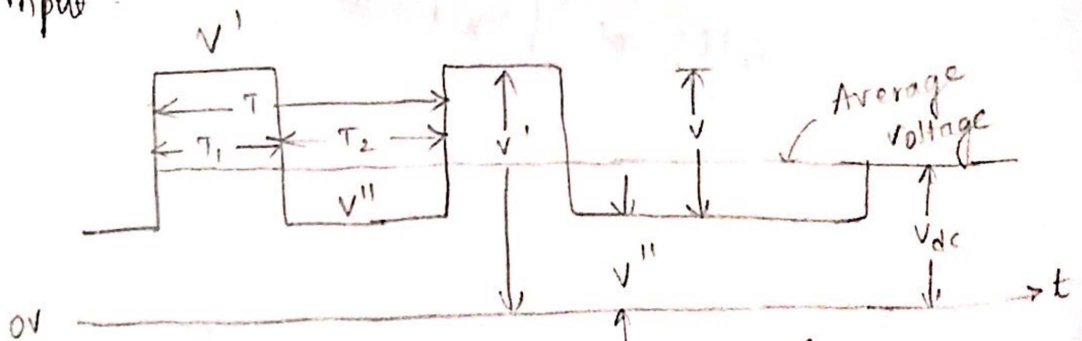
$\omega RC \geq 15$, $\phi \geq 85^\circ$

$\omega RC = 15$, $2\pi f RC = 15$

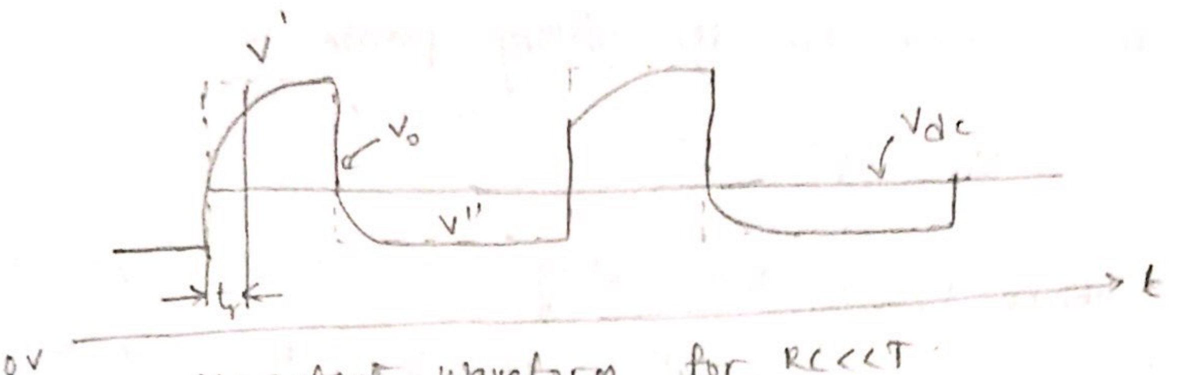
$$RC = \frac{15T}{2\pi}$$

ramp Assignment 26/07/2017

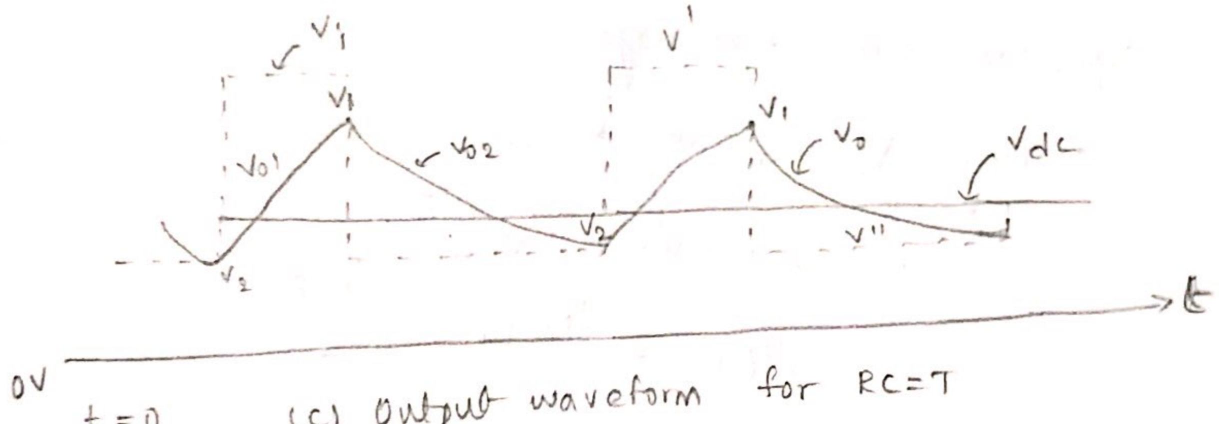
Response of LPRC circuit to a square wave input.



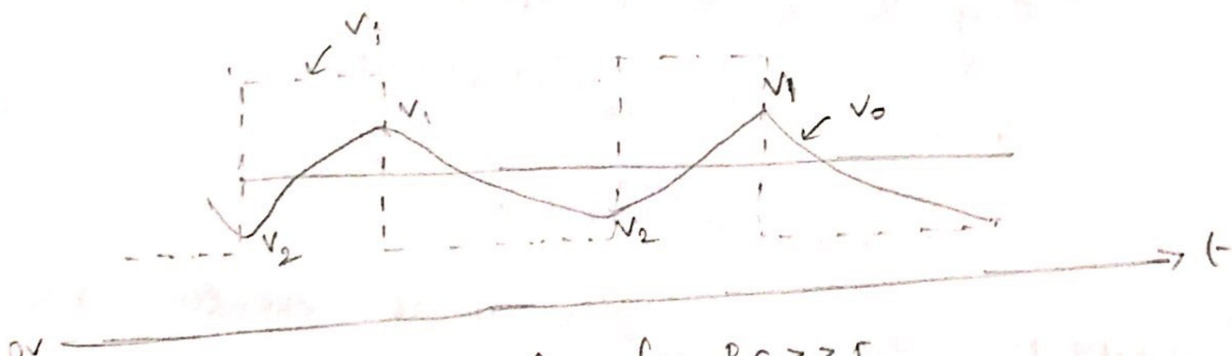
(a) Square wave input waveform



(b) output waveform for $RC \ll T$



(c) output waveform for $RC = T$



(d) output waveform for $RC \gg T$

In fig.(c) the equation for the rising portion is $v_{01} = v' - (v' - V_2)\, e^{-t/RC}$.

→ where V_2 is the voltage across the capacitor at $t=0$, and v' is the level to which the capacitor can charge.

The equation for the falling portion is

$$V_{o2} = V'' - (V'' - V_1)\, e^{-(t - T_1)/RC}$$

→ where V_1 is the voltage across the capacitor at $t = T_1$ and V'' is the level to which the capacitor can discharge.

Setting $V_{o1} = V_1$ at $t = T_1$

$$V_1 = V' - (V' - V_2)\, e^{-T_1/RC}$$

$$= V'(1 - e^{-T_1/RC}) + V_2 \cdot e^{-T_1/RC}$$

Setting $V_{o2} = V_2$ at $t = T_1 + T_2$

$$V_2 = V'' - (V'' - V_1)\, e^{-(T_1 + T_2 - T_1)/RC}$$

$$= V''(1 - e^{-T_2/RC}) + V_1 \cdot e^{-T_2/RC}$$

Substituting this value of V_2 in the expression for V_1.

$$V_1 = V'(1 - e^{-T_1/RC}) + \left[V''(1 - e^{-T_2/RC}) + V_1\, e^{-T_2/RC}\right] e^{-T_1/RC}$$

ie, $$V_1 = \frac{V'(1 - e^{-T_1/RC}) + V''(1 - e^{-T_2/RC})\, e^{-T_1/RC}}{1 - e^{-(T_1+T_2)/RC}}$$

Similarly substituting of V_1 in the expression for V_2

$$V_2 = \frac{V''(1 - e^{-T_2/RC}) + V'(1 - e^{-T_1/RC})\, e^{-T_2/RC}}{1 - e^{-(T_1+T_2)/RC}}$$

For a symmetrical square wave with zero average value.

$$T_1 = T_2 = \frac{T}{2} \quad \text{and} \quad V' = -V'' = \frac{V}{2}$$

So, V_2 will be equal to $-V_1$.

$$V_1 = \frac{\frac{V}{2}\left(1-e^{-T/2RC}\right) - \frac{V}{2}\left(1-e^{-T/2RC}\right)e^{-T/2RC}}{1-e^{-T/2RC}}$$

$$= \frac{V}{2} \cdot \frac{1-e^{-T/2RC} - e^{-T/2RC} + e^{-T/RC}}{1-e^{-T/RC}}$$

$$= \frac{V}{2} \cdot \frac{\left(1-e^{-T/2RC}\right)^2}{\left(1-e^{-\frac{T}{2RC}}\right)\left(1+e^{-\frac{T}{2RC}}\right)}$$

$$= \frac{V}{2} \cdot \left[\frac{1-e^{-T/2RC}}{1+e^{-T/2RC}}\right]$$

$$= \frac{V}{2} \left[\frac{e^{+T/2RC}-1}{e^{T/2RC}+1}\right]$$

$$\left(\because \frac{e^{2x}-1}{e^{2x}+1} = \tanh x\right)$$ where $x = \frac{T}{4RC}$ and T is the period of square wave

Now

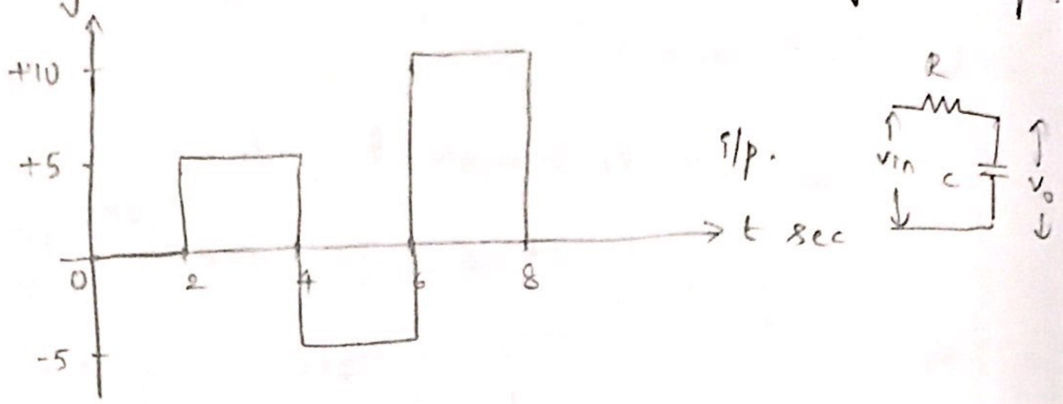
$$V_2 = -V_1 = \frac{-V}{2}\left[\tanh \frac{T}{4RC}\right]$$

OR

$$V_2 = \frac{V}{2}\left[\frac{1-e^{+T/2RC}}{1+e^{T/2RC}}\right]$$

27/07/2017

1. Draw the output response of LPRC circuit with $\tau = 1$ sec. Assume capacitor is initially uncharged.



Sol:- $0 < t < 2 \Rightarrow V_f = 0, \; V_i = 0, \; t_1 = 0$

$$V_c(t) = V_o = 0 \text{ V}.$$

$2 < t < 4 \Rightarrow V_f = 5, \; V_i = 0, \; t_i = 2, \; t = 4$

$$V_c(t) = V_o = 5 - (5-0)\, e^{-(4-2)/RC}$$

$$= 5 - 5\, e^{-2/\tau}$$

$$= 5 - 5\, e^{-2/1}$$

$$= 4.32 \text{ V}$$

$4 < t < 6 \Rightarrow V_f = -5, \; V_i = 0, \; t_i = 4, \; t = 6$

$$V_c(t=6) = V_o = -5 - (-5-0)\, e^{-(6-4)/\tau}$$

$$= -5 + 5\, e^{-\frac{2}{1}}$$

$$= -4.32 \text{ V}$$

$6 < t < 8 \Rightarrow V_f = 10, \; V_i = 0, \; t_i = 6, \; t = 8$

$$V_c(t) = V_o = 10 - (10-0)\, e^{-(8-6)/1}$$

$$= 10 - 10\, e^{-2} = 8.64 \text{ V}$$

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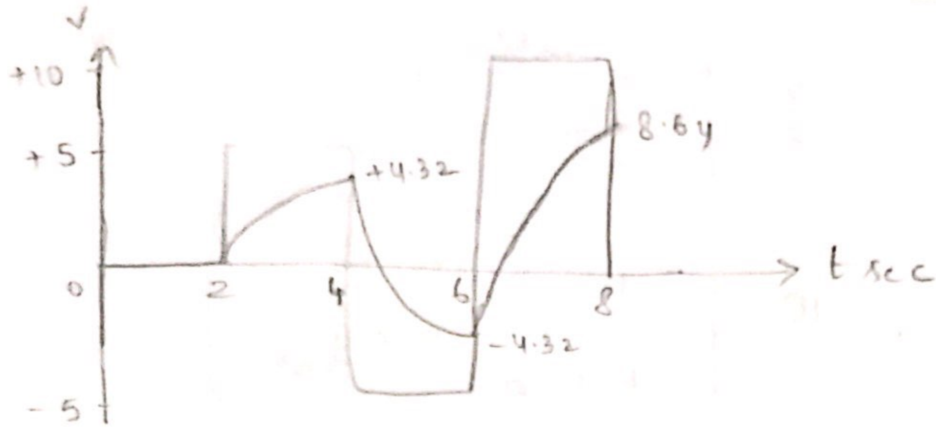
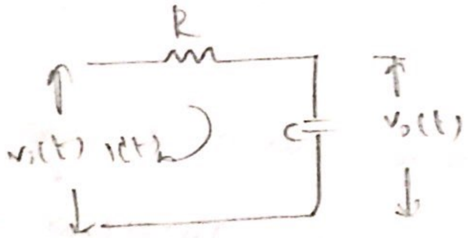


2. A Symmetrical square wave of Amplitude ± 5V and frequency 2 kHz is impressed on an RC LOW pass circuit if $R = 5k\Omega$, $C = 0.1\mu F$ calculate and plot steady state output w.r.t time.

Sol:-



Given $f = 2kHz$

$$\frac{1}{T} = 2 \times 10^3$$

$$T = 0.5 \text{ ms.}$$

$$T_1 = T_2 = \frac{T}{2} = 0.25 \text{ ms}$$

$$\tau = RC = 5 \times 10^3 \times 0.1 \times 10^{-6} = 0.5 \text{ ms}$$

Since the input is a symmetrical square wave

so,

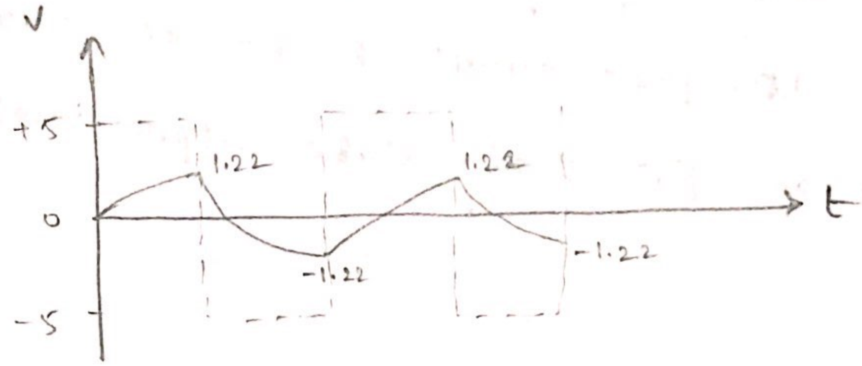
Scanned by CamScanner

$$V_1 = \frac{V}{2}\left[\frac{1-e^{-T/2RC}}{1+e^{-T/2RC}}\right]$$

$$= \frac{10}{2}\left[\frac{1-e^{-\frac{0.5\times10^{3}}{2\times0.5\times10^{3}}}}{1+e^{-\frac{0.5\times10^{3}}{2\times0.5\times10^{3}}}}\right]$$

$$V_1 = 1.22\ V.$$

$$\therefore\ V_2 = -V_1 = -1.22\ V.$$



3. A symmetrical square wave whose average value is zero. has a peak to peak amplitude of 20 V. and a period of 2 μs. This waveform is applied to a circuit whose upper 3 dB frequency is $\frac{1}{2\pi}$ MHz. calculate and sketch the steady state output waveform. and peak to peak output amplitude

Sol:-

$$T = 2\,\mu s \quad , \quad f_H = \frac{1}{2\pi}$$

$$\frac{1}{2\pi RC} = \frac{1}{2\pi}\times10^{6}$$

$$RC = \frac{1}{1\times10^{6}} = 1\times10^{-6}$$

$$= 1\ \mu s.$$

$$T_1 = T_2 = \frac{T}{2} = \frac{20\,\mu s}{2} = 10\,\mu s.$$

$$V_1 = \frac{V}{2}\left[\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}}\right]$$

$$= \frac{20}{2}\left[\frac{1 - e^{-20\mu s/2\times 1\mu s}}{1 + e^{-20\mu s/2\times 1\mu s}}\right]$$

$$= 10\left[\frac{1 - e^{-1}}{1 + e^{-1}}\right]$$

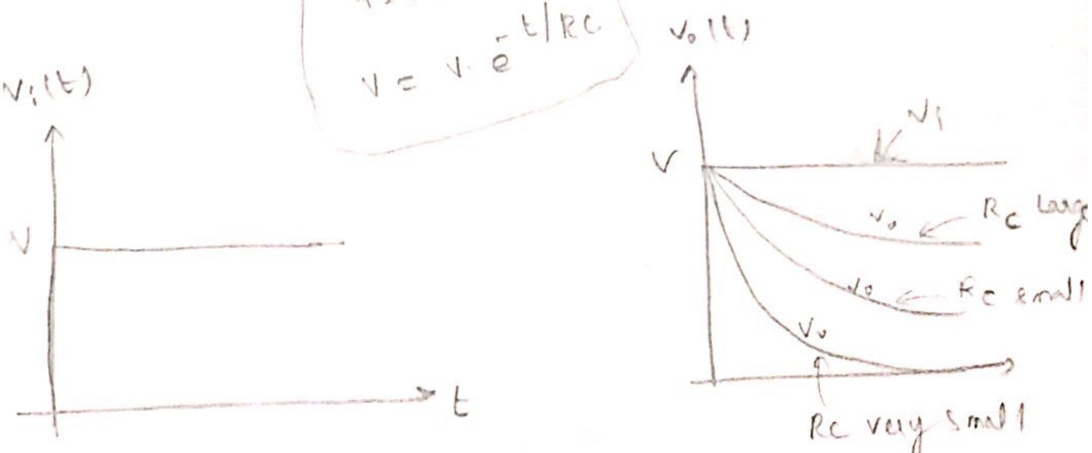
$$= 4.62\ V.$$

RESPONSE OF HPRC circuit FOR STEP INPUT:-

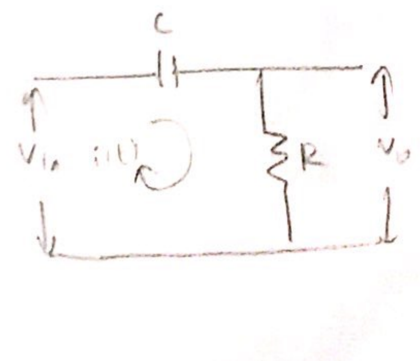
When a step signal of Amplitude 'V' volts

$V_{f} = 0,\ V_i = V$

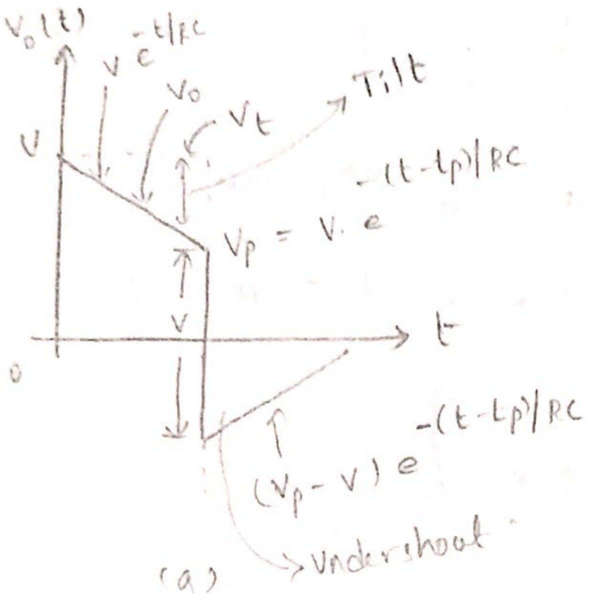
$V = V \cdot e^{-t/RC}$



Step Response for different time constants

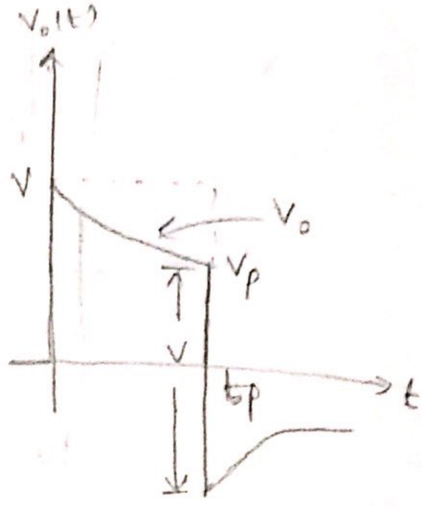


PULSE INPUT FOR HPRC CIRCUIT:- 28/07/2017



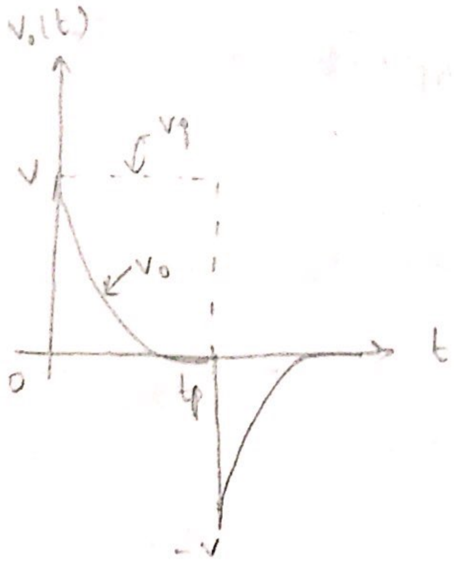
(a)

RC >> tp



(b)

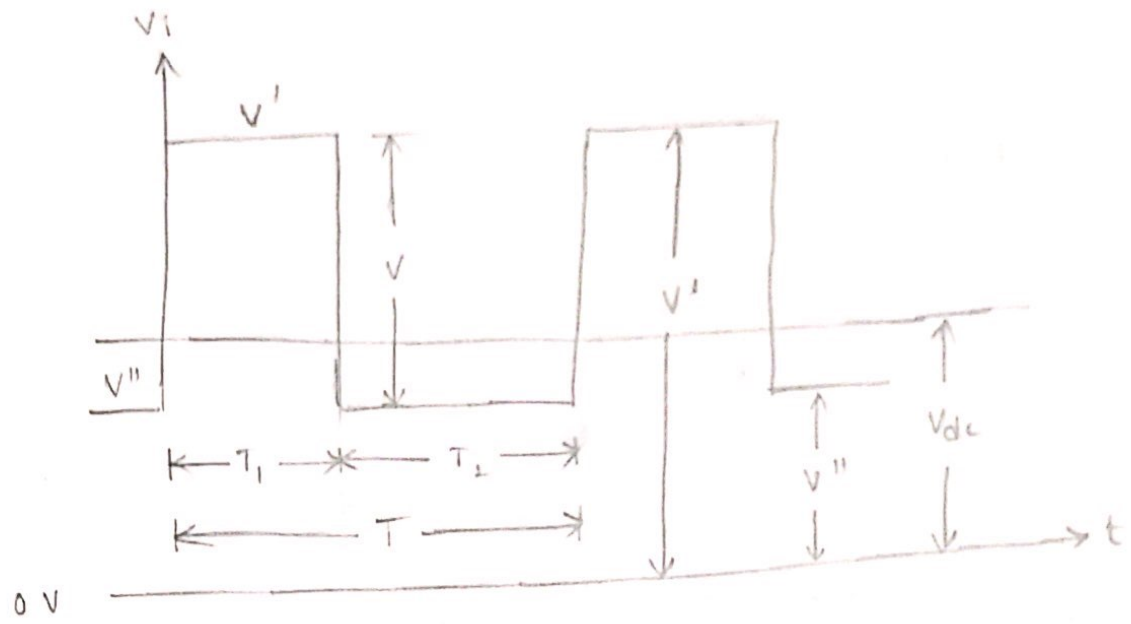
RC Comparable to tp



(c)

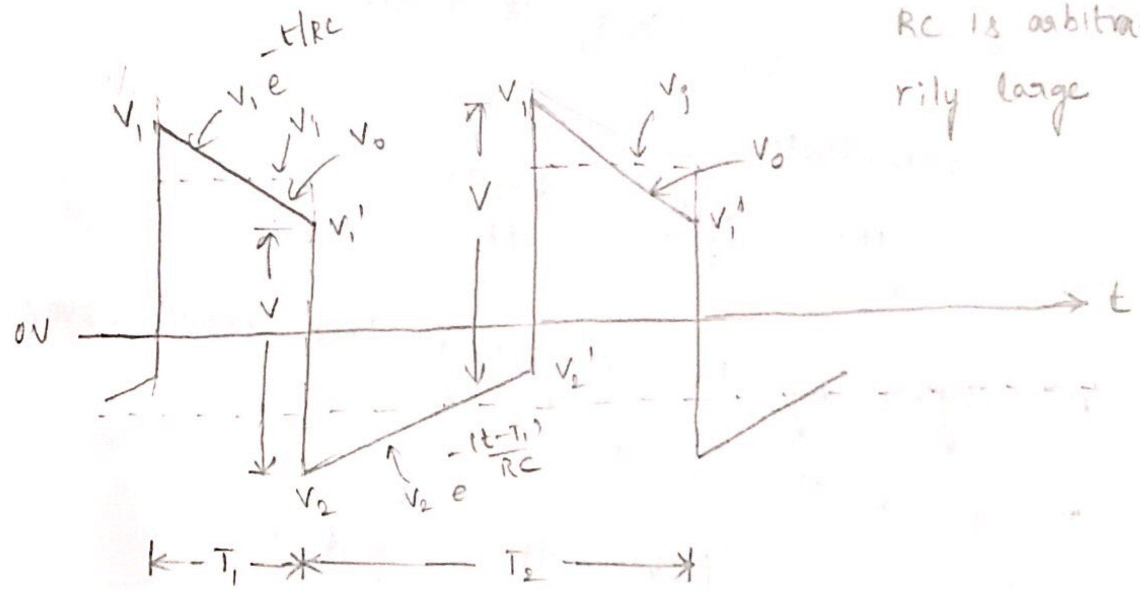
RC << tp

SQUARE INPUT FOR HPRC CIRCUIT :-

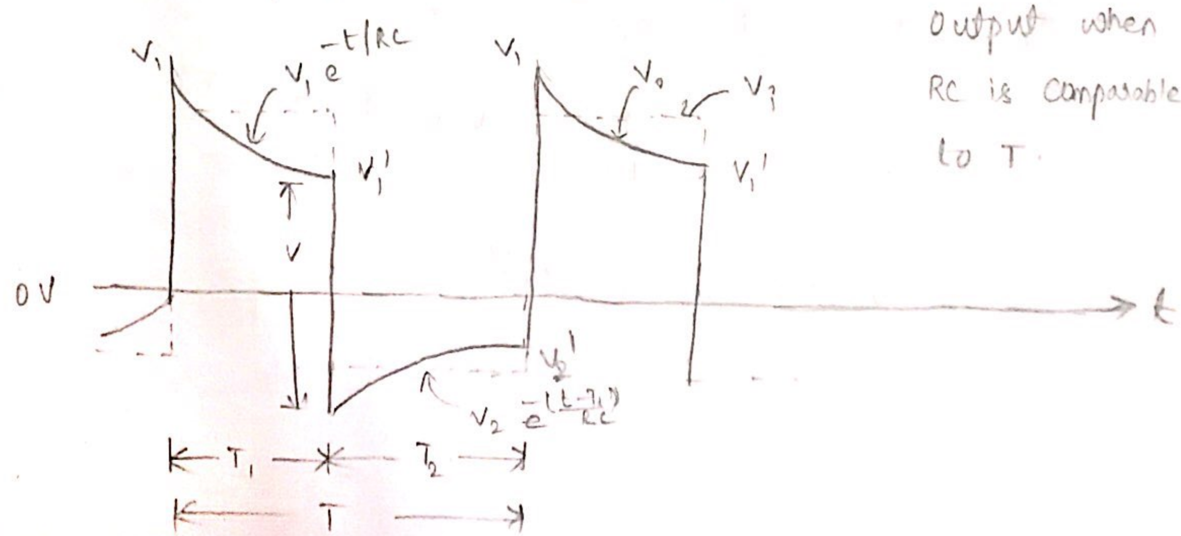


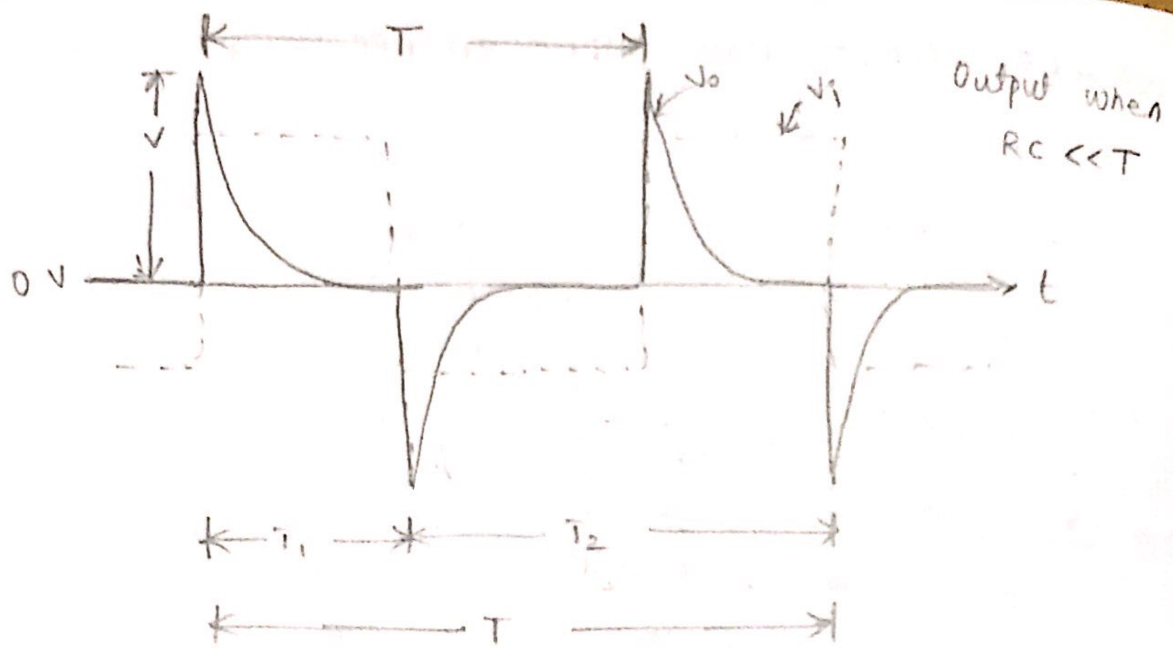
(a) A Square wave input

Output when RC is arbitrarily large



Output when RC is comparable to T





$$V_i(t) = \frac{1}{C}\int i(t)\,dt + V_o(t)$$

$$= \frac{1}{RC}\int V_o(t)\,dt + V_o(t)$$

Differentiate

$$\frac{d.\,V_i(t)}{dt} = \frac{V_o(t)}{RC} + \frac{d\,V_o(t)}{dt}$$

Multiplying by dt integrating this equation over one period of time T

$$\int_0^T d\,V_i(t) = \int_0^T \frac{V_o(t)\,dt}{RC} + \int_0^T d\,V_o(t)$$

$$V_i(T) - V_i(0) = \frac{1}{RC}\int_0^T V_o(t)\,dT + V_o(T) - V_o(0)$$

$$\Rightarrow V_i(T) = 0 \quad \& \quad V_i(t) = 0.$$

Hence $\int_0^T V_o(t)\,dt = 0.$

31/07/2017

Expression for the percentage tilt :-

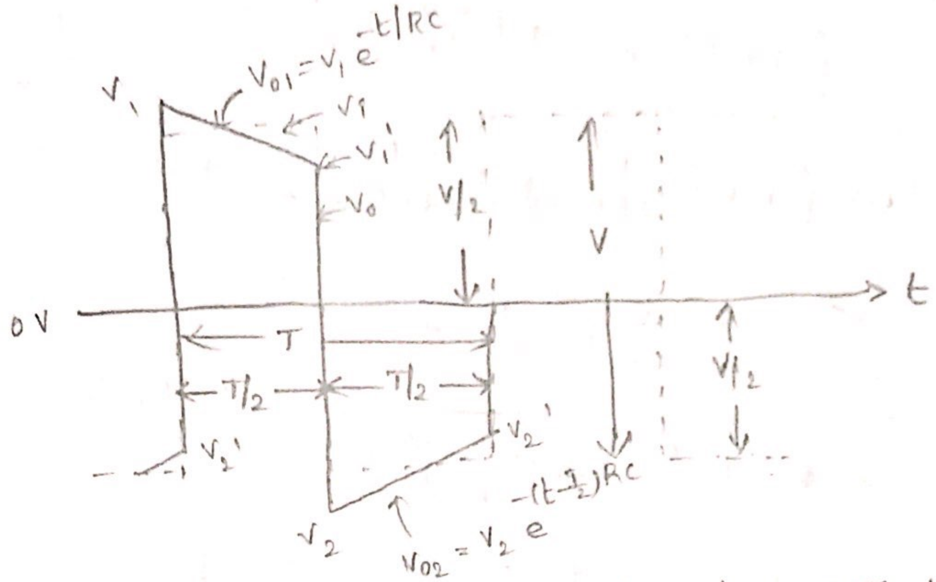


fig. Linear tilt of a symmetrical square wave when $R \gg T$

For $RC \gg T$.

For a symmetrical square wave with zero input

$V_1 = -V_2$ ie, $V_1 = |V_2|$, $V' = -V_2'$ ie, $V_1' = |V_2|$

and $T_1 = T_2 = \frac{T}{2}$

Here $V_1' = V_1 e^{-T/2RC}$

$V_2' = V_2 e^{-T/2RC}$

$V_1 - V_2' = V$

ie, $V_1 - V_2 e^{-T/2RC} = V_1 + V_1 e^{-T/2RC} = V$

$$\% \text{ Tilt}, P = \frac{V_1 - V_1'}{V/2} \times 100\%$$

$$= \frac{V_1 - V_1 e^{-T/2RC}}{V_1(1 + e^{-T/2RC})} \times 200\%$$

When $\frac{T}{RC} << 1$

$$P = \frac{1 - \left[1 + \left(-\frac{T}{2RC}\right) + \left(-\frac{T}{2RC}\right)^2 \frac{1}{2!} + \cdots\right]}{1 + \left[1 + \left(-\frac{T}{2RC}\right) + \left(-\frac{T}{2RC}\right)^2 \frac{1}{2!} + \cdots\right]} \times 200\%$$

$$= \frac{\frac{T}{2RC} - \left(\frac{-T}{2RC}\right)^2 \frac{1}{2!} + \cdots}{2 - \left(\frac{T}{2RC}\right) + \left(\frac{-T}{2RC}\right)^2 \frac{1}{2!}} \times 200\%$$

$$= \frac{\frac{T}{2RC}\left[1 - \left(\frac{T}{2RC}\right)^2 \frac{1}{2!} + \cdots\right]}{2 - \frac{T}{2RC}\left[1 + \left(\frac{T}{2RC}\right) \frac{1}{2!} + \cdots\right]} \times 200\%$$

where $RC >> T$

$$= \frac{\frac{T}{2RC}(1)}{2 - 0} \times 200\%$$

$$= \frac{T}{2RC} \times 100\%$$

$\therefore$ Percentage tilt $P = \frac{T}{2RC} \times 100\%$

(OR)

$$P = \frac{\pi f_L}{f} \times 100 \qquad \left(\because f_L = \frac{1}{2\pi RC}\right)$$

PROBLEMS.

1) Calculate the lowest square wave frequency that can be passed by an amplifier with a lower cutt off frequency of 10 Hz. with the output tilt is not to exceed 2%.

Sol:- $f_1 = 10$ Hz , tilt = 2%.

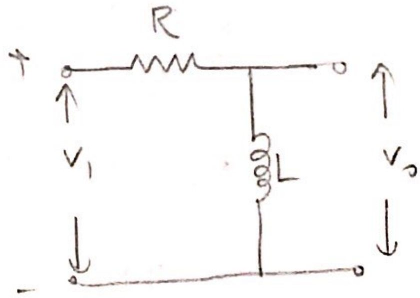
$$P = \frac{\pi f_L}{f} \times 100$$

$$2 \times 100 = \frac{\pi f_L}{\times 10 \times} \times 100$$

$$f_L = \frac{20}{\pi}$$

$$f_L = 6.36 \text{ Hz}.$$

RL CIRCUITS :-



RL High pass circuit

Applying KVL.

$$V_o - IR - E = 0.$$

$$V_o - IR - L \cdot \frac{dI}{dR} = 0$$

where I = current Inside circuit at some time constant.

$$IR + L.\frac{dI}{dt} = V_0$$

$$L.\frac{dI}{dt} = V_0 - IR$$

$$\frac{dI}{V_0 - IR} = \frac{dt}{L}$$

Integrating the Linear D.E

$$\int_{I=0}^{I} \frac{dI}{V_0 - IR} = \int_0^t \frac{dt}{dL}$$

$$\left(-\frac{\ln(V_0 - IR)}{R}\right)_0^I = \left(\frac{t}{L}\right)_0^t$$

$$\frac{-\ln(V_0 - IR) + \ln(V_0)}{R} = \frac{t}{L}$$

$$-\ln\left(\frac{V_0 - IR}{V_0}\right) = \frac{Rt}{L}$$

$$\ln\left(\frac{V_0 - IR}{V_0}\right) = -\frac{Rt}{L}$$

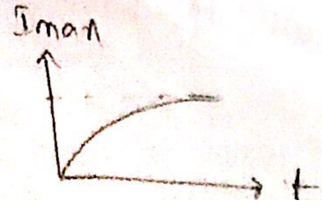
$$\frac{V_0 - IR}{V_0} = e^{-\frac{Rt}{L}}$$

$$V_0 - IR = V_0\left(e^{-\frac{Rt}{L}}\right)$$

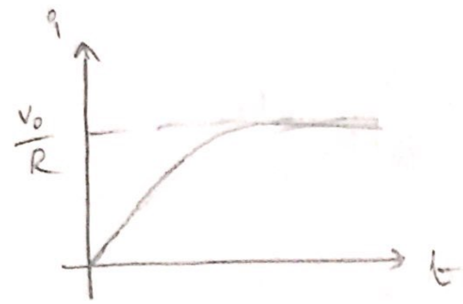
$$IR = V_0\left(1 - e^{-\frac{t}{T}}\right) \quad \text{where } T = \frac{L}{R}$$

$$I = \frac{V_0}{R}\left(1 - e^{-t/T}\right)$$

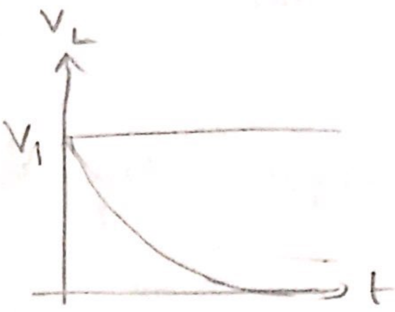
Growth of current Instab LR circuit is given by the fig



Across Inductor

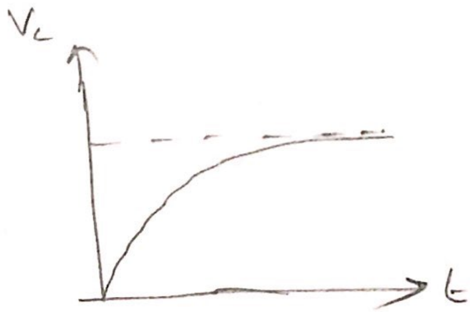


charging



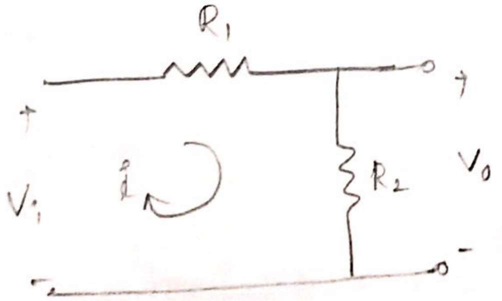
discharging

Across Resistor



01/08/2017

Attenuators :-



(a)

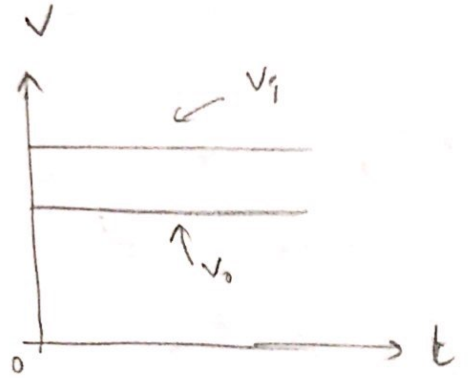
Resistance Potential Divider

To reduce the amplitude of a signal waveform the signal is transmitted through an ~~attenuator~~ attenuator circuit.

we have $i = \frac{V_i}{R_1 + R_2}$

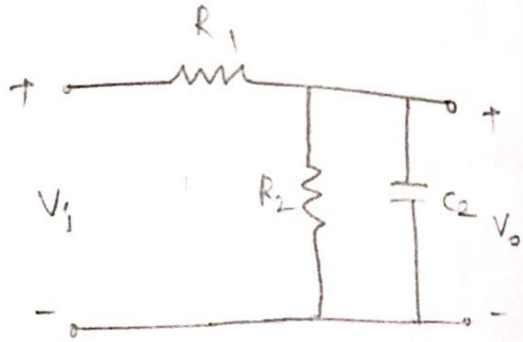
output voltage $V_o = i R_2 = \left(\frac{V_i}{R_1+R_2}\right) R_2$

or $V_o = \left(\frac{R_2}{R_1+R_2}\right) V_i$



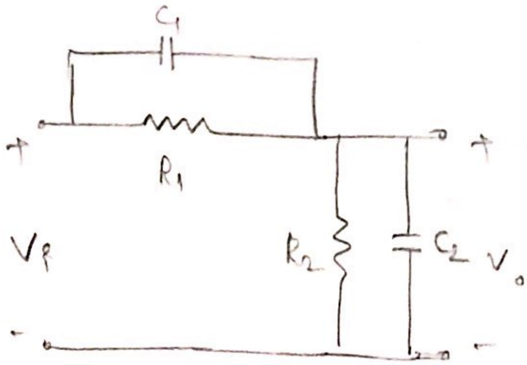
(b)

input & output waveform for Resistive attenuator



(c)

Modified attenuator circuit with C_2 shunting R_2



compensated Attenuator.

The undesirable effect of the distributed Capacitance C_2 can be neutralized and distortion of signal waveform prevented.

if $R_1 C_1 = R_2 C_2$.

we have $R_1C_1 = R_2C_2$

PROOF :-

1) At $t = (0+)$ immediately after the input signal is applied, the capacitors acts as short circuit and hence the resistor go out of circuit.

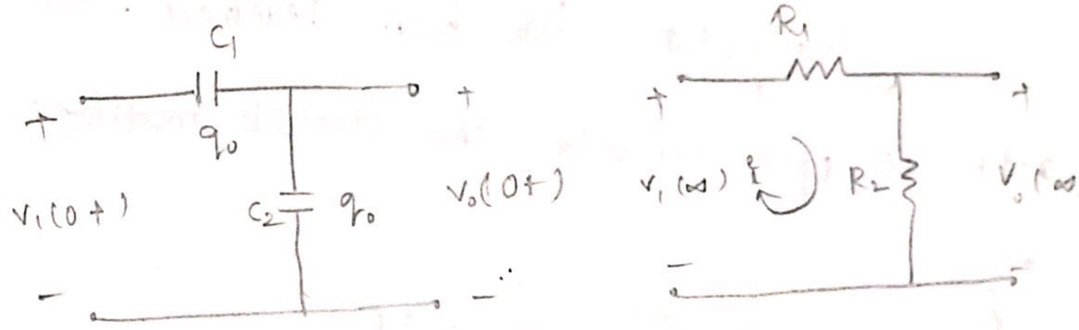


fig (a) fig (b)

The capacitor gets charged. Let q_0 denote the charge on cell each capacitor at $t = t(0+)$. In general $q = CV$

$$q_0 = C_2 V_0(0+) \quad \text{or} \quad V_0(0+) = \frac{q_0}{C_2}$$

$$\text{Also } V_1(0+) = \frac{q_0}{C_1} + \frac{q_1}{C_2} = q_0\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$V_i(0+) = q_0\left(\frac{C_1 + C_2}{C_1 C_2}\right)$$

$$\text{Initial attenuation} = \frac{V_0(0+)}{V_i(0+)}$$

$$= \frac{q_0/C_2}{q_0\left(\frac{C_1+C_2}{C_1C_2}\right)}$$

$$= \frac{C_1}{C_1+C_2} \quad \ldots\ldots (1)$$

(2) At $t=\infty$:

Steady state has been reached. The capacitor acts as open circuits the circuit modifies as fig(b).

We have output $V_o(\infty) = i R_2$

$$\text{current } i = \frac{V_i(\infty)}{R_1+R_2}$$

$$V_o(\infty) = \left(\frac{V_i(\infty)}{R_1+R_2}\right) R_2$$

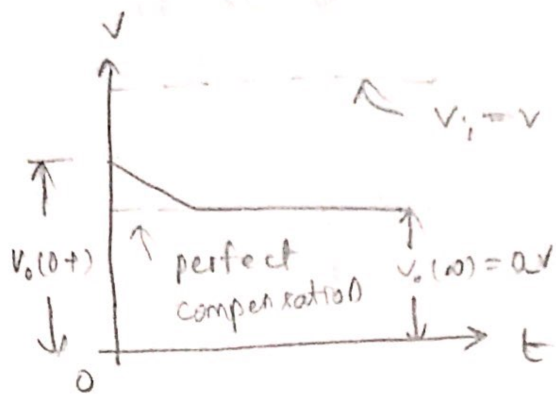
(or) $\frac{V_o(\infty)}{V_i(\infty)} = \frac{R_2}{R_1+R_2}$ But $\frac{V_o(\infty)}{V_i(\infty)}$ = Final attenuation

$$\therefore \text{Final attenuation} = \frac{R_2}{R_1+R_2} \longrightarrow (2)$$

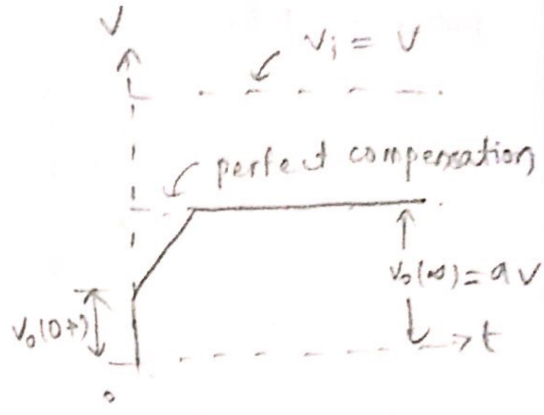
For compensation to be complete it is essential that initial attenuation = Final attenuation.

$$\therefore \frac{C_1}{C_1+C_2} = \frac{R_2}{R_1+R_2} \quad (\because \text{from eqn (1) \& eqn (2)})$$

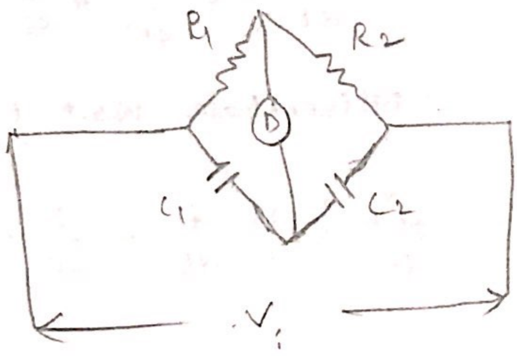
By simplifying we get $\boxed{C_1 R_1 = C_2 R_2}$



(a) Over compensation



(b) Under compensation



De sauty Bridge

Impedance of opposite ratio arms are equal for balance of bridge

$$R_1 \left(\frac{1}{j\omega C_1} \right) = R_2 \left(\frac{1}{j\omega C_2} \right)$$

$$\frac{R_1}{C_2} = \frac{R_2}{C_1}$$

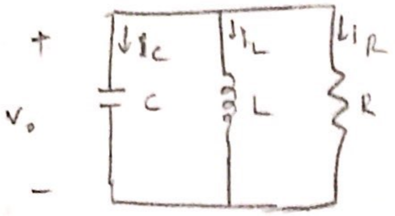
$$R_1 C_1 = R_2 C_2$$

A perfectly compensated attenuator network is equivalent to a balanced "De sauty Bridge".

02/08/2017

RLC circuit:-

Parallel RLC



$$i_C + i_L + i_R = 0$$

$$C\cdot\frac{dv}{dt} + \frac{1}{L}\int_0^t v\cdot d\tau + \frac{v}{R} = 0$$

Differentiate w.r.t t

$$C\cdot\frac{d^2v}{dt^2} + \frac{v}{L} + \frac{1}{R}\frac{dv}{dt} = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0$$

Apply Laplace Transform

$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$

where $s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

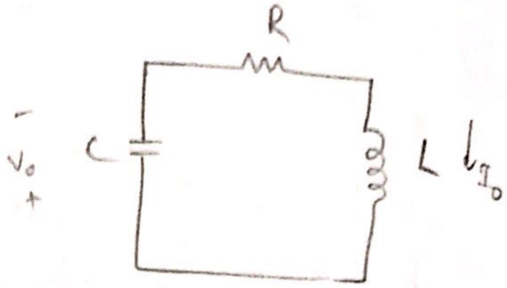
where ω_0 = Resonant frequency

$= \frac{1}{\sqrt{LC}}$

α = Neper frequency

$= \frac{1}{2RC}$

Series RLC



$$iR + L\cdot\frac{di}{dt} + \frac{1}{C}\int_0^t i\,d\tau = 0$$

$$R\frac{di}{dt} + L\cdot\frac{d^2i}{dt^2} + \frac{1}{C} = 0$$

Differentiate w.r.t t

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

Apply Laplace Transform

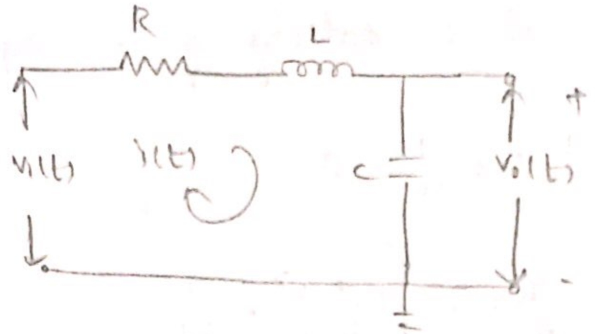
$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Here $s_{1,2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where $\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

Response of step input for RLC circuit



For a step input of amplitude

$$V_i(s) = \frac{V}{s}$$

By KVL

$$v_i(t) = R.i(t) + L.\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt$$

Taking Laplace Transform on both sides.

$$V_i(s) = I(s)\left[R + L \cdot s + \frac{1}{Cs}\right]$$

$$V_o(s) = \frac{I(s)}{Cs}$$

Transfer function of circuit is

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{I(s)}{Cs}}{I(s)\left(R + Ls + \frac{1}{Cs}\right)}$$

$$= \frac{1}{sCR + LCs^2 + 1}$$

$$= \frac{1}{LC\left(s^2 + \frac{R}{Ls} + \frac{1}{LC}\right)}$$

Scanned by CamScanner

The roots of characteristic equation s_1 and s_2 are the values of s satisfying the equation.

$$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$$

$$s_{1,2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$\therefore$ for step input, $V_i(s) = \frac{V}{s}$

$$V_o(s) = \left(\frac{V}{LC}\right)\left(\frac{1}{s\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}\right)$$

and $$I(s) = \frac{V}{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

Current Response :-

Case(a) :- Overdamped circuit

$$i(t) = \frac{V}{2AL}\left[e^{-s_1 t} - e^{-s_2 t}\right] \quad \text{here } s_1 > s_2$$

where $A = \sqrt{\left(\frac{R}{L}\right)^2 - \frac{1}{LC}}$

Case(b) :- critically damped circuit

$$R = \sqrt{\frac{L}{C}}$$

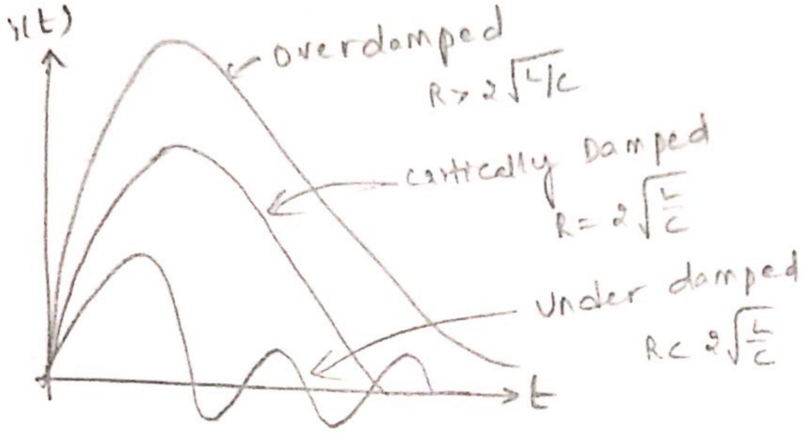
$$i(t) = \frac{V}{L} t . e^{\frac{-Rt}{2L}}$$

case(c) :- Under damped circuit

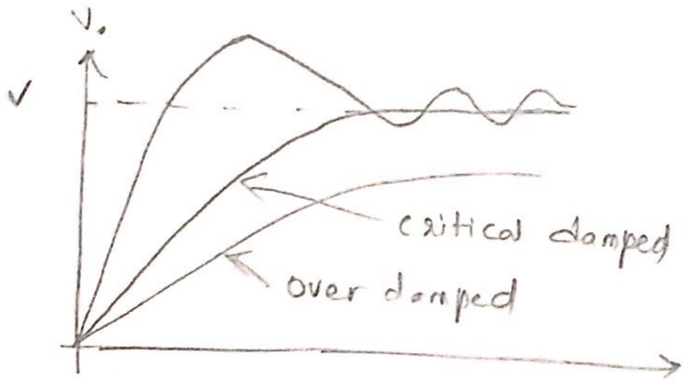
$$R < 2\sqrt{\frac{L}{C}}$$

$$i(t) = \frac{V}{BL} e^{-\frac{Rt}{2L}} \sin Bt$$

where $B = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$



(a) current response



(b) Voltage response of series RLC circuit to a step voltage