Mathematical operations on Continuous Signals

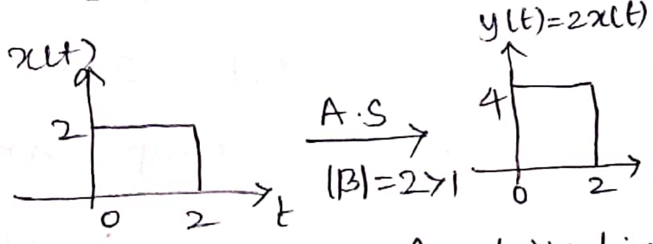
(1) Scaling operation

(i) Amplitude Scaling :

$$x(t) \xrightarrow[\beta]{A.S} y(t) = \beta\, x(t) \qquad \beta \neq 0.$$

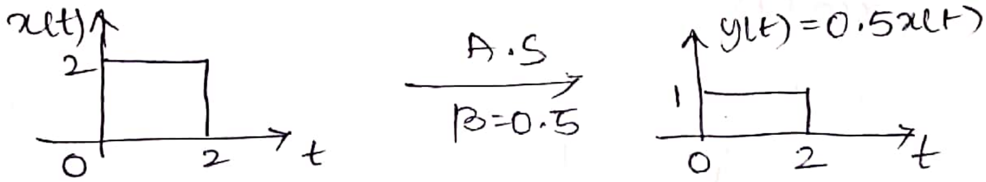
Case (i) : $|\beta| > 1$ $\quad \beta \in (-\infty, -1) \cup (1, \infty)$

$$x(t) = \begin{cases} 0 , & t < 0 \\ 2 , & 0 \leq t \leq 2 \\ 0 , & t > 2 \end{cases}$$



* Amplification

Case (ii): $|\beta| < 1$ $\quad \beta \in (-1, 0) \cup (0, 1)$



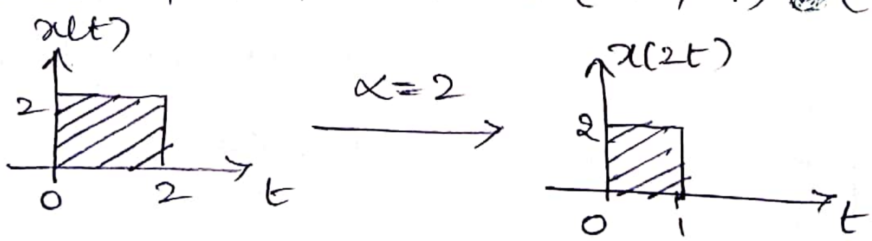
* Reduction (Attenuation)

(ii) Time Scaling

The compression or expansion of a signal in time.

$$x(t) \xrightarrow{T.S} y(t) = x(\alpha t) , \quad \alpha \neq 0.$$

Case(i): $|\alpha| > 1$ $\quad \alpha \in (-\infty, -1) \cup (1, \infty)$



Compression

Case (ii) $\quad |\alpha| < 1 , \alpha \neq 0.$ $\quad x(2t) = 2$ for $0 \leq t \leq 1$

$\alpha \in (-1, 0) \cup (0, -1)$

$\alpha = 0.5$

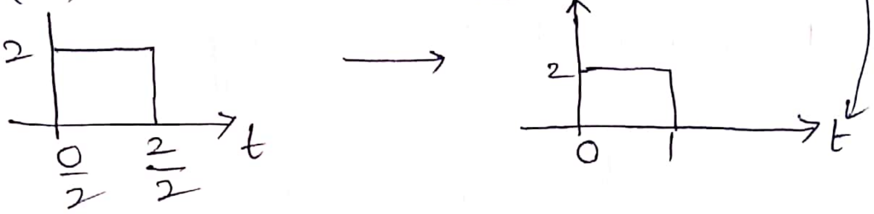
expansion

Short cut Method

$x(t) \xrightarrow{TS} y(t) = x(\alpha t)\ , \alpha \neq 0$

$\frac{\alpha t}{\alpha}$ $\quad x(2t) \Rightarrow \frac{2t}{2}$

Case(i) $\quad x(t)$



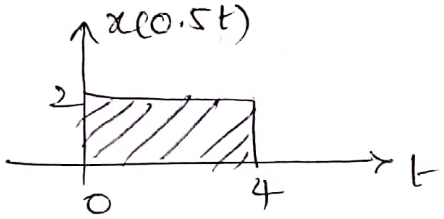
Case(ii) $\quad \alpha = 0.5$

$x(t) = 2, 0 \leq t \leq 2$

(i) Amp. Same $\quad$ (ii) $\frac{\alpha t}{\alpha} \Rightarrow \alpha = 0.5 \Rightarrow \frac{0.5t}{0.5} = t.$

$\rightarrow x(0.5t) = 2\ , \frac{0}{0.5} \leq t \leq \frac{2}{0.5}$

i.e. $x(0.5t) = 2,\ 0 \leq t \leq 4.$



(2)

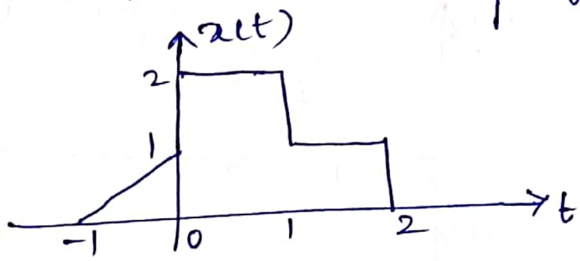
1) For the signal $x(t)$ find the following signals

i) $x(t-2)$

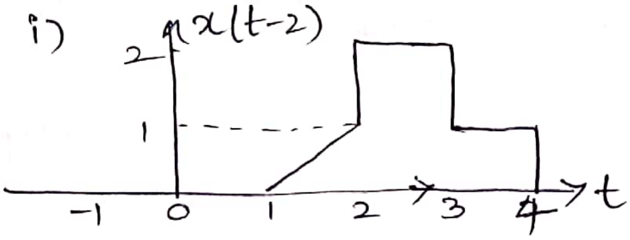
ii) $x(2t+3)$

iii) $x(\frac{3}{2}t)$

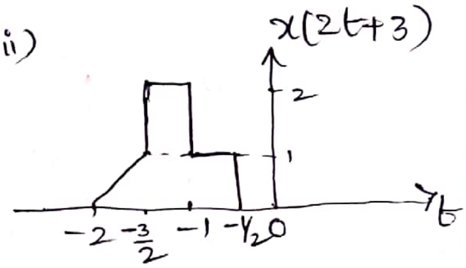
iv) $x(-t+1)$



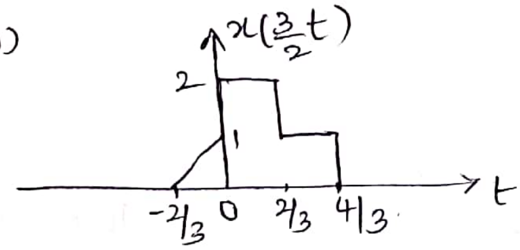
Sol i)



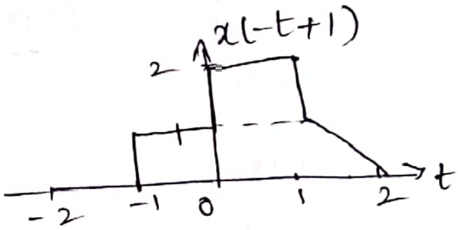
ii)



iii)

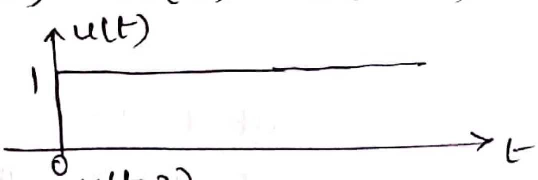


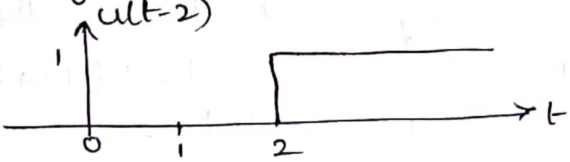
iv)

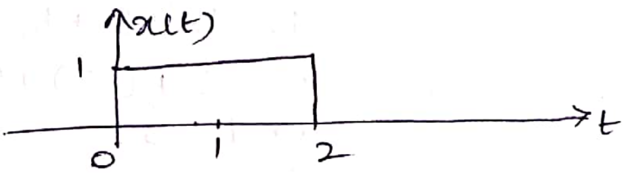


2) Sketch the following signals

i) $u(t) - u(t-2) = x(t)$

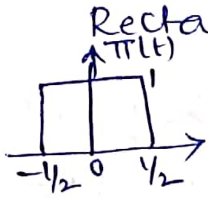






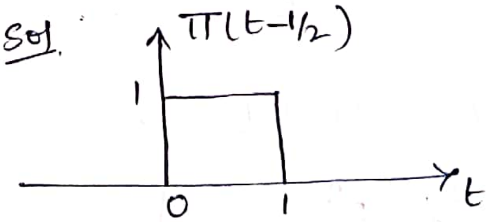
(ii) $\Pi(t-\frac{1}{2})$

Rectangular pulse



$\Pi(t) = 1, \ |t| \leq \frac{1}{2}$
$\quad\quad\ \ 0, \text{ else}$

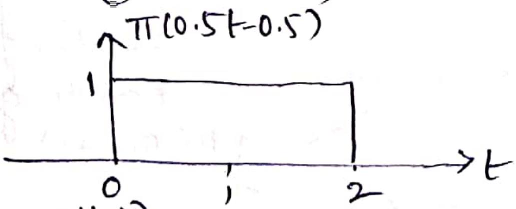
Sol.

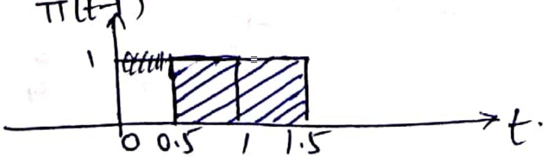


(iii) $\Pi\left(\frac{t-1}{2}\right) + \Pi(t-1)$

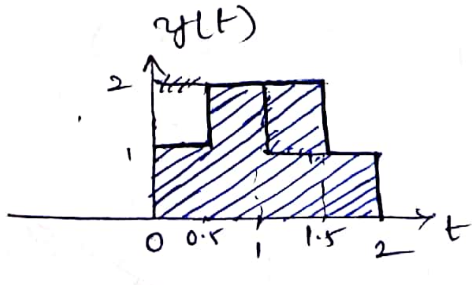
Sol

$\Pi(0.5t-0.5) + \Pi(t-1) = y(t)$





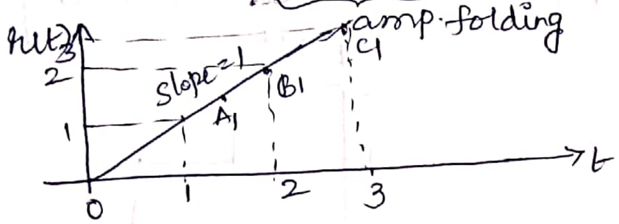
$\Rightarrow$



iv) $r(t) - \underbrace{2r(t-1)}_{\text{ramp folding}} + r(t-2)$

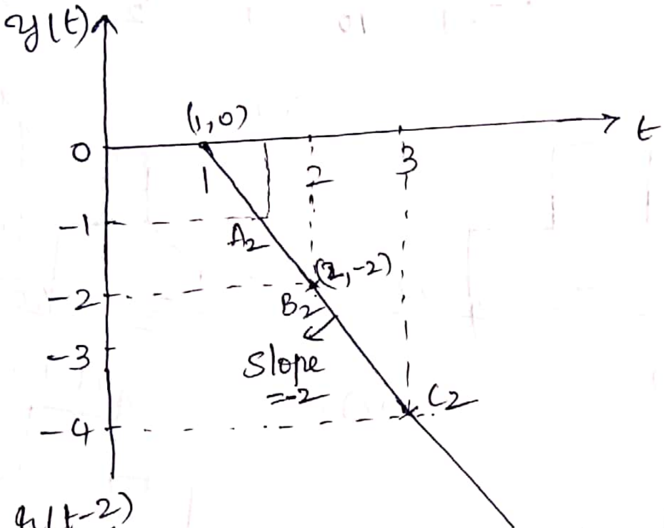
$r(t) = t$, for $t \geq 0$
$\quad 0$, else

$r(t-1) = t-1$, for $t-1 \geq 0$
i.e. $t \geq 1$
$\quad 0$, else

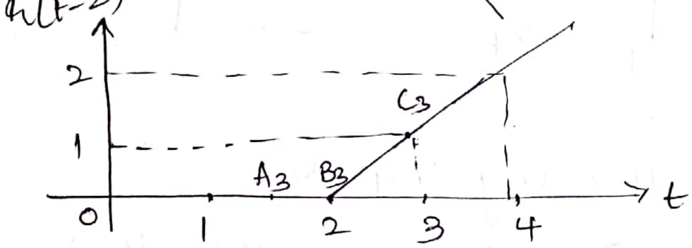


$y(t) = -2r(t-1) = -2(t-1)$, for $t \geq 1$
$= -2t+2$, for $t \geq 1$

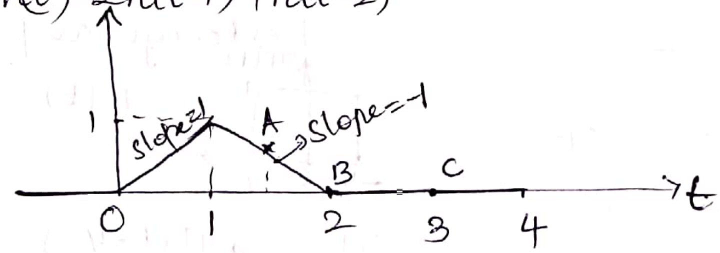
$\text{slope} = \frac{-2-0}{2-1} = -2$



$r(t-2) = t-2$, $t \geq 2$
$\quad 0$, else



$r(t) - 2r(t-1) + r(t-2)$



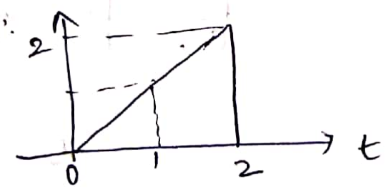
at $t = 1.5$
$A = A_1 + A_2 + A_3$
$= 1.5 + (-1) + 0 = 0.5$

at $t = 2$
$B = B_1 + B_2 + B_3$
$= 2 + (-2) + 0 = 0.$

at $t = 3$
$C = C_1 + C_2 + C_3$
$= 3 + (-4) + 1 = 0.$

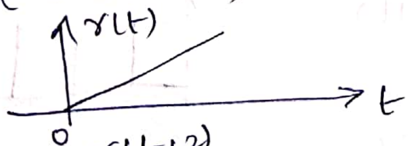
v) $r(t)\,u(2-t)$

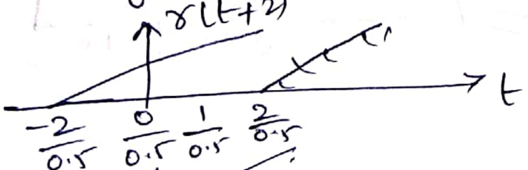
Ans:

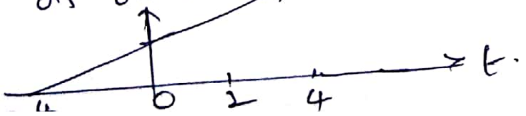


vi) $r(-0.5t + 2)$

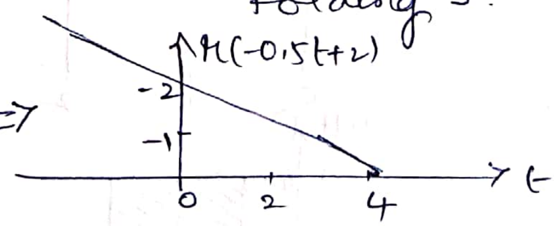
Shifting, Scaling, Folding







$\Rightarrow$



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Problems

1) Sketch the following signals

i) $u\left(\frac{t-1}{4}\right)$ ii) $\Pi(-2t+2)$ iii) $r\left(\frac{t+1}{3}\right)$

iv) $e^{-2t}u(-2+t)$

2) Find whether the following signals are periodic or not.

i) $x(t) = 4\cos 5\pi t$ Ans: periodic with period $\frac{2}{5}$ sec

ii) $x(t) = \sin 10\pi t\, u(t)$ Ans: Aperiodic

iii) $x(t) = e^{-|t|}$ Ans: Aperiodic

iv) $x(t) = 2\cos(10t+1) - \sin(4t-1)$ Ans: periodic with period π sec

v) $\cos 60\pi t + \sin 50\pi t$ Ans: periodic with $\frac{1}{5}$ sec

vi) $2u(t) + 2\sin 2t$ Ans: Aperiodic

vii) $3\cos 4t + 2\sin 2\pi t$ Ans: Aperiodic

viii) $u(t) - \frac{1}{2}$ Ans: Aperiodic

ix) $3\cos\left(17\pi t + \frac{\pi}{3}\right) + 2\sin\left(19\pi t - \frac{\pi}{3}\right)$ Ans: periodic $T = 2$ Sec

x) $u(t) - u(t-10)$ Ans: Aperiodic

xi) $\cos\left(\frac{1}{3}t\right) + \sin\left(\frac{1}{4}t\right)$ Ans: Periodic. $T = 24\pi$

3) Find even and odd component of signals

i) $x(t) = \sin t + 2\sin t + 2\sin^2 t \cos t$

Ans: $x_e(t) = 2\sin^2 t \cos t$

$x_o(t) = \sin t + 2\sin t$

ii) $x(t) = \cos t + \sin t + \cos t \sin t$

Ans: $x_e(t) = \cos t$

$x_o(t) = \sin t + \cos t \sin t$

4) Sketch the following signals and calculate their energies.

i) $e^{-10t}u(t)$ Ans: $E = \frac{1}{20}$ Joules.

ii) $u(t) - u(t-15)$ Ans: $E = 15\,J$

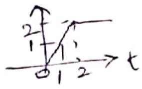
iii) $\cos(10\pi t)\,u(t)\,u(2-t)$ Ans: $E = 1\,J$

iv) Unit step signal Ans: power signal $E=\infty$, $P = 1/2$

v) Unit ramp signal Ans: neither energy nor power ($E=\infty, P=\infty$).

5) Whether the following signals are energy or power signals and calculate them?

i) $u(t) - u(t-1)$ Ans: energy signal. $E = 1J$, $P = 0$.

ii) $r(t) - r(t-2)$  Ans: $E = \infty$, $P = 2W$ power signal

iii) $(1+e^{-5t})\,u(t)$ Ans: $E = \infty$, $P = \frac{1}{2}W$ power signal

iv) $x_1(t) = e^{-3t}u(t)$ Ans: $E = \frac{1}{6}$, $P = 0$ energy signal

v) $x_2(t) = e^{j(2t+\frac{\pi}{4})}$ Ans: $E = \infty$, $P = 1$ power "

vi) $x_3(t) = \cos t$ Ans: $E = \infty$, $P = \frac{1}{2}$ ∴ power signal

~~vii)~~

6) Find which of the following are causal or non causal?

i) $x_1(t) = e^{at}u(t)$ Ans: causal

ii) $x_2(t) = e^{-2t}u(-t)$ Ans: non causal

iii) $x_3(t) = \text{sinc}\,t$ Ans: non causal

iv) $e^{-2t}u(t-2)$ Ans: causal

v) $\sin t\, u(t)$ Ans: causal

7) Unit step power signal. Ans: $E = \infty$, $P = 1/2$

8) Ramp is neither energy nor power signal.

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Classification of Signals :-

1. Deterministic & Non-Deterministic
2. Periodic & Aperiodic
3. Symmetric (even) & Asymmetric (odd)
4. Causal & Non causal signal
5. Energy & power signal.

1. Deterministic & Non-Deterministic :-

A signal $x(t)$ can be expressed by a mathematical expression is called Deterministic signal.

Ex :- sinusoidal, ramp, pulse, step signal etc.

A signal $x(t)$ cannot be expressed by a mathematical expression is called Non-Deterministic signal.

Ex :- Noise, Hum sound coming from amplifiers, oscillators, transformers etc.

2. Periodic & Aperiodic :-

A signal is said to be periodic if it satisfies

$x(t) = x(t+T)$

where, T = fundamental period.

A signal is said to be Aperiodic if it satisfies

$x(t) \neq x(t+T)$

where, T = fundamental period.

$F_0 = \frac{1}{T}$ where, F_0 = fundamental frequency (cycles/second)

$2\pi F_0 = \omega_0$ (or) Ω_0 → Angular frequency (rad/sec).

Examples

① Determine whether the following signals are periodic or not; if it is periodic find the fundamental period.

Sol:- (i) $x(t) = \sin(t)$

$x(t+T) = \sin(t+T)$ for $(T = 2\pi, 4\pi, 6\pi \ldots)$

$= x(t) = \sin(t)$

$\therefore \boxed{T = 2\pi}$ it is periodic signal

(ii) $x(t) = 2\cos\left(\frac{t}{4}\right)$

$x(t+T) = 2\cos\left(\frac{t+T}{4}\right)$

$= 2\cos\left(\frac{t}{4} + \frac{T}{4}\right)$ for $\frac{T}{4} = 2\pi, 4\pi \ldots$

$\Rightarrow \frac{T}{4} = 2\pi$

$= 2\cos\left(\frac{t}{4}\right)$ $\quad T = 8\pi$

$\therefore \boxed{T = 8\pi}$ periodic signal.

(iii) $x(t) = e^{\alpha t}$; $t > 1$

$x(t+T) = e^{\alpha(t+T)}$

$= e^{\alpha t} e^{\alpha T}$

for any values of T $x(t) \neq x(t+T)$

$\therefore$ The given signal is not periodic

(iv) $x(t) = \cos\left(\frac{2\pi}{3}t\right)$

~~$x(t) =$~~

$x(t+T) = \cos\left(\frac{2\pi}{3}t + \frac{2\pi}{3}T\right)$

$= \cos\left(\frac{2\pi}{3}t\right)$ for $\frac{2\pi}{3}T = 2\pi$

$\boxed{T = 3}$

$\therefore \boxed{T = 3}$ it is periodic signal.

NOTE:- If is a signal is a mixture of 2 periodic signals with fundamental periods T_1 & T_2 then the signal is periodic if ratio of $\frac{T_1}{T_2}$ is a rational number.

$$y(t) = x_1(t) + x_2(t)$$

$$\downarrow \qquad \downarrow$$

$$T_1 \qquad T_2$$

(1) $T_1, T_2 \rightarrow \frac{T_1}{T_2}$ (rational)

(2) LCM of T_1, T_2

Take the LCM of T_1, T_2 to get fundamental periods of $y(t)$.

Examples

(1.) $x(t) = 5\cos 4\pi t + 3\sin 8\pi t$

$x(t+T) = 5\cos(4\pi t + 4\pi T_1) + 3\sin(8\pi t + 8\pi T_2)$

$x_1(t) = 5\cos 4\pi t$

$x_1(t+T) = 5\cos(4\pi(t+T_1))$

$= 5\cos(4\pi t + \underbrace{4\pi T_1}_{2\pi})$ for $4\pi T_2 = 2\pi$

$= x_1(t)$

$\therefore$ periodic $4\pi T_1 = 2\pi$

$T_1 = \frac{2\pi}{4\pi} = \frac{1}{2}$

$x_2(t) = 3\sin 8\pi t$

$\cancel{x_2(t)} = x_2(t+T_2) = 3\sin(8\pi(t+T_2))$

$= 3\sin(8\pi t + \underbrace{8\pi T_2}_{2\pi})$

$= 3\sin 8\pi t$

$= x_2(t)$.

$\therefore$ periodic $8\pi T_2 = 2\pi$

$$T_2 = \frac{1}{4}$$

$\Rightarrow \frac{T_1}{T_2} = \frac{1/2}{1/4} = 2$ (rational)

$\Rightarrow$ LCM of $(T_1, T_2) = (1/2, 1/4)$

$$\begin{array}{r|l} 1/2 & 1/2,\ 1/4 \\ \hline 1/2 & 1,\ 1/2 \\ \hline & 1,\ 1 \end{array}$$

LCM $= 1/2$

$\therefore$ $\boxed{T = 1/2}$

③ Even & odd signal :-

A signal is said to be symmetric (or) even signal if it satisfies the condition

$$x(t) = x(-t)$$

A signal is said to be Asymmetric (or) odd signal if it satisfies the condition

$$x(t) = -x(-t)$$

Even part of signal

$$x_e(t) = \frac{1}{2}[x(t) + x(-t))$$

Odd part of signal

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Examples

Q) Determine whether following signals are odd or even, if not find their even & odd part

(1) $x(t) = e^{-t}$

$x(-t) = e^{-(-t)} = e^{t}$

$\Rightarrow x(t) \neq x(-t)$

$\therefore$ It is not even signal

$\Rightarrow x(t) = -x(-t)$

$e^{-t} = -e^{t}$

$\therefore$ It is not odd signal

$\therefore x_e(t) = \frac{1}{2}(x(t) + x(-t))$

$= \frac{1}{2}(e^{-t} + e^{t})$

$x_o(t) = \frac{1}{2}(x(t) - x(-t))$

$= \frac{1}{2}(e^{-t} - e^{t})$

(2) $x(t) = \sin(t)$

$x(-t) = \sin(-t)$

$= -\sin(t)$

$x(t) \neq x(-t)$

$\therefore$ It is not even signal.

$x(t) = -x(-t)$

$\sin(t) = -(-\sin t)$

$\sin t = \sin t$

$\therefore$ It is odd signal.

③ $x(t) = \sin 2t + \cos t + \sin t \cos 2t$

$x(-t) = -\sin 2t + \cos(t) - \sin t \cos 2t$

$-x(-t) = \sin 2t - \cos t - \sin t \cos 2t$

$\because$ $x(t) \neq x(-t)$ it is not even signal

$x(t) \neq -x(-t)$ it is not odd signal.

$$x_e(t) = \frac{1}{2}\left(\cancel{\sin 2t} + \cos t + \cancel{\sin t \cos 2t} - \cancel{\sin 2t} + \cos t - \cancel{\sin t \cos 2t}\right)$$

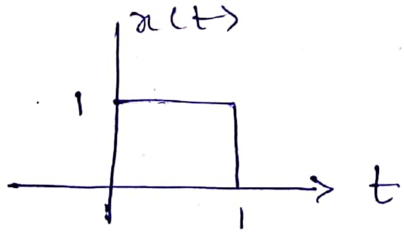
$$= \frac{1}{2}(2\cos 2t)$$

$$= \cos 2t$$

$$x_o(t) = \frac{1}{2}(2\sin 2t + 2\sin t \cos 2t)$$

$$= \sin 2t + \sin t \cos 2t$$

④

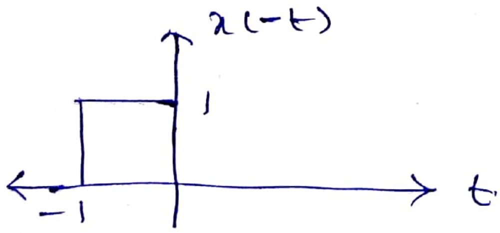


$x(t) = 1 \; ; \; 0 \leq t \leq 1$

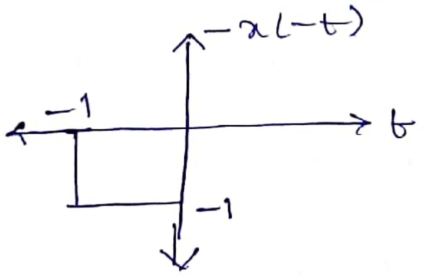
$\quad\;\; 0 \; ; \; \text{else}$

$x(-t) = 1 \; ; \; 0 \leq -t \leq 1 \Rightarrow 0 \geq t \geq -1.$

$\quad\;\; 0 \; ; \; \text{else}$



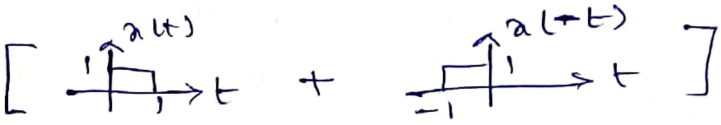
$\therefore$ $x(t) \neq x(-t)$ it is not an even signal.



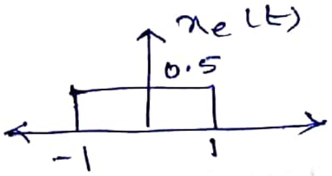
$\therefore x(t) \neq -x(-t)$

It is not an odd signal

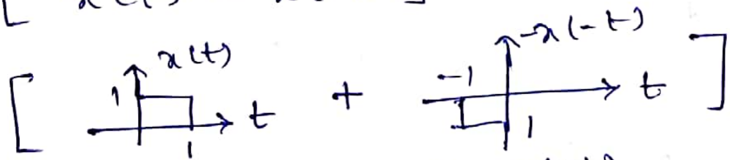
$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

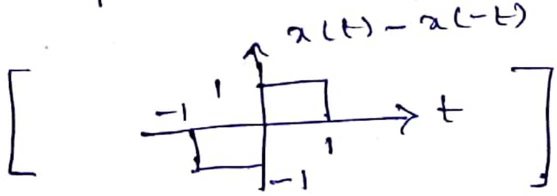
$= \frac{1}{2}$ 

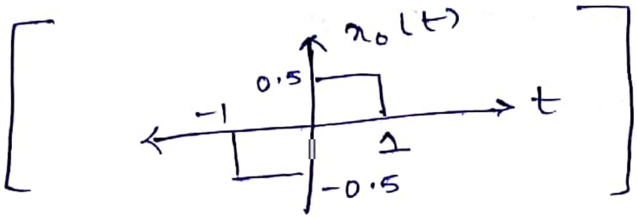
$= \frac{1}{2}$ 

$=$ 

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

$= \frac{1}{2}$ 

$= \frac{1}{2}$ 

$=$ 

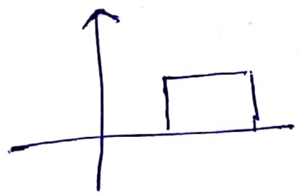
(4) Causal & Non - Causal signals :-

A signal $x(t)$ is defined only for $t \geq 0$ is said to be causal signal.

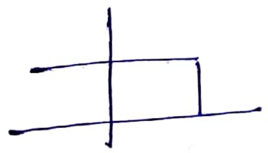
A signal $x(t)$ is defined for either for $t \leq 0$ & $t \geq 0$ is known as Non - causal signal.

If a non - causal signal which is defined only for $t \leq 0$ is known as Anti causal.

Ex :-



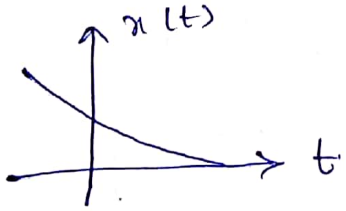
— causal



— Non - causal.

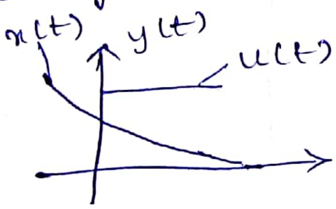
(1) $x(t) = e^{-t}$; for all t.

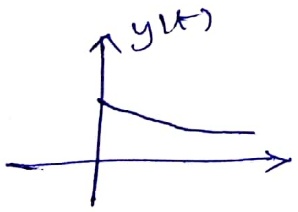
Sol:



— Non - causal

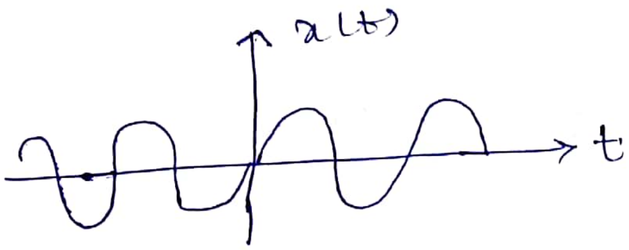
(2) $y(t) = e^{-t} u(t)$; for all t





— causal.

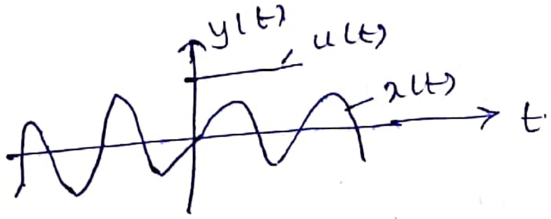
(3) $x(t) = \sin t$; for all t

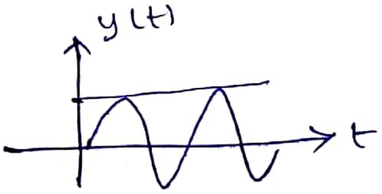


– Non causal.

(4) $y(t) = \sin t \; u(t)$ for all t

sol





– causal.

(5) Energy & Power signal :-

$$E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

A signal is said to be energy signal if energy (E) is finite & power is zero.

[E = finite ; P = 0] – Energy signal.

Ex: All non periodic signals are energy signal.

A signal is said to be power signal if power is finite & energy is infinite.

Ex: Periodic signal are power signals

[E = ∞ ; P = finite] – Power signal.

Q.7 Check whether the following signals are energy or power signals & calculate them.

<u>sol:</u> $x(t) = e^{-2t} u(t)$ for all 't'

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} |e^{-2t}|^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} e^{-4t} dt$$

$$= \lim_{T\to\infty} \left[\frac{e^{-4t}}{-4} \right]_0^T$$

$$= \lim_{T\to\infty} \left[\frac{e^{-4T} - 1}{4} \right]$$

$$= \frac{e^{-\infty} - 1}{-4}$$

$$= \frac{\frac{1}{\infty} - 1}{-4}$$

$$= \frac{1}{4} \quad \text{(finite)}$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} |e^{-2t}|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{e^{-4t}}{4} \right]_0^T$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{e^{-4T} - 1}{-4} \right]$$

$$= \frac{1}{\infty} \left[\frac{0-1}{-4} \right]$$

$$= 0$$

∴ Energy is finite & power is zero it is energy signal.

Q) Check whether the following signals are energy or power signals. calculate energy or power.

(i) Is step signal energy or power signal.
(ii) Is ramp signal energy or power signal.

Sol: (i) $u(t) = 1 \; ; \text{ for } t \geq 0$
$\qquad\qquad 0 \; ; \text{ for } t < 0$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} |1|^2 \, dt$$

$$= \lim_{T\to\infty} t\Big]_0^T$$

$$= \lim_{T\to\infty} T$$

$$\boxed{E = \infty}$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} |1|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \; t\Big]_0^T$$

$$= \lim_{T\to\infty} \frac{1}{2\cancel{T}} [\cancel{T}] \Rightarrow \boxed{\frac{1}{2} = P}$$

∴ Energy is infinite & power is finite it is power signal.

(ii) $x(t) = t$; for $t \geq 0$

$= 0$; for $t < 0$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} |t|^2 dt$$

$$= \lim_{T\to\infty} \left.\frac{t^3}{3}\right]_0^T$$

$$= \lim_{T\to\infty} \frac{T^3}{3}$$

$$= \frac{\infty^3}{3}$$

$$\boxed{E = \infty}$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} |t|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left.\frac{t^3}{3}\right]_0^T$$

$$= \lim_{T\to\infty} \frac{1}{2\cancel{T}} \frac{T^{\cancel{3}}}{3}$$

$$= \lim_{T\to\infty} \frac{T^2}{3}$$

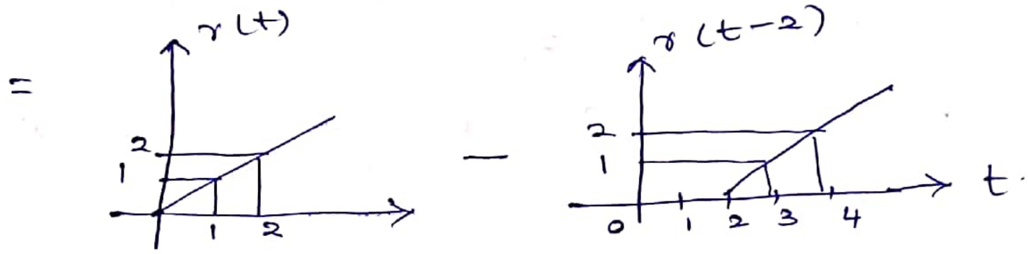
$$\boxed{P = \infty}$$

$\because$ Both energy & power are infinite the conditions for energy & power signal are not satisfied.

$\therefore$ It is neither energy nor power signal.

Q7 $x(t) = r(t) - r(t-2)$; check whether it is energy or power signal.

Sol: $x(t) = r(t) - r(t-2)$



$x(t) = t; \; 0 \le t \le 2$

$= 2; \; t \ge 2$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_0^2 |r(t)|^2 dt + \int_2^T |u(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_0^2 t^2 dt + \int_2^T |2|^2 dt$$

$$= \lim_{T\to\infty} \left[\frac{t^3}{3}\right]_0^2 + 4 \left[t\right]_2^T$$

$$= \lim_{T\to\infty} \left\{ \left[\frac{8}{3}\right] + 4\left[T-2\right] \right\}$$

$$= \left\{ \frac{8}{3} + 4(\infty - 2) \right\}$$

$$\boxed{E = \infty}$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{2} t^2 \, dt + \int_{2}^{T} |2|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left\{ \frac{t^3}{3} \Big]_0^2 + 4 \; t \Big]_2^T \right\}$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left\{ \left(\frac{8}{3}\right) + 4\,(T-2) \right\}$$

$$= \lim_{T\to\infty} \frac{1}{\cancel{2}T}\,\frac{\cancel{8}^{\,4}}{3} + \frac{\cancel{4}^{\,2}\cancel{T}}{\cancel{2T}} - \frac{\cancel{8}^{\,4}}{\cancel{2}T}$$

$$= \lim_{T\to\infty} \frac{4}{T3} + 2 - \frac{4}{T}$$

$$= \frac{4}{3\infty} + 2 - \frac{4}{\infty}$$

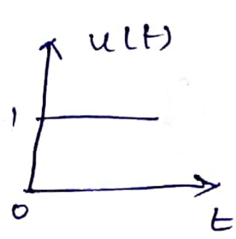
$$= 0 + 2 - 0$$

$$\boxed{P = 2.}$$

$\therefore$ Energy is infinite & power is finite it is power signal.

Q) $x(t) = u(t) - u(t-5)$

=



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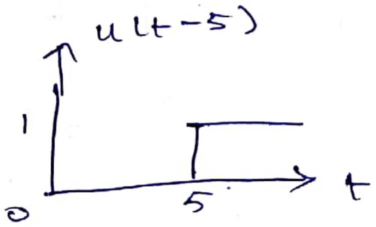

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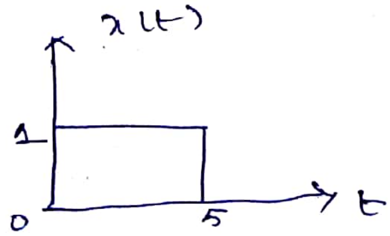

$x(t) = 1 \; ; \; 0 \le t \le 5$

$\quad\;\; = 0 \; ; \; \text{else}$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \int_0^5 (1)^2 \, dt$$

$$= \lim_{T\to\infty} [t]_0^5$$

$$= \lim_{T\to\infty} 5$$

$$\boxed{E = 5}$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_0^5 (1)^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} [t]_0^5$$

$$= \lim_{T\to\infty} \frac{1}{2T} [5]$$

$$= \frac{1}{2\times\infty} 5$$

$$= \frac{1}{0} 5$$

$$\boxed{P = 0}$$

$\therefore$ Energy is finite & power is zero.

It is an energy signal.

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Classification of continuous systems :-

① Dynamic & static system
② Time invariant (TIV) & Time variant system (TV)
③ Linear & Non linear. system.
④ Causal & Non causal system.
⑤ stable & unstable system.
⑥ feedback & Non feedback system.

① **static & Dynamic system :-** A system is said to be static or memory less if its output depends only on present input but not on future or past inputs.

In any other case the system is said to be dynamic or to have memory.

Ex :- Q.) $y(t) = a\,x(t)$

$$\left.\begin{aligned} t=0 &\rightarrow y(0) = a\,x(0) \\ t=2 &\rightarrow y(2) = a\,x(2) \end{aligned}\right\} \text{static}$$

Q.) $y(t) = a\,x(t^2)$

$$\left.\begin{aligned} t=0 &\rightarrow y(0) = a\,x(0) \\ t=1 &\rightarrow y(1) = a\,x(1) \\ t=2 &\rightarrow \underbrace{y(2)}_{\text{present}} = \underbrace{a\,x(4)}_{\text{future}} \end{aligned}\right\} \text{Dynamic}$$

Q.) $y(t) = x(t) + x(t-2)$

$$\left.\begin{aligned} y(1) &= x(1) + x(-1) \\ y(2) &= x(2) + x(0) \end{aligned}\right\} \text{Dynamic}$$

(2) Time Invariant & Time Variant :

A system is said to be time invariant if its input and output characteristics does not change with time.

$$\underset{\text{i/p}}{x(t)} \longrightarrow \boxed{\text{Delay}} \xrightarrow[\text{i/p}]{\substack{x(t-m)\\ \text{Delayed}}} \boxed{\text{system}} \longrightarrow y_1(t)$$

$y_1(t)$ Delayed i/p response.

$$x(t) \longrightarrow \boxed{\text{System}} \xrightarrow[\substack{\text{o/p}\\ \text{response}}]{y(t)} \boxed{\text{Delay}} \longrightarrow y_2(t)$$

$y_2(t)$ Delayed o/p response

If $y_1(t) = y_2(t) \longrightarrow$ Time Invariant

$y_1(t) \neq y_2(t) \longrightarrow$ Time Variant.

Ex: (1) $y(t) = 3x^2(t)$

$$\underset{\text{i/p}}{x(t)} \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{\text{system}} \longrightarrow y_1(t) = 3x^2(t-m)$$

$$x(t) \longrightarrow \boxed{\text{system}} \xrightarrow[= 3x^2(t)]{y(t)} \boxed{\text{Delay}} \longrightarrow y_2(t) = 3x^2(t-m)$$

$\therefore y_1(t) = y_2(t)$

It is time invariant.

(2) $y(t) = 3x(t^2)$

$$\underset{\text{i/p}}{x(t)} \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{\text{System}} \longrightarrow y_1(t) = 3x((t-m)^2)$$

$$x(t) \longrightarrow \boxed{\text{System}} \xrightarrow[= 3x(t^2)]{y(t)} \boxed{\text{Delay}} \longrightarrow y_2(t) = 3x(t^2-m)$$

$\therefore y_1(t) \neq y_2(t)$ It is time variant.

(3) $y(t) = t\,x(t)$

$\underset{i/p}{x(t)} \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{\text{system}} \longrightarrow y_1(t) = t\,x(t-m)$

$x(t) \longrightarrow \boxed{\text{system}} \xrightarrow{t\,x(t)} \boxed{\text{Delay}} \longrightarrow y_2(t) = (t-m)\,x(t-m)$

$\therefore y_1(t) \neq y_2(t)$

It is time variant system.

(4) $y(t) = x(-t)$

$x(t) \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{\text{system}} \longrightarrow y_1(t) = x(-t+m)$

$x(t) \longrightarrow \boxed{\text{system}} \xrightarrow{x(-t)} \boxed{\text{Delay}} \longrightarrow y_2(t) = x(-t-m)$

$\therefore y_1(t) \neq y_2(t)$

It is time variant system.

(5) $y(t) = x(t+c)$

$x(t) \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{\text{system}} \longrightarrow y_1(t) = x(t+c-m)$

$x(t) \longrightarrow \boxed{\text{system}} \xrightarrow{x(t+c)} \boxed{\text{Delay}} \longrightarrow y_2(t) = x(t+c-m)$

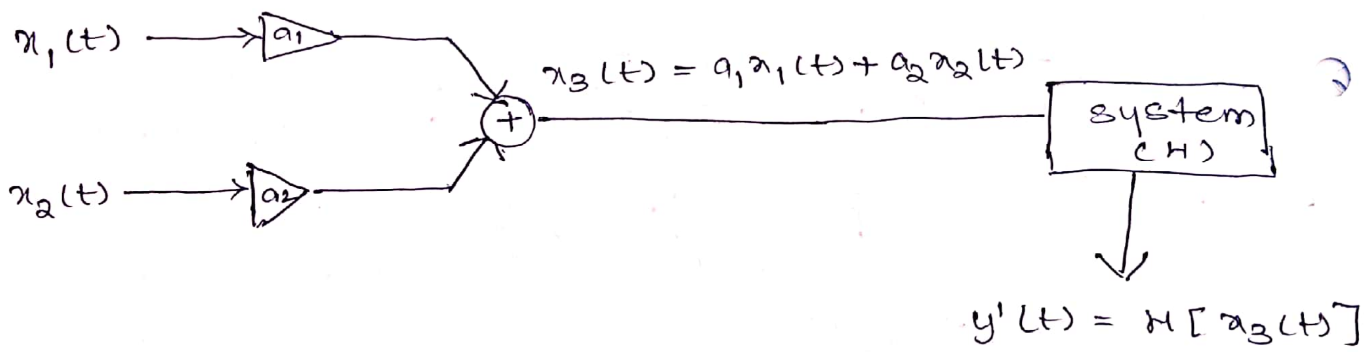
$\therefore y_1(t) = y_2(t)$

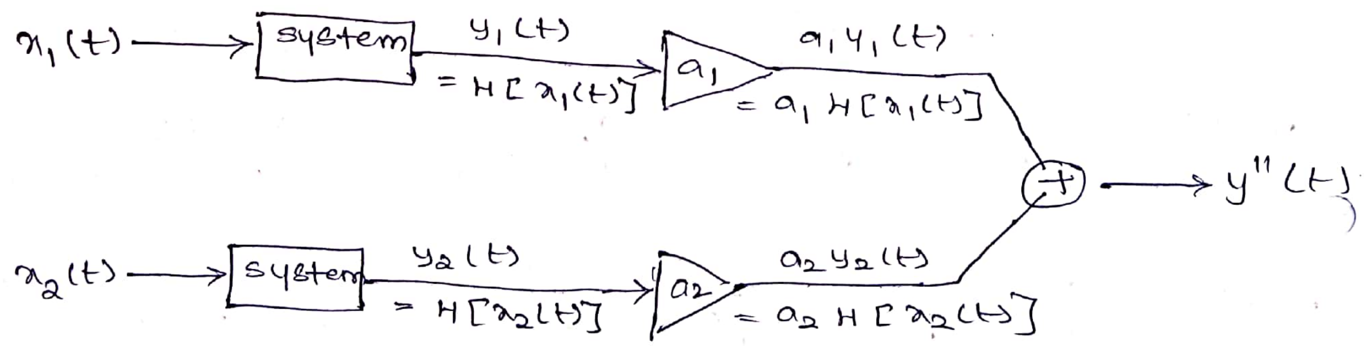
It is time invariant.

3 Linear & Non-linear System :-

A system is said to be linear system if it satisfies superposition principle.

The superposition principle states that the response of the weighted sum of the signals is equal to the corresponding weighted sum of the responses of individual signals.





$$y'(t) = y''(t) \longrightarrow \text{Linear}$$

$$H[a_1x_1(t) + a_2x_2(t)] = a_1 H[x_1(t)] + a_2 H[x_2(t)]$$

$$y'(t) \neq y''(t) \longrightarrow \text{Non-linear.}$$

Q.7 Check whether the following systems are linear or not.

(1) $y(t) = 3x^2(t)$

$y'(t) = H[x_3(t)] = 3x_3^2(t)$

$= 3[a_1x_1(t) + a_2x_2(t)]^2$ — (1)

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = 3x_1^2(t)$

$y_2(t) = H[x_2(t)] = 3x_2^2(t)$

$y''(t) = a_1y_1(t) + a_2y_2(t)$

$= a_1 3x_1^2(t) + a_2 3x_2^2(t)$

$\neq y'(t)$

$\therefore$ It is non linear.

(2) $y(t) = 3x(t^2)$

$y'(t) = H[x_3(t)] = 3x_3(t^2)$

$= 3[a_1x_1(t^2) + a_2x_2(t^2)]$ — (1)

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = 3x_1(t^2)$

$y_2(t) = H[x_2(t)] = 3x_2(t^2)$

$y''(t) = a_1y_1(t) + a_2y_2(t)$

$= a_1 3x_1(t^2) + a_2 3x_2(t^2)$

$= 3[a_1x_1(t^2) + a_2x_2(t^2)]$

$= y'$

$\therefore$ It is linear.

③ $y(t) = t\,x(t)$

$y'(t) = H[x_3(t)] = t\,x_3(t)$

$= t[a_1x_1(t) + a_2x_2(t)]$ — ①

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = t\,x_1(t)$

$y_2(t) = H[x_2(t)] = t\,x_2(t)$

$y'' = a_1y_1(t) + a_2y_2(t)$

$= a_1\,t\,x_1(t) + a_2\,t\,x_2(t)$

$= t[a_1x_1(t) + a_2x_2(t)]$

$= y'$

∴ It is linear system.

④ $y(t) = x(t) + c$

$y'(t) = H[x_3(t)] = x_3(t) + c$

$= [a_1x_1(t) + a_2x_2(t)] + c$

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = x_1(t) + c$

$y_2(t) = H[x_2(t)] = x_2(t) + c$

$y'' = a_1y_1(t) + a_2y_2(t)$

$= a_1x_1(t) + c + a_2x_2(t) + c$

$= a_1x_1(t) + a_2x_2(t) + 2c.$

$\neq y'$

∴ It is non-linear. system.

5) $y(t) = e^{x(t)}$

$y'(t) = H[x_3(t)] = e^{x_3(t)}$

$= e^{(a_1x_1(t)+a_2x_2(t))}$

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = e^{x_1(t)}$

$y_2(t) = H[x_2(t)] = e^{x_2(t)}$

$y'' = a_1 e^{x_1(t)} + a_2 e^{x_2(t)}$

$\neq y'$

∴ It is non linear system.

6) $y(t) = 2x(t) + \frac{dx(t)}{dt}$

$y'(t) = H[x_3(t)] = 2x_3(t) + \frac{dx_3(t)}{dt}$

$= 2(a_1x_1(t) + a_2x_2(t))$

$+ \frac{d(a_1x_1(t) + a_2x_2(t))}{dt}$

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = 2x_1(t) + \frac{dx_1(t)}{dt}$

$y_2(t) = H[x_2(t)] = 2x_2(t) + \frac{dx_2(t)}{dt}$

$y''(t) = a_1y_1(t) + a_2y_2(t)$

$= a_1\left(2x_1(t) + \frac{dx_1(t)}{dt}\right) + a_2\left(2x_2(t) + \frac{dx_2(t)}{dt}\right)$

$= y'(t)$

∴ It is linear system.

(7) $y(t) = x(t) \sin 200\pi t$

$y'(t) = H[x_3(t)] = x_3(t) \sin 200\pi t$

$= (a_1 x_1(t) + a_2 x_2(t)) \sin 200\pi t.$

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = x_1(t) \sin 200\pi t$

$y_2(t) = H[x_2(t)] = x_2(t) \sin 200\pi t$

$y''(t) = a_1 y_1(t) + a_2 y_2(t)$

$= a_1 x_1(t) \sin 200\pi t + a_2 x_2(t) \sin 200\pi t$

$= y'$

$\therefore$ It is linear system.

(8) $y(t) = \cos x(t)$

$y'(t) = H[x_3(t)] = \cos(x_3(t))$

$= \cos(a_1 x_1(t) + a_2 x_2(t))$

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = \cos x_1(t)$

$y_2(t) = H[x_2(t)] = \cos x_2(t)$

$y''(t) = a_1 y_1(t) + a_2 y_2(t)$

$= a_1 \cos x_1(t) + a_2 \cos x_2(t)$

$\neq y'(t)$

$\therefore$ It is non linear system.

(4) Causal & Non Causal system :-

A system is said to be causal if the output of the system depends only on present input, past input and past output. But not on future input & output.

A system is said to be non-causal if the O/P depends on future input & future output is known as Non-causal system.

Examples

(1) $y(t) = x(t) + x(t-2)$

$t = -3 \Rightarrow y(-3) = x(-3) + x(-5)$

$t = 0 \Rightarrow y(0) = x(0) + x(-2)$

$t = 2 \Rightarrow y(2) = x(2) + x(0)$

$\therefore$ It depends only on past & present

it is causal system.

(2) $y(t) = x(t) - x(3-t)$

$t = -2 \Rightarrow y(-2) = x(-2) - x(5)$

$\therefore$ It depends on future it is non causal system.

(3) $y(t) = 3x(t^2)$

$t = -2 \Rightarrow y(-2) = 3x(4)$

$t = 0 \rightarrow y(0) = 3x(0)$

$t = 2 \Rightarrow y(2) = 3x(4)$

$\therefore$ It is non causal system

(4) $y(t) = 3x^2(t)$

$t = 2 \rightarrow y(2) = 3x^2(2)$

$t = 0 \Rightarrow y(0) = 3x^2(0)$

$t = -2 \Rightarrow y(-2) = 3x^2(-2)$

(5) $y(t) = x(-t)$

$t=-2 \Rightarrow y(-2) = x(2)$

$t=0 \Rightarrow y(0) = x(0)$

$t=2 \Rightarrow y(2) = x(-2)$

$\therefore$ It is non causal system.

(6) $y(t) = t\,x(t)$

$t=-2 \Rightarrow y(t) = -2x(-2)$

$t=0 \Rightarrow y(0) = 0\,x(0)$

$t=2 \Rightarrow y(2) = 2x(2)$

$\therefore$ It is causal system.

(7) $y(t) = x(2t)$

$t=-2 \Rightarrow y(-2) = x(-4)$

$t=0 \Rightarrow y(0) = x(0)$

$t=2 \Rightarrow y(2) = x(4)$

$\therefore$ It is non causal system.

(8) $y(t) = x(t) + \int_0^t x(\lambda)\,d\lambda$

Sol: $t=-2 \Rightarrow y(-2) = x(-2) + \int_0^{-2} x(\lambda)\,d\lambda$

$\Rightarrow y(-2) = x(-2) + [\cancel{x}(\lambda)]_0^{-2}$

$\cancel{x}(-2) - \cancel{x}(0)$

$\cancel{x}(-2) + \cancel{x}(2)$

$\therefore$ It is causal system.

(9) $y(t) = x(t) + \int_0^{3t} x(\lambda)\,d\lambda$

$= x(t) + [z(\lambda)]_0^{3t}$

$t=1 \Rightarrow y(1) = x(1) + \cancel{z}(3) - \cancel{z}(0)$

$\therefore$ It is non causal system.

(10) $y(t) = x(t) \sin 200\pi t$

$y(0) = x(0) \sin 200\pi t$

$y(2) = x(2) \sin 200\pi t$

$\therefore$ It is causal system.

(5) <u>Stable and Unstable system</u> :-

A system is said to be BIBO stable if and only if for every bounded input produces bounded output.

(BIBO – bounded input and bounded output).

$$\boxed{\int_{-\infty}^{\infty} |h(t)\, dt| < \infty} \longrightarrow \text{stable}$$

else unstable.

Q) Test the stability of LTI system

1) $h(t) = e^{-5|t|}$

<u>sol</u>:- $\int_{-\infty}^{\infty} e^{-5|t|}\, dt$

$$= \int_{-\infty}^{0} |e^{-5|t|}\, dt| + \int_{0}^{\infty} e^{-5t}\, dt$$

$$= \int_{-\infty}^{0} e^{5t}\, dt + \int_{0}^{\infty} e^{-5t}\, dt$$

$$= \left.\frac{e^{5t}}{5}\right]_{-\infty}^{0} + \left[\frac{e^{-5t}}{-5}\right]_{0}^{\infty}$$

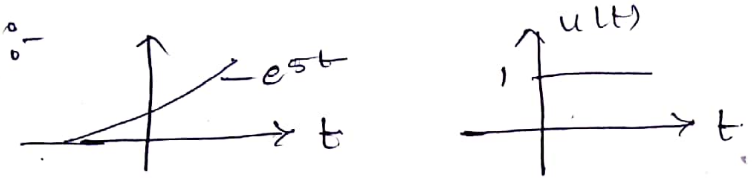
$$= \left[\frac{1-0}{5}\right] + \left[\frac{0-1}{-5}\right]$$

$$= \frac{1}{5} + \frac{1}{5} \Rightarrow \boxed{\frac{2}{5} < \infty}$$

$\therefore$ It is stable.

(2) $h(t) = e^{5t}\, u(t)$

Sol :-



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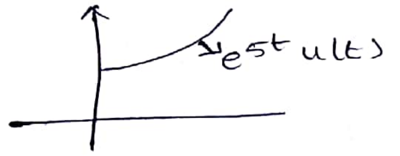

$$= \int_{-\infty}^{\infty} |h(t)|\, dt$$

$$= \int_{-\infty}^{\infty} |e^{5t}\, u(t)|\, dt$$

$$= \cancelto{0}{\int_{-\infty}^{0} e^{5t}\, dt} + \int_{0}^{\infty} e^{5t}\, (1)\, dt$$

$$= \int_{0}^{\infty} e^{5t}\, dt$$

$$= \left.\frac{e^{5t}}{5}\right]_0^{\infty}$$

$$= \frac{1}{5}\left[e^{\infty} - 1\right]$$

$$= \frac{1}{5}\left[\infty - 1\right]$$

$$= \infty$$

$\therefore$ It is unstable.

(3) $h(t) = te^{-3t} u(t)$

Sol: $\int_0^\infty te^{-3t} dt$

$$= \left[t\frac{e^{-3t}}{3} - \frac{e^{-3t}}{9} \right]_0^\infty$$

$$= [0 - 0]$$

$$= 0 < \infty$$

$\therefore$ It is stable.

(4) $h(t) = t \cos t \, u(t)$

Sol: $= \int_0^\infty t \cos t$

$$= [t \sin t + \cos t]_0^\infty$$

$$= \infty$$

(6) Feedback & Non feedback system :-

- The O/P of the system at any time 't' depends on past O/P, past I/P & present I/P is called a feedback system.

The O/P depends only on present & past input is called Non-feedback system

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Classification of Continuous time Systems

1) Static and Dynamic systems
2) Time invariant and Time variant System
3) Linear and non linear system
4) Causal and non-causal system
5) Stable and unstable system
6) Feedback and non feedback system.

① Static and dynamic system

A system is said to be static (or) memory-less if its o/p depends on i/p signal at the same time but not on past or future i/p. In any other case the system is said to be dynamic (or) to have memory.

Ex:

$$\left.\begin{aligned} y(t) &= a\,x(t) \\ y(t) &= t\,x(t) + 6x^3(t) \end{aligned}\right\} \text{Static System}$$

$$\left.\begin{aligned} y(t) &= t\,x(t) + 3x(t^2) \\ y(t) &= x(t) + 3x(t-2) \end{aligned}\right\} \text{Dynamic system.}$$

② Time invariant and Time variant system

A system is said to be TIV if its i/p-o/p characteristics does not change with time.

$$x(t) \xrightarrow{H} y(t)$$

$$x(t-m) \xrightarrow{H} y(t-m)$$

$$\underset{i/p}{x(t)} \longrightarrow \boxed{\text{Delay}} \xrightarrow[\text{delayed i/p}]{x(t-m)} \boxed{\text{System } H} \longrightarrow \underset{\text{Response for delayed i/p}}{y_1(t)}$$

$$\underset{i/p}{x(t)} \longrightarrow \boxed{\text{System}} \xrightarrow{y(t)} \boxed{\text{Delay}} \longrightarrow \underset{\text{Delayed response.}}{y_2(t)}$$

$$\boxed{\begin{array}{l} y_1(t) = y_2(t) \longrightarrow \text{TIV} \\ y_1(t) \neq y_2(t) \longrightarrow \text{TV.} \end{array}}$$

State whether the following systems are Time invariant or not.

① $y(t) = 3x^2(t)$

$$x(t) \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{\text{System}} \longrightarrow y_1(t) = 3x^2(t-m)$$

$$x(t) \longrightarrow \boxed{H} \xrightarrow{3x^2(t)} \boxed{\text{Delay}} \longrightarrow y_2(t) = 3x^2(t-m)$$

$$y_1(t) = y_2(t) \longrightarrow \therefore \text{TIV}$$

② $y(t) = 3x(t^2)$

$$x(t) \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{H} \longrightarrow y_1(t) = 3x(t^2-m)$$

$$x(t) \longrightarrow \boxed{H} \xrightarrow{3x(t^2)} \boxed{\text{Delay}} \longrightarrow y_2(t) = 3x((t-m)^2)$$

$$y_1(t) \neq y_2(t) \longrightarrow \text{TV.}$$

③ $y(t) = 2t\,x(t)$

$$x(t) \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{H} \longrightarrow y_1(t) = 2t\,x(t-m)$$

$$x(t) \longrightarrow \boxed{H} \xrightarrow{2t\,x(t)} \boxed{\text{Delay}} \longrightarrow y_2(t) = 2(t-m)x(t-m)$$

$$y_1(t) \neq y_2(t)$$

T~~I~~V

(4) $y(t) = x(t) \sin 20\pi t$ Ans: TV

(5) $y(t) = x(-t)$ Ans: TV.

(6) $y(t) = 2e^{x(t)}$ Ans: TIV

(7) $y(t) = x(t) + C$ Ans: TIV.

(8) $y(t) = \text{Odd}[x(t)]$

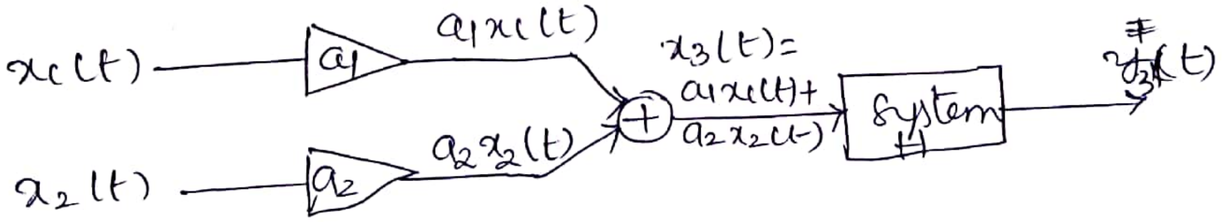
Sol $y(t) = \dfrac{x(t) - x(-t)}{2}$ Ans: TV

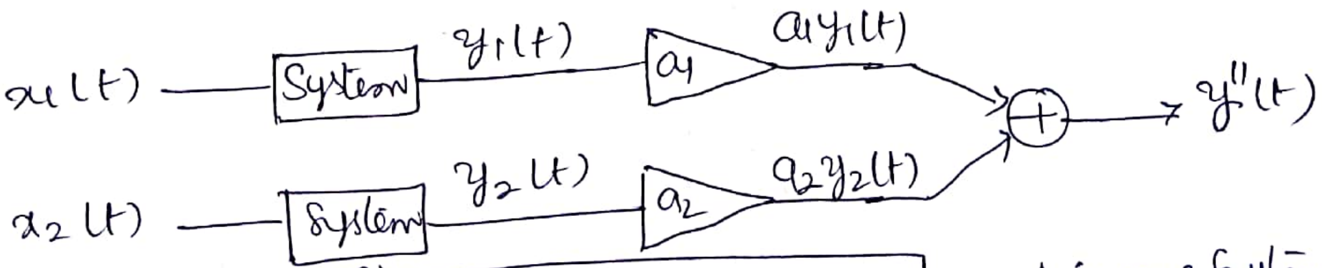
(3) Linear and Non linear System

- A linear system is the one that satisfies superposition principle.

The principle of superposition requires that the response of a system to a weighted sum of the signals is equal to the corresponding weighted sum of the responses to each of the ~~following~~ individual signals.

- $$H[a_1 x_1(t) + a_2 x_2(t)] = a_1 H[x_1(t)] + a_2 H[x_2(t)]$$





(OR) $\boxed{y_3(t) = y''(t)}$ → Linear System

i.e. $y_3(t) = a_1 y_1(t) + a_2 y_2(t)$

$y_3(t) \neq y''(t)$ → Non linear

Q: Whether the following systems are linear or not.

① $y(t) = 3x^2(t)$

Sol Let two signals $x_1(t)$ & $x_2(t)$

$$y_1(t) = H[x_1(t)] = 3x_1^2(t)$$

$$y_2(t) = H[x_2(t)] = 3x_2^2(t)$$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$$\therefore y_3(t) = H[x_3(t)]$$

$$= 3[a_1 x_1(t) + a_2 x_2(t)]^2$$

$$\therefore y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$$

Non linear system.

② $y(t) = 3x(t^2)$

Sol
$$y_1(t) = 3x_1(t^2)$$
$$y_2(t) = 3x_2(t^2)$$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$$\therefore y_3(t) = H[x_3(t)]$$

$$= 3[a_1 x_1(t^2) + a_2 x_2(t^2)]$$

$$= 3a_1 x_1(t^2) + 3a_2 x_2(t^2) \quad \text{--- (1)}$$

$$y_3(t) = a_1 y_1(t) + a_2 y_2(t)$$

$\therefore$ Linear system

③ $y(t) = t\,x(t)$

Sol
$$y_1(t) = t\,x_1(t)$$
$$y_2(t) = t\,x_2(t)$$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$$\therefore y_3(t) = t[a_1 x_1(t) + a_2 x_2(t)]$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

$\therefore$ linear system

(4) $y(t) = x(t) + c$

Sol $y_1(t) = x_1(t) + c$

$y_2(t) = x_2(t) + c$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$\therefore y_3(t) = [a_1 x_1(t) + a_2 x_2(t)] + c$

$= a_1 x_1(t) + a_2 x_2(t) + c + c - c$

$= a_1 y_1(t) + a_2 y_2(t) \mp c$

$\therefore y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$

Non linear System.

(5) $y(t) = e^{x(t)}$

Sol $y_1(t) = e^{x_1(t)}$

$y_2(t) = e^{x_2(t)}$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$\therefore y_3(t) = H[x_3(t)] = e^{a_1 x_1(t) + a_2 x_2(t)}$

$\therefore y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$

Non linear System.

(6) $y(t) = 2x(t) + \frac{dx(t)}{dt}$ Ans: Linear

(7) $y(t) = x(t) \sin 20\pi t$ Ans: linear

(8) $y(t) = \cos(x(t))$ Ans: Non linear

4) Causal and non causal system

A system is said to be causal if the o/p of the system depends only on the present i/p, past i/p and past o/p but not on future i/p & o/p's

If the system o/p depends on future i/p & o/p's then the system is called a non causal system.

Q: Test the causality of the following systems.

1) $y(t) = x(t) + x(t-2)$ Ans: Causal

2) $y(t) = x(t) - x(3-t)$ Ans: non causal

3) $y(t) = 3x^2(t)$ Ans: Causal

4) $y(t) = 3x(t^2)$ Ans: non causal

5) $y(t) = x(-t)$ Ans: Non Causal

6) $y(t) = x(t) + C$ Ans: Causal

7) $y(t) = x(2t)$ Ans: Non causal

8) $y(t) = x(t) + \int_0^t x(\lambda)\,d\lambda$ Ans: causal

9) $y(t) = x(t) + \int_0^{3t} x(\lambda)\,d\lambda$ Ans: non causal.

Sol $y(t) = x(t) + \left[Z(\lambda)\right]_0^{3t}$ let $\int x(\lambda)\,d\lambda = Z(\lambda)$

$y(t) = x(t) + Z(3t) - Z(0)$

$t = 1 \Rightarrow y(1) = \underset{\text{Present}}{x(1)} + \underset{\text{Future}}{Z(3)} - \underset{\text{past}}{Z(0)}$

$\therefore$ Non causal.

10) $y(t) = x(t)\sin 20\pi t \Rightarrow$ Ans: causal.

5) Stable and Unstable System:

Def: A system is said to be BIBO (Bounded i/p - Bounded o/p) if and only if every bounded i/p produces bounded o/p.

Ex: Bounded i/p signal → step, decaying exponential, Impulse.

Unbounded i/p " → ramp, increasing exponential.

Condition for stability

$$\boxed{\int_{-\infty}^{\infty} |h(t)|\, dt < \infty}$$

Q. Test the stability of LTI system.

1) $h(t) = e^{-5|t|}$

For stability $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

$$\therefore \int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} e^{-5|t|}\, dt$$

$$= \int_{-\infty}^{0} e^{5t}\, dt + \int_{0}^{\infty} e^{-5t}\, dt$$

$$= \left[\frac{e^{5t}}{5}\right]_{-\infty}^{0} + \left[\frac{e^{-5t}}{-5}\right]_{0}^{\infty}$$

$$= 1/5 + 0 + 0 = \frac{1}{5} < \infty$$

$\therefore$ Stable.

2) $h(t) = e^{5t} u(t)$

Sol $\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} e^{5t} u(t)\, dt$

$$= \int_{0}^{\infty} e^{5t}\, dt = \left[\frac{e^{5t}}{5}\right]_{0}^{\infty} = \infty$$

$\therefore$ Unstable

3) $h(t) = t e^{-3t} u(t)$ Ans: Stable.

4) $h(t) = t \cos t\, u(t)$

Sol $= \int_{-\infty}^{\infty} |t \cos t\, u(t)|\, dt$

$\int u\,dv = uv - \int v\, du$.

$= \int_0^{\infty} \underset{u}{t}\, \underset{dv}{\cos t\, dt}$

$= [t \sin t]_0^{\infty} - \int_0^{\infty} \sin t \cdot 1\, dt$

$= (\infty - 0) + [\cos t]_0^{\infty} = \infty$

$\therefore$ unstable

5) $h(t) = e^{-t} \sin t\, u(t)$ Ans: Stable.

6) Determine the range of values of 'a' & 'b' for the stability of LTI system with impulse response.

Ans: $a < 0$ & $b > 0$

$$h(t) = e^{at} u(t) + e^{-bt} u(t)$$

6) Feedback and non Feedback System

The o/p of the system at any time 't' depends on past o/p, past i/p and present i/p is called a feedback system.

The o/p depends only on present and past i/p is called non feedback system.

$a_0 \frac{dy(t)}{dt}$