PC221CE: SOLID MECHANICS

UNIT I: SIMPLE STRESS AND STRAINS

Introduction, Mechanical Properties of Materials, Concept of Stress, Types of Stress

Introduction to Mechanics of Materials

Strength of Materials or Mechanics of Materials is a branch of applied mechanics that deals with the behaviour of deformable solid bodies subjected to various types of loadings. While studying Engineering Mechanics it is assumed that all bodies are either rigid or point particles. In this course on Strength of Materials, the bodies are considered deformable and subjected to failure or breakage. The focus is more on the internal effects in a body due to externally applied loads. This helps in determining the safe loads on a structure and is essential in the safe design of all types of structures like airplanes, antennas, buildings, bridges, ships, automobiles, spacecrafts, etc. This course forms the foundation for most engineering disciplines.

Mechanical Properties of Engineering Materials

The mechanical properties of a material are those which affect the mechanical strength and ability of material to be engineered into a suitable shape or application. Some of the typical mechanical properties of a material are as follows.

Strength: The *strength of a material* is its ability to withstand an applied load without failure. Failure is the state of the material in which it is no longer able to bear the applied load.

Elasticity: The property of a material by the virtue of which it returns to its original shape and size after removal of the applied load is called elasticity. The materials which follow such behaviour are said to be elastic.

Plasticity: The property of a material by the virtue of which it undergoes permanent deformations, even after removal of the applied loads is known as plasticity. The materials which are not elastic are said to be plastic.

Ductility: Ductility is a property which allows the material to be deformed longitudinally to a reduced section under tensile stress. Ductility is often categorized by the ability of material to get stretched into a wire by pulling or drawing. This mechanical property is also an aspect of plasticity of material.

Brittleness: Brittleness means lack of ductility. A brittle material cannot be deformed longitudinally to a reduced section under tensile stress. It fails or breaks without significant deformation and without any warning. It is an undesirable property from structural engineering point of view.

Malleability: Malleability is property of the material which allows the material to get easily deformed into any shape under compressive stress. Malleability is often categorized by the ability of material to be formed in the form of a thin sheet by hammering or rolling. This mechanical property is an aspect of plasticity of material.

Toughness: Toughness is the ability of material to absorb energy and gets plastically deformed without fracturing. Its numerical value is determined by the amount of energy per unit volume. It unit is Joule/ m3. Value of toughness of a material can be determines by stress-strain characteristics of material. For good toughness material should have good strength as well as ductility. For example: brittle materials, having good strength but limited ductility are not tough enough. Conversely, materials having good ductility but low strength are also not tough enough. Therefore, to be tough, material should be capable to withstand with both high stress and strain.

Hardness: Hardness is the ability of a material to resist indentation or surface abrasion. Hardness measures are categorized into scratch hardness, indentation hardness and rebound hardness.

Concept of Stress and Strain

Stress: There are no engineering materials which are perfectly rigid and hence when material is subjected to external loads, it undergoes deformation. While undergoing deformation the particles of the material exert a resisting force. When this resisting force becomes equal to the applied load, an equilibrium condition takes place and deformation stops. This internal resistance is called stress.

Definition of Stress: Thus, we can define stress as follows:

When a body is subjected to an external load, the **internal resistance** developed in the material per unit area of a chosen plane of cross-section is called intensity of stress (σ).

Its SI unit is N/m^2 or Pascal (Pa).

It is common in civil engineering practices is to specify the units of stress in N/mm^2 or MPa.

Consider a uniform cross-section bar under an axial load P. Let us pass an imaginary plane perpendicular to the bar along the middle so that the bar is divided into two halves. What holds one part of the bar with the other part is the internal molecular forces, which arise due to the external load P. In other words, due to the external load there is an internal resistive force that is generated which the holds the body together. This internal resistive force per unit area is defined as stress. If A is the area of cross-section of the bar, then the average stress (σ) on a given cross-sectional area (A) of a material, which is subjected to load P, is given by

$$\sigma = \frac{P}{A} \tag{1}$$

Saint Venant's principle:

We must note that the above expression for stress is the average value of the stress over the entire surface. In reality, the stress varies along the cross-section, particularly at the ends of the bar carrying axial load, as shown in Figure 1.

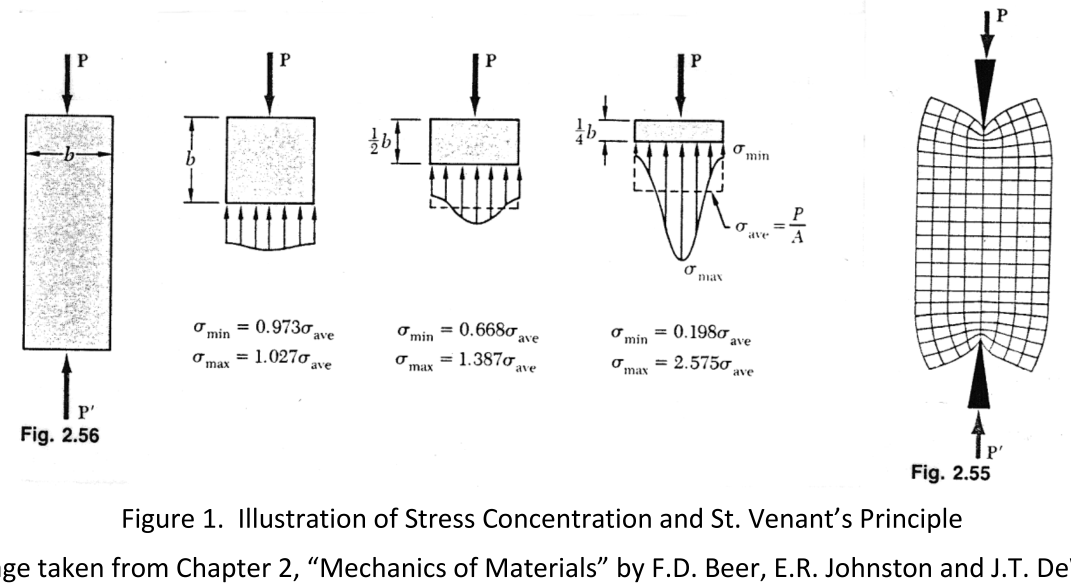


Figure 1. Illustration of Stress Concentration and St. Venant's Principle

(Image taken from Chapter 2, "Mechanics of Materials" by F.D. Beer, E.R. Johnston and J.T. DeWolf)

The stresses are highly concentrated in the immediate vicinity of the point of application of the load and reduce is magnitude as we move away from it along the cross-section. However, as we move away from the end of the bar towards the middle portion of the bar, the stress distribution becomes more uniform throughout the cross-section. Thus, away from the ends, the cross-sections can be assumed to have uniform stress, as given in equation (1). This is called the St. Venant's principle, which is more formally stated as:

The stresses in a deformable solid body at a point sufficiently remote from the point of application of the load depend only on the static resultant of the loads and not on the local distribution of the loads.

Figures (1) and (2) depict this principle.

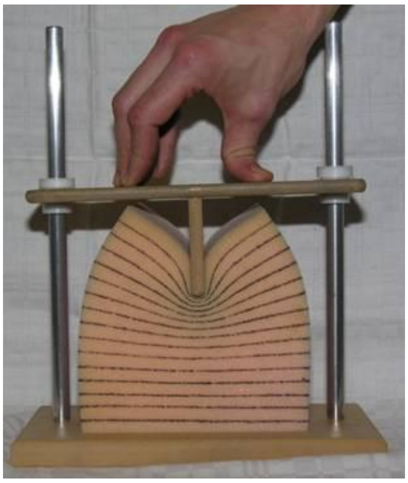


Figure 2. Illustration of St. Venant's Principle

Image taken from the website University of Manchester, U.K.:

http://www.mace.manchester.ac.uk/project/teaching/civil/structuralconcepts/Statics/stress/stress_mod3.php

Types of Stress

A) Normal/Direct Stress

1. Tensile Stress

The stress induced in a body, when subjected to two equal and opposite pull, as a result of which there is an increase in length, is known as tensile stress. Tensile stress tends to elongate the body.

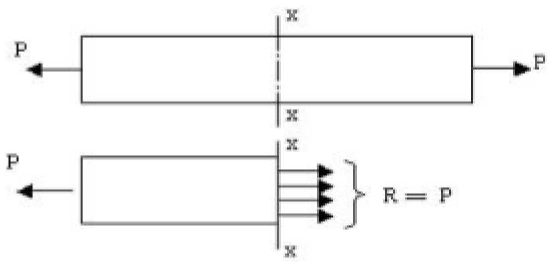


Figure 3. A bar subjected to tensile (axial) loading

Consider a uniform bar of cross-section area A subjected to an axial force P. The stress at any section, x-x, normal to the line of action of tensile force P is shown in the figure. The internal resistance R at x-x is equal to applied force P.

$$\text{Tensile Stress} = \frac{\text{Resisting Force (R)}}{\text{Cross Sectional Area (A)}} = \frac{P}{A} \qquad (2)$$

Under tensile stress, bar suffer stretching or elongation.

2. Compressive Stress

The stress induced in a body, when subjected to two equal and opposites pushes, as a result of which there is decrease in length, is known as compressive stress.

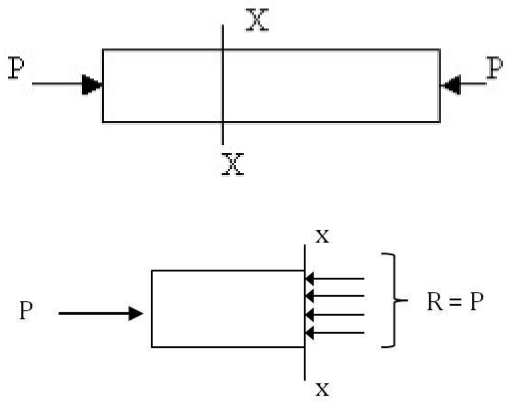


Figure 4. A bar subjected to compressive (axial) loading

Consider a uniform bar of cross-section area A subjected to an axial compressive load P. The stress at any section x-x normal to the line of action of compressive force P is shown in the figure. The internal resistance R at x-x is equal to applied load P.

$$\text{Compressive Stress} = \frac{\text{Resisting Force (R)}}{\text{Cross Sectional Area (A)}} = \frac{P}{A} \qquad (3)$$

Under compressive stress, bar suffers shortening.

Tensile Stresses are considered positive and compressive stresses are considered negative, as per general numerical sign convention for stresses.

B) Shear/Tangential Stress

A shear stress, symbolized by the Greek letter tau, τ, results when a member is subjected to a force that is parallel or tangent to the surface. The average shear stress in the member is obtained by dividing the magnitude of the resultant shear force V by the cross sectional area A. Shear stress is:

$$\text{Shear Stress, } \tau = \frac{\text{Shear Force (V)}}{\text{Cross Sectional Area (A)}} = \frac{V}{A} \qquad (4)$$

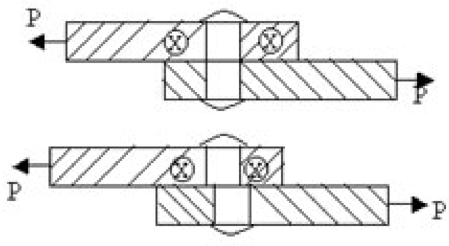


Figure 5. A rivet subjected to shear force

Consider a section of rivet is subjected to equal and opposite force P acting in a direction parallel to the resisting section. Such forces which are equal and opposite and act tangentially across the section, causing sliding of particles one over the other, are called shearing forces and corresponding stress induced in the rivet is called shearing stress.

Consider another example of a Clevis Joint as shown in Figure 6

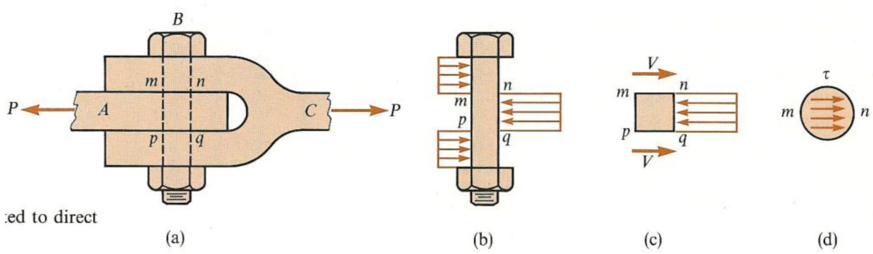


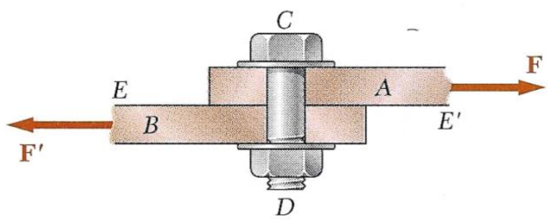
Figure 6. A rivet in a Clevis Joint subjected to shear

a) Typical clevis joint b) Free body diagram of bolt

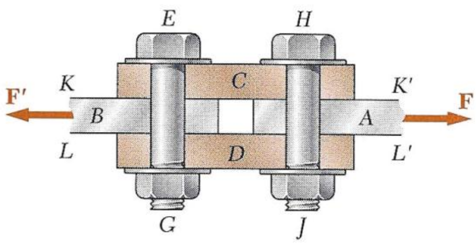
c) Free body of section mnqp d) Shear stresses on section mn

It should be noted that the distributions of shear stresses is not uniform across the cross section. Shear stress will be highest near the center of the section and become zero at the edge. This will be dealt in greater detail in Unit III.

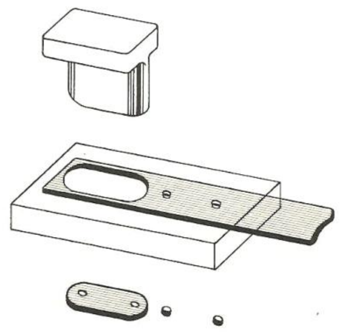
Direct or simple shear arises in the design of bolts, pins, rivets, keys, welds and glued joints.



(a) Single Shear Joint, Shear Stress = F/A



(b) Double Shear Joint, Shear Stress = F/2A



(c) Punching Shear = Punching force / Area

Figure 7. Examples of Single, Double and Punching Shear

(Image taken from Chapter 1, "Mechanics of Materials" by F.D. Beer, E.R. Johnston and J.T. DeWolf)

Concept of complementary shear stresses

Consider an element ABCD from a material subjected to shearing stress (τ) on a faces AB and CD as shown in the Figure 8 (a), due to equal and opposite forces F applied onto the two faces. Since the element is in static equilibrium, it is not just the horizontal forces that are in balance, but the moment also has to be balanced. This unbalanced moment is balanced by counter couple on the two perpendicular faces BC and AD, as shown in figure 8 (b).

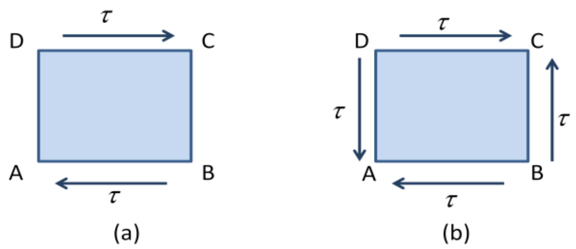


Figure 8. (a) Unbalanced Moments due to Shear

(b) Complementary Shear exists for Moment balance

To understand the existence of complementary shear, consider the following illustration. Suppose that a material block is divided into a number of rectangular elements, as shown by the full lines of Figure 9.

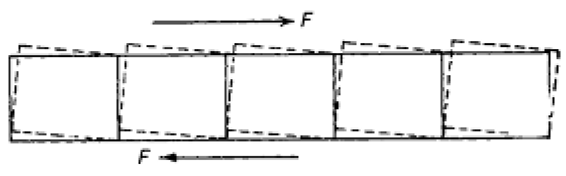


Figure 9. Illustration of existence of complimentary shear

(Image and concept taken from Chapter 3, "Strength of Materials and Structures (4th Edn.)"

by John Case, Lord Chilver and Carl Ross – Arnold Publishers, London)

Under the actions of the shearing forces F, which together constitute a couple, the elements will tend to take up the positions shown by the dotted lines in Figure 9. It will be seen that there is a tendency for the vertical faces of the elements to slide over each other. Actually the ends of the elements do not slide over each other in this way, but the tendency to so do shows that the shearing stress in horizontal planes is accompanied by shearing stresses in vertical planes perpendicular to the applied shearing forces. This is true of all cases of shearing action a given shearing stress acting on one plane is always accompanied by a complementary shearing stress on planes at right angles to the plane on which the given stress acts.

C) Bearing Stress

A bearing stress, symbolized by the Greek letter sigma σ_b, is a compressive normal stress that occurs on the surface of contact between two interacting members. The average normal stress in the member is obtained by dividing the magnitude of the bearing force F by the area of interest. Bearing stress for the situation in Figure 10 is

$$\sigma_b = \frac{\text{Punching Force}}{\text{Contact Area}} = \frac{P}{A_b} = \frac{P}{td} \qquad (5)$$

Bolts, pins and rivets create bearing stresses along the surface of contact.

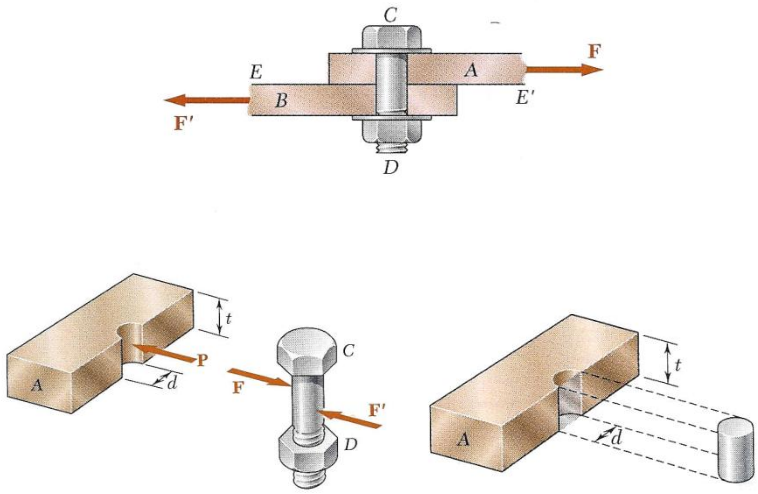


Figure 10. Demonstration of Bearing Stress

(Image taken from Chapter 1, “Mechanics of Materials” by F.D. Beer, E.R. Johnston and J.T. DeWolf)

PC221CE: SOLID MECHANICS

UNIT I: SIMPLE STRESS AND STRAINS

Concept of Strain, Types of Strain, Hooke's Law, Simple Problems

Strain

Strain is a measure of deformation produced by the application of external force. It is the ratio of change in length to original length. It is denoted by Epsilon (ε). Strain is dimensionless. Strain in direction of applied load is known as linear or longitudinal strain.

$$\text{Strain}\,(\varepsilon) = \frac{\text{Change of length}\,(\delta l)}{\text{Original length}\,(l)} \tag{6}$$

Types of Strain

1. Tensile strain

Let initial length of bar before applied load be l, when tensile load P is applied. Let the bar be elongated by δl.

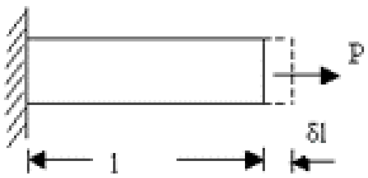


Figure 11. Tensile Strain

$$\text{Tensile strain}\,(\varepsilon) = \frac{\text{Extension in length}\,(\delta l)}{\text{Original length}\,(l)} \tag{7}$$

2. Compressive strain

Let initial length of bar before applied load be l, when compressive load P is applied. Its length gets decrease by δl.

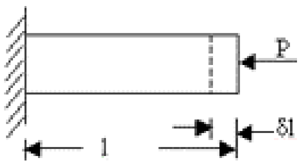


Figure 12. Compressive Strain

$$\text{Compressive Strain}\,(\varepsilon) = \frac{\text{Shortening in length}\,(\delta l)}{\text{Original length}\,(l)} \tag{8}$$

3. Shear Strain (γ)

To get a proper definition of strain, it is important to understand the concept of complementary shear stresses in a material as discussed in the previous section.

Consider again the element ABCD from a material subjected to shearing stress (τ) on a faces AB and CD as shown in Figure 13 (a). We may assume that the deformation occurs as shown in Figure 13 (b). However, this is possible only when the base AB is glued to the bottom. If the element ABCD is the portion of the material as shown in Figure 14 (a) subject to shear forces, then its deformation will be as shown in Figure 14 (b). This deformation is more common case of shear deformation in materials.

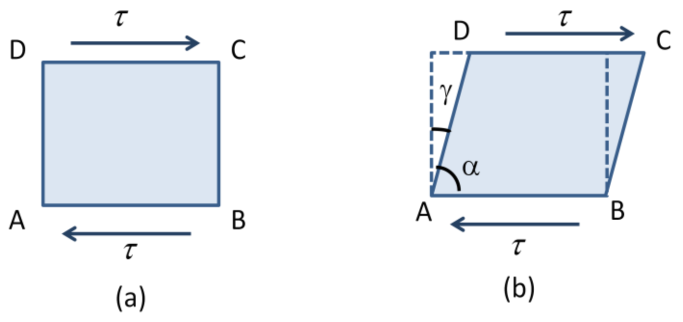


Figure 13. Shear deformation when edge AB is glued at the bottom

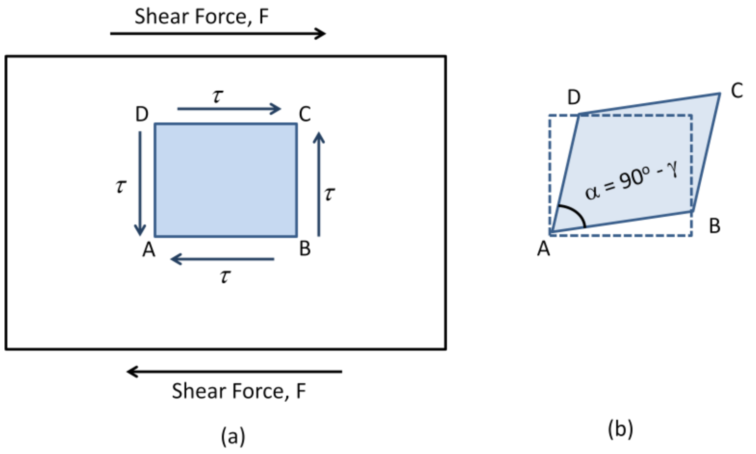


Figure 14. Shear deformation in a plane in a general case

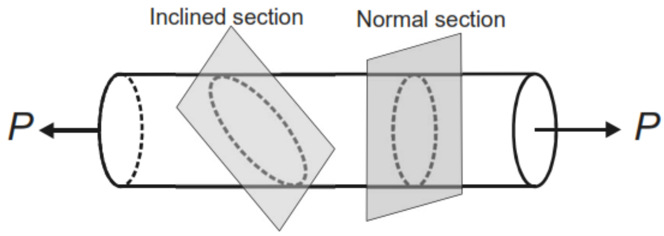
The shear strain is best defined as the change in the angle between the originally perpendicular edges of the rectangular element of the material, upon application of the shear stress. Original angle between AB and AD is 90°. After deformation it is α. Thus, shear strain is defined as

$$\gamma = 90^{o} - \alpha \tag{9}$$

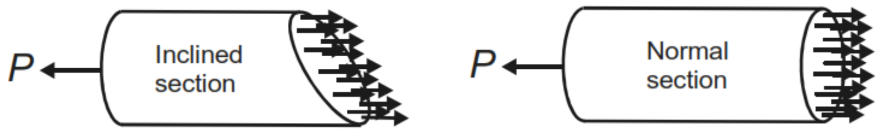
This definition is valid for both cases as shown in Figures 13 and 14.

Existence of Normal and Shear Stresses due to Axial Loading

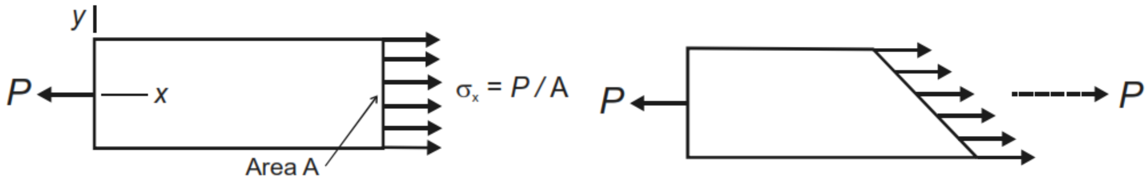
To obtain a complete picture of the stresses in a bar, we must consider the stresses acting on an "inclined" (as opposed to a "normal") section through the bar as shown in the Figure below. Because the stresses are the same throughout the entire bar, the stresses on the sections are uniformly distributed.



(a)



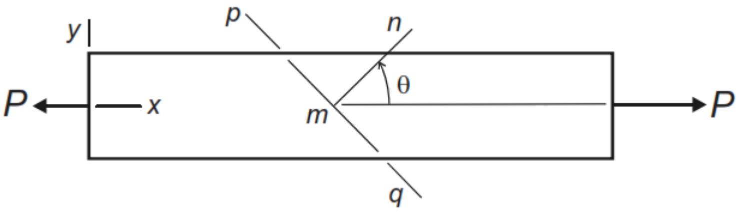
(b)



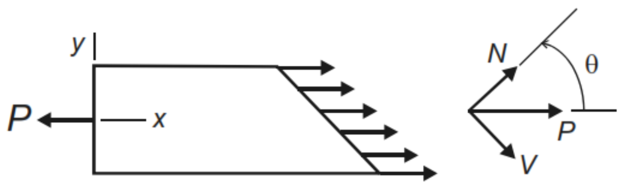
(c) Two-dimensional view of the previous image

Figure 15. Stress Distribution on an inclined section in axial loading

Specify the orientation of the inclined section *pq* by the angle θ between the x axis and the normal to the plane.



(a)



(b)

Figure 16. Resolution of the axial stress into normal and shear components

The force P can be resolved into components:

Normal force N perpendicular to the inclined plane, N = P cos θ

Shear force V tangential to the inclined plane V = P sin θ

If we know the areas on which the forces act, we can calculate the associated stresses. The area of the inclined plane section of the bar, *pq*, is A/ cos θ.

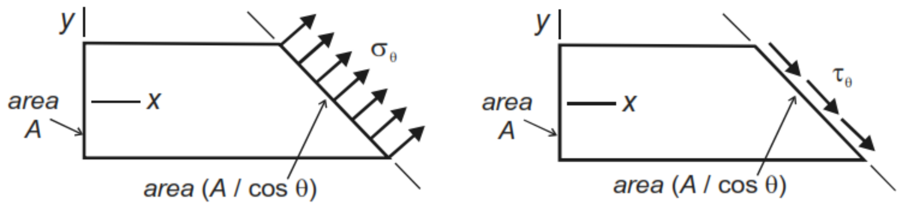


Figure 17. Normal and shear stresses on the inclined plane

Thus,

$$\sigma_\theta = \frac{Force}{Area} = \frac{N}{Area} = \frac{P\cos\theta}{A/\cos\theta} = \frac{P}{A}\cos^2\theta$$

$$\sigma_\theta = \sigma_x \cos^2\theta = \frac{\sigma_x}{2}(1+\cos 2\theta) \quad ☆$$

(10 a)

$$\tau_\theta = \frac{Force}{Area} = \frac{-V}{Area} = \frac{-P\sin\theta}{A/\cos\theta} = -\frac{P}{A}\sin\theta\cos\theta$$

$$\tau_\theta = -\sigma_x \sin\theta\cos\theta = -\frac{\sigma_x}{2}(\sin 2\theta) \quad ☆$$

(10 b)

We can thus see that even a normal force offering axial load to a bar will produce both normal and shear stresses in the internal material of the bar. We may also note that when $\theta = 0^o$, that is when the plane pq is normal to the load P, we get normal stress as maximum and equal to P/A, while the shear stress on plane pq is zero. Another very important observation is that when $\theta = 45^o$, the shear stress is maximum and is equal to P/2A (in magnitude). The maximum shear stress produced is half the value of maximum normal stress.

Hooke's Law

Within elastic limit or more accurately up to the proportional limit of the material, the stress is directly proportional to strain.

For normal stress, the Hooke's law gives $\sigma \propto \varepsilon$ or

$$\sigma = E\varepsilon \qquad (11a)$$

Where σ = Axial/Normal Stress

ε = Axial/Normal Strain

E is known as Modulus of Elasticity or Young's Modulus or Elastic Modulus.

Young's Modulus (E) = $\frac{\text{Tensile/Compressive stress } (\sigma)}{\text{Tensile/compressive strain } (\varepsilon)}$

For shear stress, the Hooke's law gives $\tau \propto \gamma$ or

$$\tau = G\gamma \qquad (11b)$$

Where τ = Shear Stress

γ = Shear Strain

G is known as Modulus of Rigidity or Shear Modulus.

Modulus of Rigidity (G) = $\frac{\text{Shear Stress } (\tau)}{\text{Shear Strain } (\gamma)}$

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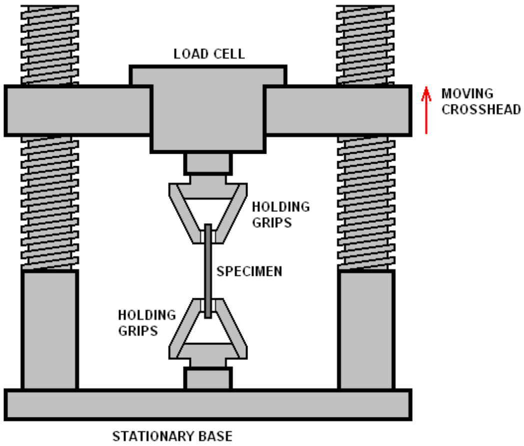
UNIT I: SIMPLE STRESS AND STRAINS

Simple Tension Test, Stress-Strain Curve for Mild-Steel, Definitions of Critical Points on the Stress-strain curve, Strain-Hardening, Necking, Engineering Stress-Strain Vs True Stress-Strain

Simple Tension Test for Mild Steel Specimen on Universal Testing Machine (UTM)

To study the behaviour of ductile materials in tension, a standard mild steel specimen is used for tensile test on universal testing machine (UTM).



- On the UTM more than one test can be performed like Tension, Compression, Bending and Shear etc.
- The end of specimens is gripped in UTM and one of the grips is moved apart by hydraulic jack or system, thus exerting tensile load on the specimen.
- The load applied is indicated on dial and the extension in the initial stages is measured by using an extensometer fixed on specimen itself and later stages by scale fixed on machine.
- Almost all machines are provided with an autographic recorder which is directly records the load vs deformation curve (or) stress vs strain curve.
- To fix the extensometer on specimen, two points are marked on a portion of specimen. The distance between these points over which the extension is marked is called **gauge length**.

The **load vs deformation curve** is not unique, even for the specimen of the same material. As the geometry (either length or cross-sectional area) of the specimen changes, the load-deformation curve also changes.

On the other hand, the **Stress vs Strain curve** for a material is unique, irrespective of the geometric dimensions of the material specimen. Thus, for studying engineering properties of a material, Stress-Strain curve is commonly used.

The following is an example of a Stress-Strain curve for mild steel specimen obtained by performing simple tension test.

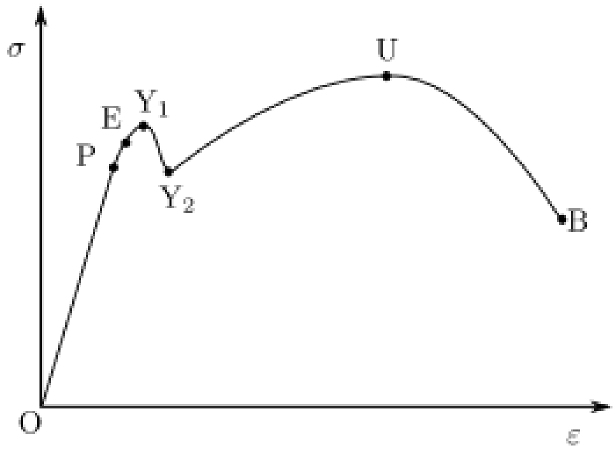


Figure 18. Stress-Strain curve for mild steel

Procedure for testing is to be done as per the following IS code:

IS 1608: 2005 Title: Metallic Materials – Tensile Testing at Ambient Temperature

Important Points on the Stress-Strain Curve

• P, Proportionality Limit:

The point up to which stress is linearly proportional to strain and hence Hooke's law is valid up to P. i.e. linear elasticity is valid. The slope of this line OP is nothing but the modulus of elasticity or Young's modulus.

• E, Elastic Limit:

The maximum stress that may be developed in a simple tension test such that there is no permanent or residual deformation when the load is entirely removed. Between P and E material is non-linearly elastic

• Y_1, upper yield point:

Point around which dislocations break through interstitial carbon atoms and relieve lateral strains. This phenomenon is particular to mild steels.

• Y_2, lower yield point:

Once carbon atoms are overcome by the dislocation, relatively lower stresses are required to keep the dislocation moving. This happens around Y_2. This phenomenon is also specific to mild carbon steel. The stress corresponding to this point is called as yield stress or yield strength.

Note: Yield stress is also defined, in other words, as the stress at which the material begins to deform plastically. That means, once stressed beyond the yield point the specimen will not gain back the original length even after the load is removed.

• U, Ultimate Stress

Maximum stress in a tensile test is reached at this point. The stress corresponding to this point is called as the Ultimate Strength.

• B or R, Breaking point or Rupture point

Point at which specimen fails, breaking into two. The stress corresponding to this point is called as the breaking strength or the rupture strength.

Note: Typical values of strains of mild steel test specimen are:

(i) ε_y = 0.0014 at yield point

(ii) ε_u = 0.16 at ultimate stress

(iii) ε_r = 0.35 at rupture

Further Observations from Stress-Strain Curve

Strain Hardening:

Strain Hardening is a phenomenon occurs in certain materials, particularly ductile metals, when it is strained beyond the yield point. An increased stress is required to produce additional plastic deformation and the metal apparently becomes stronger and more difficult to deform.

Thus, **Strain Hardening**, also known as **Work hardening** or **cold working**, is defined as the strengthening of a metallic material by plastic deformation. This strengthening occurs because of dislocation movements and dislocation generation within the crystal structure of the material.

In the Stress-Strain curve of Figure 18, the zone between the lower yield point (Y_2) and the Ultimate Stress (U) is called as the **strain-hardening zone**.

In Figure 19, the phenomenon of strain-hardening is more clearly illustrated.

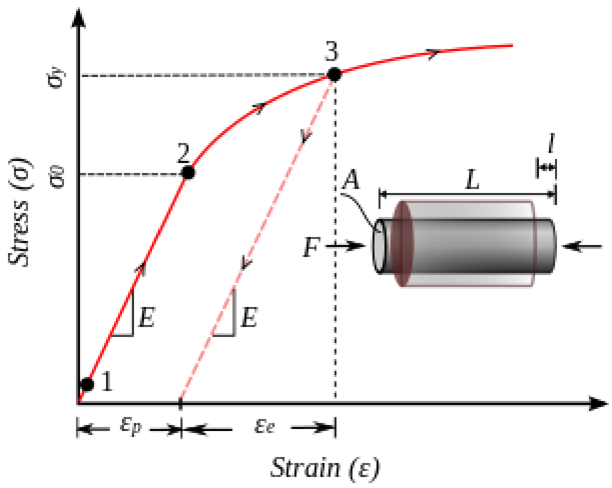


Figure 19: Illustration of the Phenomenon of Strain hardening

(Courtesy: Wikipedia)

For work hardening materials the yield stress increases with increasing plastic deformation. The strain can be decomposed into a recoverable elastic strain, ε_e and an inelastic strain, ε_p. The stress at initial yield is σ_0. But, once the materials is stressed beyond σ_0 and up to the point 3, at stress σ_y, and unloaded, it has a **permanent strain** ε_p at zero stress. This permanent strain at zero stress develop due to onset of plasticity in the material is also called as the **permanent set** or **residual strain**. If once again the material is gradually stressed its stress-strain curve follows the straight line path till it reaches σ_y after which it again deforms plastically. This means that the yield stress is σ_y for the second cycle of loading, which is greater than the yield stress in the first cycle of loading, σ_0. This phenomenon is called as strain-hardening.

Necking

Necking is a mode of tensile deformation in ductile materials where relatively large amounts of strain localize disproportionately in a small region of the material. This results in prominent decrease in local cross-sectional area and the material specimen assumes a shape in the form of a "neck". This phenomenon is called necking.

In the Stress-Strain curve of Figure 18, the zone between the Ultimate Stress (U) and the point of Rupture (B) is called as the **necking zone**.

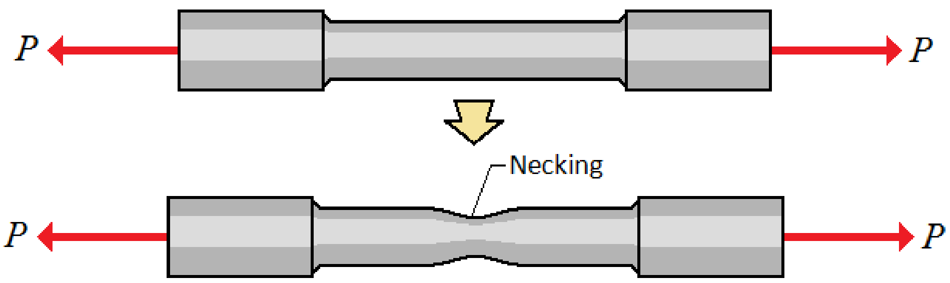


Figure 20: The phenomenon of necking

(courtesy: http://www.engineeringarchives.com/les_mom_necking.html)

Engineering Stress and Engineering Strain

(also called as Nominal Stress and Nominal Strain)

To plot Stress Vs Strain curve for the test specimen used in the simple tension test, the stress and the strain can be calculated in two ways.

Commonly, the Stress is calculated as the ratio of the applied load P measured on the test equipment (Universal Testing Machine) and the original area of cross-section A_0 of the specimen. This is called **engineering stress or nominal stress**.

$$\text{Engineering Stress} = \frac{\text{Applied load}}{\text{Original Area of cross-section}}; \qquad \sigma = \frac{P}{A_0}$$

Engineering strain is the amount that a material deforms per unit length in a tensile test. The reference length for this purpose is taken as the original guage length, before the material is tested. This is also known as nominal strain.

$$\text{Engineering Strain} = \frac{\text{Change in the guage length of the specimen}}{\text{Original guage length of the specimen}}; \qquad \varepsilon = \frac{\Delta L}{L_0}$$

True Stress and True Strain

In tensile test, since the cross-sectional area of the specimen decreases as applied load P increases, the stress plotted in the stress-strain diagram may not represent the actual stress in the specimen. This is observed particularly for ductile materials after the onset of the yield.

Thus, **true stress** is defined as the applied load divided by the actual cross-sectional area (the changing area with respect to time) of the specimen at that load.

$$\text{True Stress} = \frac{\text{Applied load}}{\text{cross-section area at the time of load applied}}; \qquad \sigma_t = \frac{P}{A}.$$

True strain at any particular instance of the loading of the specimen is equal to the natural log of the quotient of current length over the original length as given by the equation: $\varepsilon = \ln \frac{L}{L_0}$.

This expression for true strain is obtained as follows:

Instead of using the total elongation ΔL and the original value L_0 of the gage length, all the successive values of L are used that have been recorded. Dividing each increment ΔL of the distance between the gauge marks, by the corresponding value of L, the elementary strain is obtained:

$$\Delta\varepsilon = \frac{\Delta L}{L}$$

Adding the successive values of Δε, the true strain, ε_t, is defined:

$$\varepsilon_t = \sum \Delta\varepsilon = \sum \frac{\Delta L}{L}$$

With the summation replaced by an integral, the true strain can also be expressed as follows:

$$\varepsilon_t = \int_{L_0}^{L} \frac{dL}{L} = \ln \frac{L}{L_0}$$

Note: The diagram obtained by plotting true stress versus true strain reflects more accurately the behavior of the material. There is no decrease in true stress during the necking phase. Also, the results obtained from tensile and from compressive tests will produce essentially the same plot when true stress and true strain are used. This is not the case for large values of the strain when the engineering stress is plotted versus the engineering strain. However, engineers, whose responsibility is to determine whether a load, P, will produce an acceptable stress and an acceptable deformation in a given member, will want to use a diagram based on the engineering stress and the engineering strain, since their respective expressions involve data that are available to them, namely the cross-sectional area A_0 and the length L_0 of the member in its undeformed state.

PC221CE: SOLID MECHANICS

UNIT I: SIMPLE STRESS AND STRAINS

Stress-Strain Curves for Other Materials - comparison, Proof Stress, Elasticity Modulus for Non-Linear Elasticity, Ductility, and Permanent Set – Slip and Creep.

STRESS-STRAIN CURVES FOR MATERIALS OTHER THAN MILD STEEL

In Figure 18, the Stress-Strain Curve for mild steel was presented and discussed. Now we consider the stress-strain curves for other materials.

(A) High Strength Steels

The behaviour of high strength steels, which are sometimes also referred to as High Strength Deformed (HSD) steel bars are much different from mild steel as indicated by the stress-strain curves in Figures 21 and 22.

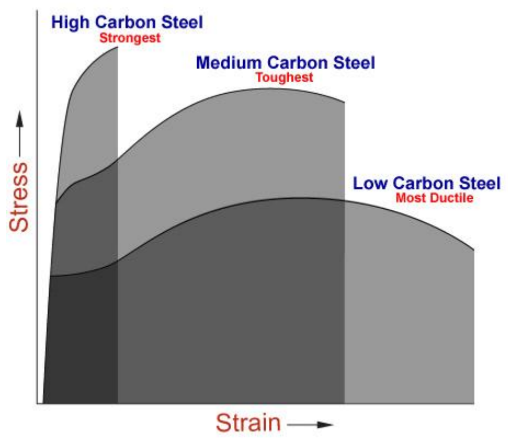


Fig 21: Mild Steel Vs High Strength Steels

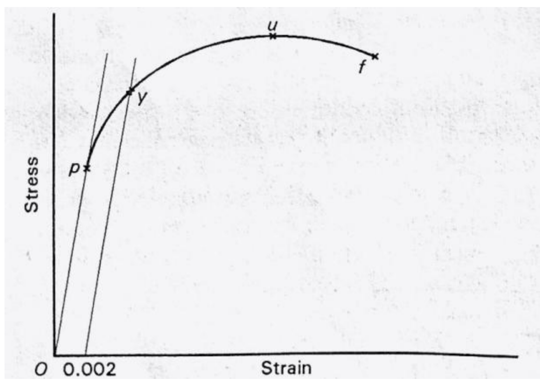


Fig 22: Stress-Strain curve for High Strength Steels

Firstly, high strength steels are less ductile but of higher strength as compared to mild steel. This is indicated by shorter strains and high rupture stresses in the stress-strain curves (Fig.21).

Secondly, the stress-strain curves for high strength steels (HSD steel bars) do not have a well defined yield point as in the case of mild steel (see Fig 21 and 22).

The HSD material displays linear characteristics of the stress-strain curve up to proportional limit, say p. Beyond 'p', a smooth transition of curvature takes place without a well-defined yield point.

Off-set method to determine the yield point and Proof Stress:

For such materials, the yield stress (σ_y) is determined by 0.2% off-set method. A line parallel to the initial straight line part of the curve is drawn from 0.2 percent strain on the x-axis (as shown in Fig 22) to intersect the stress-strain curve at y. This intersection point y is considered as the yield point and the stress corresponding to this point is taken as the yield stress. This yield stress determined by 0.2% off-set method is called as the **Proof Stress**, and sometimes called as 0.2 percent proof stress.

Thus, **Proof Stress** is defined as the stress which induces a specific residual strain (usually 0.2 percent) in the material.

(B) Concrete

The strength of concrete in tension is about one-tenth of its strength in compression. Thus, concrete is usually tested in compression, as per the procedure specified by IS: 516-1959. The stress-strain curve for concrete is shown in Figure 23.

Concrete shows a non-linear behaviour in the stress-strain curve even for small stress.

(**NOTE:** There is difference between non-linear elasticity and plasticity)

The strain at failure for concrete is about 0.002 to 0.004 which is about 100 times smaller than the strain at failure for mild steel (which is about 0.35). Thus, concrete structures fails abruptly without showing significant deformations to the naked eye. However, steel structures, being much more ductile, show significant deformations before actual failure.

Initial modulus, Secant modulus and Tangent Modulus

In mild steel and other materials with initial linear characteristics of the stress-strain curves, the slope of the initial linear portion of the curve is the Young's Modulus of Elasticity (E).

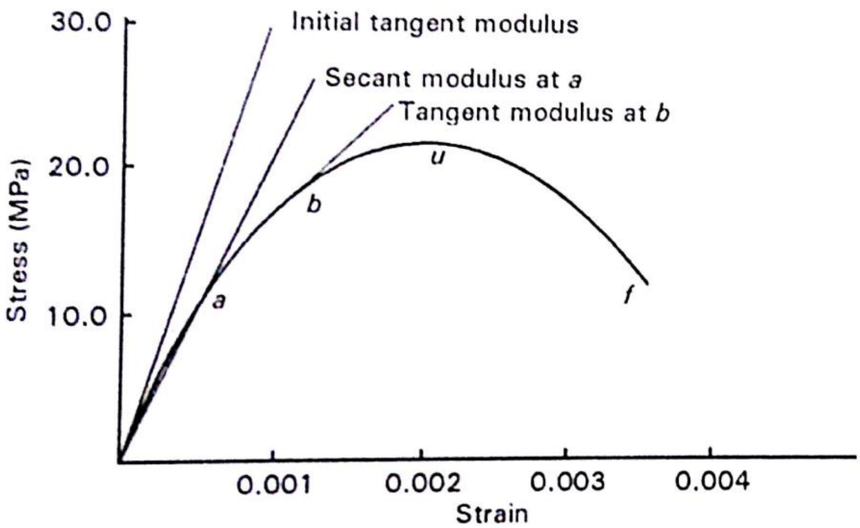


Figure 23: Stress-Strain Characteristics of Concrete in Compression

However, due to non-linear elasticity, the slope in the stress-strain curve in concrete is varying. Thus, following are the methods to determine elasticity modulus in concrete.

Initial Modulus: The elasticity modulus of concrete is usually defined by the slope of the stress-strain curve at the origin. This slope is called as the initial tangent modulus or simply **initial modulus**.

Secant Modulus: If the elasticity modulus of concrete is defined by the slope of the line joining the origin and a point (say a) on the stress-strain curve, then it is known as **secant modulus**, corresponding to the stress at that point.

Tangent Modulus: If the elasticity modulus of concrete is defined by the slope of the tangent to the stress-strain curve at some point (say b), then it is known as **tangent modulus**, corresponding to the stress at that point.

(C) OTHER MATERIALS

Typical values of material and elastic properties of few metals and alloys are given below:

Table 1.1 *Physical and elastic properties of a few metals and alloys (gage length = 50.0 mm)*

S. No	Metal/alloy	Unit weight (kN/m^3)	Proportionality limit (MPa)	Ultimate strength (MPa)	E (GPa)	Poisson's ratio	Percentage elongation
1	Mild steel	78.5	240.0	410.0	200.0	0.30	35.0
2	HSD steel	78.5	420.0	550.0	200.0	0.30	18.0
3	Cast iron	72.0	40.0	140.0	100.0	0.25	3.0
4	Wrought iron	77.0	210.0	370.0	170.0	0.33	35.0
5	Aluminium (cast)	26.5	60.0	90.0	70.0	0.22	20.0
6	Aluminium (alloy)	27.0	220.0	390.0	70.0	0.22	25.0
7	Brass (rolled)	85.0	170.0	380.0	100.0	0.25	30.0
8	Bronze (cast)	82.0	140.0	230.0	80.0	0.14	10.0
9	Copper (hard drawn)	88.0	260.0	380.0	120.0	0.48	4.0

[Courtesy: Taken from "Strength of Materials – A Practical Approach Vol.1" by D. S. Prakash Rao.]

The stress-strain curves for various other materials are shown below in Figures 24.

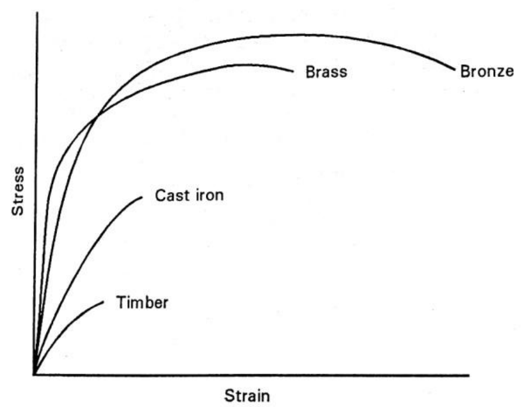


Figure 24 (a)

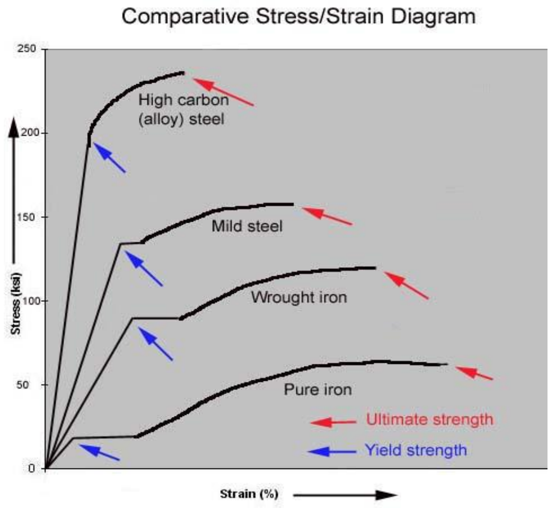


Figure 24 (b)

DUCTILE VS BRITTLE MATERIALS

The contrasting examples between the ductile materials like mild steel and brittle materials like cast iron and concrete is illustrated through comparing their stress-strain curves in Figures 24(a), (b) and Figure 25.

The stress-strain curves of the ductile materials are elongated. Before rupture, there is sufficient elongation of the test specimen in the plastic zone. The stress-strain curves for brittle materials end abruptly. That is to say that the brittle materials rupture suddenly, even before giving visual indications of any significant plastic deformation.

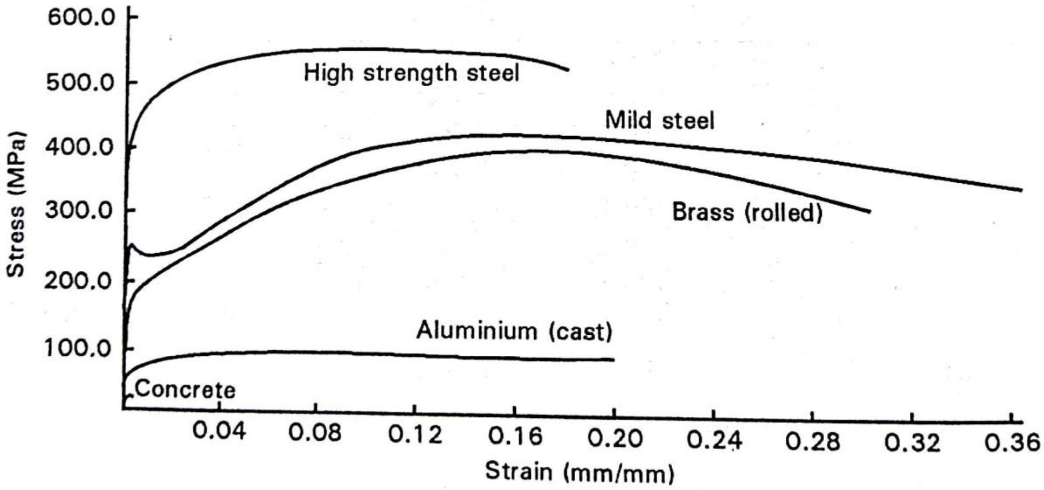
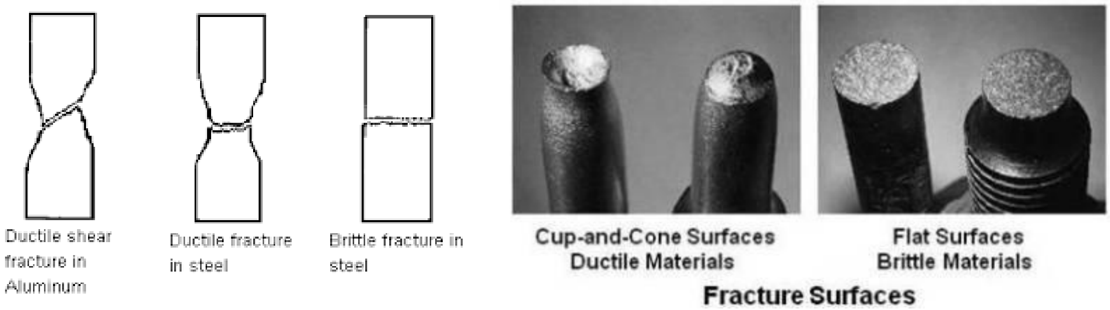


Figure 25. Stress-Strain curve for ductile and brittle materials

It should be notes that for ductile materials fail primarily due to shear stress, and their typical fracture mechanism is cup-cone fracture mechanism. While the brittle materials fails, primarily due to normal stress and failure surface is flat, as shown in the following figures.



(a) (b)

Figure 27. Ductile and brittle failure patterns in materials

Ductility of a material is usually expressed in terms of **percentage elongation** of gage length of the specimen at fracture or **percentage contraction** of cross-section area at fracture. These are calculated as follows:

Percentage elongation = $\frac{l_u - l}{l} \times 100$, where l = original gage length and l_u = gage length at fracture

Percentage contraction = $\frac{A - A_u}{A} \times 100$, where A = original c.s. area and A_u = c.s. area at fracture

Concepts of Permanent Plastic Deformation (Slip and Creep)

If the material has a well-defined yield point, the elastic limit, the proportional limit and the yield point are essentially equal. In other words, the material behaves elastically and linearly as long as the stress is kept below the yield point.

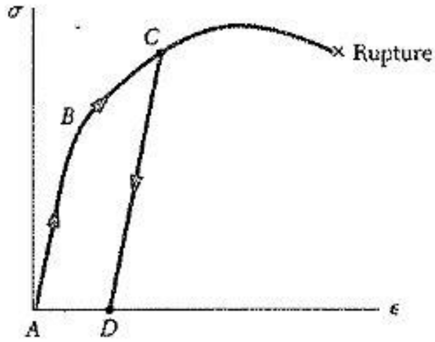


Figure 28: Concept of Permanent Set

If the yield point is reached, however, yield takes place as described and, when the load is removed, the stress and strain decrease in a linear fashion, along a line CD parallel to the straight-line portion AB of the loading curve (Fig. 28).

The fact that ε does not return to zero after the load has been removed indicates that a **permanent set** or **plastic deformation** of the material has taken place. For most materials, the plastic deformation depends not only upon the maximum value reached by the stress, but also upon the time elapsed before the load is removed.

The stress-dependent part of the plastic deformation is referred to as **slip**, and the time-dependent part is referred to as **creep**.

Creep usually occurs as a result of long-term exposure of the material to high levels of stress that are still below the yield strength of the material. Creep is more severe in materials that are subjected to heat for long periods, and generally increases as they near their melting point. Unlike brittle fracture, creep deformation does not occur suddenly upon the application of stress. Instead, strain accumulates as a result of long-term stress. Therefore, **creep is a "time-dependent" deformation**.

PC221CE: SOLID MECHANICS

UNIT I: SIMPLE STRESS AND STRAINS

Working Stress, Factor of Safety, Deformation of bars under axial loads – uniform sections, abruptly varying sections.

Working Stress and Factor of Safety

In real life structural systems, there are uncertainties in
(a) load conditions,
(b) material characteristics at site,
(c) non-linearity of elastic modulus.

Thus, the predicted strength and behavior characteristics of materials in well controlled laboratory set-up may differ at the actual application site. Therefore, as a safety measure, while designing a structural member, we need to take into consideration the above mentioned uncertainties. For this purpose, the structural member is assumed to bear stress which is much lower than the actual yield stress.

Working Stress (σ_w): Maximum stress that a material is allowed to bear in design practices is called **working stress**, **allowable stress** or **maximum permissible stress**.
It is necessary that the working stress should be well below the elastic limit and to achieve this condition, the yield stress is divided by factor of safety to obtain working stress.

$$\text{Working Stress}\,(\sigma_w) = \frac{\text{Yield Stress}\,(\sigma_y)}{\text{F.O.S.}}$$

Factor of Safety (F.O.S): Ratio of yield stress to working stress is called factor of safety.

$$\text{Factor of Safety (F.O.S.)} = \frac{\text{Yield Stress}\,(\sigma_y)}{\text{Working Stress}\,(\sigma_w)}$$

Sometimes, factor of safety is taken as the ratio of ultimate stress to working stress, in some special design methodologies.

Deformation of Axially Loaded Bars

Consider a bar of length L and cross-sectional area A, subjected to an axial load of P. If ΔL is the elongation of the bar due to this load then it follows from Hooke's law

$$\varepsilon = \frac{\sigma}{E}$$

Substituting for stress and strain as $\sigma = \frac{P}{A}$ and $\varepsilon = \frac{\Delta L}{L}$ in the above equation, we get the elongation of the bar as

$$\Delta L = \frac{PL}{AE}.$$

This expression will be used for all the deformation problems in bars.

Principle of Superposition

The principle of superposition simply states that on a linear elastic structure, the combined effect of several loads acting simultaneously is equal to the algebraic sum of the effects of each load acting individually.

In other words, the total displacement or internal loadings (stress) at a point in a structure subjected to several external loads, which can be determined by adding together the displacements or internal loadings (stress) caused by each external load acting separately.

Problems on Deformation of Bars under Axial Loads

Categories within Determinate Problems:

1. Based on Simple Tension Test and related characteristics
2. Design problems of a single bar or ropes of uniform cross-section
3. Bars of abruptly varying cross-section and materials
4. Bars of uniformly varying cross-section
5. Deformation of bars under self- weight

PC221CE: SOLID MECHANICS

UNIT I: SIMPLE STRESS AND STRAINS

Lateral Strain, Poisson's Ratio, Generalized State of Stresses in 3D, Volumetric Strain, Bulk Modulus, and Relationship between various Elastic Modulii.

Lateral Strain and Poisson's Ratio

In all engineering materials, the longitudinal deformation (elongation/compression) produced by an axial force along the longitudinal axis is accompanied by a deformation (contraction/elongation) in the transverse directions. This deformation of length in the transverse direction is referred to as lateral deformation. Lateral deformation per unit length is called as lateral strain.

Strain at right angles to the direction of applied load is known as lateral strain.
δl = Increase in length
δb = Decrease in breadth
δd = Decrease in depth

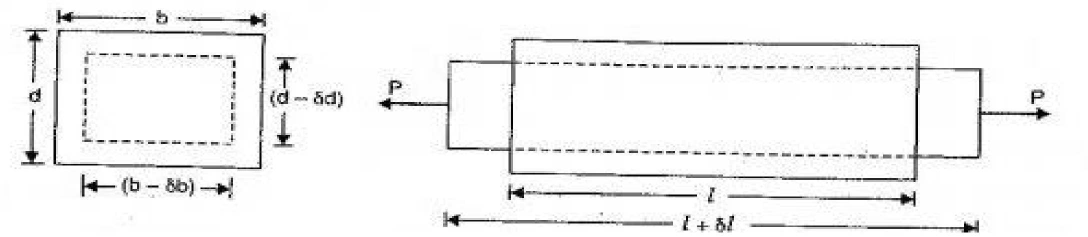


Figure 29: Transverse deformation of a bar

Lateral Strain = $-\dfrac{\delta b}{b}$ or $-\dfrac{\delta d}{d}$

- If longitudinal strain is tensile, lateral strain will be compressive.
- If longitudinal strain is compressive, lateral strain will be tensile.

This lateral strain differs from material to material. For every material the amount of lateral strain developed due to axial stresses is determined by an intrinsic characteristic of the material called as the Poisson's ratio.

Poisson's Ratio: Ratio of lateral strain to longitudinal strain is called Poisson's ratio. It is constant for given material, when the material is stressed within elastic limit. It is denoted by ν (Greek letter Nu) or 1/m.

Poisson's ratio (ν) = $\frac{1}{m} = -\frac{Lateral\ strain}{Longitudinal\ strain}$

Poisson's Ratio varies from 0.25 to 0.33 for steel and 0.45 to 0.50 for rubber. The value of ν lies between 0 and 0.5.

Material Characteristics:

The following are some of the important characteristics of a material in terms of studying their mechanical properties:

A. **Homogeneous Materials** are materials of uniform composition throughout their geometry that cannot be mechanically separated into different materials. Example: Metals
B. **Isotropic Materials** are materials whose mechanical properties remain same in all directions. That is to say that Elastic modulus-E, Poisson's ratio-ν, ductility, strength, etc. remain same in all directions. Example: Most Metals, glass.
C. **Anisotropic materials** are materials whose mechanical properties differ in different directions. Materials that are not isotropic are called as anisotropic materials. That is to say that Elastic modulus-E, Poisson's ratio-ν, ductility, strength, etc. differ in different directions of loading. Example: Wood, layered minerals such as slate, carbon fibre reinforced composites
D. **Orthotropic materials** are a class of anisotropic materials whose mechanical properties vary along three mutually perpendicular directions.

Three Dimensional Stresses and Strains – Stress Tensor – Generalized Hooke's Law

So far we discussed stresses that are acting along one direction particularly for a member subjected to axial loading.
Now we shall consider a case of generalized loading, that is, a material is loaded from all three mutually perpendicular directions, both normal and shear loads. Consider a 3D rectangular Cartesian coordinate system.

In this case, the stress developed at a point will have nine components as follows:
Normal stresses: σ_{xx}, σ_{yy}, σ_{zz} (Most commonly denoted as σ_x, σ_y, σ_z)
Shear stresses: τ_{xy}, τ_{yz}, τ_{zx}, τ_{yx}, τ_{zy}, τ_{xz}.

By the principle complimentary shear for body under static equilibrium,
$\tau_{xy} = \tau_{yx}$, $\tau_{yz} = \tau_{zy}$, $\tau_{zx} = \tau_{xz}$

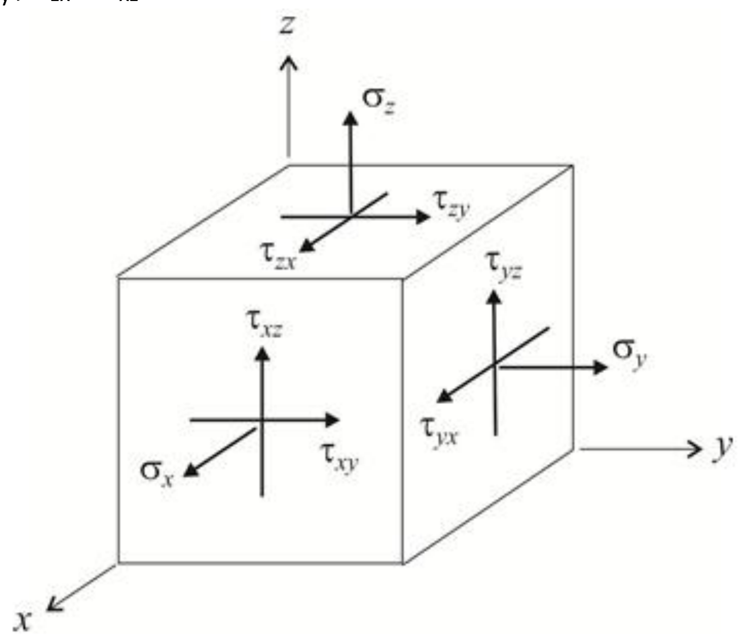


Figure 29: Generalized State of Stress

Stress at a point in a material has 9 components and is a second order **Tensor.**
The Stress Tensor is given by

$$S = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

Similarly, strain has 9 components, and is given as

$$e = \begin{bmatrix} \varepsilon_x & \gamma_{yx} & \gamma_{zx} \\ \gamma_{xy} & \varepsilon_y & \gamma_{zy} \\ \gamma_{xz} & \gamma_{yz} & \varepsilon_z \end{bmatrix}$$

Let us consider that the material is **homogeneous** and **isotropic**. That means that the Young's modulus and Poisson's ratio are the same in all directions.

Now, for the stress σ_x the corresponding axial strain is ε_x, and the lateral strains will be

$\varepsilon_y = \varepsilon_z = - \nu \varepsilon_x$ ....(i)

Similarly, for the stress σ_y the corresponding axial strain is ε_y, and the lateral strains will be

$\varepsilon_x = \varepsilon_z = - \nu \varepsilon_y$ ....(ii)

And, for the stress σ_z the corresponding axial strain is ε_z, and the lateral strains will be

$\varepsilon_x = \varepsilon_y = - \nu \varepsilon_z$ ....(iii)

Now if all the three stresses are acting simultaneously then by the principle of superposition

(a) total strain in the x-direction is

$$\varepsilon_x = (\varepsilon_x)_{(i)} + (\varepsilon_x)_{(ii)} + (\varepsilon_x)_{(iii)}$$

$$\varepsilon_x = \varepsilon_x - \nu\varepsilon_y - \nu\varepsilon_z = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y - \nu\sigma_z\right)$$

Similarly in y and z directions the total strains are

(b) $\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x - \nu\sigma_z\right)$

(c) $\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\sigma_x - \nu\sigma_y\right)$

Now, when we also include shear stress and shear strain relations then

(d) $\tau_{xy} = G\gamma_{xy}\,;\ \tau_{yz} = G\gamma_{yz};\ \tau_{zx} = G\gamma_{zx}$

Together the six equations in (a), (b), (c) and (d) together are called generalized Hooke's law.

Generalized Hooke's Law:

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y - \nu\sigma_z\right);$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x - \nu\sigma_z\right);$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\sigma_x - \nu\sigma_y\right);$$

$$\tau_{xy} = G\gamma_{xy}\,;\ \tau_{yz} = G\gamma_{yz};\ \tau_{zx} = G\gamma_{zx}$$

Volumetric Strain and Bulk Modulus

Volumetric Strain: Change in dimensions of body will cause some change in its volume. Volumetric strain is defined as the ratio of change in volume to original volume.

$$\varepsilon_v = \frac{\delta V}{V}$$

Consider a unit cube whose edges are along the coordinate axes. Let the cube be subjected to normal stresses only in all three directions. Normal stresses are σ_x, σ_y, and σ_z. Let the corresponding normal strains be ε_x, ε_y, and ε_z . Shear stresses are considered to be absent.

Thus the elongated lengths of the edges of the cube after deformation will be
$(1 + \varepsilon_x)$, $(1 + \varepsilon_y)$ and $(1 + \varepsilon_z)$.

Therefore, the change in volume of the cube is $\delta V = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z) - 1$.

Neglecting the higher order terms of strain, we get the change in volume as

$$\delta V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Therefore, expression for volumetric strain is

$$\boxed{\varepsilon_v = \frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z}$$

Substituting the expressions for the three strains from the Generalized Hooke's law we get,

$$\varepsilon_v = \frac{\left(\sigma_x + \sigma_y + \sigma_z\right)}{E} - \frac{2\nu\left(\sigma_x + \sigma_y + \sigma_z\right)}{E} \quad \text{or}$$

$$\boxed{\varepsilon_v = \frac{\left(1-2\nu\right)}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)}$$

BULK MODULUS:

A case of special interest is that of a body subjected to a uniform hydrostatic pressure. Hydrostatic pressure at a point is a compressive stress that acts at a point equally from all directions.

If the hydrostatic pressure is denoted by *p*, then the components of the stresses are
$\sigma_x = \sigma_y = \sigma_z = -p$, where negative sign indicates compressive stress.

Substituting the above in the expression for volumetric strain, we get

$$\boxed{\varepsilon_v = \frac{-3\left(1-2\nu\right)}{E}p}$$

We now introduce the constant known as Bulk Modulus or the Modulus of Compression.

Bulk Modulus, *K*, is defined as ratio of the hydrostatic pressure acting at a point in a material to the volumetric strain of the material about that point. Thus, Bulk Modulus is given by

$$K = -\frac{p}{\varepsilon_v} = \frac{E}{3(1-2\nu)}$$

Bulk Modulus, K, is expressed in the same units as the modulus of elasticity E, that is, in Pascals. Like the modulus of elasticity, bulk modulus is also an intrinsic material property.

The reciprocal of bulk modulus gives the compressibility of a material, that is, it gives a measure of how much a material can be compressed upon an applied pressure.

With applied compressive pressure *p*, the volume of the material will tend to decrease, and thus volumetric strain ε_v will be negative. Since we want a positive sign for bulk modulus, a negative sign in introduced in the definition.

From the above expression for bulk modulus in terms of E and ν, since K and E are positive values, we find that $1-2\nu > 0 \Rightarrow \nu < \frac{1}{2}$. Since the Poisson's ratio is positive for all engineering materials, we get $0 < \nu < \frac{1}{2}$.

We note that:

(a) When ν = 0, it represents an ideal material which could be stretched in one direction without any lateral contraction. Bulk Modulus K will be exactly one-third of E.

(b) When ν = ½ , we get K = ∞, or 1/K = 0. Thus, it represents a perfectly incompressible material, with volumetric strain as zero.

Elastic Constants:

1) **Modulus of Elasticity:** Ratio of longitudinal stress to longitudinal strain or linear stress to linear strain. It is denoted by **E**.

$$\text{Modulus of elasticity (E)} = \frac{\text{Longitudnal stresss } (\sigma)}{\text{Longitudnal strain } (\varepsilon)}$$

2) **Modulus of Rigidity:** Ratio of shear stress to shear strain. It is denoted by **G**.

$$\text{Modulus of Rigidity (G)} = \frac{\text{Shear stress } (\tau)}{\text{Shear strain } (\gamma)}$$

3) **Bulk Modulus:** When a body is subjected to three mutually perpendicular stresses of equal intensity, the ratio of direct stress to volumetric strain is known as bulk modulus. It is denoted by **K**.

$$\text{Bulk Modulus (K)} = \frac{\text{Direct (hydrostatic) stress}}{\text{Volumetric Strain}} = -\frac{p}{(\frac{\delta V}{V})}$$

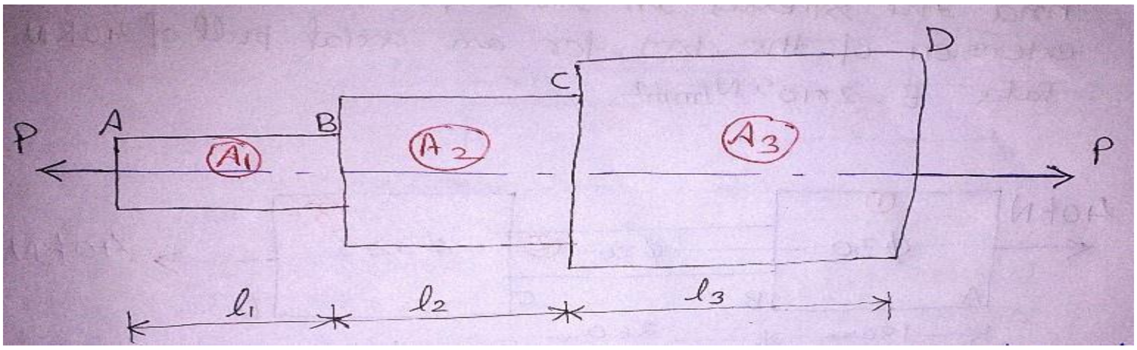
Relations between E, G and K

a. E = 2G (1 + ν)

b. E = 3K (1 - 2ν)

c. $E = \frac{9KG}{3K+G}$ or $\frac{9}{E} = \frac{3}{G} + \frac{1}{K}$

Bars of varying sections: Figure shows a bar which consist of three lengths l_1 l_2 & l_3 with sectional area A_1, A_2 & A_3 and subjected to an axial load P.
Even though the total force on each section is the same, the intensities of stress will be different for three sections.



Intensity of stress on Section AB = $\sigma 1 = \frac{P}{\sigma 1}$

Intensity of stress on Section BC = $\sigma_2 = \frac{P}{\sigma 2}$

Intensity of stress on Section CD = $\sigma 3 = \frac{P}{\sigma 3}$

Now let E, be the Young's Modulus

Strain on Section AB = $\frac{\sigma 1}{E}$

Strain on Section BC = $\frac{\sigma 2}{E}$

Strain on Section CD = $\frac{\sigma 3}{E}$

Change in length on Section AB =
$\delta l_1 = \varepsilon_1 \, l_1$

Change in length on Section BC =
$\delta l_2 = \varepsilon_2 \, l_2$

Change in length on Section CD = $\delta l_3 = \varepsilon_3 \, l_3$

Total Change in length of bar = $\delta l = {}_1 + \delta l_2 + \delta l_3$

$$\delta l = \frac{Pl_1}{A_1 E} + \frac{Pl_2}{A_2 E} + \frac{Pl_3}{A_3 E}$$

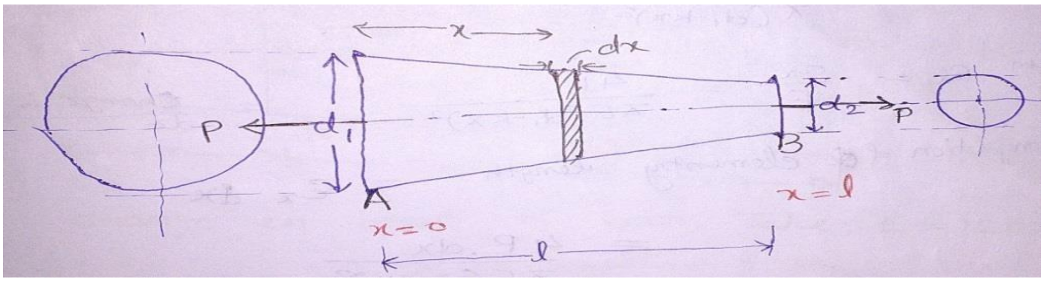
$$\delta l = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]$$

Note: When a body is subjected to a number of forces acting on its outer edge as well as at some other sections, along the length of the body. In such case, the forces are split up and their effects are considered on individual sections. The resulting deformation of the body is equal to the algebraic sum of the deformations of the individual sections, called principle of superposition.

Stress in the bars of uniformly tapering circular sections:

Consider a circular bar of uniformly tapering circular sections

Let P = Pull of bar, l = length of bar, d_1 = diameter of bigger end, d_2 = diameter of smaller end. Now consider a small element of length dx of the bar, at a distance x from the bigger end.



We can find out diameter of bar at a distance x from the left end A by using polynomial equation

D = a+bx …………………………………….(i)

Where D = diameter of taper section at a distance x from left side In figure

At x = 0   at $x = l$

D = d_1 D = d_2

a = d_1 $d_2 = a + bl$ or $d_2 = d_1 + bl$ so $b = \frac{d_2 - d_1}{l}$

put in equation (i)

$D = d_1 + \left(\frac{d_2 - d_1}{l}\right) x$ or $D = d_1 + (d_2 - d_1)\frac{x}{l}$

$D = d_1 - (d_1 - d_2)\frac{x}{l}$ or   D = Bigger end dia (major dia – minor dia) $\frac{x}{l}$

$D = d_1 - kx$ Where $k = \left(\frac{d_1 - d_2}{l}\right)$

Cross section are of the bar at this section $A_x = \frac{\pi}{4}(d_1 - kx)^2$

Induced stress at this section $\sigma_x = \frac{4P}{\pi(d_1 - kx)^2}$

Strain at this section $= \varepsilon_x = \frac{\sigma_x}{E} = \frac{4P}{\pi(d_1 - kx)^2}$

Elongation of elementary length = . dx $= \frac{4P}{\pi E(d_1 - kx)^2}.dx$

Total extension of bar may be found out by integrating the above equation between the limit 0 to l.

$$\delta l = \int_0^l \frac{4P}{\pi E(d_1 - kx)^2}\,.\,dx$$

$$\delta l = \frac{4P}{\pi E}\int_0^l \frac{dx}{(d_1 - kx)^2}$$ let $d_1 - kx = M$

$$\delta l = \frac{4P}{\pi E}\left(-\frac{1}{k}\right)\int_{d_1}^{d_1 - kl} \frac{dM}{M^2}$$

0 –k.dx = dM

$dx = -\frac{dM}{k}$

Limit changing if $x = 0$ then $d_1 = M$

if x = l the $M = d_1 - kl$

$\delta l = -\frac{4P}{\pi E k}\left[-\frac{1}{M}\right]_{d_1}^{d_1-kl}$

$\delta l = +\frac{4P}{\pi E k}\left[\frac{l}{d_1-kl} - \frac{1}{d_1}\right]$ Put the value of k and by solving

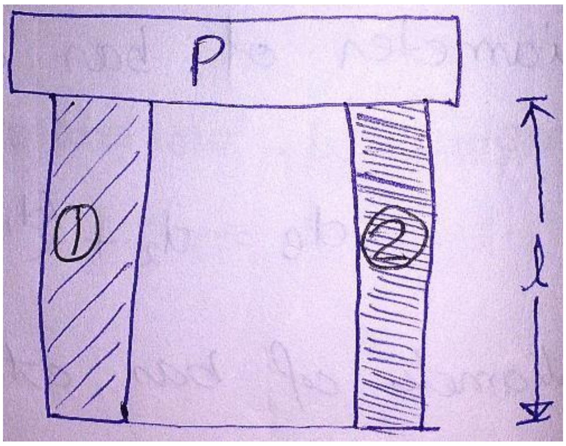
$$\delta l = \frac{4Pl}{\pi E d_1 d_2}$$

Note: Same method & fundamental will apply for all tapering sections like square, Rectangular tapering section.

Bars of Composite section or Composite Bars:

A bar made up of two or more than two different materials, joined together is called a composite bar.

1) The total external load on the composite bar is equal to sum of load carried by each different material
2) The extension or compression in each bar is equal. Hence deformation per unit length, i.e. strain in each bar is equal



P = Total load on composite bar

l = Length of composite bar and also length of bars of different materials

A_1 = Cross-sectional area of bar 1

A_2 = Cross-sectional area of bar 2

E_1 = Young's Modulus of bar 1

E_2 = Young's Modulus of bar 2

P_1 = Load shared by bar 1

P_2 = Load shared by bar 2

σ_1 = Stress induced in bar 1

σ_2 = Stress induced in bar 2

ε_1= Strain in bar 1

ε_2= Strain in bar 2

By first point discussed above $P = P_1 + P_2$

$P = \sigma_1 A_1 + \sigma_2 A_2$

By second point discussed above that strain is same for both bar

Strain in bar 1 (ε_1) = Strain in bar 2 (ε_2)

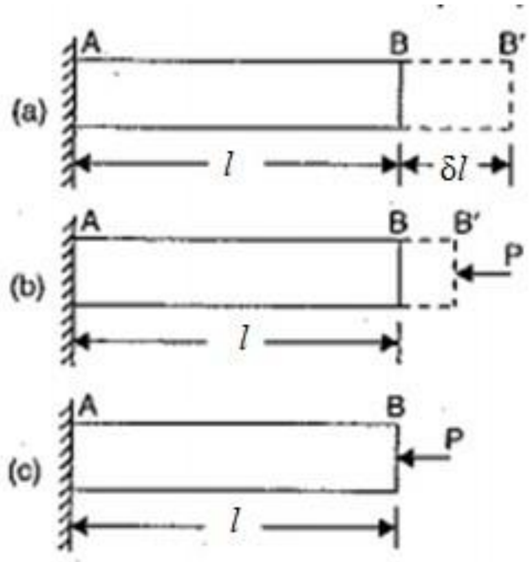
$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$ $\sigma_1 = \frac{E_1}{E_2}\sigma_2$

Where $\frac{E_1}{E_2}$ is known as Modular Ratio.

Thermal stresses or Temperature stresses:

Thermal stresses are the stresses induced in a body due to change in temperature. Thermal stresses are setup in a body. When the temperature of body is raised or lowered and the body is not allowed to expand or contract freely. But if the body is allowed to expand or contract freely, no stress will be setup in the body.

You must have notice that in many times is structure will provide a gap between two structural elements. We allow the structural member to expand or contract due to variation in temperature. You must have noticed in the railway tracks are not continuous; some gaps are maintained at a certain distance travel. If this not, then rail track will be subjected to tremendous amount of stress.



Consider a body which is heated at a certain temperature. l = original length of the body Δt = variation in temperature E = young's modulus

α = Co-efficient of linear expansion/ thermal expansion

δl = extension of rod due to rise of temperature

δl is proportional to strain $\Delta t.l$, Δt = + for expansion

Δt = − for contraction

If the rod is free to expand **$\delta l = \alpha\, \Delta t.l$**

AB represents the original length and BB' represents the increase in length due to temperature rise, Now suppose that an external compressive load P is applied at B' so that the rod is decreased in its length from (l+ α $\Delta t.l$) to l = (l+ α $\Delta t.l$) – δl = l

Total compressive strain $= \frac{Decrease\ in\ length}{Original\ length}$

Thermal strain

$\varepsilon_T = \frac{\propto \Delta t.l}{l} = \propto \Delta t$

Thermal stress

$\sigma_T = \varepsilon_T.E = \alpha\, \Delta t E$

If ends of body are fixed to rigid supports, so that its expansion is prevented then compressive stress and strain will be set up in the rod. These are known as thermal stress and thermal strain. If supports yield by an amount equal to d,

Then the actual expansion = expansion due to rise in temperature − δ

$= \alpha\, \Delta t.l - \delta$

Actual Strain = $\frac{Actual\ Expansion}{Original\ length} = \frac{\alpha\, \Delta t.l - \delta}{l}$; Actual Stress = $\frac{\alpha\, \Delta t.l - \delta}{l} \times E$

5.4. Thermal Stresses in Bars of Varying Section

Consider a bar ABC fixed at its ends A and C and subjected to an increase of temperature as shown in Fig. 5.2.

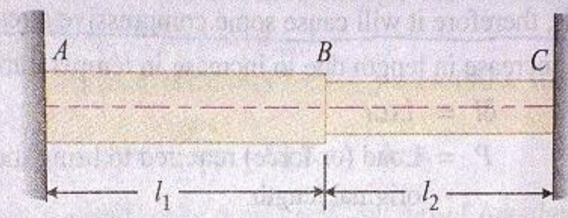


Fig. 5.2. Bar of varying section

Let l_1 = Length of portion AB,

σ_1 = Stress in portion AB,

A_1 = Cross-sectional area of portion AB,

l_2, σ_2, A_2 = Corresponding values for the portion BC,

α = Coefficient of linear expansion and

t = Increase in temperature

We know that as a result of the increase in temperature, the bar ABC will tend to expand. Bu since it is fixed at its ends A and C, therefore it will cause some compressive stress in the bod Moreover, as the thermal stress is shared equally by both the portions, therefore

$$\sigma_1 A_1 = \sigma_2 A_2$$

Moreover, the total deformation of the bar (assuming it to be free to expand),

$$\delta l = \delta l_1 + \delta l_2 = \frac{\sigma_1 l_1}{E} + \frac{\sigma_2 l_2}{E} = \frac{l}{E}(\sigma_1 l_1 + \sigma_2 l_2)$$

Note. Sometimes, the modulus of elasticity is different for different sections. In such cases, the total deformatio

$$\delta l = \left(\frac{\sigma_1 l_1}{E_1} + \frac{\sigma_2 l_2}{E_2}\right)$$

5.5. Thermal Stresses in Composite Bars

Whenever there is some increase or decrease in the temperature of a bar, consisting of two or more different materials, it causes the bar to expand or contract. On account of different coefficients of linear expansions the two materials do not expand or contract by the same amount, but expand or contract by different amounts.

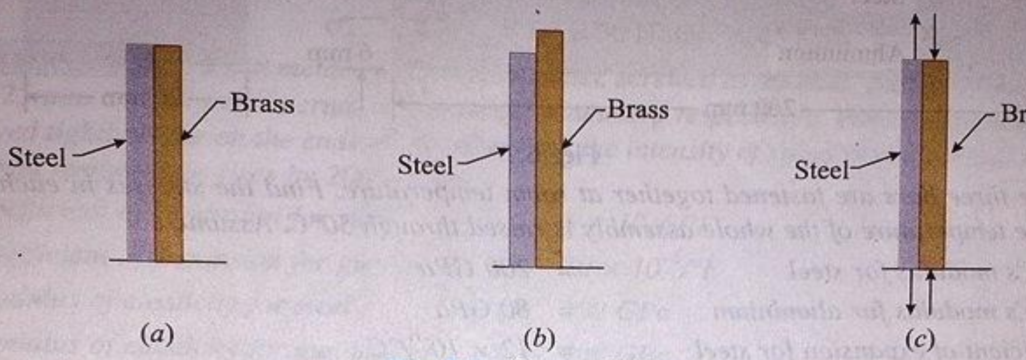


Fig. 5.6. Composite bars

Now consider a composite bar consisting of two members, a bar of steel and another of brass as shown in Fig. 5.6 (*a*).

Let the bar be heated through some temperature. If the component members of the bar (*i.e.*, steel and brass) could have been free to expand, then no internal stresses would have induced. But, since the two members are rigidly fixed, therefore the composite bar, as a whole, will expand by the same amount. We know that the brass expands more than the steel (because the coefficient of linear expansion of the brass is greater than that of the steel). Therefore the free expansion of the brass will be more than that of the steel. But since both the members are not free to expand, therefore the expansion of the composite bar, as a whole, will be less than that of the brass; but more than that of the steel as shown in Fig. 5.6 (*b*). It is thus obvious that the brass will be subjected to compressive force, whereas the steel will be subjected to tensile force as shown in Fig. 5.6 (*c*).

Now let σ_1 = Stress in brass

ε_1 = Strain in brass,

α_1 = Coefficient of linear expansion for brass,

A_1 = Cross-sectional area of brass bar,

$\sigma_2, \varepsilon_2, \alpha_2\ A_2$ = Corresponding values for steel, and

ε = Actual strain of the composite bar per unit length.

As the compressive load on the brass is equal to the tensile load on the steel, therefore

$$\sigma_1.A_1 = \sigma_2.A_2$$

Now strain in brass,

$$\varepsilon_1 = \alpha_1.t - \varepsilon \quad ...(i)$$

and strain in steel,

$$\varepsilon_2 = \alpha_2.t - \varepsilon \quad ...(ii)$$

Adding equation (*i*) and (*ii*), we get

$$\varepsilon_1 + \varepsilon_2 = -t\,(\alpha_1 + \alpha_2)$$

NOTES : 1. In the above equation the value of α_1 is taken as greater of the two values of α_1 and α_2.

2. The values of strain (ε_1 and ε_2) may also be found out from the relation $\frac{\text{Stress}}{\text{Modulus of elasticity}}$ or $\frac{\delta l}{l}$.