UNIT-I

Mathematical operations on Continuous Signals

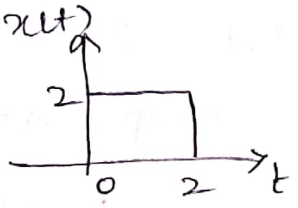
(1) Scaling operation

(i) Amplitude Scaling :

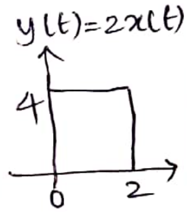
$$x(t) \xrightarrow[\beta]{A.S} y(t) = \beta\, x(t) \qquad \beta \neq 0.$$

Case (i) : $|\beta| > 1$ $\quad \beta \in (-\infty, -1) \cup (1, \infty)$

$$x(t) = \begin{cases} 0, & t < 0 \\ 2, & 0 \leq t \leq 2 \\ 0, & t > 2 \end{cases}$$

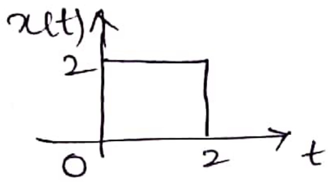


$\xrightarrow[|\beta| = 2 > 1]{A.S}$

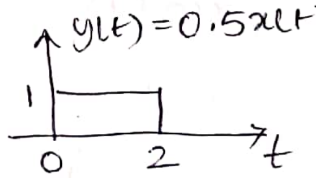


* Amplification

Case (ii): $|\beta| < 1$ $\quad \beta \in (-1, 0) \cup (0, 1)$



$\xrightarrow[\beta = 0.5]{A.S}$



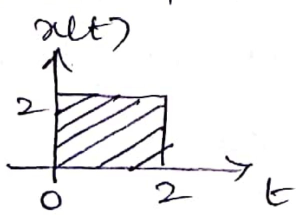
* Reduction (Attenuation)

(ii) Time Scaling

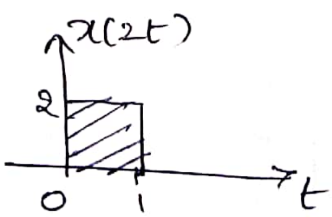
The compression or expansion of a signal in time.

$$x(t) \xrightarrow{T.S} y(t) = x(\alpha t), \quad \alpha \neq 0.$$

Case (i): $|\alpha| > 1$ $\quad \alpha \in (-\infty, -1) \cup (1, \infty)$



$\xrightarrow{\alpha = 2}$



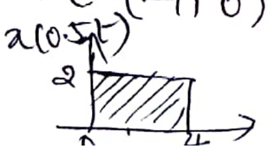
Compression

Case (ii) $\quad$ $x(2t) = 2$ for $0 \leq t \leq 1$

$|\alpha| < 1, \alpha \neq 0.$

$\alpha \in (-1, 0) \cup (0, 1)$

$\alpha = 0.5$



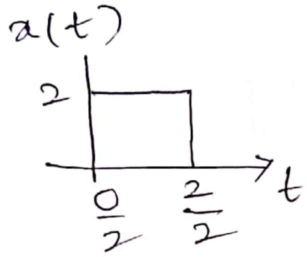
expansion

Short cut Method

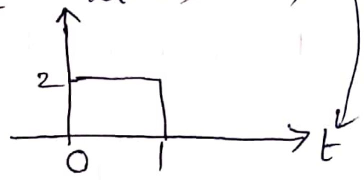
$$x(t) \xrightarrow{TS} y(t) = x(\alpha t), \ \alpha \neq 0$$

$\frac{\alpha t}{\alpha}$ $\quad x(2t) \Rightarrow \frac{2t}{2}$

Case(i)



$\longrightarrow$



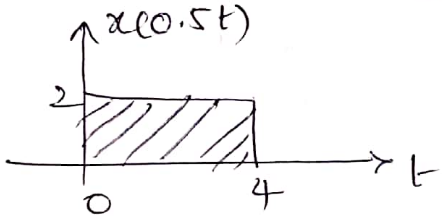
Case(ii) $\alpha = 0.5$

$x(t) = 2, \ 0 \leq t \leq 2$

(i) Amp. Same (ii) $\frac{\alpha t}{\alpha} \Rightarrow \alpha = 0.5 \Rightarrow \frac{0.5t}{0.5} = t.$

$\rightarrow x(0.5t) = 2, \ \frac{0}{0.5} \leq t \leq \frac{2}{0.5}$

i.e. $x(0.5t) = 2, \ 0 \leq t \leq 4.$



(2)

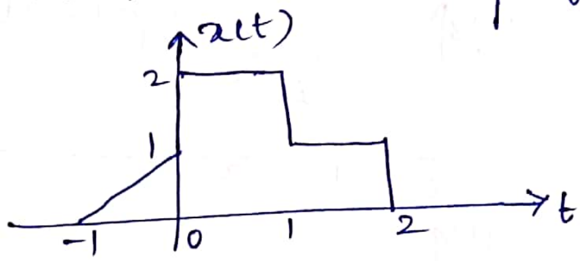
1) For the signal $x(t)$ find the following signals

i) $x(t-2)$

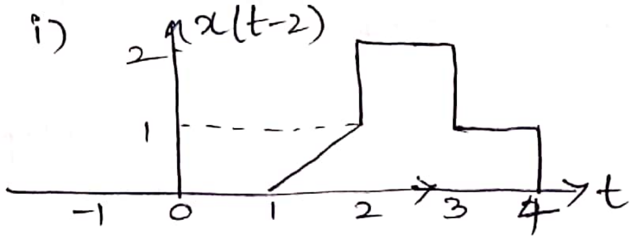
ii) $x(2t+3)$

iii) $x(\frac{3}{2}t)$

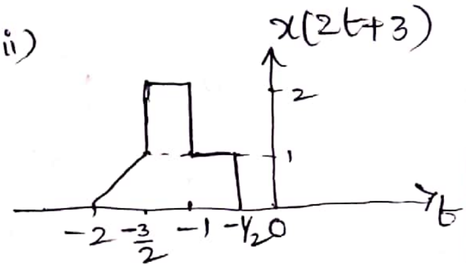
iv) $x(-t+1)$



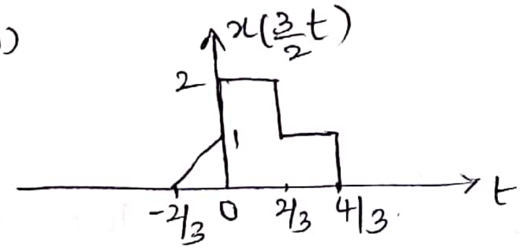
Sol i)



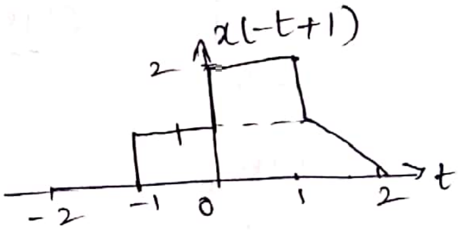
ii)



iii)

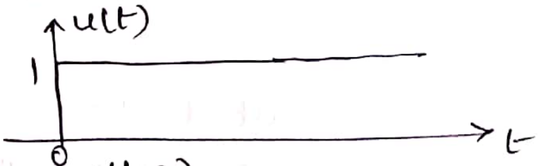


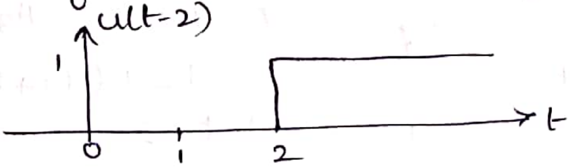
iv)

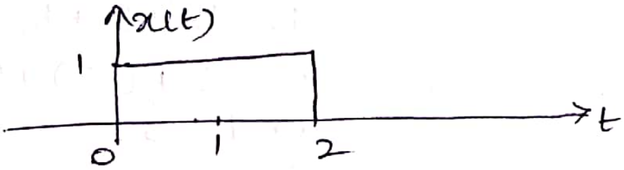


2) Sketch the following signals

i) $u(t) - u(t-2) = x(t)$

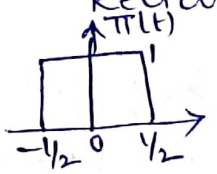






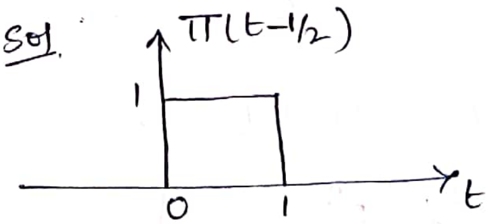
(ii) $\Pi(t - 1/2)$

Rectangular pulse



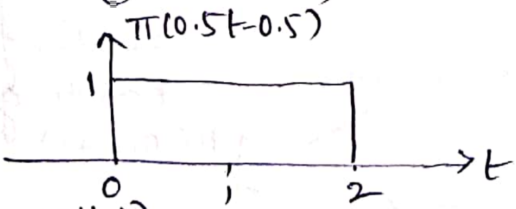
$\Pi(t) = 1, \ |t| \leq 1/2$
$\quad\quad\ \ 0, \text{ else}$

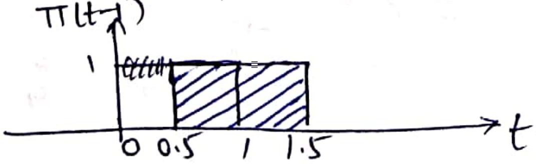
Sol.



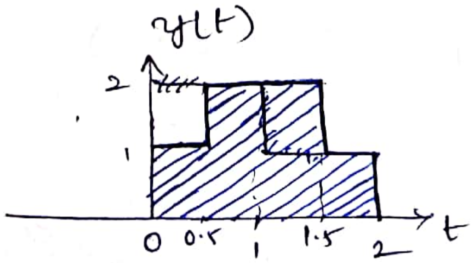
(iii) $\Pi\left(\frac{t-1}{2}\right) + \Pi(t-1)$

Sol $\Pi(0.5t - 0.5) + \Pi(t-1) = y(t)$

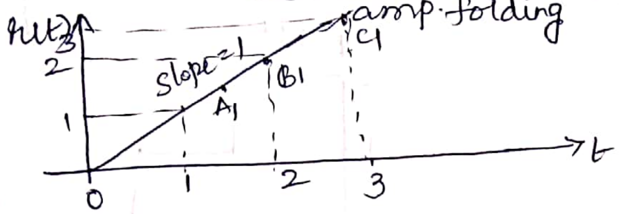




$\Rightarrow$

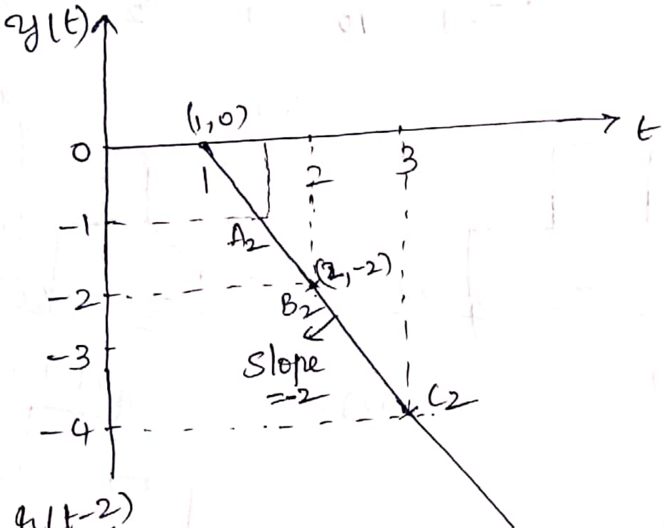


iv) $r(t) - \underbrace{2r(t-1)}_{\text{ramp folding}} + r(t-2)$



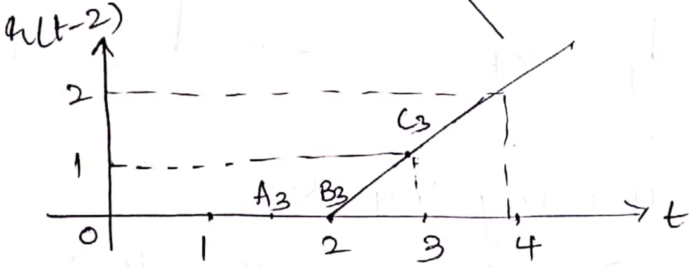
$r(t) = t$, for $t \geq 0$
$\quad\;\; 0$, else

$r(t-1) = t-1$, for $t-1 \geq 0$
i.e. $t \geq 1$
0, else

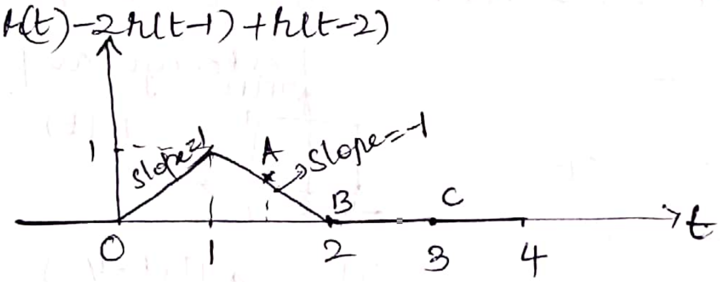


$y(t) = -2r(t-1) = -2(t-1)$, for $t \geq 1$
$= -2t+2$, for $t \geq 1$

$\text{slope} = \frac{-2-0}{2-1} = -2$



$r(t-2) = t-2$, $t \geq 2$
0, else



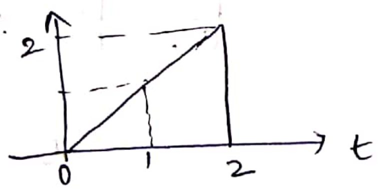
at $t = 1.5$
$A = A_1 + A_2 + A_3$
$= 1.5 + (-1) + 0 = 0.5$

at $t = 2$
$B = B_1 + B_2 + B_3$
$= 2 + (-2) + 0 = 0.$

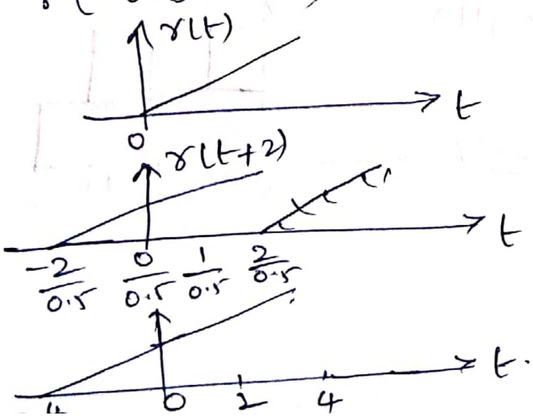
at $t = 3$
$C = C_1 + C_2 + C_3$
$= 3 + (-4) + 1 = 0.$

v) $r(t)\,u(2-t)$

Ans:

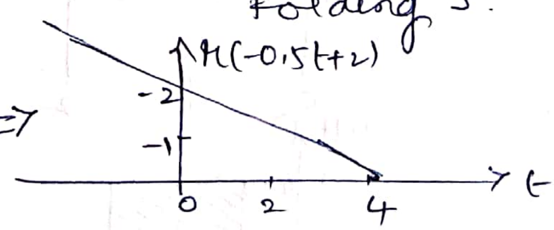


vi) $r(-0.5t+2)$



Shifting
Scaling
Folding

$\Rightarrow$



Problems

1) Sketch the following signals

i) $u\left(\frac{t-1}{4}\right)$ ii) $\Pi(-2t+2)$ iii) $r\left(\frac{t+1}{3}\right)$

iv) $e^{-2t}u(-2+t)$

2) Find whether the following signals are periodic or not.

i) $x(t) = 4\cos 5\pi t$ Ans: periodic with period $\frac{2}{5}$ sec

ii) $x(t) = \sin 10\pi t\, u(t)$ Ans: Aperiodic

iii) $x(t) = e^{-|t|}$ Ans: Aperiodic

iv) $x(t) = 2\cos(10t+1) - \sin(4t-1)$ Ans: periodic with period π sec

v) $\cos 60\pi t + \sin 50\pi t$ Ans: periodic with $\frac{1}{5}$ sec

vi) $2u(t) + 2\sin 2t$ Ans: Aperiodic

vii) $3\cos 4t + 2\sin 2\pi t$ Ans: Aperiodic

viii) $u(t) - \frac{1}{2}$ Ans: Aperiodic

ix) $3\cos\left(17\pi t + \frac{\pi}{3}\right) + 2\sin\left(19\pi t - \frac{\pi}{3}\right)$ Ans: periodic $T = 2$ Sec

x) $u(t) - u(t-10)$ Ans: Aperiodic

xi) $\cos\left(\frac{1}{3}t\right) + \sin\left(\frac{1}{4}t\right)$ Ans: Periodic. $T = 24\pi$

3) Find even and odd component of signals

i) $x(t) = \sin t + 2\sin t + 2\sin^2 t\cos t$

Ans: $x_e(t) = 2\sin^2 t\cos t$

$x_o(t) = \sin t + 2\sin t$

ii) $x(t) = \cos t + \sin t + \cos t\sin t$

Ans: $x_e(t) = \cos t$

$x_o(t) = \sin t + \cos t\sin t$

4) Sketch the following signals and calculate their energies.

i) $e^{-10t}u(t)$ — Ans: $E = \frac{1}{20}$ Joules.

ii) $u(t) - u(t-15)$ — Ans: $E = 15$ J

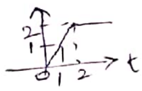
iii) $\cos(10\pi t)\, u(t)\, u(2-t)$ — Ans: $E = 1$ J

iv) Unit step signal — Ans: power signal $E=\infty$, $P = 1/2$

v) Unit ramp signal — Ans: neither energy nor power ($E=\infty, P=\infty$)

5) Whether the following signals are energy or power signals and calculate them?

i) $u(t) - u(t-1)$ — Ans: energy signal. $E=1$J, $P=0$.

ii) $r(t) - r(t-2)$  Ans: $E = \infty$, $P = 2$W power signal

iii) $(1+e^{-5t})\, u(t)$ — Ans: $E=\infty$, $P = \frac{1}{2}$W power signal

iv) $x_1(t) = e^{-3t}u(t)$ — Ans: $E = \frac{1}{6}$, $P=0$ energy signal

v) $x_2(t) = e^{j(2t+\frac{\pi}{4})}$ — Ans: $E=\infty$, $P=1$ power "

vi) $x_3(t) = \cos t$ — Ans: $E=\infty$, $P=\frac{1}{2}$ ∴ power signal

~~vii)~~

6) Find which of the following are causal or non causal?

i) $x_1(t) = e^{at}u(t)$ — Ans: causal

ii) $x_2(t) = e^{-2t}u(-t)$ — Ans: non causal

iii) $x_3(t) = \text{sinc}\, t$ — Ans: non causal

iv) $e^{-2t}u(t-2)$ — Ans: causal

v) $\sin t\; u(t)$ — Ans: causal

7) Unit step power signal. — Ans: $E=\infty$, $P = 1/2$

8) Ramp is neither energy nor power signal.

Classification of Signals :-

1. Deterministic & Non-Deterministic
2. Periodic & Aperiodic
3. Symmetric (even) & Asymmetric (odd)
4. Causal & Non causal signal
5. Energy & power signal.

1. Deterministic & Non-Deterministic :-

A signal $x(t)$ can be expressed by a mathematical expression is called Deterministic signal.

Ex :- sinusoidal, ramp, pulse, step signal etc.

A signal $x(t)$ cannot be expressed by a mathematical expression is called Non - Deterministic signal.

Ex :- Noise, Hum sound coming from amplifiers, oscillators, transformers etc.

2. Periodic & Aperiodic :-

A signal is said to be periodic if it satisfies

$x(t) = x(t+T)$

where, T = fundamental period.

A signal is said to be Aperiodic if it satisfies

$x(t) \neq x(t+T)$

where, T = fundamental period.

$F_0 = \frac{1}{T}$ where, F_0 = fundamental frequency (cycles/second)

$2\pi F_0 = \omega_0$ (or) Ω_0 → Angular frequency (rad/sec).

Examples

① Determine whether the following signals are periodic or not; if it is periodic find the fundamental period.

Sol:- (i) $x(t) = \sin(t)$

$x(t+T) = \sin(t+T)$ for ($T = 2\pi, 4\pi, 6\pi \ldots$)

$= x(t) = \sin(t)$

$\therefore \boxed{T = 2\pi}$ it is periodic signal

(ii) $x(t) = 2\cos\left(\frac{t}{4}\right)$

$x(t+T) = 2\cos\left(\frac{t+T}{4}\right)$

$= 2\cos\left(\frac{t}{4} + \frac{T}{4}\right)$ for $\frac{T}{4} = 2\pi, 4\pi \ldots$

$\Rightarrow \frac{T}{4} = 2\pi$

$= 2\cos\left(\frac{t}{4}\right)$ $T = 8\pi$

$\therefore \boxed{T = 8\pi}$ periodic signal.

(iii) $x(t) = e^{\alpha t}$; $t > 1$

$x(t+T) = e^{\alpha(t+T)}$

$= e^{\alpha t} e^{\alpha T}$

for any values of T $x(t) \neq x(t+T)$

$\therefore$ The given signal is not periodic

(iv) $x(t) = \cos\left(\frac{2\pi}{3} t\right)$

~~$x(t) =$~~

$x(t+T) = \cos\left(\frac{2\pi}{3} t + \frac{2\pi}{3} T\right)$

$= \cos\left(\frac{2\pi}{3} t\right)$ for $\frac{2\pi}{3} T = 2\pi$

$\boxed{T = 3}$

$\therefore \boxed{T = 3}$ it is periodic signal.

NOTE:- If is a signal is a mixture of 2 periodic signals with fundamental periods T_1 & T_2 then the signal is periodic if ratio of $\frac{T_1}{T_2}$ is a rational number.

$$y(t) = x_1(t) + x_2(t)$$

$$\begin{array}{cc} \downarrow & \downarrow \\ T_1 & T_2 \end{array}$$

① $T_1, T_2 \rightarrow \frac{T_1}{T_2}$ (rational)

② LCM of T_1, T_2

Take the LCM of T_1, T_2 to get fundamental periods of $y(t)$.

Examples

① $x(t) = 5\cos 4\pi t + 3\sin 8\pi t$

$x(t+T) = 5\cos(4\pi t + 4\pi T_1) + 3\sin(8\pi t + 8\pi T_2)$

$x_1(t) = 5\cos 4\pi t$

$x_1(t+T) = 5\cos(4\pi(t+T_1))$

$= 5\cos(4\pi t + \underbrace{4\pi T_1}_{2\pi})$ for $4\pi T_2 = 2\pi$

$= x_1(t)$

∴ periodic $4\pi T_1 = 2\pi$

$T_1 = \frac{2\pi}{4\pi} = \frac{1}{2}$

$x_2(t) = 3\sin 8\pi t$

$\cancel{x_2(t)} = x_2(t+T_2) = 3\sin(8\pi(t+T_2))$

$= 3\sin(8\pi t + \underbrace{8\pi T_2}_{2\pi})$

$= 3\sin 8\pi t$

$= x_2(t)$.

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$\therefore$ periodic $8\pi T_2 = 2\pi$

$$T_2 = \frac{1}{4}$$

$\Rightarrow \frac{T_1}{T_2} = \frac{1/2}{1/4} = 2$ (rational)

$\Rightarrow$ LCM of $(T_1, T_2) = (1/2, 1/4)$

$$\begin{array}{r|l} 1/2 & 1/2,\ 1/4 \\ \hline 1/2 & 1,\ 1/2 \\ \hline & 1,\ 1 \end{array}$$

LCM $= 1/2$

$\therefore \boxed{T = 1/2}$

③ Even & odd signal :-

A signal is said to be symmetric (or) even signal if it satisfies the condition

$$x(t) = x(-t)$$

A signal is said to be Asymmetric (or) odd signal if it satisfies the condition

$$x(t) = -x(-t)$$

Even part of signal

$$x_e(t) = \frac{1}{2}[x(t) + x(-t))$$

Odd part of signal

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Examples

Q) Determine whether following signals are odd or even, if not find their even & odd part

(1) $x(t) = e^{-t}$

$x(-t) = e^{-(-t)} = e^{t}$

$\Rightarrow x(t) \neq x(-t)$

$\therefore$ It is not even signal

$\Rightarrow x(t) = -x(-t)$

$e^{-t} = -e^{t}$

$\therefore$ It is not odd signal

$\therefore x_e(t) = \frac{1}{2}(x(t) + x(-t))$

$= \frac{1}{2}(e^{-t} + e^{t})$

$x_o(t) = \frac{1}{2}(x(t) - x(-t))$

$= \frac{1}{2}(e^{-t} - e^{t})$

(2) $x(t) = \sin(t)$

$x(-t) = \sin(-t)$

$= -\sin(t)$

$x(t) \neq x(-t)$

$\therefore$ It is not even signal.

$x(t) = -x(-t)$

$\sin(t) = -(-\sin t)$

$\sin t = \sin t$

$\therefore$ It is odd signal.

3) $x(t) = \sin 2t + \cos t + \sin t \cos 2t$

$x(-t) = -\sin 2t + \cos(t) - \sin t \cos 2t$

$-x(-t) = \sin 2t - \cos t - \sin t \cos 2t$

$\because$ $x(t) \neq x(-t)$ it is not even signal

$x(t) \neq -x(-t)$ it is not odd signal.

$x_e(t) = \frac{1}{2}(\cancel{\sin 2t} + \cos t + \cancel{\sin t \cos 2t}$
$\quad - \cancel{\sin 2t} + \cos t - \cancel{\sin t \cos 2t})$

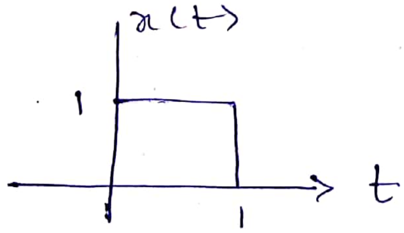
$= \frac{1}{2}(2\cos 2t)$

$= \cos 2t$

$x_o(t) = \frac{1}{2}(2\sin 2t + 2\sin t \cos 2t)$

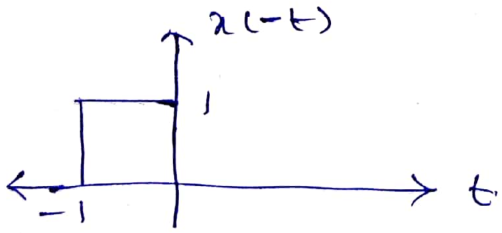
$= \sin 2t + \sin t \cos 2t$

4)

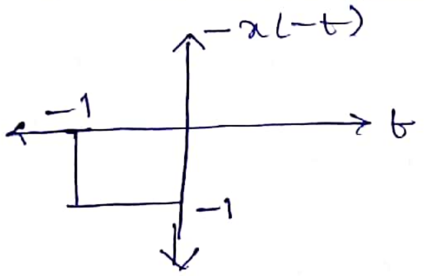


$x(t) = 1 \;;\; 0 \le t \le 1$
$\quad\quad 0 \;;\; \text{else}$

$x(-t) = 1 \;;\; 0 \le -t \le 1 \Rightarrow 0 \ge t \ge -1.$
$\quad\quad 0 \;;\; \text{else}$



$\therefore$ $x(t) \neq x(-t)$ it is not an even signal.

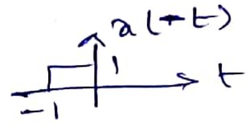


$\therefore x(t) \neq -x(-t)$

It is not an odd signal

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$= \frac{1}{2}\Big[$ 

$+$ 

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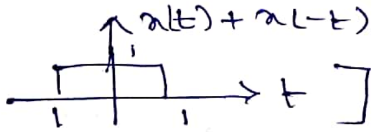
$= \frac{1}{2}\Big[$ 

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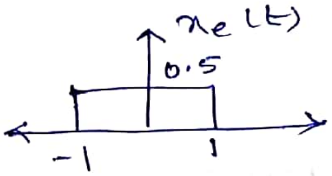
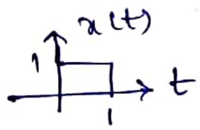
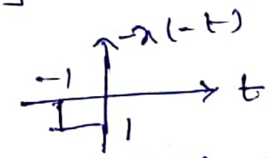
$=$ 

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

$= \frac{1}{2}\Big[$ 

$+$ 

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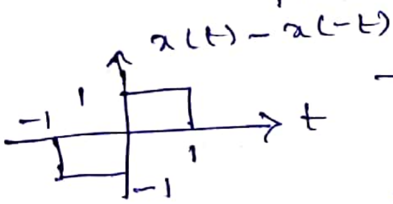
$= \frac{1}{2}\Big[$ 

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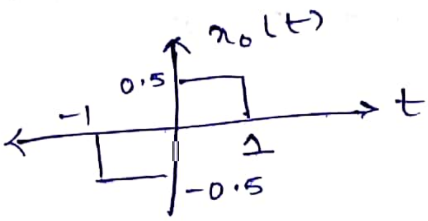
$= \Big[$ 

$\Big]$

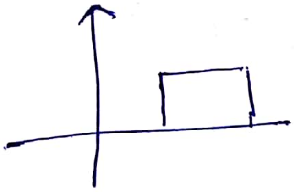
(4) Causal & Non - Causal signals :-

A signal $x(t)$ is defined only for $t \geq 0$ is said to be causal signal.

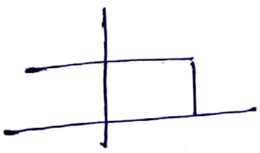
A signal $x(t)$ is defined for either for $t \leq 0$ & $t \geq 0$ is known as Non - causal signal.

If a non - causal signal which is defined only for $t \leq 0$ is known as Anti Causal.

Ex :-



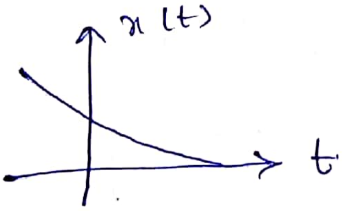
— causal



— Non - causal.

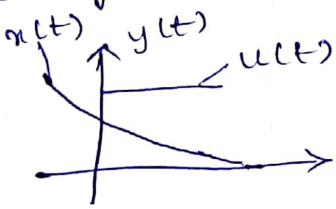
(1) $x(t) = e^{-t}$; for all t.

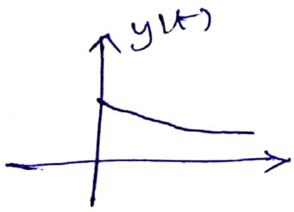
Sol :-



— Non - causal

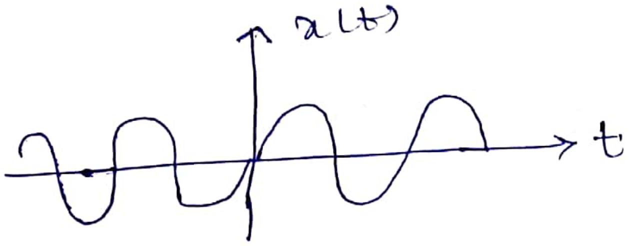
(2) $y(t) = e^{-t} u(t)$; for all t





— causal.

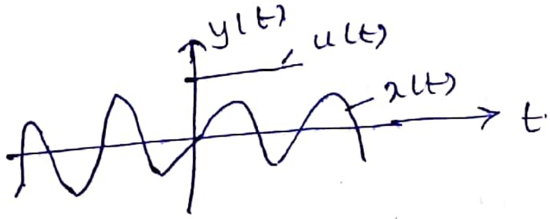
(3) $x(t) = \sin t$; for all t

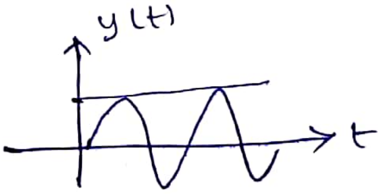


– Non causal.

(4) $y(t) = \sin t \; u(t)$ for all t

<u>sol</u>





– causal.

(5) <u>Energy & Power signal :-</u>

$$E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

A signal is said to be energy signal if energy (E) is finite & power is zero.

[E = finite ; P = 0] – Energy signal.

Ex: All non periodic signals are energy signal.

A signal is said to be power signal if power is finite & energy is infinite.

Ex: Periodic signal are power signals

[E = ∞ ; P = finite] – Power signal.

Q.7 Check whether the following signals are energy or power signals & calculate them.

Sol: $x(t) = e^{-2t} u(t)$ for all 't'

$$E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T \to \infty} \int_{0}^{T} |e^{-2t}|^2 dt$$

$$= \lim_{T \to \infty} \int_{0}^{T} e^{-4t} dt$$

$$= \lim_{T \to \infty} \left[\frac{e^{-4t}}{-4} \right]_0^T$$

$$= \lim_{T \to \infty} \left[\frac{e^{-4T} - 1}{4} \right]$$

$$= \frac{e^{-\infty} - 1}{-4}$$

$$= \frac{\frac{1}{\infty} - 1}{-4}$$

$$= \frac{1}{4} \quad \text{(finite)}$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{0}^{T} |e^{-2t}|^2 dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[\frac{e^{-4t}}{4} \right]_0^T$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[\frac{e^{-4T}-1}{-4}\right]$$

$$= \frac{1}{\infty}\left[\frac{0-1}{-4}\right]$$

$$= 0$$

$\therefore$ Energy is finite & power is zero it is energy signal.

Q) check whether the following signals are energy or power signals. calculate energy or power.

(i) Is step signal energy or power signal.
(ii) Is ramp signal energy or power signal.

Sol: (i) $u(t) = 1$; for $t \geq 0$
$\quad\quad\quad\quad 0$; for $t < 0$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\, dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} |1|^2\, dt$$

$$= \lim_{T\to\infty} t\Big]_0^T$$

$$= \lim_{T\to\infty} T$$

$$\boxed{E = \infty}$$

$\therefore$ Energy is infinite & power is finite it is power signal.

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{0}^{T} |1|^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\, t\Big]_0^T$$

$$= \lim_{T\to\infty} \frac{1}{2\cancel{T}}[\cancel{T}] \Rightarrow \boxed{\frac{1}{2} = P}$$

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(ii) $x(t) = t$; for $t \geq 0$

$= 0$; for $t < 0$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} |t|^2 dt$$

$$= \lim_{T\to\infty} \left. \frac{t^3}{3} \right]_0^T$$

$$= \lim_{T\to\infty} \frac{T^3}{3}$$

$$= \frac{\infty^3}{3}$$

$$\boxed{E = \infty}$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_0^T |t|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left. \frac{t^3}{3} \right]_0^T$$

$$= \lim_{T\to\infty} \frac{1}{2\cancel{T}} \frac{T^{\cancel{3}}}{3}$$

$$= \lim_{T\to\infty} \frac{T^2}{3}$$

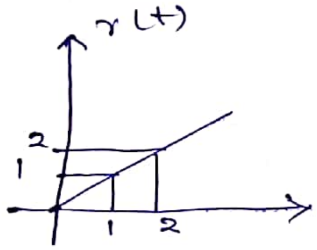
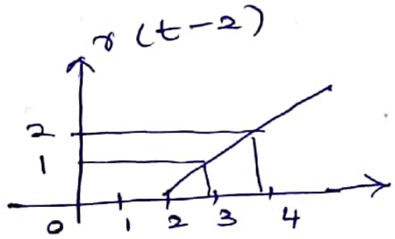
$$\boxed{P = \infty}$$

∵ Both energy & power are infinite the conditions for energy & power signal are not satisfied.

∴ It is neither energy nor power signal.

Q7 $x(t) = r(t) - r(t-2)$; check whether it is energy or power signal.

Sol: $x(t) = r(t) - r(t-2)$

$=$  $-$ 

$=$ 

$x(t) = t \; ; \; 0 \le t \le 2$
$\quad\;\; = 2 \; ; \; t \ge 2$

$$E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \int_0^2 |r(t)|^2 \, dt + \int_2^T |u(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \int_0^2 t^2 \, dt + \int_2^T |2|^2 \, dt$$

$$= \lim_{T \to \infty} \left. \frac{t^3}{3} \right]_0^2 + 4 \left. t \right]_2^T$$

$$= \lim_{T \to \infty} \left\{ \left[\frac{8}{3} \right] + 4 \left[T - 2 \right] \right\}$$

$$= \left\{ \frac{8}{3} + 4 (\infty - 2) \right\}$$

$$\boxed{E = \infty}$$

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$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{2} t^2 \, dt + \int_{2}^{T} |2|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left\{ \left. \frac{t^3}{3} \right]_0^2 + 4 \left. t \right]_2^T \right\}$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left\{ \left(\frac{8}{3}\right) + 4\,(T-2) \right\}$$

$$= \lim_{T\to\infty} \frac{1}{\cancel{2}T} \frac{\cancel{8}^{4}}{3} + \frac{\cancel{4}^{2}\cancel{T}}{\cancel{2T}} - \frac{\cancel{8}^{4}}{\cancel{2}T}$$

$$= \lim_{T\to\infty} \frac{4}{T3} + 2 - \frac{4}{T}$$

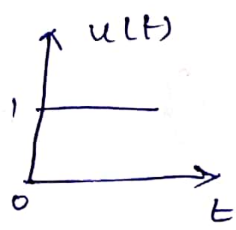
$$= \frac{4}{3\infty} + 2 - \frac{4}{\infty}$$

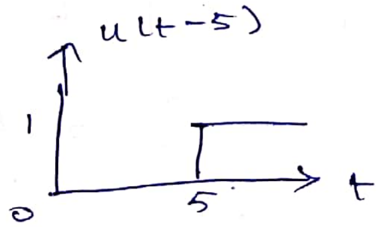
$$= 0 + 2 - 0$$

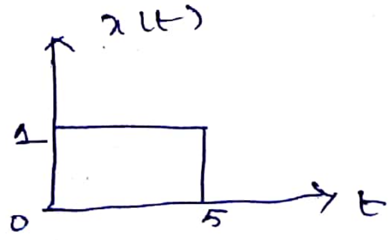
$$\boxed{P = 2.}$$

$\therefore$ Energy is infinite & power is finite it is power signal.

Q) $x(t) = u(t) - u(t-5)$

$=$ 

$-$ 

$=$ 

$x(t) = 1 \; ; \; 0 \le t \le 5$
$\quad\;\; = 0 \; ; \text{ else}$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \int_{0}^{5} (1)^2 \, dt$$

$$= \lim_{T\to\infty} [t]_0^5$$

$$= \lim_{T\to\infty} 5$$

$$\boxed{E = 5}$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{5} (1)^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} [t]_0^5$$

$$= \lim_{T\to\infty} \frac{1}{2T} [5]$$

$$= \frac{1}{2\times\infty} 5$$

$$= \frac{1}{\infty} 5$$

$$\boxed{P = 0}$$

$\therefore$ Energy is finite & power is zero.

It is an energy signal.

Classification of continuous systems :-

① Dynamic & static system
② Time invariant (TIV) & Time variant system (TV)
③ Linear & Non linear. system.
④ Causal & Non causal system.
⑤ stable & unstable system.
⑥ feedback & Non feedback system.

① **static & Dynamic system** :- A system is said to be static or memory less if its output depends only on present input but not on future or past inputs.

In any other case the system is said to be dynamic or to have memory.

Ex :- Q.) $y(t) = a x(t)$

$$t=0 \rightarrow y(0) = a x(0)$$
$$t=2 \rightarrow y(2) = a x(2)$$
} static

Q.) $y(t) = a x(t^2)$

$$t=0 \rightarrow y(0) = a x(0)$$
$$t=1 \rightarrow y(1) = a x(1)$$
$$t=2 \rightarrow \underset{\text{present}}{y(2)} = a x(\underset{\text{future}}{4})$$
} Dynamic

Q.) $y(t) = x(t) + x(t-2)$

$$y(1) = x(1) + x(-1)$$
$$y(2) = x(2) + x(0)$$
} Dynamic

② Time Invariant & Time Variant :-

A system is said to be time invariant if its input and output characteristics does not change with time.

$x(t) \longrightarrow \boxed{\text{Delay}} \xrightarrow[\text{i/p}]{x(t-m) \text{ Delayed}} \boxed{\text{system}} \longrightarrow y_1(t)$

i/p — Delayed i/p response.

$x(t) \longrightarrow \boxed{\text{system}} \xrightarrow[\text{o/p response}]{y(t)} \boxed{\text{Delay}} \longrightarrow y_2(t)$

Delayed o/p response

If $y_1(t) = y_2(t) \longrightarrow$ Time Invariant

$y_1(t) \neq y_2(t) \longrightarrow$ Time Variant.

Ex:- ① $y(t) = 3x^2(t)$

$x(t) \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{\text{system}} \longrightarrow y_1(t) = 3x^2(t-m)$

i/p

$x(t) \longrightarrow \boxed{\text{system}} \xrightarrow[= 3x^2(t)]{y(t)} \boxed{\text{Delay}} \longrightarrow y_2(t) = 3x^2(t-m)$

$\therefore y_1(t) = y_2(t)$

It is time invariant.

② $y(t) = 3x(t^2)$

$x(t) \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{\text{system}} \longrightarrow y_1(t) = 3x((t-m)^2)$

i/p

$x(t) \longrightarrow \boxed{\text{system}} \xrightarrow[= 3x(t^2)]{y(t)} \boxed{\text{Delay}} \longrightarrow y_2(t) = 3x(t^2-m)$

$\therefore y_1(t) \neq y_2(t)$ It is time variant.

③ $y(t) = t\,x(t)$

$\underset{i/p}{x(t)} \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{\text{system}} \longrightarrow y_1(t) = t\,x(t-m)$

$x(t) \longrightarrow \boxed{\text{system}} \xrightarrow{t\,x(t)} \boxed{\text{Delay}} \longrightarrow y_2(t) = (t-m)\,x(t-m)$

$\therefore y_1(t) \neq y_2(t)$

It is time variant system.

④ $y(t) = x(-t)$

$x(t) \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{\text{system}} \longrightarrow y_1(t) = x(-t+m)$

$x(t) \longrightarrow \boxed{\text{system}} \xrightarrow{x(-t)} \boxed{\text{Delay}} \longrightarrow y_2(t) = x(-t-m)$

$\therefore y_1(t) \neq y_2(t)$

It is time variant system.

⑤ $y(t) = x(t+c)$

$x(t) \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{\text{system}} \longrightarrow y_1(t) = x(t+c-m)$

$x(t) \longrightarrow \boxed{\text{system}} \xrightarrow{x(t+c)} \boxed{\text{Delay}} \longrightarrow y_2(t) = x(t+c-m)$

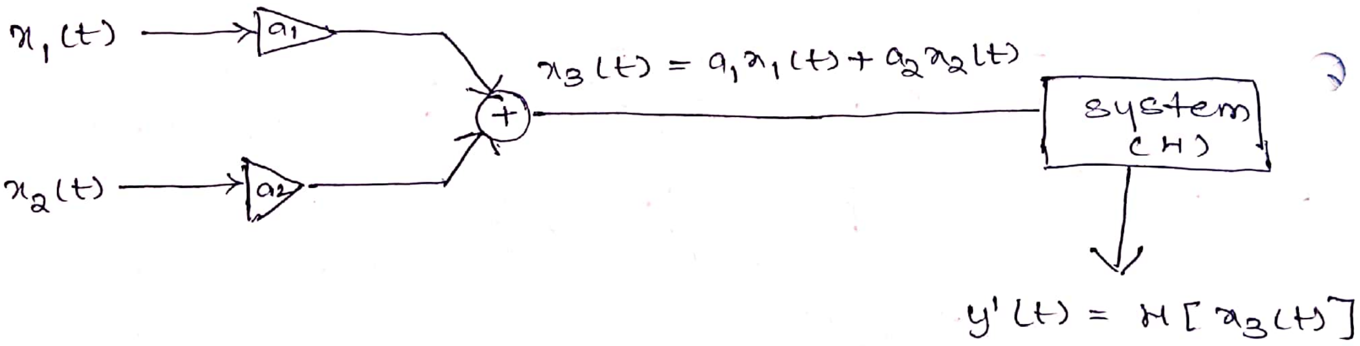
$\therefore y_1(t) = y_2(t)$

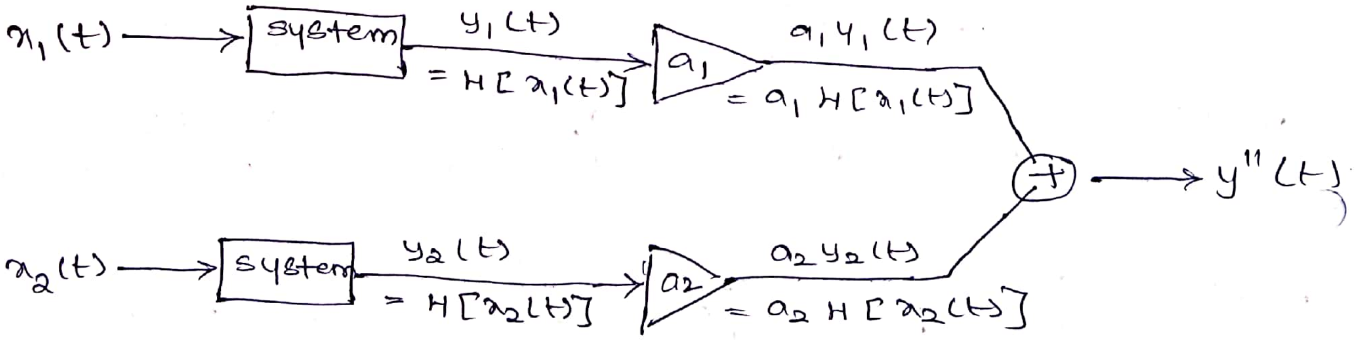
It is time invariant.

(3) Linear & Non-linear System :-

A system is said to be linear system if it satisfies superposition principle.

The superposition principle states that the response of the weighted sum of the signals is equal to the corresponding weighted sum of the responses of individual signals.





$$y'(t) = y''(t) \longrightarrow \text{Linear}$$

$$H[a_1x_1(t) + a_2x_2(t)] = a_1 H[x_1(t)] + a_2 H[x_2(t)]$$

$$y'(t) \neq y''(t) \longrightarrow \text{Non-linear.}$$

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Q.7 Check whether the following systems are linear or not.

① $y(t) = 3x^2(t)$

$y'(t) = H[x_3(t)] = 3x_3^2(t)$

$= 3[a_1x_1(t) + a_2x_2(t)]^2$ — ①

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = 3x_1^2(t)$

$y_2(t) = H[x_2(t)] = 3x_2^2(t)$

$y''(t) = a_1y_1(t) + a_2y_2(t)$

$= a_1 3x_1^2(t) + a_2 3x_2^2(t)$

$\neq y'(t)$

∴ It is non linear.

② $y(t) = 3x(t^2)$

$y'(t) = H[x_3(t)] = 3x_3(t^2)$

$= 3[a_1x_1(t^2) + a_2x_2(t^2)]$ — ①

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = 3x_1(t^2)$

$y_2(t) = H[x_2(t)] = 3x_2(t^2)$

$y''(t) = a_1y_1(t) + a_2y_2(t)$

$= a_1 3x_1(t^2) + a_2 3x_2(t^2)$

$= 3[a_1x_1(t^2) + a_2x_2(t^2)]$

$= y'$

∴ It is linear.

③ $y(t) = t\,x(t)$

$y'(t) = H[x_3(t)] = t\,x_3(t)$

$= t[a_1x_1(t) + a_2x_2(t)]$ — ①

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = t\,x_1(t)$

$y_2(t) = H[x_2(t)] = t\,x_2(t)$

$y'' = a_1y_1(t) + a_2y_2(t)$

$= a_1 t\,x_1(t) + a_2 t\,x_2(t)$

$= t[a_1x_1(t) + a_2x_2(t)]$

$= y'$

∴ It is linear system.

④ $y(t) = x(t) + c$

$y'(t) = H[x_3(t)] = x_3(t) + c$

$= [a_1x_1(t) + a_2x_2(t)] + c$

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = x_1(t) + c$

$y_2(t) = H[x_2(t)] = x_2(t) + c$

$y'' = a_1y_1(t) + a_2y_2(t)$

$= a_1x_1(t) + c + a_2x_2(t) + c$

$= a_1x_1(t) + a_2x_2(t) + 2c.$

$\neq y'$

∴ It is non-linear. system.

5) $y(t) = e^{x(t)}$

$y'(t) = H[x_3(t)] = e^{x_3(t)}$

$= e^{(a_1 x_1(t) + a_2 x_2(t))}$

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = e^{x_1(t)}$

$y_2(t) = H[x_2(t)] = e^{x_2(t)}$

$y'' = a_1 e^{x_1(t)} + a_2 e^{x_2(t)}$

$\neq y'$

∴ It is non linear system.

6) $y(t) = 2x(t) + \frac{dx(t)}{dt}$

$y'(t) = H[x_3(t)] = 2x_3(t) + \frac{dx_3(t)}{dt}$

$= 2(a_1 x_1(t) + a_2 x_2(t))$

$+ \frac{d(a_1 x_1(t) + a_2 x_2(t))}{dt}$

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = 2x_1(t) + \frac{dx_1(t)}{dt}$

$y_2(t) = H[x_2(t)] = 2x_2(t) + \frac{dx_2(t)}{dt}$

$y''(t) = a_1 y_1(t) + a_2 y_2(t)$

$= a_1\left(2x_1(t) + \frac{dx_1(t)}{dt}\right) + a_2\left(2x_2(t) + \frac{dx_2(t)}{dt}\right)$

$= y'(t)$

∴ It is linear system.

(7) $y(t) = x(t) \sin 200\pi t$

$y'(t) = H[x_3(t)] = x_3(t) \sin 200\pi t$

$= (a_1 x_1(t) + a_2 x_2(t)) \sin 200\pi t.$

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = x_1(t) \sin 200\pi t$

$y_2(t) = H[x_2(t)] = x_2(t) \sin 200\pi t$

$y''(t) = a_1 y_1(t) + a_2 y_2(t)$

$= a_1 x_1(t) \sin 200\pi t + a_2 x_2(t) \sin 200\pi t$

$= y'$

$\therefore$ It is linear system.

(8) $y(t) = \cos x(t)$

$y'(t) = H[x_3(t)] = \cos(x_3(t))$

$= \cos(a_1 x_1(t) + a_2 x_2(t))$

Let $x_1(t)$ & $x_2(t)$

$y_1(t) = H[x_1(t)] = \cos x_1(t)$

$y_2(t) = H[x_2(t)] = \cos x_2(t)$

$y''(t) = a_1 y_1(t) + a_2 y_2(t)$

$= a_1 \cos x_1(t) + a_2 \cos x_2(t)$

$\neq y'(t)$

$\therefore$ It is non linear system.

(4) Causal & Non Causal system :-

A system is said to be causal if the output of the system depends only on present input, past input and past output. But not on future input & output.

A system is said to be non-causal if the o/p depends on future input & future output is known as Non-causal system.

Examples

(1) $y(t) = x(t) + x(t-2)$

$t = -3 \Rightarrow y(-3) = x(-3) + x(-5)$

$t = 0 \Rightarrow y(0) = x(0) + x(-2)$

$t = 2 \Rightarrow y(2) = x(2) + x(0)$

$\therefore$ It depends only on past & present i/p

it is causal system.

(2) $y(t) = x(t) - x(3-t)$

$t = -2 \Rightarrow y(-2) = x(-2) - x(5)$

$\therefore$ It depends on future it is non causal system.

(3) $y(t) = 3x(t^2)$

$t = -2 \Rightarrow y(-2) = 3x(4)$

$t = 0 \rightarrow y(0) = 3x(0)$

$t = 2 \Rightarrow y(2) = 3x(4)$

$\therefore$ It is non causal system

(4) $y(t) = 3x^2(t)$

$t = 2 \rightarrow y(2) = 3x^2(2)$

$t = 0 \Rightarrow y(0) = 3x^2(0)$

$t = -2 \Rightarrow y(-2) = 3x^2(-2)$

⑤ $y(t) = x(-t)$

$t=-2 \; y(-2) = x(2)$

$t=0 \Rightarrow y(0) = x(0)$

$t=2 \Rightarrow y(2) = x(-2)$

$\therefore$ It is non causal syetem.

⑥ $y(t) = t\,x(t)$

$t=-2 \Rightarrow y(t) = -2x(-2)$

$t=0 \Rightarrow y(0) = 0\,x(0)$

$t=2 \Rightarrow y(2) = 2x(2)$

$\therefore$ It is causal system.

⑦ $y(t) = x(2t)$

$t=-2 \Rightarrow y(-2) = x(-4)$

$t=0 \Rightarrow y(0) = x(0)$

$t=2 \Rightarrow y(2) = x(4)$

$\therefore$ It is non causal system.

⑧ $y(t) = x(t) + \int_0^t x(\lambda)\,d\lambda$

Sol: $t=-2 \Rightarrow y(-2) = x(-2) + \int_0^{-2} x(\lambda)\,d\lambda$

$\Rightarrow y(-2) = x(-2) + [\cancel{z}(\lambda)]_0^{-2}$

$\cancel{z}(-2) - \cancel{z}(0)$

$\cancel{z}(-2) + \cancel{z}(2)$

$\therefore$ It is causal system.

⑨ $y(t) = x(t) + \int_0^{3t} x(\lambda)\,d\lambda$

$= x(t) + [z(\lambda)]_0^{3t}$

$t=1 \Rightarrow y(1) = x(1) + z(3) - z(0)$

$\therefore$ It is non causal system.

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(10) $y(t) = x(t) \sin 200\pi t$

$y(0) = x(0) . \sin 200\pi t$

$y(2) = x(2) \sin 200\pi t$

$\therefore$ It is causal system.

(5) <u>stable and Unstable system</u> :-

A system is said to be BIBO stable if and only if for every bounded input produces bounded output.

(BIBO — bounded input and bounded output).

$$\boxed{\int_{-\infty}^{\infty} |h(t) dt| < \infty} \longrightarrow \text{stable}$$

else unstable.

Q) Test the stability of LTI system

1) $h(t) = e^{-5|t|}$

<u>sol</u>

$$\int_{-\infty}^{\infty} e^{-5|t|} dt$$

$$= \int_{-\infty}^{0} |e^{-5|t|} dt| + \int_{0}^{\infty} e^{-5t} dt$$

$$= \int_{-\infty}^{0} e^{5t} dt + \int_{0}^{\infty} e^{-5t} dt$$

$$= \left.\frac{e^{5t}}{5}\right]_{-\infty}^{0} + \left[\frac{e^{-5t}}{5}\right]_{0}^{\infty}$$

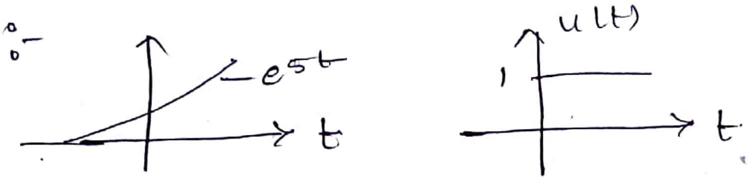
$$= \left[\frac{1-0}{5}\right] + \left[\frac{0-1}{-5}\right]$$

$$= \frac{1}{5} + \frac{1}{5} \Rightarrow \boxed{\frac{2}{5} < \infty}$$

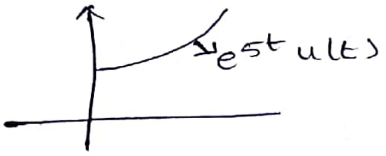
$\therefore$ It is stable.

(2) $h(t) = e^{5t}\,u(t)$

<u>sol</u> :-



$=$



$$= \int_{-\infty}^{\infty} |h(t)|\,dt$$

$$= \int_{-\infty}^{\infty} |e^{5t}\,u(t)|\,dt$$

$$= \cancel{\int_{-\infty}^{0} e^{5t}\,dt} \xrightarrow{} 0 + \int_{0}^{\infty} e^{5t}(1)\,dt$$

$$= \int_{0}^{\infty} e^{5t}\,dt$$

$$= \left.\frac{e^{5t}}{5}\right]_{0}^{\infty}$$

$$= \frac{1}{5}\left[e^{\infty} - 1\right]$$

$$= \frac{1}{5}\left[\infty - 1\right]$$

$$= \infty$$

$\therefore$ It is unstable.

(8) $h(t) = te^{-3t}u(t)$

Sol: $\int_0^\infty te^{-3t}dt$

$= \left[t\frac{e^{-3t}}{3} - \frac{e^{-3t}}{9} \right]_0^\infty$

$= [0 - 0]$

$= 0 < \infty$

$\therefore$ It is stable.

(9) $h(t) = t\cos t\, u(t)$

Sol: $= \int_0^\infty t\cos t$

$= [t\sin t + \cos t]_0^\infty$

$= \infty$

(6) Feedback & Non feedback system:

- The o/p of the system at any time 't' depends on past o/p, past i/p & present i/p is called a feedback system.

The o/p depends only on present & past input is called Non-feedback system

Classification of Continuous time Systems

1) Static and Dynamic systems
2) Time invariant and Time variant System
3) Linear and non linear system
4) Causal and non-causal system
5) Stable and unstable system
6) Feedback and non feedback system.

① Static and dynamic system

A system is said to be static (or) memory-less if its o/p depends on i/p signal at the same time but not on past or future i/p. In any other case the system is said to be dynamic (or) to have memory.

Ex:

$$\left.\begin{aligned} y(t) &= a\,x(t) \\ y(t) &= t\,x(t) + 6x^3(t) \end{aligned}\right\} \text{Static System}$$

$$\left.\begin{aligned} y(t) &= t\,x(t) + 3x(t^2) \\ y(t) &= x(t) + 3x(t-2) \end{aligned}\right\} \text{Dynamic system.}$$

② Time invariant and Time variant system

A system is said to be TIV if its i/p-o/p characteristics does not change with time.

$$x(t) \xrightarrow{H} y(t)$$

$$x(t-m) \xrightarrow{H} y(t-m)$$

$$\underset{\text{i/p}}{x(t)} \longrightarrow \boxed{\text{Delay}} \xrightarrow[\text{delayed i/p}]{x(t-m)} \boxed{\begin{array}{c}\text{System}\\ H\end{array}} \longrightarrow \underset{\text{Response for delayed i/p}}{y_1(t)}$$

$$\underset{\text{i/p}}{x(t)} \longrightarrow \boxed{\text{System}} \xrightarrow{y(t)} \boxed{\text{Delay}} \longrightarrow \underset{\text{Delayed response.}}{y_2(t)}$$

$$\boxed{\begin{array}{l} y_1(t) = y_2(t) \longrightarrow \text{TIV} \\ y_1(t) \neq y_2(t) \longrightarrow \text{TV.} \end{array}}$$

State whether the following systems are Time invariant or not.

① $y(t) = 3x^2(t)$

$$x(t) \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{\text{System}} \longrightarrow y_1(t) = 3x^2(t-m)$$

$$x(t) \longrightarrow \boxed{H} \xrightarrow{3x^2(t)} \boxed{\text{Delay}} \longrightarrow y_2(t) = 3x^2(t-m)$$

$$y_1(t) = y_2(t) \longrightarrow \therefore \text{TIV}$$

② $y(t) = 3x(t^2)$

$$x(t) \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{H} \longrightarrow y_1(t) = 3x(t^2-m)$$

$$x(t) \longrightarrow \boxed{H} \xrightarrow{3x(t^2)} \boxed{\text{Delay}} \longrightarrow y_2(t) = 3x((t-m)^2)$$

$$y_1(t) \neq y_2(t) \longrightarrow \text{TV.}$$

③ $y(t) = 2t\,x(t)$

$$x(t) \longrightarrow \boxed{\text{Delay}} \xrightarrow{x(t-m)} \boxed{H} \longrightarrow y_1(t) = 2t\,x(t-m)$$

$$x(t) \longrightarrow \boxed{H} \xrightarrow{2t\,x(t)} \boxed{\text{Delay}} \longrightarrow y_2(t) = 2(t-m)x(t-m)$$

$$y_1(t) \neq y_2(t)$$

TV

(4) $y(t) = x(t) \sin 20\pi t$ Ans: TV

(5) $y(t) = x(-t)$ Ans: TV.

(6) $y(t) = 2e^{x(t)}$ Ans: TIV

(7) $y(t) = x(t) + C$ Ans: TIV.

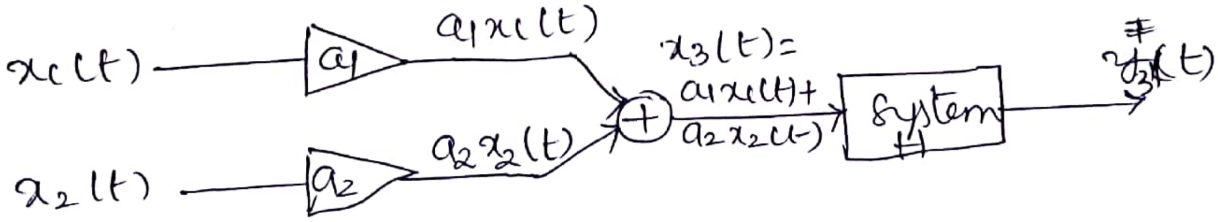
(8) $y(t) = \text{Odd}[x(t)]$

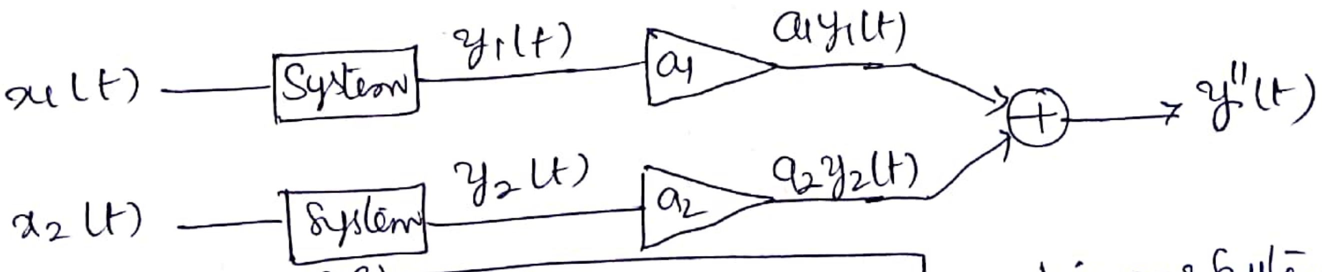
Sol $y(t) = \dfrac{x(t) - x(-t)}{2}$ Ans: TV

(3) Linear and Non linear System

- A linear system is the one that satisfies superposition principle.
  The principle of superposition requires that the response of a system to a weighted sum of the signals is equal to the correspon-ding weighted sum of the responses to each of the ~~following~~ individual signals.

- $$H[a_1 x_1(t) + a_2 x_2(t)] = a_1 H[x_1(t)] + a_2 H[x_2(t)]$$





(OR) i.e. $y_3(t) = a_1 y_1(t) + a_2 y_2(t)$

$\boxed{y_3(t) = y''(t)}$ → Linear System

$y_3(t) \neq y''(t)$ → Non linear

Q: Whether the following systems are linear or not.

① $y(t) = 3x^2(t)$

Sol Let two signals $x_1(t)$ & $x_2(t)$

$$y_1(t) = H[x_1(t)] = 3x_1^2(t)$$

$$y_2(t) = H[x_2(t)] = 3x_2^2(t)$$

Let $x_3(t) = a_1x_1(t) + a_2x_2(t)$

$$\therefore y_3(t) = H[x_3(t)]$$

$$= 3[a_1x_1(t) + a_2x_2(t)]^2$$

$$\therefore y_3(t) \neq a_1y_1(t) + a_2y_2(t)$$

Non linear system.

② $y(t) = 3x(t^2)$

Sol
$$y_1(t) = 3x_1(t^2)$$
$$y_2(t) = 3x_2(t^2)$$

Let $x_3(t) = a_1x_1(t) + a_2x_2(t)$

$$\therefore y_3(t) = H[x_3(t)]$$

$$= 3[a_1x_1(t^2) + a_2x_2(t^2)]$$

$$= 3a_1x_1(t^2) + 3a_2x_2(t^2) \text{ ---(1)}$$

$$y_3(t) = a_1y_1(t) + a_2y_2(t)$$

$\therefore$ Linear system

③ $y(t) = t\,x(t)$

Sol
$$y_1(t) = t\,x_1(t)$$
$$y_2(t) = t\,x_2(t)$$

Let $x_3(t) = a_1x_1(t) + a_2x_2(t)$

$$\therefore y_3(t) = t[a_1x_1(t) + a_2x_2(t)]$$

$$= a_1y_1(t) + a_2y_2(t)$$

$\therefore$ linear system

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④ $y(t) = x(t) + c$

Sol $y_1(t) = x_1(t) + c$

$y_2(t) = x_2(t) + c$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$\therefore y_3(t) = [a_1 x_1(t) + a_2 x_2(t)] + c$

$= a_1 x_1(t) + a_2 x_2(t) + c + c - c$

$= a_1 y_1(t) + a_2 y_2(t) \mp c$

$\therefore y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$

Non linear System.

⑤ $y(t) = e^{x(t)}$

Sol $y_1(t) = e^{x_1(t)}$

$y_2(t) = e^{x_2(t)}$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$\therefore y_3(t) = H[x_3(t)] = e^{a_1 x_1(t) + a_2 x_2(t)}$

$\therefore y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$

Non linear System.

⑥ $y(t) = 2x(t) + \frac{dx(t)}{dt}$ Ans: Linear

⑦ $y(t) = x(t) \sin 20\pi t$ Ans: linear

⑧ $y(t) = \cos(x(t))$ Ans: Non linear

4) Causal and non causal system

A system is said to be causal if the o/p of the system depends only on the present i/p, past i/p and past o/p but not on future i/p & o/p's

If the system o/p depends on future i/p & o/p's then the system is called a non causal system.

Q: Test the causality of the following systems.

1) $y(t) = x(t) + x(t-2)$ Ans: Causal

2) $y(t) = x(t) - x(3-t)$ Ans: non causal

3) $y(t) = 3x^2(t)$ Ans: Causal

4) $y(t) = 3x(t^2)$ Ans: non causal

5) $y(t) = x(-t)$ Ans: Non Causal

6) $y(t) = x(t) + C$ Ans: Causal

7) $y(t) = x(2t)$ Ans: Non causal

8) $y(t) = x(t) + \int_0^t x(\lambda)\, d\lambda$ Ans: causal

9) $y(t) = x(t) + \int_0^{3t} x(\lambda)\, d\lambda$ Ans: non causal.

Sol $y(t) = x(t) + [Z(\lambda)]_0^{3t}$ let $\int x(\lambda)\, d\lambda = Z(\lambda)$

$y(t) = x(t) + Z(3t) - Z(0)$

$t = 1 \Rightarrow y(1) = \underset{\text{Present}}{x(1)} + \underset{\text{Future}}{Z(3)} - \underset{\text{past}}{Z(0)}$

$\therefore$ Non causal.

10) $y(t) = x(t) \sin 20\pi t \Rightarrow$ Ans: causal.

5) Stable and Unstable System:

Def: A system is said to be BIBO (Bounded i/p - Bounded o/p) if and only if every bounded i/p produces bounded o/p.

Ex: Bounded i/p signal → step, decaying exponential, Impulse.

Unbounded i/p " → ramp, increasing exponential.

Condition for stability

$$\boxed{\int_{-\infty}^{\infty} |h(t)|\, dt < \infty}$$

Q. Test the stability of LTI system.

1) $h(t) = e^{-5|t|}$

For stability $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

$$\therefore \int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} e^{-5|t|}\, dt$$

$$= \int_{-\infty}^{0} e^{5t}\, dt + \int_{0}^{\infty} e^{-5t}\, dt$$

$$= \left[\frac{e^{5t}}{5}\right]_{-\infty}^{0} + \left[\frac{e^{-5t}}{-5}\right]_{0}^{\infty}$$

$$= 1/5 + 0 + 0 = \frac{1}{5} < \infty$$

$\therefore$ Stable.

2) $h(t) = e^{5t} u(t)$

Sol $\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} e^{5t} u(t)\, dt$

$$= \int_{0}^{\infty} e^{5t}\, dt = \left[\frac{e^{5t}}{5}\right]_{0}^{\infty} = \infty$$

$\therefore$ Unstable

3) $h(t) = t e^{-3t} u(t)$ Ans: Stable.

4) $h(t) = t \cos t \, u(t)$

Sol $= \int_{-\infty}^{\infty} |t \cos t \, u(t)| \, dt$

$\int u \, dv = uv - \int v \, du$.

$= \int_{0}^{\infty} \underset{u}{t} \, \underset{dv}{\cos t \, dt}$

$= [t \sin t]_0^{\infty} - \int_0^{\infty} \sin t \cdot 1 \, dt$

$= (\infty - 0) + [\cos t]_0^{\infty} = \infty$

$\therefore$ Unstable

5) $h(t) = e^{-t} \sin t \, u(t)$ Ans: Stable.

6) Determine the range of values of 'a' & 'b' for the stability of LTI system with impulse response.

Ans: $a < 0$ & $b > 0$

$h(t) = e^{at} u(t) + e^{-bt} u(t)$

6) Feedback and non Feedback System

The o/p of the system at any time 't' depends on past o/p, past i/p and present i/p is called a feedback system.

The o/p depends only on present and past i/p is called non feedback system.

$a_0 \frac{dy(t)}{dt}$

Signal Analysis & Transform Techniques

Assignment - I.

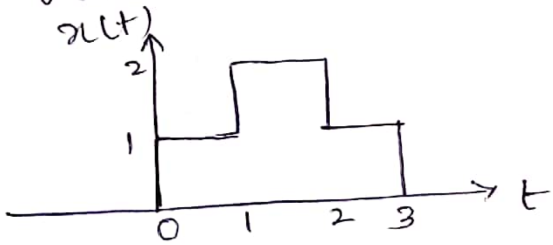
1) Sketch the following signals

a) $u(t) + u(t+2) - u(t-3)$

b) $\Pi\left(\frac{t-1}{2}\right) + \Pi(t-1)$

c) $r(-0.5t+2)$

2) For the signal $x(t)$ find the following signals.



i) $x(t+3)$

ii) $x(-2t+1)$

iii) $x(-t+3)$

3) Verify whether the following systems are Static (or) dynamic, linear or non linear, Time variant or TIV, causal or non causal.

(i) $y(t) = \sqrt{x(t)}$

(ii) $y(t) = t + x(t)$

iii) $y(t) = \cos t \cdot x(t)$

4) Verify the following systems are stable or unstable.

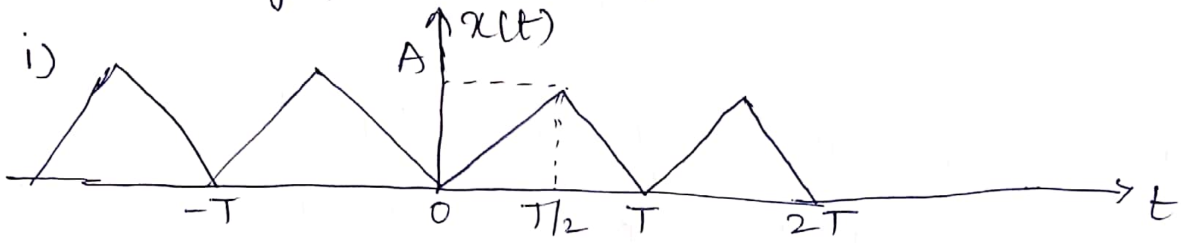
i) $y(t) = t^2 x(t)$

ii) $y(t) = e^{-t} \sin t \, u(t)$

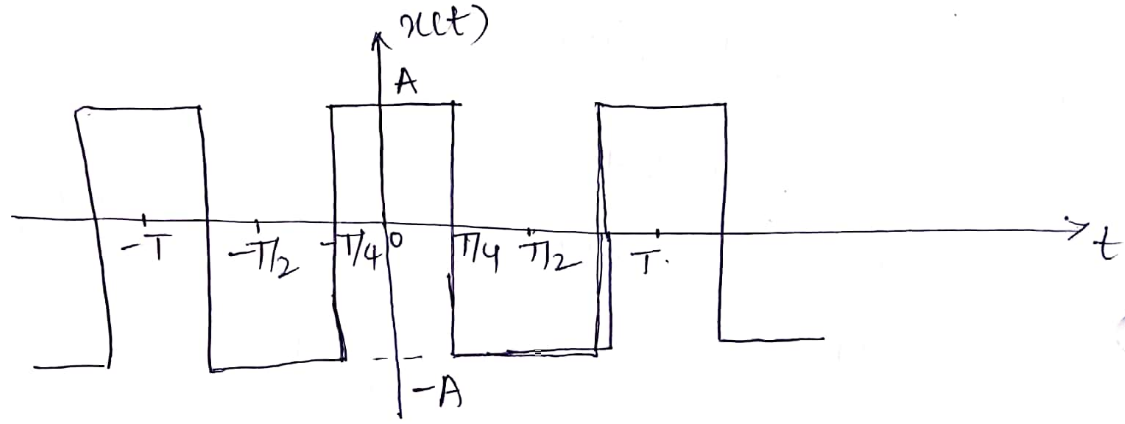
iii) $h(t) = \left[A + Be^{-ct}\right] u(t)$

5) Find the trigonometric Fourier Series & Exponential FS of the following periodic signals.

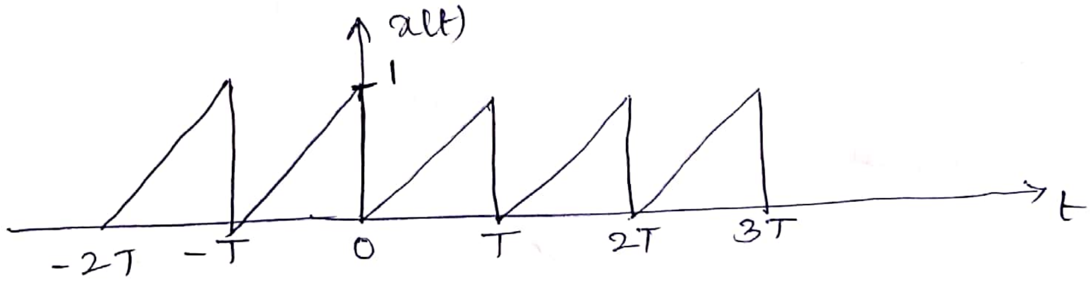
i)



ii)



iii)



Fourier Series Analysis of continuous time Signals.

A periodic signal is one which repeats itself periodically over $-\infty < t < \infty$.

For example, a sinusoidal signal $x(t) = A \sin \omega_0 t$ is a periodic signal with period $T = \frac{2\pi}{\omega_0}$

- Sum of two sinusoidal signal is periodic provided that their frequencies are integer multiples of fundamental frequency ω_0.

- Now let us consider a signal $x(t)$, a sum of sine and cosine functions whose frequencies are integer multiples of ω_0.

$$x(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + a_3 \cos 3\omega_0 t + \ldots a_K \cos(K\omega_0 t)$$
$$+ b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \ldots + b_K \sin(K\omega_0 t)$$

$$x(t) = a_0 + \sum_{n=1}^{K} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)] \quad \text{——— (1)}$$

$a_0, a_1 \ldots a_K$ and $b_0, b_1 \ldots b_K$ are constant and ω_0 is fundamental frequency.

If signal $x(t)$ is to be periodic

$$x(t) = x(t+T) \quad \text{——— (2)}$$

From (1)

$$x(t+T) = a_0 + \sum_{n=1}^{K} [a_n \cos n\omega_0 (t+T) + b_n \sin n\omega_0 (t+T)]$$

$$= a_0 + \sum_{n=1}^{K} a_n \cos(n\omega_0 t + 2n\pi) + b_n \sin(n\omega_0 t + 2n\pi)]$$

$$\left(\because T = \frac{2\pi}{\omega_0}\right).$$

$$= a_0 + \sum_{n=1}^{K} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t.$$

$$= x(t).$$

In eqn (1) if $K \to \infty$, we obtain Fourier Series representation of a periodic signals $x(t)$. Thus any periodic signal can be represented as an infinite sum of sine & cosine functions which themselves are periodic signals of angular frequencies $0, \omega_0, 2\omega_0 \ldots K\omega_0$.

This series of sine and cosine term is known as trigonometric Fourier Series and can be written as

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t] \quad \text{—— (3)}$$

a_n, b_n are constant.

Coefficient a_0 is called dc component

$a_1 \cos\omega_0 t + b_1 \sin\omega_0 t$ the first harmonic

$a_2 \cos 2\omega_0 t + b_2 \sin 2\omega_0 t \rightarrow$ second harmonic.

$a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \rightarrow$ nth harmonic

Evaluation of Fourier Coefficients

$\left.\begin{matrix} a_0, a_1, a_2 \ldots a_n \\ b_0, b_1, b_2 \ldots b_n \end{matrix}\right\}$ Fourier Coefficients.

To evaluate a_0 we shall integrate both sides of eqn (3) over one period (t_0, t_0+T) of $x(t)$ an arbitrary time t_0.

$$\int_{t_0}^{t_0+T} x(t)\,dt = a_0 \int_{t_0}^{t_0+T} dt + \int_{t_0}^{t_0+T} \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]\,dt.$$

$$= a_0 T + \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_0+T} \cos(n\omega_0 t)\,dt + \sum_{n=1}^{\infty} b_n \int_{t_0}^{t_0+T} \sin(n\omega_0 t)\,dt$$

$$\int_{t_0}^{t_0+T} x(t)\,dt = a_0 T$$

$$\boxed{a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} x(t)\,dt.}$$

To evaluate a_n & b_n we can use the following results.

$$\int_{t_0}^{t_0+T} \cos(n\omega_0 t)\cos(m\omega_0 t)\,dt = \begin{cases} 0, & m \neq n \\ T/2, & m = n \neq 0. \end{cases} \quad \text{—— (4)}$$

$$\int_{t_0}^{t_0+T} \sin(n\omega_0 t)\cos(m\omega_0 t)\,dt = 0 \quad \text{for all } m, n. \quad \text{—— (5)}$$

$$\int_{t_0}^{t_0+T} \sin(n\omega_0 t)\sin(m\omega_0 t)\,dt = \begin{matrix} 0, & m \neq n \\ T/2, & m = n \neq 0 \end{matrix} \quad \text{—— (6).}$$

∴ To find fourier coefficients a_n eqn ③ by $\cos m\omega_0 t$ and integrate over one period

i.e. $$\int_{t_0}^{t_0+T} x(t)\cos(m\omega_0 t)\,dt = a_0\int_{t_0}^{t_0+T}\cos(m\omega_0 t)\,dt + \sum_{n=1}^{\infty} a_n\int_{t_0}^{t_0+T}\cos(n\omega_0 t)\cdot\cos(m\omega_0 t)\,dt$$

$$+\sum_{n=1}^{\infty} b_n\int_{t_0}^{t_0+T}\sin(n\omega_0 t)\cdot\cos(m\omega_0 t)\,dt$$

(7)

The first integral on the RHS of ⑦ is zero because we are integrating over an integer multiples of periods.

sub ④ ⑤ in ⑦

$$\int_{t_0}^{t_0+T} x(t)\cos(m\omega_0 t)\,dt = 0 + a_m\cdot T/2 + 0.$$

$a_m = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos(m\omega_0 t)dt.$

$$\boxed{a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos(n\omega_0 t)\,dt}$$

To find b_n multiply eqn ③ by $\sin(m\omega_0 t)$

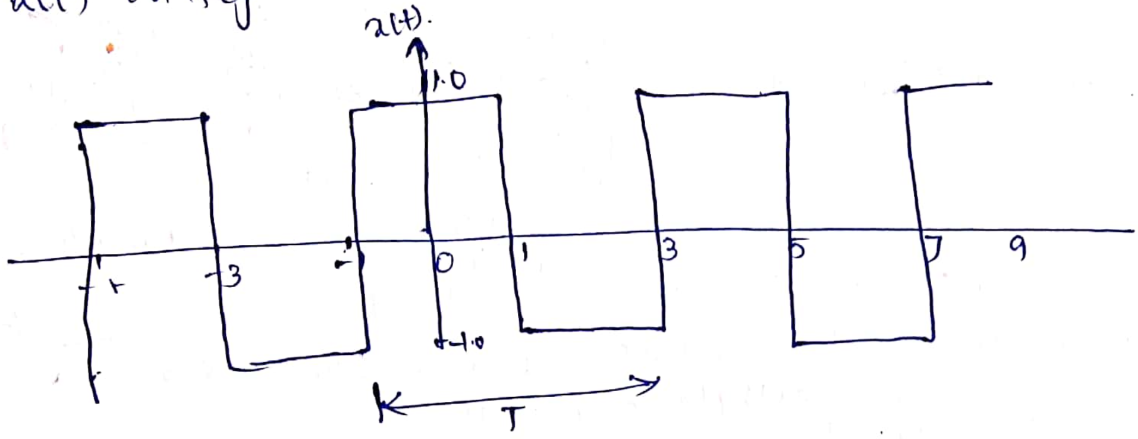
$$\int_{t_0}^{t_0+T} x(t)\sin(m\omega_0 t)\,dt = \int_{t_0}^{t_0+T} a_0\sin(m\omega_0 t)\,dt + \sum_{n=1}^{\infty} a_n\int_{t_0}^{t_0+T}\cos(n\omega_0 t)\sin(m\omega_0 t)\,dt$$

$$+\sum_{n=1}^{\infty} b_n\int_{t_0}^{t_0+T}\sin(n\omega_0 t)\sin(m\omega_0 t)\,dt$$

$$= 0 + 0 + b_n\cdot\frac{T}{2}.$$

$$\boxed{b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin(m\omega_0 t)\,dt}$$

① Find the trigonometric Fourier series for periodic signal $x(t)$ in fig.



$T = 4$.

Choose one period of signal from $t=-1$ to $t=3$

$$\text{fundamental frequency} = \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \pi/2$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t.$$

$$= a_0 + \sum_{n=1}^{\infty} \left[a_n \cos \frac{n\pi}{2} t + b_n \sin \frac{n\pi}{2} t\right]$$

Given $x(t) = 1$ for $-1 \leq t \leq 1$

-1 for $1 \leq t \leq 3$.

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\,dt$$

$$= \frac{1}{4}\int_{-1}^{3} x(t)\,dt = \frac{1}{4}\left[\int_{-1}^{1} dt + \int_{1}^{3} -dt\right]$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{4}\int_{-1}^{3} x(t)\cos \frac{n\pi}{2} t\,dt = \frac{1}{2}\left[\int_{-1}^{1} \cos\left(\frac{n\pi}{2} t\right)dt + \int_{1}^{3} (-1)\cos \frac{n\pi}{2} t\,dt\right]$$

$$= \frac{1}{2}\left[\frac{2}{n\pi} \sin \frac{n\pi}{2} t\right]_{-1}^{1} + \left(-\frac{2}{n\pi}\right)\left[\sin \frac{n\pi}{2} t\right]_{1}^{3}$$

$$a_n = \frac{4}{n\pi}\sin\frac{n\pi}{2} = \begin{cases} 0 & n\ \text{even} \\ \frac{4}{n\pi} & n = 1,5,9,13\ldots \\ -\frac{4}{n\pi} & n = 3,7,11,15\ldots \end{cases}$$

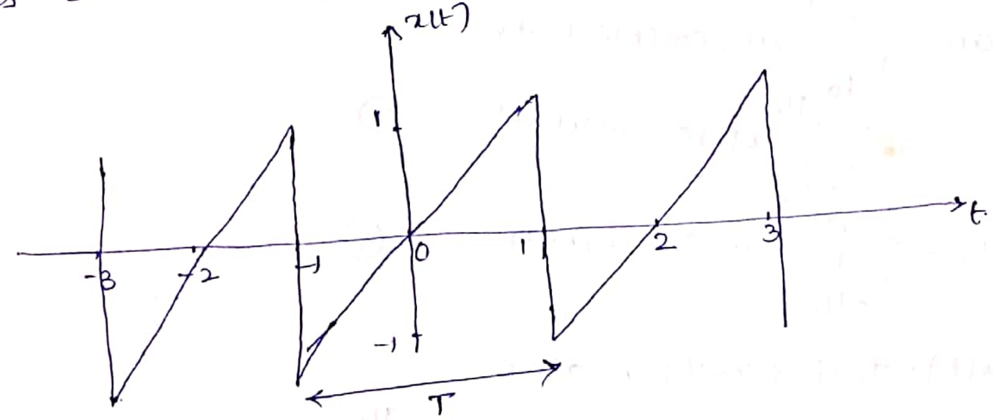
$$\therefore\ x(t) = 0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{2}t\right)$$

$$= \frac{4}{\pi}\cos\frac{\pi}{2}t + 0 - \frac{4}{3\pi}\cos\frac{3\pi}{2}t + \frac{4}{5\pi}\cos\frac{5\pi}{2}t - \frac{4}{7\pi}\cos\frac{7\pi}{2}t.$$

$$= \frac{4}{\pi}\left[\cos\frac{\pi}{2}t - \frac{1}{3}\cos\frac{3\pi}{2}t + \frac{1}{5}\cos\frac{5\pi}{2}t - \frac{1}{7}\cos\frac{7\pi}{2}t + \cdots\right]$$

---

② Find the trigonometric Fourier Series for the periodic signals $x(t)$ as shown in fig.



$T = 2$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$ $\qquad x(t) = t\ ,\ -1 \le t \le 1$

$t = -1$ to $+1$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)$$

$$= a_0 + \sum_{n=1}^{\infty} a_n\cos(n\pi t) + b_n\sin(n\pi t) \quad \text{—①}$$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\,dt = \frac{1}{2}\int_{-1}^{1} t\,dt = \frac{1}{2}\left[\frac{t^2}{2}\right]_{-1}^{1} = \frac{1}{2}\left[\frac{1}{2} - \frac{1}{2}\right] = 0.$$

$\boxed{a_0 = 0}$

$\int u\,dv = uv - \int v\,du.$

$dV = \cos n\pi t$

$V = \frac{\sin n\pi t}{n\pi}$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos n\omega_0 t\,dt = \frac{\cancel{2}}{\cancel{2}}\int_{-1}^{1} t.\cos n\pi t\,dt.$$

$$= t.\frac{\sin n\pi t}{n\pi} - \int \frac{\sin n\pi t}{n\pi}.1\,dt.$$

$$= \left[t\frac{\sin n\pi t}{n\pi}\right]_{-1}^{1} + \left[\frac{\cos n\pi t}{n^2\pi^2}\right]_{-1}^{1}$$

$$= 0 + 0 = 0.$$

$\boxed{a_n = 0}$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin(n\omega_0 t)\,dt = \frac{2}{2}\int_{-1}^{1} t.\sin(n\pi t)\,dt = -t\frac{\cos n\pi t}{n\pi} - \int -\frac{\cos n\pi t}{n\pi}.1\,dt.$$

$$= \left[-t\frac{\cos n\pi t}{n\pi} + \frac{\sin n\pi t}{n^2\pi^2}\right]_{-1}^{1}$$

$$= -\frac{2}{n\pi}\cos n\pi = -\frac{1}{n\pi}\left[\cos n\pi + \cos n\pi + (0-0)\right]$$

$$= \frac{2}{\pi}\left[\frac{-(-1)^n}{n}\right]$$

Sub a_0, a_n, b_n in (1)

$$x(t) = 0 + 0 + \sum_{n=1}^{\infty} \frac{2}{\pi}\left[-\frac{(-1)^n}{n}\right] \sin n\pi t$$

$$= \frac{2}{\pi}\left[\sin \pi t - \frac{1}{2}\sin 2\pi t + \frac{1}{3}\sin 3\pi t - \frac{1}{4}\sin 4\pi t + \cdots\right]$$

Symmetry condition

$$x(t) = x_e(t) + x_o(t). \quad \text{——(1)}$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \quad \text{—(2)}$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)] \quad \text{—(3)}$$

For convinient check interval $-T/2$ to $T/2$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cos(n\omega_0 t)\,dt \quad \text{——(4)}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\sin(n\omega_0 t)\,dt \quad \text{——(5)}$$

Sub. $x(t) = x_e(t) + x_o(t)$ in above.

$$a_n = \frac{2}{T}\left[\int_{-T/2}^{T/2} x_e(t)\cos(n\omega_0 t)\,dt + \int_{-T/2}^{T/2} x_o(t)\cos(n\omega_0 t)\,dt\right] \quad \text{——(6)}$$

lly $$b_n = \frac{2}{T}\left[\int_{-T/2}^{T/2} x_e(t)\sin(n\omega_0 t)\,dt + \int_{-T/2}^{T/2} x_o(t)\sin(n\omega_0 t)\,dt\right] \quad \text{——(7)}$$

We know that

odd function × odd function = even function

even " × even " = even "

even " × odd " = odd "

For any even function $x_e(t)$

$$\int_{-t_0}^{t_0} x_e(t)\,dt = 2\int_{0}^{t_0} x_e(t)\,dt \quad \text{——(8)}$$

For any odd function $x_o(t)$

$$\int_{-t_0}^{t_0} x_o(t)\,dt = 0 \quad \text{——(9)}$$

If $x(t)$ is an even function, then $x_o(t) = 0$. Sub in (7)

$$b_n = \frac{2}{T}\left[\int_{-T/2}^{T/2} \underbrace{x_e(t)}_{\text{even}} \underbrace{\sin(n\omega_0 t)}_{\text{odd}}\,dt\right]$$

even × odd = odd

So, resulting integral is equal to zero

$\boxed{b_n = 0}$

From (6)

$$a_n = \frac{2}{T}\left[\int_{-T/2}^{T/2} x_e(t)\cos(n\omega_0 t)\,dt\right]$$

$$\boxed{a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt}$$

and $\boxed{a_0 = \frac{2}{T}\int_0^{T/2} x(t)\,dt.}$

* The Fourier Series expansion of an even periodic function contains only cosine terms & a constant.

If $x(t)$ is an odd function, then $x_e(t) = 0$. Sub. in (6)

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} \underbrace{x_o(t)}_{\text{odd.}} \underbrace{\cos(n\omega_0 t)}_{\text{even.}}\,dt$$

$\boxed{a_n = 0}$ $\left(\because \begin{array}{l}\text{odd (even)} = \text{odd} \\ \int \text{odd function} = 0\end{array}\right).$

$$a_0 = \frac{2}{T}\int_0^{T/2} x_o(t)\,dt$$

$\boxed{a_0 = 0}$

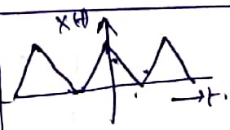
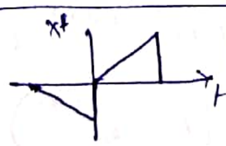
Sub. above conditions in (7)

$$b_n = \frac{2}{T}\left[\int_{-T/2}^{T/2} \underbrace{x_o(t)}_{\text{odd}}\underbrace{\sin(n\omega_0 t)}_{\text{odd.}}\,dt\right]$$

$\int_{-T/2}^{T/2}$ even function

$$\boxed{b_n = \frac{4}{T}\int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt.}$$

* Fourier series expansion of an odd periodic functions contains only sine terms.

Type of symmetry	Condition	Example	a_0	a_n	b_n	Property
Even	$x(t) = x(-t)$		$a_0 = \frac{2}{T}\int_0^{T/2} x(t)\,dt$. $\omega_0 = \frac{2\pi}{T}$	$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt$	0	Cosine terms only
Odd	$x(t) = -x(-t)$		0	0	$b_n = \frac{4}{T}\int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt$	Sine terms only
Half wave	$x(t) = -x(t \pm \frac{T}{2})$		0	$\frac{4}{T}\int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt$	$\frac{4}{T}\int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt$.	odd n only

Half wave symmetry

A periodic signal which satisfy the condition

$$\boxed{x(t) = -x(t \pm T/2)}$$

is said to have a half wave symmetry. The fourier Expansion of such a type of periodic signal contains odd harmonics only.

Cosine representation

The trigonometric Fourier series contains sine & cosine terms of the same frequency. By using trigonometric identity, we can write

$$a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) = A_n[\cos(n\omega_0 t + \theta_n)]$$

we obtain cosine representation of $x(t)$ which contains sinusoids of frequencies $\omega_0, 2\omega_0, \ldots$

i.e. $$\boxed{x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \theta_n)}$$

where $$\left.\begin{aligned} A_0 &= a_0 \\ A_n &= \sqrt{a_n^2 + b_n^2} \\ \theta_n &= -\tan^{-1}\left(\frac{b_n}{a_n}\right). \end{aligned}\right\}$$

$A_n \rightarrow$ amplitude coefficients

$\theta_n \rightarrow$ phase coefficients

Exponential Fourier Series

Exponential F.S. in another form of Fourier series. By using Euler's identity we can write

$$A_n \cos(n\omega_0 t + \theta_n) = A_n\left[\frac{e^{j(n\omega_0 t + \theta_n)} + e^{-j(n\omega_0 t + \theta_n)}}{2}\right] \quad \text{—(1)}$$

Sub (1) in definition of cosine F.S.

$$\therefore x(t) = A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2}\left[e^{j(n\omega_0 t + \theta_n)} + e^{-j(n\omega_0 t + \theta_n)}\right]$$

$$= A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2}\left[e^{jn\omega_0 t} \cdot e^{j\theta_n} + e^{-jn\omega_0 t} \cdot e^{-j\theta_n}\right]$$

$$= A_0 + \sum_{n=1}^{\infty}\left[\frac{A_n}{2} e^{jn\omega_0 t} \cdot e^{j\theta_n} + \frac{A_n}{2} e^{-jn\omega_0 t} \cdot e^{-j\theta_n}\right]$$

$$= A_0 + \sum_{n=1}^{\infty}\left(\frac{A_n}{2} e^{j\theta_n}\right) e^{jn\omega_0 t} + \sum_{n=1}^{\infty}\left(\frac{A_n}{2} e^{-j(\theta_n)}\right) e^{j(-n)\omega_0 t} \quad \text{—(2)}.$$

... for signal shown in fig. (6) (5)

Let $n = -k$ in second summation of eqn (2) then

$$x(t) = A_0 + \sum_{n=1}^{\infty}\left(\frac{A_n}{2}e^{j\theta_n}\right)e^{jn\omega_0 t} + \sum_{k=-1}^{-\infty}\left(\frac{A_k}{2}e^{j\theta_k}\right)e^{j\omega_0 k t} \quad \text{—— (3)}$$

Comparing (2) & (3)

$$\left.\begin{aligned} A_n &= A_k \\ -\theta_n &= \theta_k \end{aligned}\right\} \begin{aligned} n &> 0 \\ k &< 0 \end{aligned} \quad \text{—— (4)}$$

Let us define

$$\left.\begin{aligned} C_0 &= A_0, \\ C_n &= \frac{A_n}{2}e^{j\theta_n}, \quad n>0 \end{aligned}\right\} \text{—— (5)}$$

using (4) in (3)

$$x(t) = A_0 + \sum_{n=1}^{\infty}\left(\frac{A_n}{2}e^{j\theta_n}\right)e^{jn\omega_0 t} + \sum_{n=-1}^{-\infty}\left(\frac{A_n}{2}e^{j\theta_n}\right)e^{j\omega_0 n t}.$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}}$$

This is known as exponential Fourier series

Now we develop expressions for the coefficients of the exponential F.S.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}, \qquad \omega_0 = \frac{2\pi}{T}$$

Multiply by $e^{-jk\omega_0 t}$ & Integrate over one period

$$\int_{t_0}^{t_0+T} x(t)e^{-jk\omega_0 t}\,dt = \int_{t_0}^{t_0+T}\left[\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}\right]e^{-jk\omega_0 t}\,dt \quad \text{—— (6)}$$

Substituting the relation $\int_{t_0}^{t_0+T} e^{jn\omega_0 t}e^{-jk\omega_0 t} = \begin{cases} 0, & k \neq n \\ T & k = n \end{cases}$

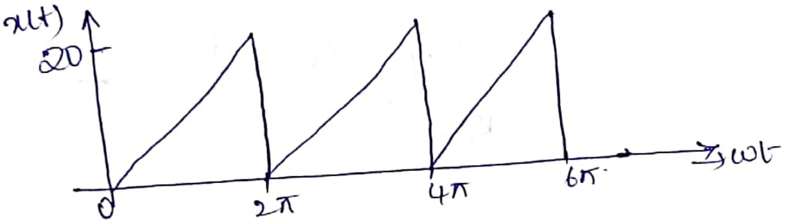
From (6) $\therefore \int_{t_0}^{t_0+T} x(t)e^{-jk\omega_0 t}\,dt = T C_k.$

$$C_k = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)e^{-jk\omega_0 t}\,dt.$$

$$\boxed{C_n = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)e^{-jn\omega_0 t}\,dt.}$$

$C_n \rightarrow$ Fourier Coefficient of Exponential F.S.

① Find the Fourier Series for the waveform shown in fig. and plot the spectrum.



$$x(t) = \frac{20}{2\pi} \cdot \omega t \quad —①$$

$T = 2\pi$.

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$.

$$a_0 = \frac{1}{T}\int_0^{2\pi} x(t)\, dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{20}{2\pi}\omega t \cdot d\omega t = \frac{20}{(2\pi)^2}\left[\frac{(\omega t)^2}{2}\right]_0^{2\pi} = \frac{20}{(2\pi)^2}\left[\frac{4\pi^2}{2} - 0\right]$$

$$= \frac{20}{4\pi^2}\cdot\frac{4\pi^2}{2} = \underline{10}$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos(n\omega_0 t)\, d\omega t.$$

$$= \frac{2}{2\pi}\int_0^{2\pi} \frac{20}{2\pi}\cdot\omega t \cdot \cos(n\omega_0 t)\, d\omega t.$$

$\int u\,dv = uv - \int v\,du$.

$dv = \cos(n\omega_0 t)$

$V = \frac{\sin(n\omega_0 t)}{n\omega_0}$.

$$= \frac{20}{2\pi^2}\left[\frac{\omega t \cdot \sin(n\omega_0 \omega t)}{n\omega_0} + \frac{\cos(n\omega_0 \omega t)}{n^2\omega_0^2}\right]_0^{2\pi}$$

$$= \frac{10}{\pi^2}\left[\frac{(\omega t)\cdot \sin n\omega t}{n} + \frac{\cos n\omega t}{n^2}\right]_0^{2\pi}$$

$$= \frac{10}{\pi^2}\left[0 + \frac{\cos n2\pi}{n^2} - \frac{1}{n^2}\right]$$

$$= \frac{10}{\pi^2 n^2}\left[\cos n2\pi - 1\right]$$

$= 0$, for all values of n.

$$b_n = \frac{2}{T}\int_0^{2\pi} x(t)\sin(n\omega_0 \omega t)\, dt$$

$$= \frac{2}{2\pi}\int_0^{2\pi} \frac{20}{2\pi}\cdot\omega t \cdot \sin(n\omega t)\, dt.$$

$dv = \sin(n\omega t)$

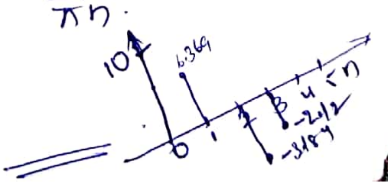
$V = -\frac{\cos(n\omega t)}{n}$.

$$= \frac{20}{2\pi^2}\left[\frac{-(\omega t)\cdot\cos n\omega t}{n} + \frac{\sin(n\omega t)}{n^2}\right]_0^{2\pi}$$

$$= \frac{20}{2\pi^2}\left[-\frac{2\pi\cos n2\pi}{n} + \underbrace{\frac{\sin n2\pi}{n^2}}_{=0} - (0+0)\right]$$

$$= \frac{20}{2n\pi^2}\left[-2\pi\cos n2\pi\right] = \frac{-10}{n\pi^2}\left[+2\pi\right] = \frac{-20}{\pi n}.$$

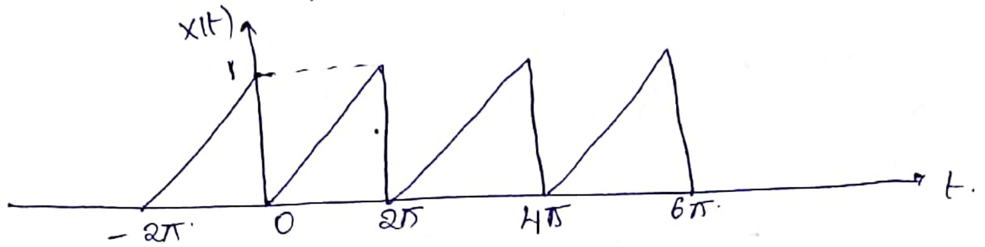
Spectrum



$$\therefore x(t) = 10 + 0 + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

$$= 10 - \frac{20}{\pi}\sin\omega t - \frac{20}{2\pi}\sin 2\omega t - \frac{20}{3\pi}\sin 3\omega t \ldots$$

①. Find the Cosine Representation Fourier Series for the signal shown in fig. ⑥).



Sol

$T = 2\pi$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1.$

$x(t) = \frac{1}{2\pi} t.$ for $0 \le t \le 2\pi$

$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\,dt = \frac{1}{2\pi}\int_0^{2\pi} \frac{t}{2\pi}\,dt = \frac{1}{4\pi^2}\left[\frac{t^2}{2}\right]_0^{2\pi} = \frac{1}{4\pi^2}\left[\frac{4\pi^2}{2}\right] = \frac{1}{2}.$

$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos(n\omega_0 t)\,dt$

$= \frac{2}{2\pi}\int_0^{2\pi} \frac{t}{2\pi}\cos(nt)\,dt = \frac{1}{2\pi^2}\int_0^{2\pi} \underset{u}{t}\, \underset{dv}{\cos(nt)\,dt}$

$dv = \cos nt$
$V = \frac{\sin nt}{n}.$

$= \frac{1}{2\pi^2}\left[\frac{t\sin(nt)}{n} + \frac{\cos nt}{n^2}\right]_0^{2\pi}$

$= \frac{1}{2\pi^2}\left[0 - \frac{1}{n^2} - 0 + \frac{1}{n^2}\right] = 0.$

$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin(n\omega_0 t)\,dt$

$= \frac{2}{2\pi}\int_0^{2\pi} \frac{t}{2\pi}\sin(n\omega_0 t)\,dt = \frac{1}{2\pi^2}\left[-\frac{t\cos(n\omega_0 t)}{n} + \frac{\sin nt}{n}\right]_0^{2\pi}$

$= \frac{1}{2\pi^2}\left[-\frac{2\pi\cos 2\pi n}{n}\right]$

$= \frac{1}{2\pi^2}\left[-\frac{2\pi}{n}\right] = -\frac{1}{n\pi}$

For Cosine Representation

$A_0 = a_0 = \frac{1}{2}.$

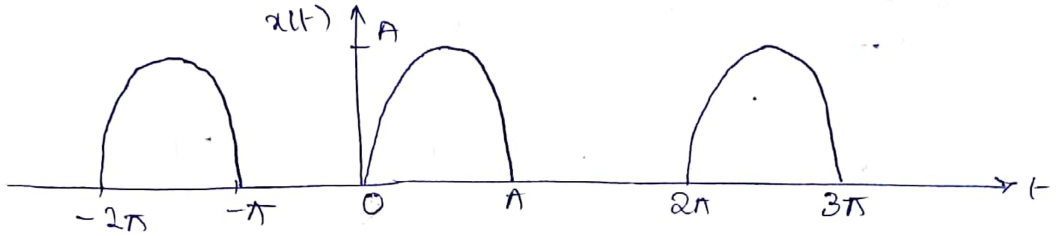
$A_n = \sqrt{a_n^2 + b_n^2} = \frac{1}{n\pi}$

$\theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right) = -\tan^{-1}\left(\frac{-1/n\pi}{0}\right) = \frac{\pi}{2}.$

$\therefore x(t) = A_0 + \sum_{n=1}^{\infty} A_n\cos(n\omega_0 t + \theta_n).$

$= \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi}\cos\left(nt + \frac{\pi}{2}\right)$

② Find Cosine Fourier Series of an half wave rectified Sin function



Sol

$x(t) = A\sin\omega_0 t$ for $0 \le t \le \pi$

$= 0$ for $\pi \le t \le 2\pi$

$T = 2\pi$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\,dt = \frac{1}{2\pi}\int_0^{\pi} A\sin t\,dt = \frac{A}{2\pi}\left[-\cos t\right]_0^{\pi} = -\frac{A}{2\pi}\left[-1-1\right]$

$= A/\pi$

$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos(nt)\,dt = \frac{2}{2\pi}\int_0^{\pi} A\sin t\cos nt$

$\because \sin A\cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$= \frac{A}{\pi}\left[\int_0^{\pi}\frac{1}{2}\left[\sin(t+nt) + \sin(t-nt)\right]dt\right]$

$= \frac{A}{2\pi}\left[-\frac{\cos(1+n)t}{1+n} - \frac{\cos(1-n)t}{1-n}\right]_0^{\pi}$

$= \frac{A}{2\pi}\left[-\frac{\cos(1+n)\pi}{1+n} - \frac{\cos(1-n)\pi}{1-n} + \frac{1}{1+n} + \frac{1}{1-n}\right]$

$= \frac{A}{2\pi}\left[\frac{2}{1+n} + \frac{2}{1-n}\right]$ where n is even.

$= \frac{A}{2\pi}\left[\frac{4}{1-n^2}\right] = \frac{2A}{\pi(1-n^2)}$ where n is even

$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin nt\,dt = \frac{2}{2\pi}\int_0^{\pi} A\sin t\sin nt\,dt$

$= \frac{A}{2\pi}\int_0^{\pi}\{\cos(t-nt) - \cos(t+nt)\}dt$

$= \frac{A}{2\pi}\int_0^{\pi}[\cos(1-n)t - \cos(1+n)t]\,dt$

$= \frac{A}{2\pi}\left[\frac{\sin(1-n)t}{1-n} - \frac{\sin(1+n)t}{1+n}\right]_0^{\pi} = 0.$

for $n=1 \Rightarrow a_n = \frac{2}{2\pi}\int_0^{\pi} A\sin t\cos t\,dt = \frac{A}{2\pi}\int_0^{\pi}\sin 2t\,dt = \frac{A}{2\pi}\left[\frac{-\cos 2t}{2}\right]_0^{\pi} = \frac{A}{4\pi}\left[-1+1\right] = 0.$

for $n=1 \rightarrow b_n = \frac{2}{2\pi}\int_0^{\pi} A\sin t\sin t = \frac{2A}{2\pi}\int_0^{\pi}\sin^2 t = \frac{A}{\pi}\int_0^{\pi}\frac{1-\cos 2t}{2} = \frac{A}{2\pi}\left[t - \frac{\sin 2t}{2}\right]_0^{\pi}$

$= \frac{A}{2\pi}[\pi] = A/2$

$A_0 = a_0 = \frac{A}{\pi}$

$A_1 = \sqrt{a_1^2 + b_1^2} = A/2$

$A_n = \sqrt{a_n^2 + b_n^2} = \frac{2A}{\pi(1-n^2)}$

$\theta_1 = -\tan^{-1} b_1/a_1 = -\pi/2$

$\theta_n = \tan^{-1}\frac{b_n}{a_n}$

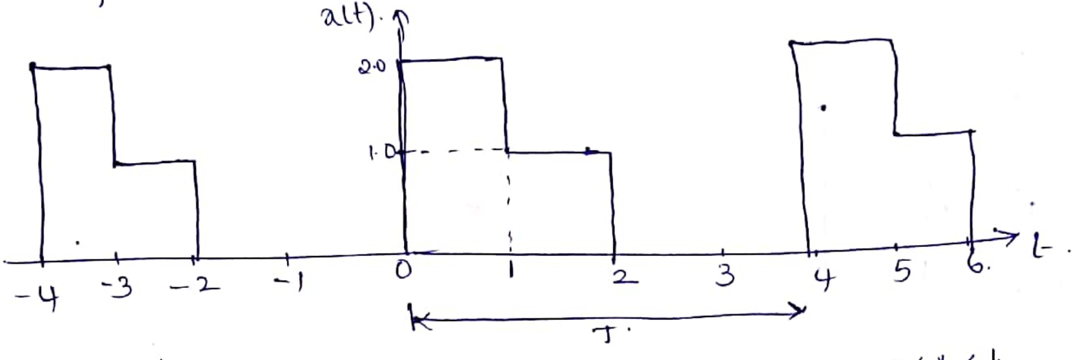
$= -\tan^{-1} 0$

$= 0.$

$\therefore x(t) = A_0 + \sum_{n=1}^{\infty} A_n\cos(nt + \theta_n).$

$= \frac{A}{\pi} + \frac{A}{2}\cos(t - \pi/2) + \sum_{n=2}^{\infty}\frac{2A}{\pi(1-n^2)}\cos nt.$

(3). Compute the exponential Fourier Series of the following signal.



Sol

$T = 4$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \pi/2.$

$x(t) = 2, \quad 0 \le t \le 1$
$\quad = 1, \quad 1 \le t \le 2.$
$\quad = 0, \quad \text{else } 2 \le t \le 4.$

$$C_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{4}\int_0^4 x(t)\,dt$$

$$= \frac{1}{4}\left[\int_0^1 2\,dt + \int_1^2 1\,dt\right] = \frac{1}{4}\left[2\cdot[t]_0^1 + 1[t]_1^2\right]$$

$$= \frac{1}{4}\left[2+1\right] = 3/4.$$

$$C_n = \frac{1}{T}\int_0^T x(t)\,\cancel{dt}\,e^{-jn\omega_0 t}\,dt.$$

$$= \frac{1}{4}\int_0^4 x(t)\,e^{-jn\frac{\pi}{2}t}\,dt.$$

$$= \frac{1}{4}\left[\int_0^1 2\cdot e^{-jn\frac{\pi}{2}t}\,dt + \int_1^2 1\cdot e^{-jn\frac{\pi}{2}\cdot t}\,dt\right]$$

$$= \frac{1}{4}\left[2\cdot\left[\frac{e^{-jn\frac{\pi}{2}t}}{-jn\frac{\pi}{2}}\right]_0^1 + \left[\frac{e^{-jn\frac{\pi}{2}t}}{-jn\frac{\pi}{2}}\right]_1^2\right]$$

$$= \frac{1}{4}\left[\frac{2}{-jn\frac{\pi}{2}}\left(e^{-jn\frac{\pi}{2}}-1\right) + \left(-\frac{1}{jn\frac{\pi}{2}}\right)\left(e^{-jn\pi} - e^{-jn\frac{\pi}{2}}\right)\right]$$

$$= -\frac{1}{jn\pi}\left[e^{-jn\frac{\pi}{2}}-1\right] - \frac{1}{2jn\pi}\left[e^{-jn\pi} - e^{-jn\frac{\pi}{2}}\right]$$

$$= \frac{1}{jn\pi}\left[1 - e^{-jn\frac{\pi}{2}} - \frac{1}{2}e^{-jn\pi} + \frac{1}{2}e^{-jn\frac{\pi}{2}}\right]$$

$$= \frac{1}{jn\pi}\left[1 - \frac{1}{2}(-1)^n - \frac{1}{2}e^{-jn\pi/2}\right]$$

The exponential F.S. of x(t)

$$= \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} C_n e^{jn\frac{\pi}{2}t}$$

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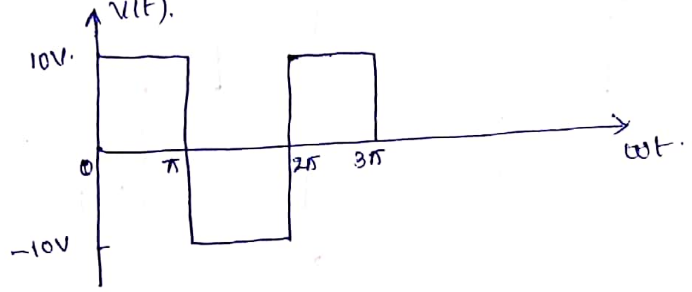
(4) Find the complex Fourier series for the square wave shown in fig and sketch the line spectrum.



$v(t) = 10$ for $0 < \omega t < \pi$

$= -10$ for $-\pi < \omega t < 0$

The avg. value of wave is zero.

$$C_n = \frac{1}{2\pi}\left[\int_{-\pi}^{0} -10\, e^{-jn\omega t}\, d\omega t + \int_{0}^{\pi} (+10)\, e^{-jn\omega t}\, d\omega t\right].$$

$$= \frac{10}{2\pi}\left[-\left[\frac{e^{-jn\omega t}}{-jn}\right]_{-\pi}^{0} + \left[\frac{e^{-jn\omega t}}{-jn}\right]_{0}^{\pi}\right]$$

$$= \frac{10}{-2\pi jn}\left[\cancel{(e^{-j\pi n} - 1)} + \cancel{(e^{-j2\pi n} - e^{-j\pi n})}\right]$$

$$= \frac{10}{-2\pi jn}\left[-(1 - e^{jn\pi}) + (e^{-jn\pi} - 1)\right]$$

$$= \frac{10}{-2\pi jn}\left[-2 + (\cos n\pi + j\cancel{\sin n\pi}) + (\cos n\pi - j\cancel{\sin n\pi})\right]$$

$$= \frac{j10}{2\pi n}\left[-2(1 - \cos n\pi)\right]$$

$$C_n = -\frac{j10}{\pi n}\left[1 - \cos n\pi\right]$$

n is even $\quad C_n = 0$

n is odd $\quad C_n = -\frac{j20}{n\pi}$

The Fourier series is

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}.$$

$$= \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}$$

$$= \ldots + j\frac{20}{3\pi} e^{-j3\omega t} + j\frac{20}{\pi} e^{-j\omega t} - j\frac{20}{\pi} e^{j\omega t} - \frac{j20}{3\pi} e^{j3\omega t} - \ldots$$

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Complex Fourier Series

The Fourier Series

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)] \quad \text{--- (1)}$$

Can be written in complex form

We know that

$$e^{jn\omega t} = \cos n\omega t + j \sin(n\omega t) \quad \text{--- (2)}$$

$$e^{-jn\omega t} = \cos n\omega t - j \sin n\omega t \quad \text{--- (3)}$$

By adding (2) & (3)

$$\cos n\omega t = \frac{1}{2}(e^{jn\omega t} + e^{-jn\omega t})$$

Subtracting (2)-(3)

$$\sin n\omega t = \frac{1}{2j}(e^{jn\omega t} - e^{-jn\omega t}) \quad \text{--- (4)}$$

$$a_n \cos n\omega t + b_n \sin n\omega t = \frac{1}{2} a_n (e^{jn\omega t} + e^{-jn\omega t}) + \frac{1}{2j} b_n (e^{jn\omega t} - e^{-jn\omega t})$$

$$= \frac{1}{2}(a_n - jb_n) e^{jn\omega t} + \frac{1}{2}(a_n + jb_n) e^{-jn\omega t}$$

Consider $a_0 = C_0$

$$\frac{1}{2}(a_n - jb_n) = C_n$$

$$\frac{1}{2}(a_n + jb_n) = C_{-n}$$

Then eqn (1) becomes

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n e^{jn\omega t} + C_{-n} e^{-jn\omega t} \quad \text{--- (5)}$$

$$C_n = \frac{1}{2}(a_n - jb_n)$$

$$= \frac{1}{2}\left[\frac{2}{T}\int_{-\pi}^{\pi} x(t) \cos n\omega t - j \int_{-\pi}^{\pi} x(t) \sin n\omega t \, dt\right]$$

$$= \frac{\cancel{2}}{\cancel{2}} \cdot \frac{1}{2\pi} \int_{-\pi}^{\pi} x(t)(\cos n\omega t - j \sin n\omega t) \, d\omega t.$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} x(t) e^{-jn\omega t} \, d(\omega t)$$

$$C_{-n} = \frac{1}{2}(a_n + jb_n)$$

$$= \frac{\cancel{2}}{\cancel{2}} \frac{1}{2\pi} \int_{-\pi}^{\pi} x(t)(\cos n\omega t + j \sin n\omega t) \, d(\omega t)$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} x(t) e^{jn\omega t} \, d(\omega t).$$

By combining two formulas & writing ~~$C_n = C_{-n}$ we get~~

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}}$$

Where $C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} x(t) e^{-jn\omega t} \, d\omega t$ for $n = 0, \pm 1, \pm 2, \ldots$

This is called Complex form of Fourier Series (or) Complex Fourier Series of $x(t)$

$C_n \rightarrow$ complex Fourier coefficient of $x(t)$

(8) Find the Relationship between Fourier coefficients of Trigonometric and exponential form.

Sol The trigonometric F.S. of $x(t)$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\Omega_0 t + b_n \sin n\Omega_0 t)$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \left[\frac{e^{jn\Omega_0 t} + e^{-jn\Omega_0 t}}{2}\right] + \sum_{n=1}^{\infty} b_n \left[\frac{e^{jn\Omega_0 t} - e^{-jn\Omega_0 t}}{2j}\right]$$

$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$

$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

$\frac{1}{j} = \frac{j}{j^2} = -j$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} \frac{a_n}{2} e^{jn\Omega_0 t} + \frac{a_n}{2} e^{-jn\Omega_0 t} - j\frac{b_n}{2} e^{jn\Omega_0 t} + j\frac{b_n}{2} e^{-jn\Omega_0 t}$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[\frac{a_n}{2} - j\frac{b_n}{2}\right] e^{jn\Omega_0 t} + \left[\frac{a_n}{2} + j\frac{b_n}{2}\right] e^{-jn\Omega_0 t}$$

Let $C_0 = \frac{a_0}{2}$, $C_n = \frac{a_n}{2} - j\frac{b_n}{2}$, $C_n^* = \frac{a_n}{2} + j\frac{b_n}{2}$

$$\therefore x(t) = C_0 + \sum_{n=1}^{\infty} C_n e^{jn\Omega_0 t} + C_n^* e^{-jn\Omega_0 t}$$

$$= C_0 + \sum_{n=1}^{\infty} C_n e^{jn\Omega_0 t} + \sum_{n=1}^{\infty} C_n^* e^{-jn\Omega_0 t}$$

$$= C_0 + \sum_{n=1}^{\infty} C_n e^{jn\Omega_0 t} + \sum_{-n=1}^{\infty} C_{-n} e^{jn\Omega_0 t} \qquad (C_n^* = C_{-n})$$

$$= \sum_{n=-1}^{-\infty} C_n e^{jn\Omega_0 t} + C_0 + \sum_{n=1}^{\infty} C_n e^{jn\Omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} C_n e^{jn\Omega_0 t}$$

$$\therefore \boxed{\begin{aligned} C_0 &= \frac{a_0}{2} \\ C_n &= \frac{1}{2}[a_n - jb_n] \\ C_{-n} &= \frac{1}{2}[a_n + jb_n] \end{aligned}}$$

C_n — for $n = 1, 2, 3 \ldots \infty$

C_{-n} — for $-n = -1, -2, -3 \ldots -\infty$.

Signal: Dependant variable or function of one or more independant variables.

$$\underset{\text{Dependent}}{F}\,[\underbrace{x_1, x_2, \ldots x_n}_{\text{independant}}]$$

ac → signal
Dc → not a s.

$f = \frac{1}{T} = \frac{1}{\infty} = 0.$

Ex: height, weight,

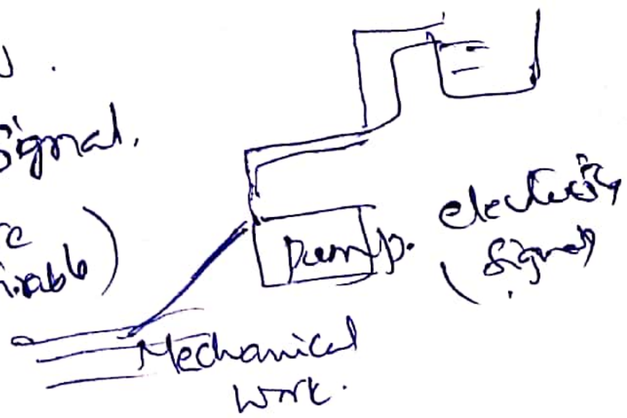
(1) Single variable signal Ex: $F(x)$, $g(t)$.

(2) Multi " " Ex: $F(x_1, x_2)$. $g_1(t_1 t_2 t_3)$.

System The meaningful interconnection

- of physical devices. and components is called as system.

i/p. Signal → [System] → o/p. Signal. (more variable)



i/p Signal → $F(x_1, x_2)$
o/p " → $g(x_1, x_2)$.

- ~~(1) Analysis problem.~~ → i/p; System is there Find o/p?.

(2) Synthesis "
i/ps ✓
System ?
o/p's ✓

Standard Continuous Time Signal

(1) Impulse Sgn

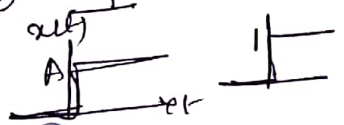
$$\int_{-\infty}^{\infty} \delta(t)\,dt = A$$

$\delta(t) = \infty,\ t=0$
$= 0,\ t \neq 0.$

Unit impulse

$\delta(t) = 1,\ t=0$
$= 0,\ t \neq 0.$

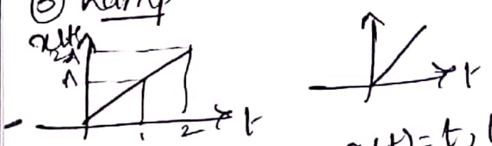
(2) Step



u(t)
$x(t) = A,\ t \geq 0$
$u(t)\ \ 0,\ t<0.$

$x(t) = 1,\ t \geq 0$
$= 0,\ t<0.$

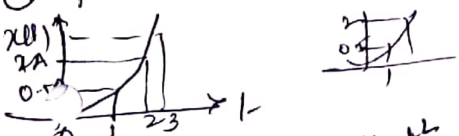
(3) Ramp



$x(t) = At,\ t \geq 0$
r(t) $0,\ t<0.$

$x(t) = t,\ t \geq 0$
$0,\ t<0.$

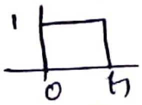
(4). parabolic Sgn



$x(t) = \frac{At^2}{2},\ t \geq 0.$
$= 0,\ t<0.$

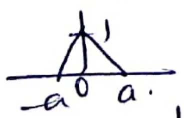
$x(t) = \frac{t^2}{2},\ t \geq 0.$
$= 0,\ t<0.$

(5) unit pulse Signal



$x(t) = 1,\ 0 \leq t \leq t_1$
$= 0,$ else.

(6) triangular pulse.

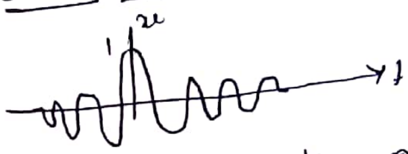


$x(t) = 1 - \frac{|t|}{a},\ |t| \leq a$
$= 0,\ |t| > a.$

(7) Signum Signal

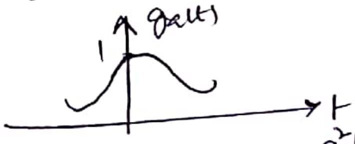
$x(t) = sgn(t) = 1,\ t>0$
$= 0,\ t=0.$
$= -1,\ t<0.$

(8) Sinc Signal



$x(t) = sinc(t) = \frac{\sin t}{t},\ -\infty < t < \infty.$

(9) Gaussian Signal



$x(t) = ga(t) = e^{-a^2t^2},\ -\infty < t < \infty.$

→ Mathematical operations on Continuous Time Signals.

i) Scaling < 1) Amp. Scaling, 2) Time Scaling.

ii) Folding (Reflection/Transpose)

iii) Time Shifting

iv) Addition

v) Multiplication.

vi) differentiation & Integration

Orthogonality in Signals

Let $f(t)$ represented in terms of $x(t)$ over interval t_1 & t_2

$$f(t) = c\,x(t) + e(t)$$

$$e(t) = f(t) - c\,x(t) \quad \text{—} ① \qquad t_1 \le t \le t_2$$

min. value of $e(t)$ will give best approximation of $f(t)$ in $x(t)$

- Rather than min. value of $e(t)$ min energy of $e(t)$ or mean square value of $e(t)$ serves approximate measure.
- Hence for min. energy of $e(t)$, representation of $f(t)$ in $x(t)$ will be better.

Energy of $e(t)$ will be

$$E_e = \int_{t_1}^{t_2} e^2(t)\,dt \quad \text{—} ②$$

and mean square value of $e(t)$ will be given as

$$\overline{e^2(t)} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} e^2(t)\,dt$$

$$= \frac{E_e}{t_2 - t_1} \quad \text{—} ③.$$

Sub (1) in (2)

$$E_e = \int_{t_1}^{t_2} [f(t) - c\,x(t)]^2\,dt$$

Hence the value of 'c' should be selected such that E_e will be min. This can be obtained by differentiating E_e w.r.t. c and equating it to zero i.e.

for min E_e

$$\frac{dE_e}{dc} = 0$$

$$\frac{d}{dc}\left[\int_{t_1}^{t_2}[f(t) - c\,x(t)]^2\,dt\right] = 0.$$

$$\frac{d}{dc}\left[\int_{t_1}^{t_2}(f^2(t) + c^2x^2(t) - 2c\,f(t)x(t))\right]$$

$$\underbrace{\frac{d}{dc}\int_{t_1}^{t_2} f^2(t)\,dt}_{=0} + \frac{d}{dc}\int_{t_1}^{t_2} c^2x^2(t) - \frac{d}{dc}\int_{t_1}^{t_2} 2c\,f(t)x(t)$$

$$0 + 2c\int_{t_1}^{t_2} x^2(t)\,dt - 2\int_{t_1}^{t_2} f(t)\,x(t)\,dt = 0$$

$$\boxed{c = \frac{\int_{t_1}^{t_2} f(t)\,x(t)\,dt}{\int_{t_1}^{t_2} x^2(t)\,dt}}$$

→ energy of $x(t)$

It is similar to $c = \frac{f \cdot x}{|x|^2}$

- The same exp can be obtained for min value of $\overline{e^2(t)}$
- Denominator is energy of $x(t)$ it cannot be zero. Hence numerator must be zero to make 'c' zero.
- If c is zero, there will be no component of $f(t)$ along $x(t)$. Then $f(t)$ and $x(t)$ are said to be orthogonal over an interval $[t_1, t_2]$ i.e.

For orthogonality $\boxed{\int_{t_1}^{t_2} f(t)\,x(t)\,dt = 0}$

IIIly if $f(t)$ & $x(t)$ are complex signals, then they are orthogonal

$\int_{t_1}^{t_2} f(t)\,x^*(t)\,dt = 0$ (or) $\int_{t_1}^{t_2} f^*(t)\,x(t)\,dt = 0$

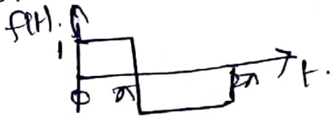
① Show that the following signals orthogonal over an interval [0,1]

$f(t) = 1$

$x(t) = \sqrt{3}(1-2t)$

So $\int_{t_1}^{t_2} f(t)\,x(t)\,dt = 0.$

② Fig. shows a square wave. Represent this signal by $\sin t$, plot an error in this representation.



Sol $f(t) \rightarrow$ square wave

$x(t) \rightarrow \sin t$

$$f(t) = c\,x(t) = c \sin t$$

Value of 'c'

$$c = \frac{\int_{t_1}^{t_2} f(t)\,x(t)\,dt}{\int_{t_1}^{t_2} x^2(t)\,dt}$$

$$\int_{t_1}^{t_2} f(t)x(t)\,dt = \int_0^{2\pi} f(t) \sin t\,dt.$$

$$= \int_0^{\pi} 1 \cdot \sin t\,dt + \int_{\pi}^{2\pi} (-1)\sin t\,dt$$

$$= [-\cos t]_0^{\pi} - [-\cos t]_{\pi}^{2\pi}$$

$$= 4$$

$$\int_{t_1}^{t_2} x^2(t)\,dt = \int_{t_1}^{t_2} \sin^2 t\,dt$$

$$= \int_{t_1}^{t_2} \frac{1-\cos 2t}{2}\,dt$$

$$= \frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\,dt$$

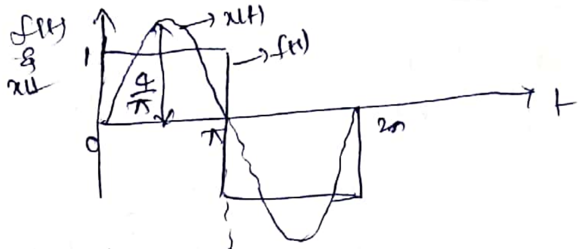
$$= \pi$$

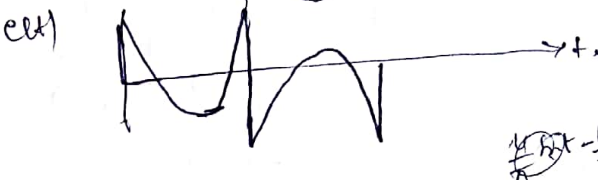
$$\therefore\ C = \frac{4}{\pi}$$

$\therefore$ approximation becomes

$$f(t) = \frac{4}{\pi}\,x(t)$$

error $e(t) = f(t) - c\,x(t)$..





$\frac{4}{\pi}\sin t - \frac{4}{\pi}$

$1 - \frac{4}{\pi}\sin t$

Signal Representation by a discrete set of orthogonal functions

So eqn for mean square error.

ε

$$\varepsilon^2(t) = \frac{1}{t_2-t_1}\left[\int_{t_1}^{t_2} x^2(t)\,dt - (C_1^2K_1 + C_2^2K_2 + \dots + C_n^2K_n)\right]$$

$$= \frac{1}{t_2-t_1}\left[\int_{t_1}^{t_2} x^2(t)\,dt - \sum_{r=1}^{n} C_r^2 K_r\right]$$

if we increase 'n' i.e. if we approximate x(t) by a larger number of orthogonal functions, the error will be smaller.

- But mean square value is positive quantity, hence in the limit as number of terms is made infinity, the sum $\sum_{r=1}^{\infty} C_r^2 K_r$ may converge to integral $\int_{t_1}^{t_2} x^2(t)\,dt$ and then 'e' vanishes.

$$\boxed{\int_{t_1}^{t_2} x^2(t)\,dt = \sum_{r=1}^{\infty} C_r^2 K_r}.$$

$x(t) = C_1 m_1(t) + C_2 m_2(t) + \dots + C_r m_r(t)$

RHS side of above eqn converges to x(t) such that mean square of error is zero. The representation of x(t) is now exact

4

Signal → A function of one (or) more independent variables which contain some information is called signal.

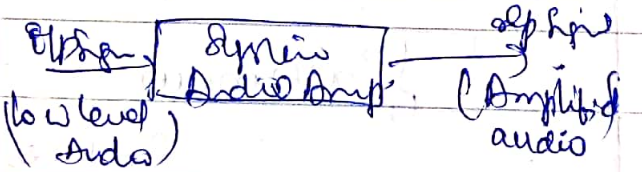
Ex: Electric, voltage (or) current, radio signal, TV signal, telephone signal.

Non-electric → Pressure signal, Sound signal.

System: A system is a set of elements (or) functional blocks that are connected together and produces an o/p in response to an i/p signal.

Ex: Audio amplifier, attenuator, Tx, Rx.

Any m/c (or) engine.

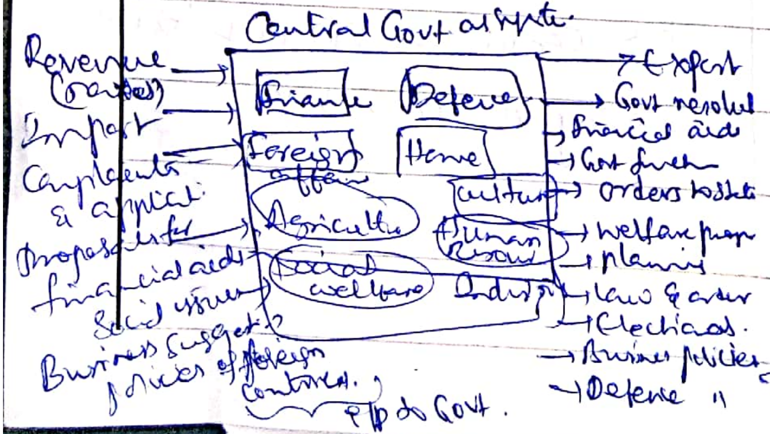


→ every system has one or more i/ps called excitation

" " o/ps called Response.

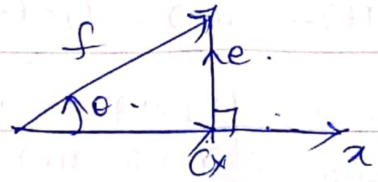
→ i/p & o/p always signals

→ Analogy of signals & system



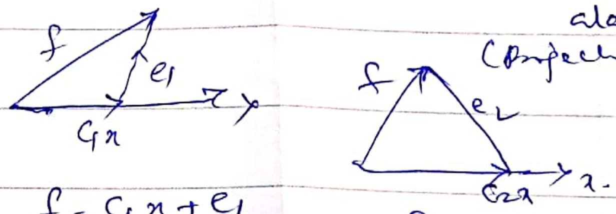
Analogy b/w vectors & signals

Orthogonality concept in vectors



$f \cdot x = |f||x| \cos\theta$ — ①

$f = Cx + e$ — ② $Cx \to$ Component of vector f along x. (Projecting f on x)



$f = C_1x + e_1$ $f = C_2x + e_2$

$e \to$ min only when it is vertical.

e_1, e_2 greater than e.

e is min only when it intersects.

The component of f along x is Cx.

It is also given as $f \cos\theta$.

i.e. $C|x| = |f|\cos\theta$.

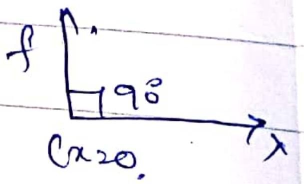
$C|x|^2 = |f||x|\cos\theta$.

$C|x|^2 = f \cdot x$

$C = \frac{f \cdot x}{|x|^2}$

Since $x \cdot x = |x|^2$

$C = \frac{f \cdot x}{x \cdot x}$.



$Cx = 0$.

$f \cdot x = |f||x|\cos\theta$.
$= 0$

f & x are said to be orthogonal if their dot product is zero.

→ Vectors are orthogonal if they are mutually perpendicular.

orthogonality in signals

16/12/13 Med

ECE-A 1, 2, 4, 5, 6, 7, 10, 11, 13, 15, 17, 18, 30, 43, 44, 46, 58, 56 3

$f(t) = Cx(t) + e(t)$

$e(t) = f(t) - Cx(t) \quad t_1 \le t \le t_2 \quad (1)$

min. value of e(t) will give best approximation of f(t) in x(t)

Completeness (or) Closed set of orthogonal functions

A set of mutually orthogonal functions $m_1(t), m_2(t) \ldots m_r(t)$ over an interval (t_1, t_2) is said to be a complete or a closed set if there exist no function $x(t)$ for which it is true that,

$$\boxed{\int_{t_1}^{t_2} x(t)\, m_k(t)\, dt = 0.}$$

for $k = 1, 2 \ldots$

If a function $x(t)$ could be found such that above integral is zero, then obviously $x(t)$ is orthogonal to each member of the set [x(t)] and as a result, itself a member of the set. So the set cannot be complete without x(t) being its member.

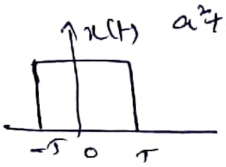
T. Sravan Kumar

1. F.T. of $x(t) = e^{-a|t|} \operatorname{sgn}(t)$.

Ans: $\dfrac{-j2\omega}{a^2+\omega^2}$.

Sampling funct. $Sa(t) = \dfrac{\sin t}{t}$.

Sinc funct. $\operatorname{sinc}(t) = \dfrac{\sin \pi t}{\pi t}$.

2. 

$= 2T \operatorname{Sinc} \omega T$

3. 

4. $x(t) = t e^{-at} u(t)$ Ans: $\dfrac{1}{(j\omega+a)^2}$

5. Det F.T. of $x(t)$ using time shifting property

$x(t) = e^{-3|t-t_0|} + e^{-3|t+t_0|}$ $F\{e^{-a|t|}\} = \dfrac{2a}{a^2+(j\omega)^2}$.

Sol $X(j\omega) = F\{e^{-3|t-t_0|}\} + F\{e^{-3|t+t_0|}\}$

$= e^{-j\omega t_0} F\{e^{-3|t|}\} + e^{j\omega t_0} F\{e^{-3|t|}\}$

$= e^{-j\omega t_0} \cdot \dfrac{2(3)}{(3)^2+(j\omega)^2} + e^{j\omega t_0} \dfrac{2(+3)}{(3)^2+(j\omega)^2} = \dfrac{12\cos\omega t_0}{3^2+(j\omega)^2}$

6. If $x(t)$ & $X(j\omega)$ are F.T. pairs, Det. the F.T. of $x(t) \sin\omega_0 t$.

Sol $F\{x(t)\sin\omega_0 t\} = F\{x(t) \left[\dfrac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right]$

$= \dfrac{1}{2j}\left[F\{x(t)e^{j\omega_0 t}\} - F\{x(t)e^{-j\omega_0 t}\}\right]$

$= \dfrac{1}{2j} X(j(\omega-\omega_0)) - \dfrac{1}{2j} X(j(\omega+\omega_0))$