

Introduction:

gas dynamics \rightarrow Fluid dynamics of compressible flows.

Compressibility is to be studied.

In applications of high speed aerodynamics

- i) rocket & missile propulsion
- ii) steam & gas turbines
- iii) high speed Turbo compressors.

Compressible fluid dynamics is used to obtain solutions to problems.

Temp (T), pr (P), density (ρ), int. energy (u)
enthalpy (h), entropy (s), viscosity (μ) symbols
are used.

Fluid — liq, gas, vapour & their mixtures.

real fluid is viscous

ideal fluid is non-viscous or inviscid ~~flow~~ fluid

Perfect fluid — inviscid as well as incompressible.
(They do not stick to the surface),

Closed System — It has a fixed qty of
or fixed mass of matter.
matter (fluid) & boundaries can change.

system can interact with its surroundings
through heat & work transfer.

no out flow or in flow of fluid from & to
the system.

open system (control vol) (fluid)

Continuous flow of matter is there.
fixed space is there. But fixed mass
is not there. There is continuous flow of mass through open
system.

isolated system — There is no flow of
mass & energy to & from the
system.

Reynolds No.

Dimensionless no.

$$Re = \frac{\text{inertia force}}{\text{viscous force}}$$

$$= \frac{\rho A c^2}{\mu c \cdot l} = \frac{\rho l^2 c^2}{\mu \cdot c \cdot l} = \frac{\rho \cdot l \cdot c}{\mu} = \frac{l \cdot c}{\nu}$$

$l = \text{length}, A = l^2 \frac{\mu}{\rho} \Rightarrow$

$$\frac{\text{vel. length, density}}{\text{dynamic vis}}$$

$$Re = \frac{\rho \cdot c \cdot l}{\mu} = \frac{c \cdot l}{\nu}$$

Mach No.

Non dimensional no.

$$M^{\sim} = \frac{\text{Inertia force}}{\text{Elastic force}}$$

$$= \frac{\rho A \cdot c^{\sim}}{k \cdot A} = \frac{\rho c^{\sim}}{k}$$

where $k = \text{Bulk Modulus of elasticity of fluid}$.

(This will be proved later)
But $k = \rho a^2$

$$M^{\sim} = \frac{\rho c^{\sim}}{\rho a^2} = \frac{c}{a}$$

$c = \text{fluid velocity}$
 $a = \text{local vel of sound}$

(Mach no is very important in compressible flows)

Incompressible flow $Ma < 0.33$
Transonic $0.8 < Ma < 1.2$
Shock wave disturbances
$M = \frac{c}{a}$
$M^{\sim} = \frac{c^{\sim}}{a^{\sim}}$
Supersonic $Ma > 1$
Hyper Sonic $Ma > 3$
$Ma < 0.33$ incompressible

Subsonic $Ma < 0.33$
Sonic $Ma = 1$
Supersonic $Ma > 1$

Laminar flow

orderly 11k layers without any fluctuating components in the three directions.
Such flows occur at lower values of Re .

WORK in flow process:

If the diff in kinetic energy terms is negligible then

$$w_s = h_1 - h_2 = - \int_1^2 dh$$

For perfect gas

$$w_s = -c_p \int_1^2 dT$$

$$w_s = c_p (T_1 - T_2)$$

For a reversible process

$$w_s = - \int_1^2 \frac{dp}{\rho} = - \int_1^2 v dp$$

✓ Adiabatic energy transformation

In Expt of gases in nozzles, converging in diffusers are adiabatic processes involving only energy transformation. (shaft work is absent in such cases).

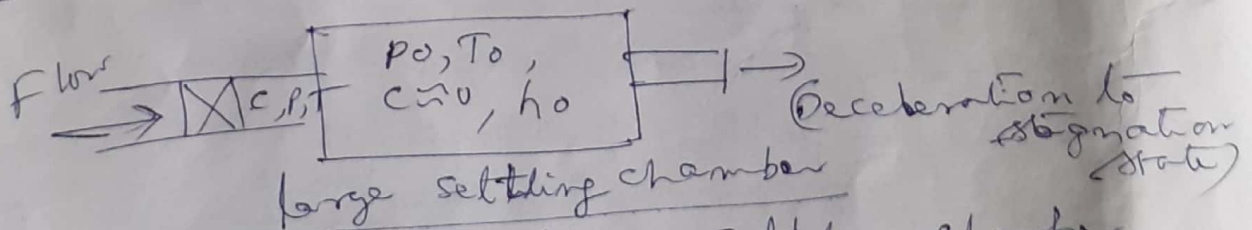
$$h_1 + g z_1 + \frac{1}{2} c_1^2 = h_2 + g z_2 + \frac{1}{2} c_2^2$$

$$h_1 + \frac{1}{2} c_1^2 = h_2 + \frac{1}{2} c_2^2$$

Change in elevation is ignored.

(c = fluid vel
 a = local vel
of sound)

Stagnation enthalpy:



In an infinitely large settling chamber when deceleration of gas stream at vel c , pr p , & Temp T to almost zero vel ($c \rightarrow 0$) is

Stagnation enthalpy of a gas or vapour is its enthalpy when it is adiabatically decelerated to zero vel, at zero elevation.

Putting $h_1 = h_2$, $z_1 = z_2$, $C_1 = C_2$.
(initial conditions)

Final state $h_2 = h_0$, $z_2 = 0$, $C_2 = 0$.

$$h_1 + gz_1 + \frac{1}{2} C_1^2 = h_2 + gz_2 + \frac{1}{2} C_2^2$$

$$\boxed{h + gz + \frac{1}{2} C^2 = h_0}$$

neglecting gz , $\boxed{h_0 = h + \frac{1}{2} C^2}$ ✓✓

For adiabatic energy transformation
stagnation enthalpy is constant

Differentiating above eqn

$$dh + C \cdot dC = 0$$

Stagnation Temp: ✓

Stagnation Temp is temp of gas when it is
adiabatically decelerated to zero vel at
zero elevation.

$$h_0 = h + \frac{1}{2} C^2$$

For perfect gas $C_p T_0 = C_p T + \frac{1}{2} C^2$

$$T_0 = T + \frac{C^2}{2C_p}$$

$$\begin{aligned} dh &= C_p dT \\ h &= C_p T \end{aligned}$$

$\frac{C^2}{2C_p}$ is known as velocity Temp (T_c) corresponding
to velocity C .

Hence $\boxed{T_0 = T + T_c}$
 $\boxed{T_c = \frac{C^2}{2C_p}}$ ✓

Again $T_0 = T + \frac{C^2}{2C_p}$

dividing by T , $\frac{T_0}{T} = 1 + \frac{C^2}{2C_p \cdot T}$

$$\frac{T_0}{T} = 1 + \frac{C^2}{2\gamma RT / (\gamma - 1)}$$

$$\begin{aligned} C_p - C_v &= R \\ \frac{C_p}{C_v} &= \gamma \\ C_p &= \frac{\gamma R}{\gamma - 1} \end{aligned}$$

But we know that $\gamma RT = \tilde{a}^2$

$$\& \frac{\tilde{c}^2}{\tilde{a}^2} = M^2$$

$$\text{Hence } \frac{T_0}{T} = 1 + \frac{\tilde{c}^2}{2\gamma RT / (\gamma - 1)}$$
$$= 1 + \frac{\tilde{c}^2 (\gamma - 1)}{2 \cdot \tilde{a}^2}$$

$$\boxed{\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} (M^2)}$$

Stagnation vel of sound

We know $\gamma RT = \tilde{a}^2$

For perfect gas and at stagnation Temp

Substituting

$$R = \frac{(\gamma - 1)}{\gamma} C_p; \quad a_0 = \sqrt{\gamma R T_0}$$
$$a_0 = \sqrt{(\gamma - 1) C_p \cdot T_0}$$
$$a_0 = \sqrt{(\gamma - 1) h_0}$$

Stagnation press:

Stagnation press is pr of a fluid which is attained when it is decelerated to zero vel at zero elevation in a reversible adiabatic (isentropic) process.

From Stagnation Temp eqns.

$$\frac{P_0}{P} = \left(\frac{T_0}{T} \right)^{\gamma / \gamma - 1}$$

Substituting above eqns

$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\gamma / \gamma - 1}$$

✓ Stagnation density

For given values of stagnation pr & stagnation Temp.

$$\text{Stagnation density} = \rho_0 = \frac{p_0}{RT_0} \quad \checkmark$$

From isentropic relation

$$\frac{p_0}{p} = \left(\frac{\rho_0}{\rho} \right)^{\frac{\gamma}{\gamma-1}} \quad \checkmark$$

$$\frac{p_0}{p} = \left(\frac{T_0}{T} \right)^{\frac{\gamma}{\gamma-1}} = \left\{ 1 + \frac{\gamma-1}{2} M^2 \right\}^{\frac{\gamma}{\gamma-1}} \quad \checkmark$$

Stagnation state:

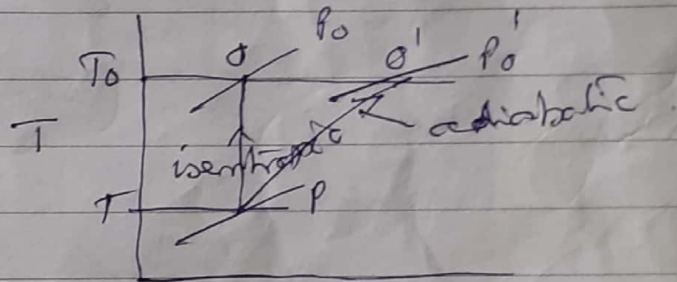


Figure depicts the deceleration of gas in both isentropic & adiabatic processes. During adiabatic process, stagnation pr decreases & entropy increases.

The state of a fluid attained by isentropically decelerating it to zero vel at ~~at~~ zero elevation is referred to as the stagnation state. (state o)

For stagnation ~~pr~~ pr & stagnation state, the deceleration process ~~is~~ needs to be isentropic only.

This condition is not necessary for stagnation enthalpy & Temp because ~~the~~ h_0, T_0 have been ~~is~~ derived from energy eqn for adiabatic flow.

Critical vel of fluid

critical vel of fluid = vel at $M=1$.

$$M_{critical} = \frac{c^*}{a^*} = 1$$

$$c^* = a^* = \sqrt{\gamma R T^*}$$

we know

$$h_0 = h + \frac{1}{2} \tilde{c}^2$$

(For adiabatic expansion from stagnation Temp to critical Temp)

$$h_0 = h^* + \frac{1}{2} c^{*2}$$

For perfect gas

$$T_0 = T + T_c$$

$$T_0 = T + \frac{c^2}{2c_p}$$

So $T_0 = T^* + \frac{c^{*2}}{2c_p}$

$$2c_p(T_0 - T^*) = c^{*2}$$

$$\text{or } c^* = \sqrt{2c_p(T_0 - T^*)}$$

we know

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$$

(For critical Mach no, $M=1, T=T^*$)

$$\frac{T_0}{T^*} = 1 + \frac{\gamma-1}{2} = \frac{\gamma+1}{2} = 1.2 \text{ for } (\gamma=1.4)$$

$$c^* = \sqrt{2c_p(T_0 - T^*)}$$

$$c^* = a^*$$

Contd

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Mach Number M^*

$M^* = \frac{\text{fluid vel}}{\text{critical fluid vel or critical sound M.}}$

$M^* = A$ 2nd kind of Mach No.

$$M^* = \frac{c}{c^*} = \frac{c}{a^*}$$

$$M^{*2} = \frac{c^2}{a^{*2}} = \frac{c^2}{a^2} \cdot \frac{a^2}{a^{*2}} = \frac{a^2}{a^{*2}} (M^2)$$

Sometimes it is more convenient to use M^* because,
 i) at high fluid velocities M approaches infinity
 ii) M is not proportional to fluid velocity alone, M^* does not mean $M=1$, This is only another type of Mach number.

We know

$$\frac{2\tilde{a}}{\gamma-1} + \tilde{c} = \left(\frac{\gamma+1}{\gamma-1} \right) \tilde{a}^2$$

$$\left[\frac{2\tilde{a}^2}{(\gamma-1)a^{*2}} + \frac{\tilde{c}^2}{a^{*2}} = \frac{\gamma+1}{\gamma-1} \right]$$

We have

$$M^{*2} = \frac{a^2}{a^{*2}} M^2, \quad \frac{a^2}{a^{*2}} = \frac{M^{*2}}{M^2}$$

$$\left[\frac{2}{\gamma-1} \cdot \frac{M^{*2}}{M^2} + M^{*2} = \frac{\gamma+1}{\gamma-1} \right]$$

$$M^{*2} \left[1 + \frac{2}{\gamma-1} \left(\frac{1}{M^2} \right) \right] = \frac{\gamma+1}{\gamma-1}$$

$$M^{*2} \left(\frac{2}{\gamma-1} \right) \left[\frac{\gamma-1}{2} + \frac{1}{M^2} \right] = \frac{\gamma+1}{\gamma-1}$$

$$M^{*2} (2) \left[\frac{(\gamma-1)M^2 + 2}{2M^2} \right] = \frac{\gamma+1}{\gamma-1}$$

$$\frac{M^{*2}}{M^2} \left\{ (\gamma-1)M^2 + 2 \right\} = \frac{\gamma+1}{\gamma-1}$$

$$M^{*2} = \frac{(\gamma+1)M^2}{(\gamma-1)M^2 + 2} = \frac{(\gamma+1)M^2/2}{(\gamma-1)M^2/2 + 1}$$

Imp.

Isentropic flow with variable Area

If flow parameters do not vary with time - it is steady flow.

If flow parameters do not vary ~~with~~ in directions normal to flow direction → one dimensional.

A Truly one dimensional flow is in stream tube.
In many cases one dimensional flow is assumed for quick rough idea of variations.

For more accurate assessment, two & 3 dimensional analysis is used which is tedious.

The problem is further simplified by taking the assumption of isentropic flow. This is justified when heat transfer is negligible and there are no irregularities due to fluid friction. The process is

occurring in real systems deviate from the idealized one dimensional isentropic flow. This is taken as a reference process for comparing with actual process. Now we will be dealing with variation due to area change ^{in compression & expansion processes}. Shaft work is absent.

Comparison of isentropic & Adiabatic processes

Figure shows the isentropic & adiabatic expansions of a perfect gas between 2 states 1 and 2.

P_{01s} = Initial stagnation pr.

Kinetic energy = $\frac{1}{2} C_1^2$

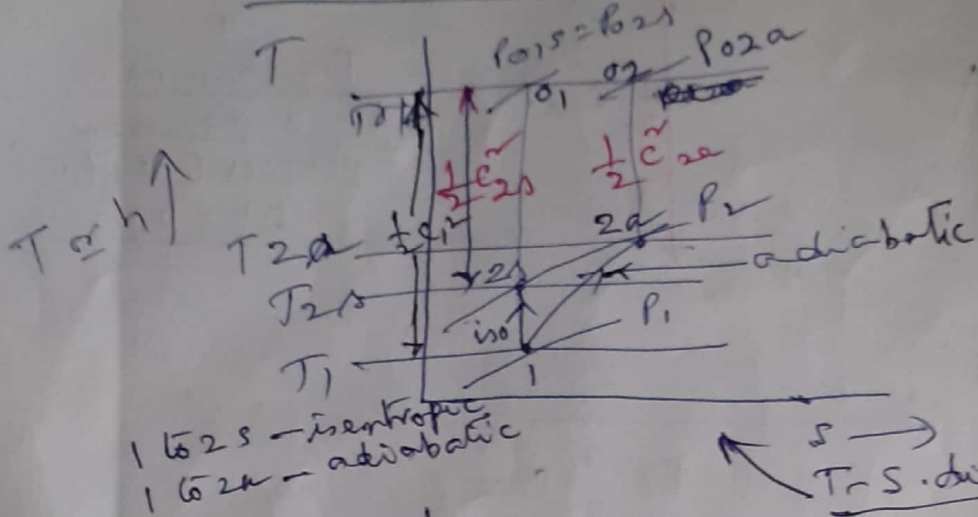
T_{01} = Stagnation Temp.

T_1 = Static Temp.

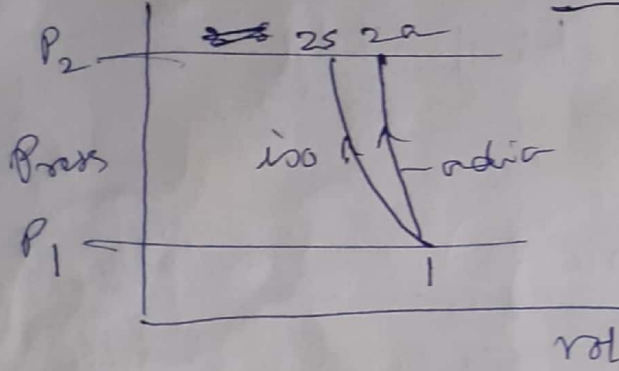
Final Temp = T_{2s} , Final KE = $\frac{1}{2} C_{2s}^2$.

- s - isentropic
- a - adiabatic
- 1 - initial
- 2 - final
- 0 - stagnat

Compression processes



$T_{2a} > T_{2s}$
 $v_{2a} > v_{2s}$
 $P_{02a} < P_{01s}$
 $\frac{1}{2} C_{2a} < \frac{1}{2} C_{2s}$



\leftarrow P. diagram for compression
 shaft work = 0

Mach no variation:

isentropic energy eqn is
 $dp = -\rho c dc$

From continuity eqn, mass flow rate is
 $m = \rho A c = \text{const.}$

$\rho A c = \text{const.}$

Take log & diff

$\ln \rho + \ln A + \ln c = -\text{const.}$

$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dc}{c} = 0$

$\frac{dc}{c} = -\left(\frac{d\rho}{\rho} + \frac{dA}{A}\right)$

$dc = -c\left(\frac{d\rho}{\rho} + \frac{dA}{A}\right)$

Substitution

$dp = +\rho c^2 \left(\frac{d\rho}{\rho} + \frac{dA}{A}\right)$

$\frac{dp}{\rho c^2} = \frac{d\rho}{\rho} + \frac{dA}{A}$

From stagnation enthalpy eqn & adiabatic eqn
 $dh + c dc = 0$
 (adiabatic energy eqn)
 for isentropic
 $\frac{dp}{\rho} + c dc = 0$
 $dp = -\rho c dc$

$$\frac{dp}{\rho c^2} - \frac{dp}{\rho} = \frac{dA}{A}$$

$$\frac{dp}{\rho c^2} \left(1 - \frac{dp}{dp} c^2\right) = \frac{dA}{A}$$

$$\frac{dp}{\rho c^2} \left(1 - \frac{c^2}{a^2}\right) = \frac{dA}{A}$$

$$\boxed{\frac{dA}{A} = \frac{dp}{\rho c^2} (1 - M^2)}$$

35

$$a = \sqrt{\left(\frac{dp}{dp}\right)} = \sqrt{\gamma RT}$$

$$\frac{dp}{dp} = a^2$$

$$\frac{dp}{dp} = \frac{1}{a^2}$$

This eqn can be considered for ^{both} accelerating & decelerating passages for various Mach Nos.

Expansion in Nozzles

gases & vapours are expanded in nozzles by providing pass ratio across them. The shape of passage depends on local Mach No.

Since the purpose of a nozzle is to accelerate the flow by providing a pressure drop, dp is -ve in above eqn. Three conditions are there

i) For $M < 1$, ~~$dA = 0$~~ $dA = -ve$,

For nozzles area decreases in the range of $M=0$ to $M=1$ (Convergent).

ii) For $M=1$ (Sonic vel)

$dA=0$, which implies that there is no change of passage area ($M=1$) (Throat)

iii) For $M > 1$, $dA = +ve$,

$M > 1$, area of nozzle increases, so divergent passage.

P.T.O.

✓ (b) Compression in diffuser:

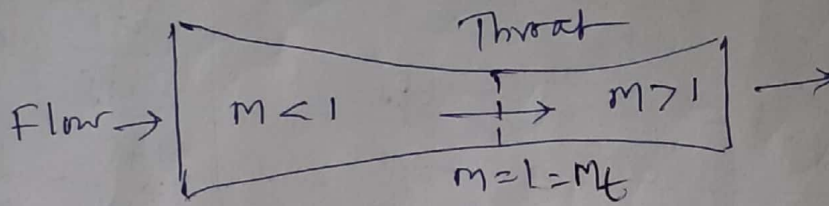


Fig:- isentropic flow of gas in a nozzle (decreasing pr).

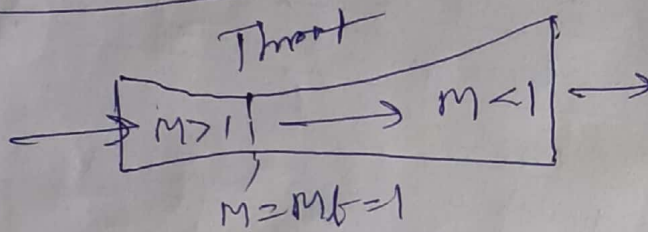


Fig:- isentropic flow of gas in a diffuser (increasing pr).

Diffusers are employed to obtain press rise in flowing gases for a given value of initial KE. The static pr rise is at the cost of deceleration of flow in diffuser, dp in above eqn is +ve. 3 cases are given

- i) For $M < 1$, $dA = +ve$
For subsonic diffusers ($M=1$ to $M=0$).
The area increases, (divergent)
- ii) For $M = 1$, $dA = 0$
no change in passage area (Throat)
- iii) For $M > 1$, $dA = -ve$.
The area decreases, (Convergent passage).

P.T.O.

Impulse Function:

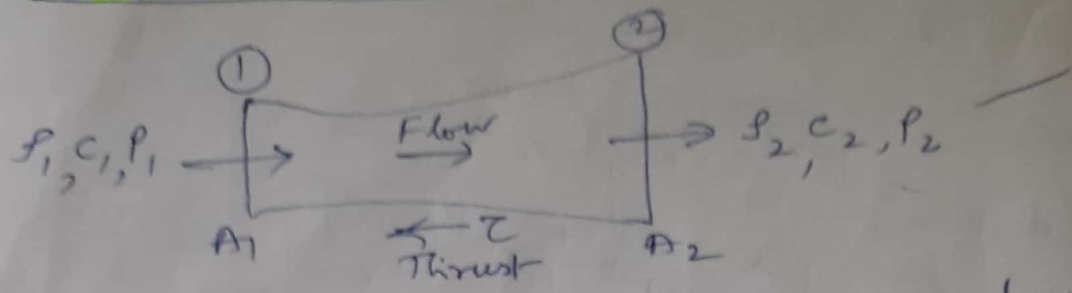


Fig. Directions of flow & thrust in a duct.

One dimensional flow through a control surface (symmetrical straight duct) is shown in fig. Thrust or wall force is a result of change in press & mach no between cross sections ① & ②

$F = \text{Impulse function} = \text{wall force function.}$
 $\rho A \dot{c}$ & $\rho A \dot{c}^2 = \text{are units of force.}$
 By momentum eqn

$$\text{Thrust } T = (\rho_2 A_2 + \rho_2 A_2 \dot{c}_2^2) - (\rho_1 A_1 + \rho_1 A_1 \dot{c}_1^2)$$

For perfect gas

$$\rho \dot{c} = \frac{\rho}{RT} \cdot \dot{c} = \gamma \cdot \rho \frac{\dot{c}^2}{\gamma \cdot RT}$$

$$= \gamma \cdot \rho \cdot M^2$$

$$\therefore a = \sqrt{\gamma RT}$$

$$\dot{c}^2 = \gamma RT$$

$$M = \frac{\dot{c}}{a}$$

$$\text{Impulse Function} = F$$

$$= \rho A + \rho A \dot{c}^2$$

Substituting $F = \rho A + \gamma \cdot \rho \cdot A \cdot M^2$
 $= \rho A (1 + \gamma M^2)$

Again substituting

$$\text{Thrust } T = \rho \cdot F_2 - F_1 =$$

$$= \rho_2 A_2 (1 + \gamma M_2^2) - \rho_1 A_1 (1 + \gamma M_1^2)$$

These eqns are independent of flow process.

Mass flow rate in terms of Area ratio

(44)

From Continuity eqn

$$\dot{m} = \rho A C = \rho^* A^* C^*$$

$$\frac{\dot{m}}{A} = \rho C = \rho^* C^* \frac{A^*}{A}$$

Now $\rho^* = \frac{p^*}{R T^*}$, $C^* = a^* = \sqrt{\gamma R T^*}$ where $\gamma = 1.4$

$$\therefore \frac{\dot{m}}{A} = \frac{p^*}{R T^*} \sqrt{\gamma R T^*} \frac{A^*}{A} \quad \left[\begin{array}{l} \frac{T^*}{T_0} = \frac{2}{1.4+1} = 0.833 \\ \frac{p^*}{p_0} = \left(\frac{2}{2.4}\right)^{\frac{1.4}{0.4}} = 0.528 \end{array} \right]$$

$$\frac{\dot{m}}{A} = \frac{p^*}{\sqrt{T^*}} \cdot \sqrt{\frac{\gamma}{R}} \cdot \frac{A^*}{A}$$

We know $\frac{T^*}{T} = \frac{2}{\gamma+1}$, $\frac{p^*}{p} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$ $\frac{\rho^*}{\rho} = \left(\frac{2}{2.4}\right)^{\frac{1}{0.4}} = 0.628$

~~$\frac{\dot{m}}{A} =$~~ By simplifying, we get,

non dimensional form of mass flow parameter

$$\frac{\dot{m} \sqrt{T_0}}{A \cdot p_0} \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(\frac{A^*}{A}\right)$$

Mass flow rate in terms of Mach. No.

We know $\frac{A}{A^*} = \frac{1}{M} \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} M^2 \right]^{\frac{\gamma+1}{2(\gamma-1)}}$

Substituting this in the above eqn

$$\frac{\dot{m} \sqrt{T_0}}{A \cdot p_0} \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \cdot \frac{M}{\left[\left(\frac{2}{\gamma+1}\right) + \frac{\gamma-1}{\gamma+1} M^2 \right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

By rearranging we get

$$\frac{\dot{m} \sqrt{T_0}}{A \cdot p_0} \sqrt{\frac{R}{\gamma}} = \frac{M}{\left[1 + \left(\frac{\gamma-1}{2}\right) M^2 \right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

For Max Mass flow condition

$A = A^*$, $M = 1$,

$$\boxed{\frac{\dot{m}_{\max} \sqrt{T_0}}{A^* p_0} \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

Flow through Nozzle

Convergent nozzles are used for subsonic & sonic flows. They can be used as flow measuring & flow regulating devices.

Convergent-divergent nozzles are used for supersonic flows. It is difficult to obtain C-D passages in compressors & turbine blades.

Convergent Nozzle:

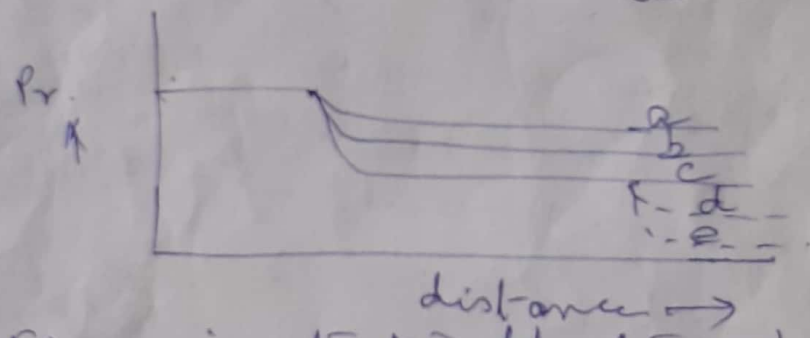
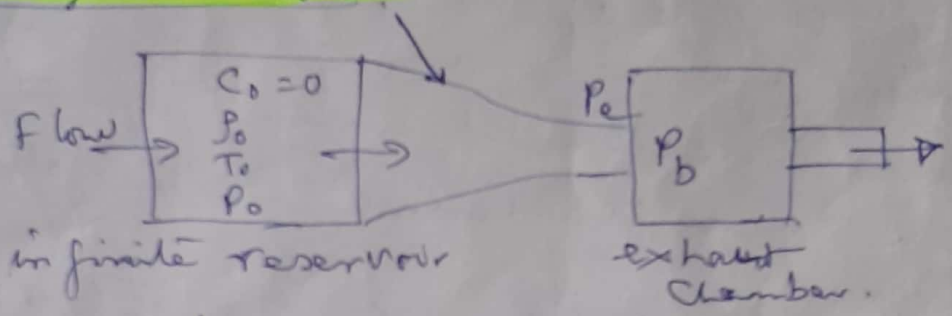
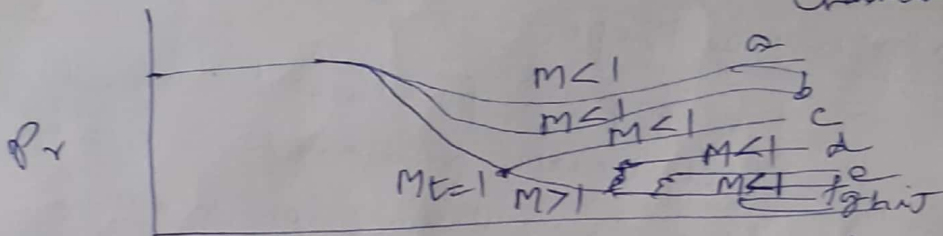
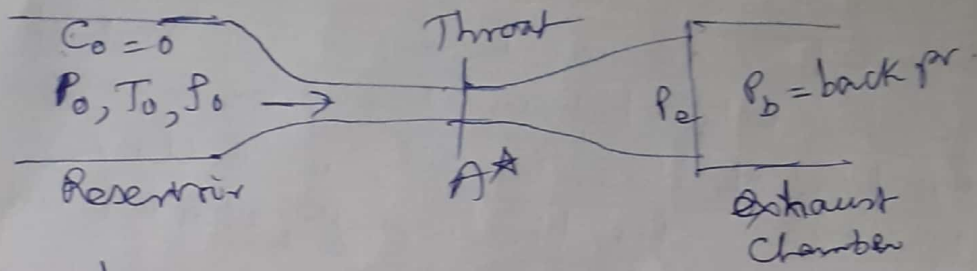


Fig: isentropic flow through convergent nozzle.

Pressure distribution along the nozzle for various values of pressure ratio (P_e/P_0) are shown in curves abcde.

- a, b → Press ratio more than critical.
- Curve c → critical pr ratio $P_b/P_0 = 0.528$ and $P_b = P_e$ ($\gamma = 1.4$)
- d, e → P_b is further reduced. Nozzle exit pr = P_e^* , $P_b < P_e^*$

Convergent divergent nozzle



dist along the nozzle axis,
 a to j are curves for press distribution
 for various values of P_b/P_0 .

- a, b → flow is accelerating upto throat, pressure rises upto P_b in diverging part (diffuser)
- c → $P_t = P^*$, $M_t = 1$ at throat, but diverging part still acts as diffuser
- h → curve for design value of back pr. Here, flow is supersonic in divergent part.

After curve c, when P_b is further lowered, expansion takes place to supersonic velocity, ~~occurs~~ beyond ^{the} throat to a point where a discontinuity in the flow occurs. (d and e). In these cases at some point the minimum press reached in supersonic expansion has to rise to the back press. But earlier discussion says for deceleration of supersonic flow the passage should be ~~be~~ convergent. But the down stream shape is divergent. & hence incompatible, so the flow readjusts & becomes subsonic.

The press, temp, & density suddenly rise to values compatible with subsonic flow. Such sudden change of supersonic to subsonic flow occurs through a plane of discontinuity between the flow regions.

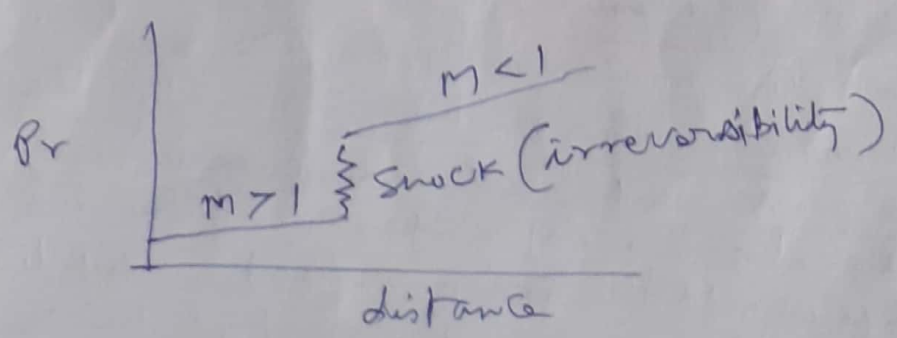
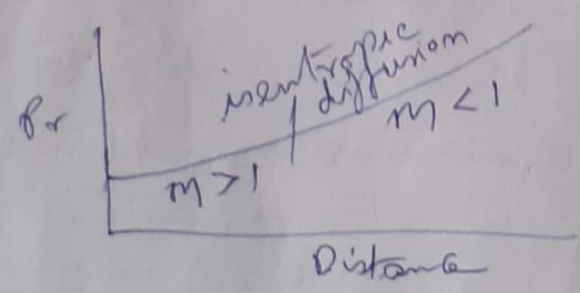
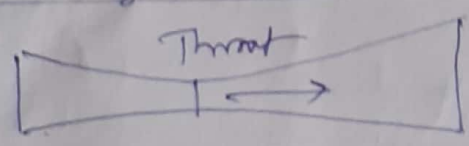
This plane of discontinuity is called Shock wave. The flow through shock wave is no longer isentropic.

f, g → when P_b is further lowered, shock wave moves downward.

i, j → $P_b < P_c$, expansion wave outside the nozzle.

Max^m mass flow conditions are reached at pt \odot when $P_t = P^*$. There is no further increase in mass flow after this pt. This condition is called 'Choking'.

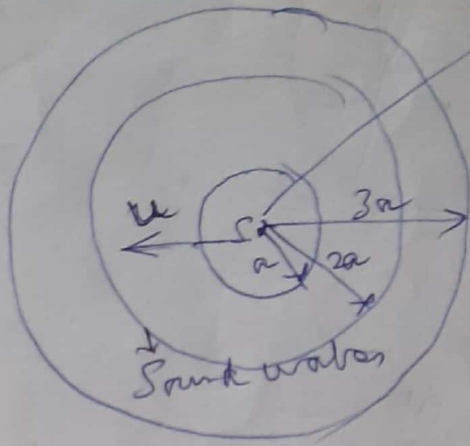
Flow through diffuser



$P_t = \text{Throat Press.}$
 Choking $P_t = P^*$
 $M_t = 1$

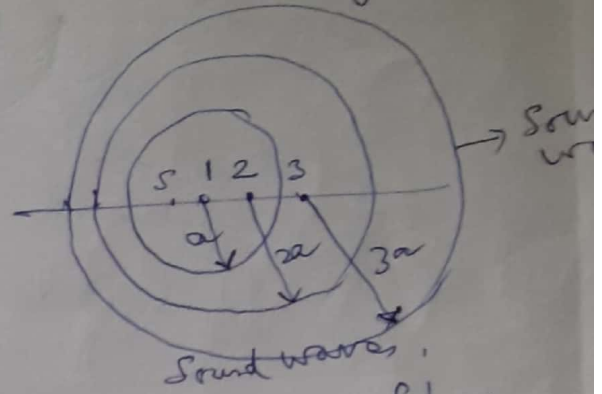
Mach Angle

(59)



1, 2, 3

Movement of a row of the distur



Sound waves

a) Incompressible flow model
 $u \ll a \ll \infty, M \ll 0$

b) Subsonic flow
 $M = 0.5$

a) disturbance = S, ~~with~~ vel of disturbance = u (right to left)

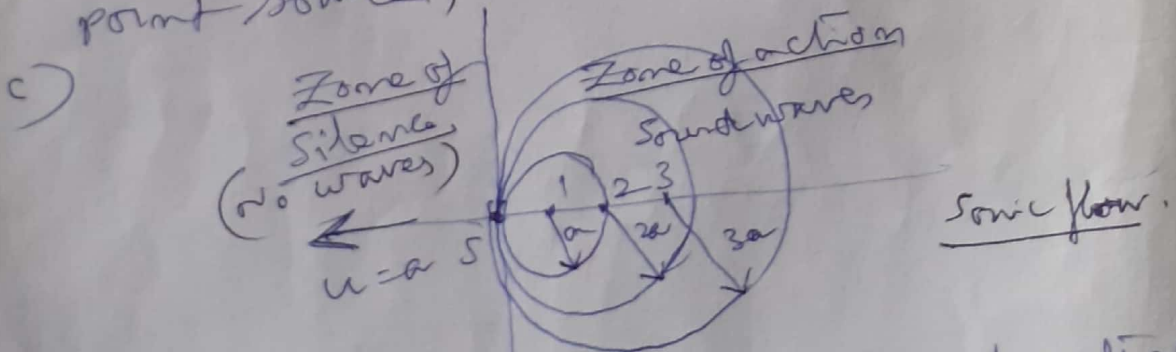
positions 1, 2, 3 shows after 1, 2, 3 seconds.

The disturbance travels distances, a, 2a, 3a

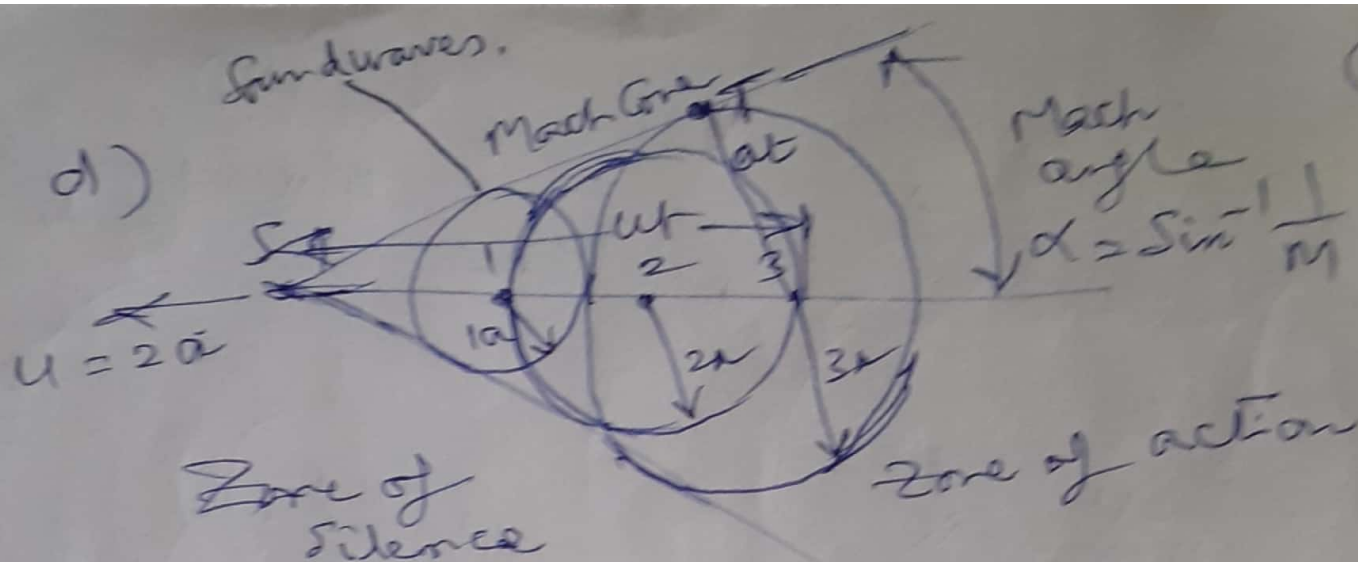
velocity of sound = a (Infinitesimal spherical or point source, intensity is not symmetrical)

b) Subsonic flow, $M = \frac{u}{a} = 0.5$.

Source of disturbance travels at half the velocity of wave. Spherical sound waves are shown. wave fronts move ahead of point source, intensity is not symmetrical



Point source travels with the same vel as that of wave. ($M = 1$, Sonic). wave fronts exist at the present position of pt source. left side is Zone of Silence. Right side



Supersonic flow, $\alpha = \frac{1}{2} u$.

$M = 2$

$M = \frac{u}{a} = 2$, point source is always ahead of the wave fronts; Tangents drawn from point S on spheres define a conical surface called as Mach Cone.

Zone of action is in Mach Cone,
 out of Mach cone is Zone of Silence.
 Semiangle of cone is Mach angle.

$$\alpha = \sin^{-1} \frac{at}{ut} = \sin^{-1} \frac{1}{u/a}$$

$$= \sin^{-1} \left(\frac{1}{M} \right)$$

Rarefaction wave: (expn wave)

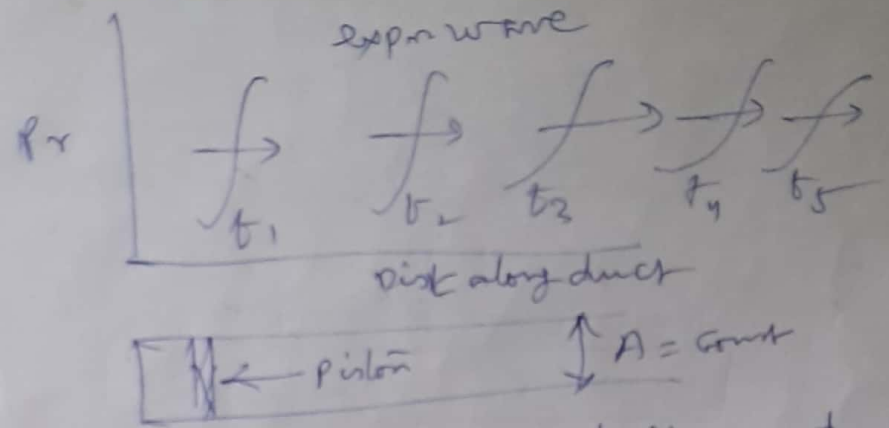


Fig: Wave generated by leftward movement of piston in duct. rarefaction waves are generated. wave becomes weaker (flatter) as it moves further as shown in fig. rarefaction waves are not possible.

Eqns.

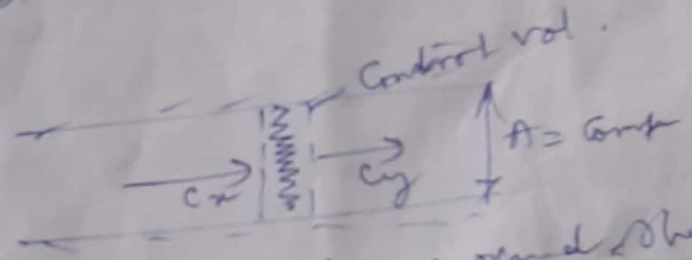


Fig: Flow through a normal shock in a const area duct. Control volume is studied.

Continuity eqn $\rho_x A_x C_x = \rho_y A_y C_y$
 A is const $\frac{\rho_x}{\rho_y} = \frac{C_y}{C_x} = \frac{A_y}{A_x}$

If heat transfer is negligible & also no shaft work, then adiabatic energy eqn for the C. volume

$h_{0x} = h_{0y} = h_0 = \text{const}$
 $T_{0x} = T_{0y} = T_0 = \text{const}$
 $h_x + \frac{1}{2} C_x^2 = h_y + \frac{1}{2} C_y^2$

By momentum eqn

$$(P_x - P_y)A = \dot{m} (C_{y2} - C_{x2})$$

$$P_x - P_y = \frac{\dot{m}}{A} (C_{y2} - C_{x2})$$

Substituting

$$P_x - P_y = \rho_a C_x (C_{y2} - C_{x2})$$

$$= \rho_a C_x C_{y2} - \rho_a C_x^2$$

$$P_x - P_y = \rho_a C_x C_{y2} - \rho_a C_x^2$$

$$\boxed{P_x + \rho_a C_x^2 = P_y + \rho_a C_x C_{y2}}$$

$$\therefore \rho_a C_x = \rho_a C_{y2}$$

By Impulse function eqn

$$F_x = F_y = \text{const}$$

Eqn of state gives

$$h = f(s, p)$$

$$s = f(p, p)$$

These eqns are used to define Fanno & Rayleigh lines.

Fanno line

Fanno line describes a diabatic flow process in a const area duct with friction. The stagnation enthalpy & flow rate per unit area remain constant.

On account of friction, the process is irreversible.

We know $\frac{\dot{m}}{A} = \rho_a C_y$

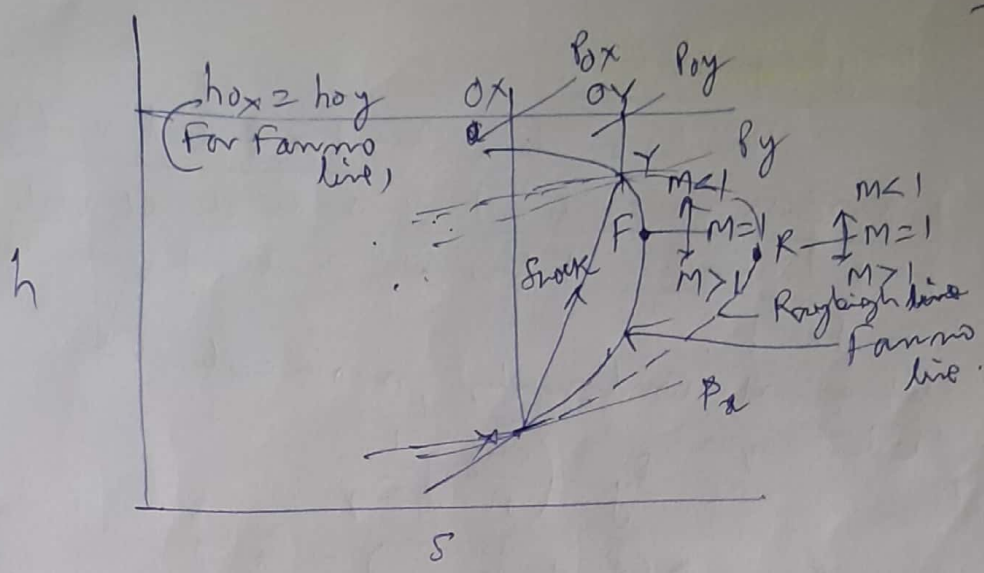
$$\rho_a = \left(\frac{\dot{m}}{A}\right) \frac{1}{C_y}$$

$$h_y = h_0 - \frac{1}{2} C_y^2$$

$$S_y = f(P_{0y}, P_y)$$

on h-s diagram draw fannolines or Fanno curve for diff values of C_y .

P.T.O



Max^m entropy point = F
 At this pt, infinitesimal Fanno process is considered. The change being small it is considered as reversible
 (ΔS = 0) near F.

$$h + \frac{1}{2} \tilde{c}^2 = \text{const}$$

$$\boxed{dh = -c dc} \text{ energy eqn, } dc = -\frac{dh}{c}$$

Also $\rho_x c_x = \rho_y c_y$
 So $\rho c = \text{constant}$

diffg

$$\rho \cdot dc + c \cdot d\rho = 0$$

$$\boxed{dc = -\frac{c}{\rho} d\rho} \text{ Continuity eqn}$$

For isentropic process

$$\boxed{dh = \frac{1}{\rho} d\rho}$$

~~by substituting the eqns~~

where $dc = -\frac{c}{\rho} d\rho$

$$dc \cdot dc = -\frac{dh}{c} = -\frac{c}{\rho} d\rho$$

$$-\frac{1}{\rho} \cdot \frac{(d\rho)}{(c)} = -\frac{c}{\rho} d\rho$$

$$\therefore dh = \frac{1}{\rho} d\rho$$

$$\frac{d\rho}{d\rho} = \tilde{c}$$

$$\text{So } \tilde{c} = \tilde{a} = \frac{d\rho}{d\rho} = \left[\frac{\partial \rho}{\partial p} \right]_s \checkmark$$

So at point F, M = 1, (Sonic)
 upper side is subsonic, lower side is supersonic.

Rayleigh line

This line describes Frictionless flow process in a constant area duct with heat transfer. Flow rate per unit area is constant.

Let us put continuity, momentum & state eqns.

$$\rho y = \left(\frac{m}{A}\right) \frac{1}{c_y}$$

$$p_y = \rho (p_x + \rho x c_x^2) - \rho y c_y^2$$

$$p_y = f(p_x, p_y)$$

For different values of c_y , Rayleigh line is drawn on h-s diagram.

R = Max^m entropy point on this line.

momentum eqn $p + \rho c^2 = \text{const.}$

differentiate

$$dp + c^2 dp + 2\rho c \cdot dc = 0.$$

$$\left. \begin{aligned} \rho \frac{dc}{dp} = \frac{c}{\rho} \end{aligned} \right\}$$

$$dp + c^2 dp - 2\rho c \left(\frac{c}{\rho}\right) dp = 0$$

$$dp + c^2 dp - 2c^2 dp = 0.$$

$$dp - c^2 dp = 0.$$

$$dp = c^2 dp, \quad c = \frac{dp}{dp}$$

Change at point R is small, so it is considered as isentropic $\Delta S = 0$.

$$\left[S_0 \quad c = a = \left[\frac{\partial p}{\partial \rho} \right]_S \right]$$

Hence at R, $M = 1$, upper side is subsonic and lower side is supersonic.

Shock occurs from X to Y, for Fanno & Rayleigh lines i.e. (Supersonic to subsonic).

Prandtl-Meyer relation:

$$h = c_p \cdot T$$

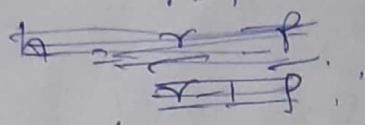
$$= \frac{\gamma}{\gamma - 1} \cdot R \cdot T$$

$$a = \sqrt{\gamma R T}$$

$$R T = \frac{P}{\rho}$$

$$a^2 = \gamma R T$$

$$a^2 = \gamma \cdot \frac{P}{\rho}$$



$$h = \frac{a^2}{\gamma - 1}$$

Applying adiabatic energy eqn before & after shock:

$$\frac{a_x^2}{\gamma - 1} + \frac{1}{2} c_x^2 = \frac{a_y^2}{\gamma - 1} + \frac{1}{2} c_y^2$$

$$= \frac{1}{2} \frac{\gamma + 1}{\gamma - 1} a^2$$

(already known) (2:47 eqn)

$$\frac{a_x^2}{\gamma - 1} + \frac{1}{2} c_x^2 = \frac{1}{2} \frac{\gamma + 1}{\gamma - 1} a^2$$

Solving:

$$\frac{a_x^2}{\gamma - 1} = \frac{1}{2} \frac{\gamma + 1}{\gamma - 1} a^2 - \frac{1}{2} c_x^2$$

$$a_x^2 = \frac{1}{2} (\gamma + 1) a^2 - \frac{1}{2} c_x^2 (\gamma - 1)$$

divide by c_x

$$\frac{a_x^2}{c_x} = \frac{\gamma + 1}{2} \frac{a^2}{c_x} - \frac{1}{2} (\gamma - 1) c_x$$

Similarly other part gives

$$\frac{a_y^2}{c_y} = \frac{\gamma + 1}{2} \frac{a^2}{c_y} - \frac{\gamma - 1}{2} c_y$$

But $P_x = P_y = \frac{m}{A} (c_y - c_x)$

$$(P_x - P_y) \frac{A}{m} = (c_y - c_x)$$

$$\frac{P_x - P_y}{\rho_x \cdot c_x} = c_y - c_x$$

$$\frac{P_x}{\rho_x \cdot c_x} - \frac{P_y}{\rho_x \cdot c_x} = c_y - c_x$$

Density Ratio across the shock

(Rankine - Hugoniot eqn)

eqn of state for perfect gas

$$\frac{p_y}{p_x} = \frac{\rho_y}{\rho_x} \frac{T_x}{T_y} \quad \checkmark$$

Substituting the Pr ratio & Temp ratio eqns

$$\frac{p_y}{p_x} = \left[\frac{2\gamma}{\gamma-1} M_x^2 - \frac{\gamma-1}{\gamma+1} \right] \frac{1 + \frac{\gamma+1}{2} M_x^2}{\left(1 + \frac{\gamma-1}{2} M_x^2 \right) \left(\frac{2\gamma}{\gamma-1} M_x^2 - 1 \right)}$$

$$\frac{p_y}{p_x} = \frac{\left(\frac{\gamma+1}{2} \right) M_x^2}{1 + \frac{\gamma-1}{2} M_x^2}$$

eqn of continuity for constant flow gives

$$\frac{c_y}{c_x} = \frac{\rho_x}{\rho_y}$$

$$\frac{c_y}{c_x} = \frac{1 + \frac{\gamma-1}{2} M_x^2}{\frac{\gamma+1}{2} M_x^2}$$

Another expression for density ratio across the shock can be derived in terms of pressure ratios.

$$\frac{p_y}{p_x} = \frac{1 + \left(\frac{\gamma+1}{\gamma-1} \right) \frac{p_y}{p_x}}{\frac{\gamma+1}{\gamma-1} + \frac{p_y}{p_x}} \quad \checkmark$$

$$\checkmark \frac{p_y}{p_x} = \frac{\left(\frac{\gamma+1}{\gamma-1} \right) \frac{p_y}{p_x} - 1}{\frac{\gamma+1}{\gamma-1} - \frac{p_y}{p_x}} \quad \checkmark \text{ Imp.}$$

These eqns are known as Rankine Hugoniot eqn.

for isentropic process-

$$\frac{p_y}{p_x} = \left(\frac{\rho_y}{\rho_x} \right)^\gamma$$

Flow in constant area ducts with friction

Flow in constant area duct with friction in the absence of work and heat transfer is known as Fanno flow. We know that at maximum entropy point "F" on Fannolines, the gas has sonic velocity. Practical situations are flow process in gas ducts of air-craft engines, A/c systems etc. Flow parameters vary due to duct wall friction.

The Fanno curves.

Assumptions :- (a) perfect gas (b) constant area duct (c) one dimensional steady frictional flow (d) absence of heat and work transfer and body forces.

Continuity eqn is $m = \rho A c$

Mass flow density is defined by

$$G = \frac{m}{A} = \rho c, \quad c = \frac{G}{\rho}$$

Energy eqn is $h + \frac{1}{2} c^2 = h_0$

or $h + \frac{1}{2} \frac{G^2}{\rho^2} = h_0$

$$h = h_0 - \frac{1}{2} \frac{G^2}{\rho^2}$$

Eqn of state can be written as

$$p = f(\rho, h)$$

$$h = h_0 - \frac{1}{2} \frac{G^2}{[f(\rho, h)]^2}$$

These eqns yield Fannolines eqn for given values of h_0 & G , a curve can be drawn in h - ρ plane

P.T.O

Change of entropy

$$\frac{P_{01}}{P_{0x}} = \frac{(P_0/P_0^*)_1}{(P_0/P_0^*)_x} = \frac{1.773}{1.189} = 1.491$$

$$\frac{(AS)_{1-x}}{R} = \ln \frac{P_{01}}{P_{0x}} = \ln 1.491 = 0.399$$

$$\frac{(AS)_{x-y}}{R} = \ln \frac{P_{0x}}{P_{0y}} = \ln \frac{1}{0.926} = 0.077$$

$$\frac{P_{02}}{P_{0y}} = \frac{(P_0/P_0^*)_2}{(P_0/P_0^*)_y} = \frac{1}{1.097}$$

$$\frac{(AS)_{y-2}}{R} = \ln \frac{P_{0y}}{P_{02}} = \ln 1.097 = 0.0925 \text{ Am}$$

Unit-II

Flow in constant area ducts with heat transfer
(chapter 9).

Frictionless flow process in a constant area duct with heat transfer is referred to as Rayleigh flow.

Ex: Heat Transfer processes in Heat Exchangers and Combustion ~~chambers~~ chambers are considered ignoring frictional effects. Fairly good values can be obtained using Rayleigh flow Model.

P.T.O
detained.
323 } 4B.
331 }
332 }

The Rayleigh line

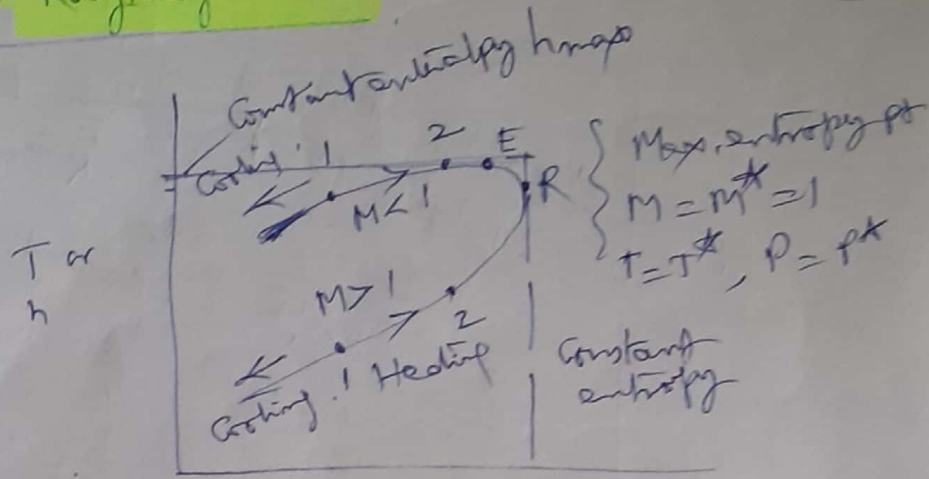


Fig: Rayleigh line or Curve

Assumptions in the process

- a) Perfect gas
- b) Constant area duct
- c) One dimensional, steady frictionless flow
- d) Absence of body forces.

Rayleigh line is the locus of all state points during the Rayleigh process and satisfies the eqns of state, continuity and momentum.

We know that velocity at max^m entropy point is sonic.

$$\text{Mass flow density} = G = \frac{m}{A} = \rho c$$

$$c = \frac{G}{\rho}$$

Momentum eqn for frictionless flow

$$P + \rho c^2 = \text{constant}$$

$$P + \rho \cdot \frac{G^2}{\rho^2} = \text{const}, \quad P + \frac{G^2}{\rho} = \text{const}$$

$$P + G^2 \cdot \rho = \text{const}$$

This eqn represents a straight line on P-ρ plane (See fig)

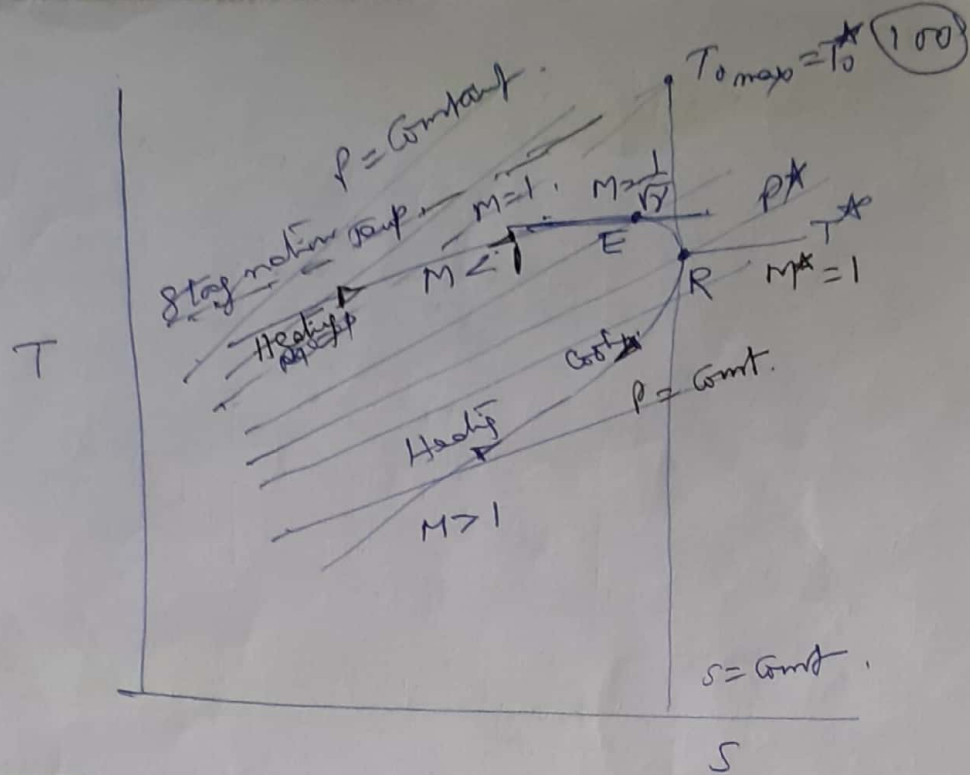


Fig:- Various flow parameters on Rayleigh line

Slope of Rayleigh line

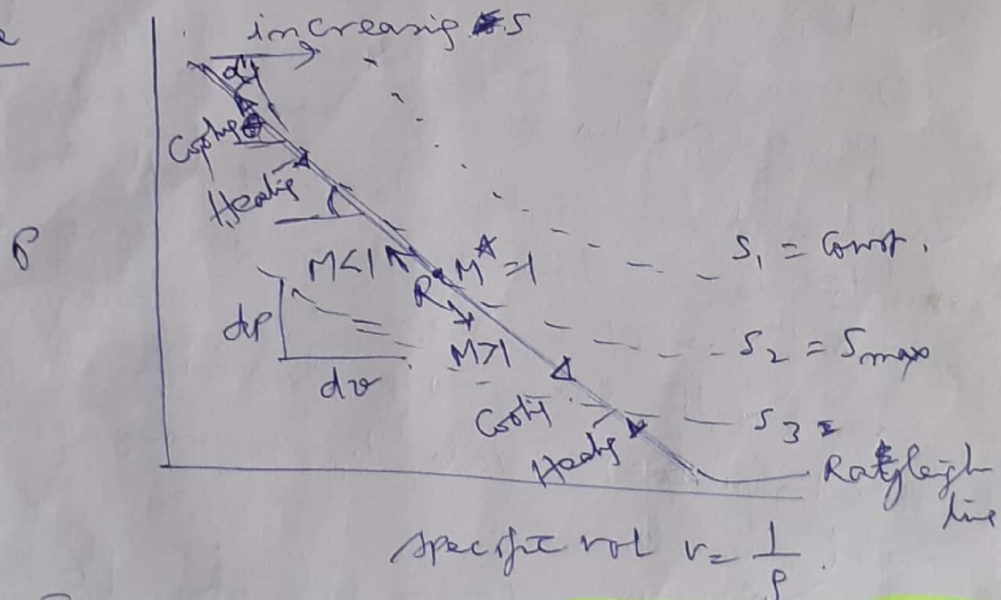


Fig:- Constant entropy lines & Rayleigh line on P-v diagram

Gradient of this line is $-G^*$

equation of state $P = P(h, s)$
 $\rho = \rho(h, s)$

$$P(h, s) + \frac{G^*}{\rho(h, s)} = \text{const.}$$

This eqn represents the Rayleigh line on h-s plane.

IIIrd unit Compressors

It is driven by a prime mover.

Some energy is converted to work. Some will be lost to radiation & coolant (if used).

But for compressor, the energy available at shaft of prime mover, is the energy which it receives.

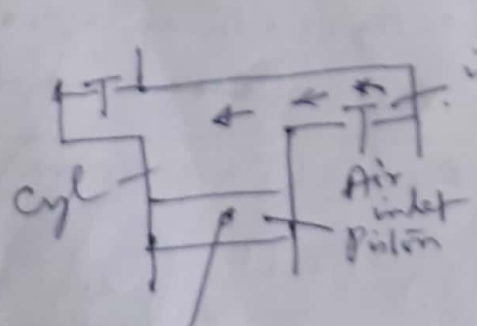
Classification:

- 1) Reciprocating
- 2) Rotary.
- i) If one unit, then it is single stage & more than one unit, Multistage.
- ii) Reciprocating $\left\{ \begin{array}{l} \text{Single acting} \\ \text{Double acting} \end{array} \right.$
- iii) Centrifugal - It is rotary type only. It can be single entry or double entry.

Air compressors are also classified according to the use.

- i) To produce vacuum (Pumps, exhausts)
- ii) Blowers, Superchargers (low pressure)
- iii) Booster (air or gas compressor, used in higher pressure requirement)

Reciprocating (Single Stage):



Crank arrangement.
Fig I

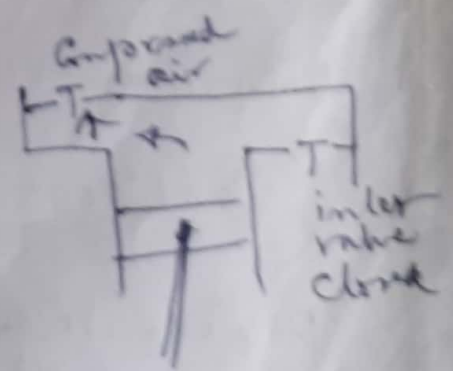
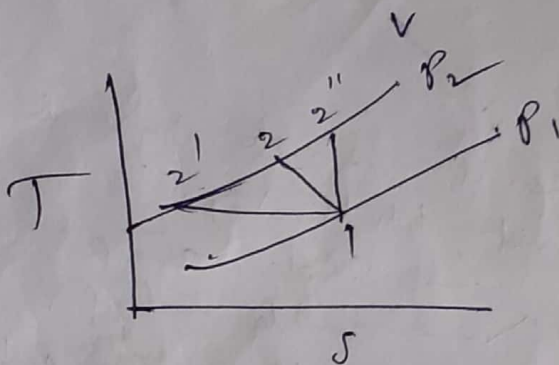
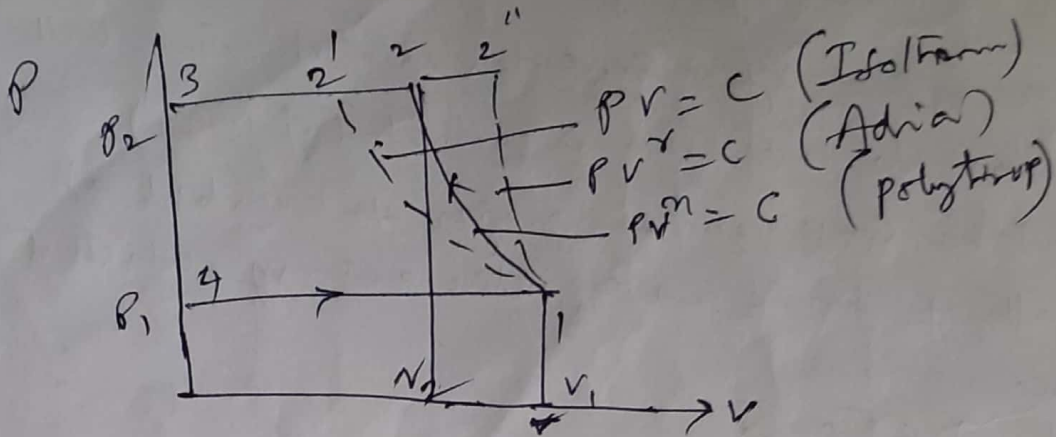


Fig II.

Single stage (neglecting clearance)



i) operation 4-1 :- volume of air (v_1) expanded into comp at p_1, T_1 .

ii) opn 1-2 \rightarrow Air compressed according to $p v^m = \text{const.}$ (from p_1 to p_2)
Vol reduces from v_1 to v_2 .
Temp increases from T_1 to T_2

iii) opn 2-3 \rightarrow compressed air v delivered at p_2, v_2 , & T_2 .

Minimum work is done by isothermal process.

$$\text{Isothermal } \eta = \frac{\text{Isothermal work done}}{\text{Actual work done}}$$

$$\begin{aligned} \text{Total work done} &= W = 41234 \\ &= \text{Area under } (4-1) - \text{Area under } (1-2) - \text{Area under } (2-3) \\ &= p_1 v_1 - \frac{p_2 v_2 - p_1 v_1}{n-1} - p_2 v_2 \end{aligned}$$

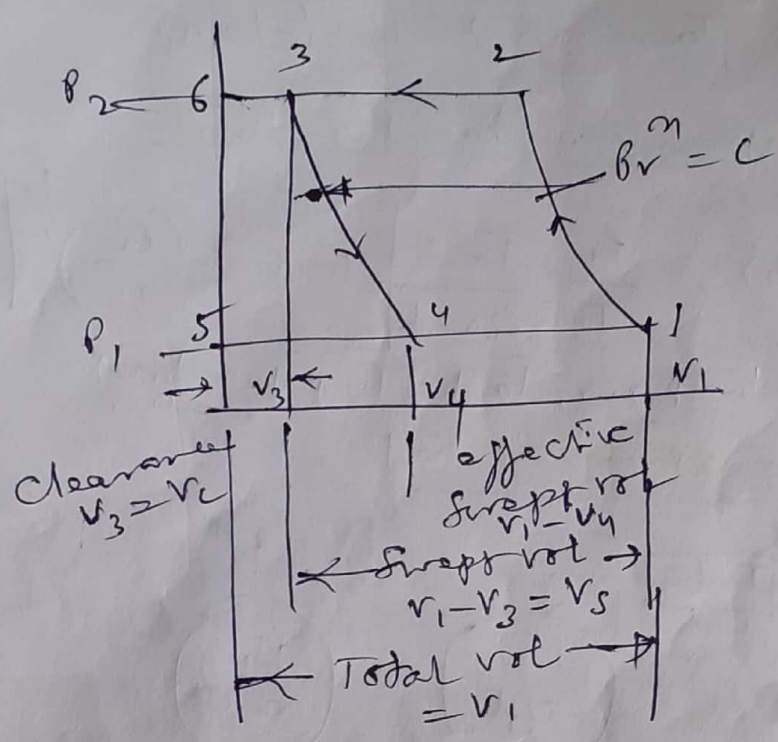
$$W = \left(\frac{n}{n-1} \right) \left(p_1 v_1 - p_2 v_2 \right) = \frac{n}{n-1} p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]$$

The work will be done on compressor

$P_1, V_1 = nRT_1$, can be used

Also $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$

work done (with clearance)



$$W = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$- \frac{n}{n-1} P_4 V_4 \left[\left(\frac{P_3}{P_4}\right)^{\frac{n-1}{n}} - 1 \right]$$
 But $P_4 = P_1, P_3 = P_2$
Simplify

Work done/cycle = $\frac{\text{area } 12341}{2} - \frac{\text{area } (54365)}{2}$

$$= \frac{n}{n-1} P_1 (V_1 - V_4) \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

Volume ratio:

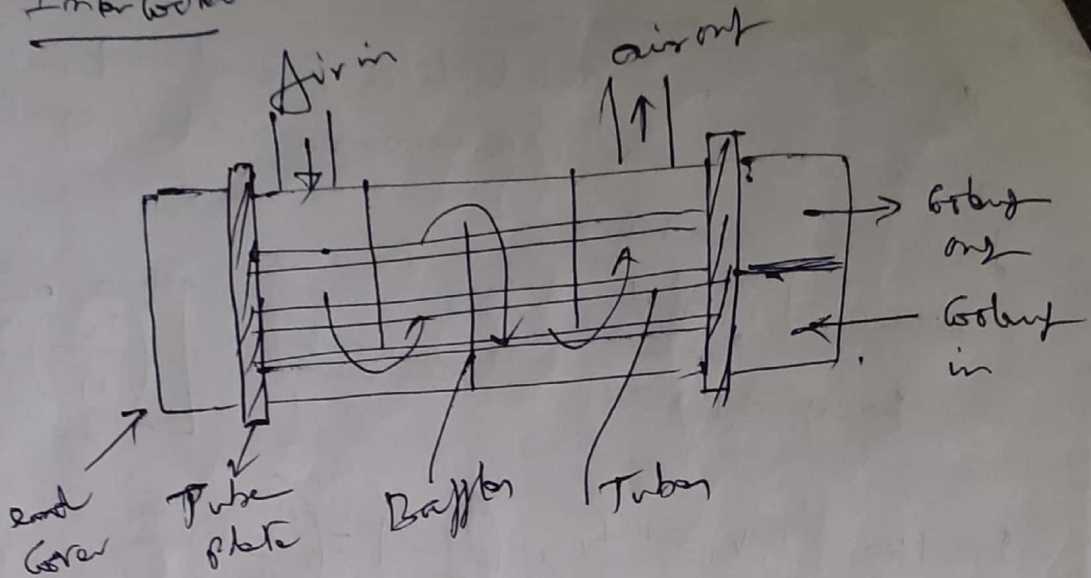
Free air delivered
Compressor displacement

Also
$$\frac{\text{effective swept vol}}{\text{swept vol}} = \frac{v_1 - v_4}{v_1 - v_3}$$

Clearance ratio = $\frac{\text{Clearance vol}}{\text{swept vol}}$

$$= \frac{v_3}{v_1 - v_3} = \frac{v_c}{v_s} = k$$

Inter cooler



Cooler which is placed in between stages is called inter cooler.

Coolers fitted after last stage, are ~~called~~ for removing moisture are called after coolers. After coolers cannot influence the work done. They are Heat Exchangers.

Efficiency of Compressor

$$\begin{aligned} \text{Isothermal workdone/cycle} &= \text{Area of } p-v \text{ diagram.} \\ &= P_1 V_1 \log_e r \end{aligned}$$

$$\text{Isothermal power} = \frac{P_1 V_1 \log_e r \times N}{60 \times 1000} \text{ K.W.}$$

Indicated power is the power obtained from a chart indicator diagram.

$$\text{Compressor } \eta = \frac{\text{Isothermal H.P.}}{\text{Indicated H.P.}}$$

$$\text{Isothermal } \eta = \frac{\text{Isothermal H.P.}}{\text{Shaft H.P.}}$$

$$\text{Shaft HP} = \text{Brake H.P.} = \text{B.H.P.}$$

$$\text{Adiabatic } \eta = \frac{\text{B.H.P.}}{\text{area of indicator diagram}}$$

$$\text{Mech } \eta = \frac{\text{~~HP~~ output}}{\text{Input}}$$

assuming adiabatic compression.

Free Air delivered (FAD) & displacement.

FAD is actual volume delivered at the stated pressure reduced to intake pressure & temperature & expressed in m^3/min .

Displacement = Actual vol swept out per minute by L.P piston during suction stroke (m^3/min).

FAD < displacement of compressor because of fluid resistance, losses, etc.

Rotary Compressors:

Whenever large quantities of air or gas are required at relatively low pressure rotary compressors are employed.

Classification

(i) Displacement compressors

- a) Roots blower
- b) Sliding Vane Compr

2) Steady flow compressors

- a) Centrifugal
- b) Axial flow.

Displacement Compressors

Displacement compressors are those compressors in which air is compressed by being trapped in the reduced space formed by two sets of engaging surfaces.

Roots blower

In the Figure 2 lobe type is shown. but 3 lobe, 4 lobe types are in use for higher pressure ratios. One of the rotors is connected to drive and 2nd rotor is gear driven by first. Profile of lobes is of cycloidal or involute form so that a seal is formed between inlet & delivery sides.

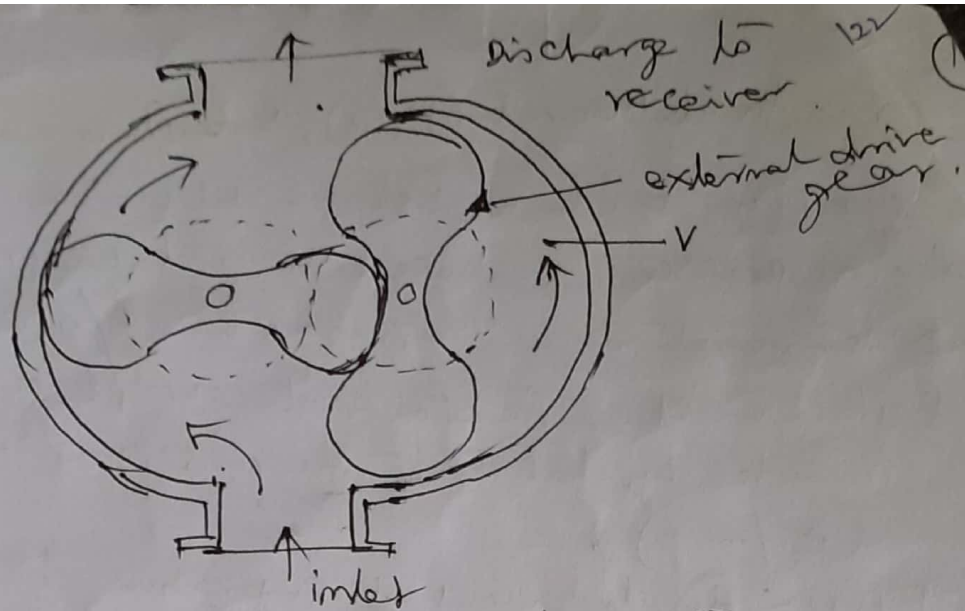


Fig:- Roots blower, 2 lobe rotors.

There must be some clearance between lobes and between casing & lobes, to reduce wear. This clearance forms a leakage path & reduces efficiency.

As each side of each lobe faces its side of the casing, a volume of gas V , at press P_1 , is displaced towards delivery side at constant pressure. At the receiver it collects, ^{after irreversible compression} and pressure P_2 is obtained. This process is carried out 4 times/revolution of driving shaft.

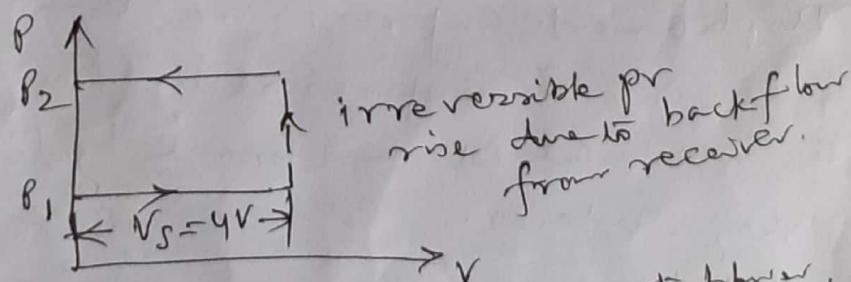


Fig:- P-v diagram of roots blower.

$$\text{Work done/cycle} = (P_2 - P_1) V$$

$$\text{Work done/revolution} = 4 (P_2 - P_1) V$$

If V_s is the volume dealt with per minute at P_1 & T , then $(P_2 - P_1) V_s$.

The ideal Compression process is a reversible adiabatic (ie isentropic) process.

work done/minute ideally is given by

$$\text{work done/min} = \frac{\gamma}{\gamma-1} \cdot P_1 V_1 \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}$$

Roots $\eta = \frac{\text{work done isentropically}}{\text{Actual work done}}$

$$= \frac{\frac{\gamma}{\gamma-1} \cdot P_1 V_s \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}}{V_s (P_2 - P_1)}$$

$$= \frac{\frac{\gamma}{\gamma-1} \cdot P_1 V_s \left\{ (r)^{\frac{\gamma-1}{\gamma}} - 1 \right\}}{P_1 V_s \cdot (\gamma - 1)}$$

$$= \frac{C_p}{R} \left\{ \frac{(r)^{\frac{\gamma-1}{\gamma}} - 1}{(\gamma - 1)} \right\}$$

where $r = \frac{P_2}{P_1}$ ratio.
but $\frac{\gamma}{\gamma-1} = \frac{C_p}{R}$

The η decreases as pressure ratio increases, so is well suited for scavenging, & super charging of I. c. engines.

Vane type blower:

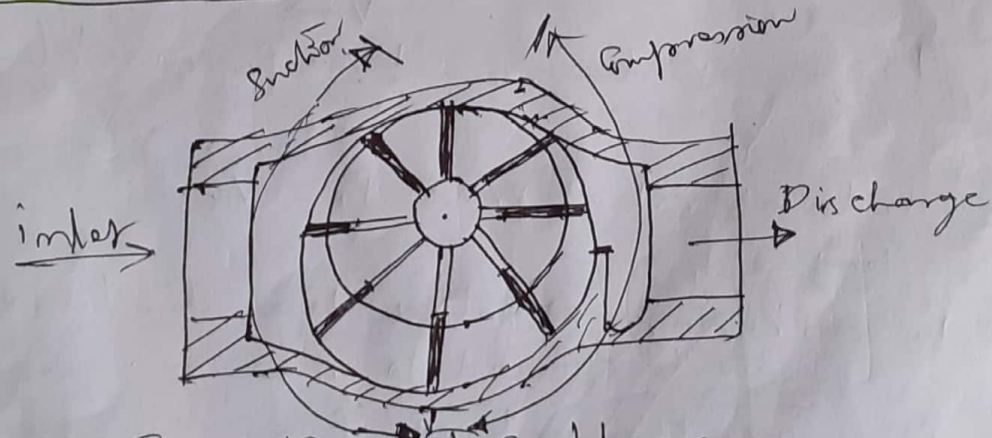


Fig:- Vane type blower

Vane Type blower consists of a rotor mounted eccentrically in the body & supported by ball & roller bearings in the end covers of the body.

Steady flow Compressors

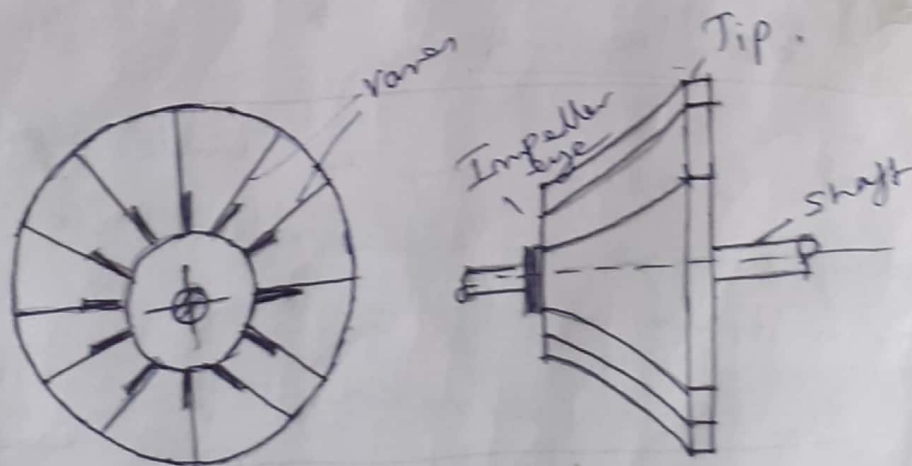
Steady flow compressors are those compressors in which compression occurs by transfer of kinetic energy from a rotor.

Centrifugal compressor is lighter and is able to operate effectively over a wide range of mass flows at any speed. For larger units with higher pressure ratios, the axial flow compressor is more efficient and is usually preferred. For air craft, the trend has been to higher pressure ratios & axial flow type is preferred.

Centrifugal type have following advantages over axial flow type

- 1) cheaper
- 2) More robust
- 3) less prone to icing troubles at high altitudes
- 4) Possess a wider operating range

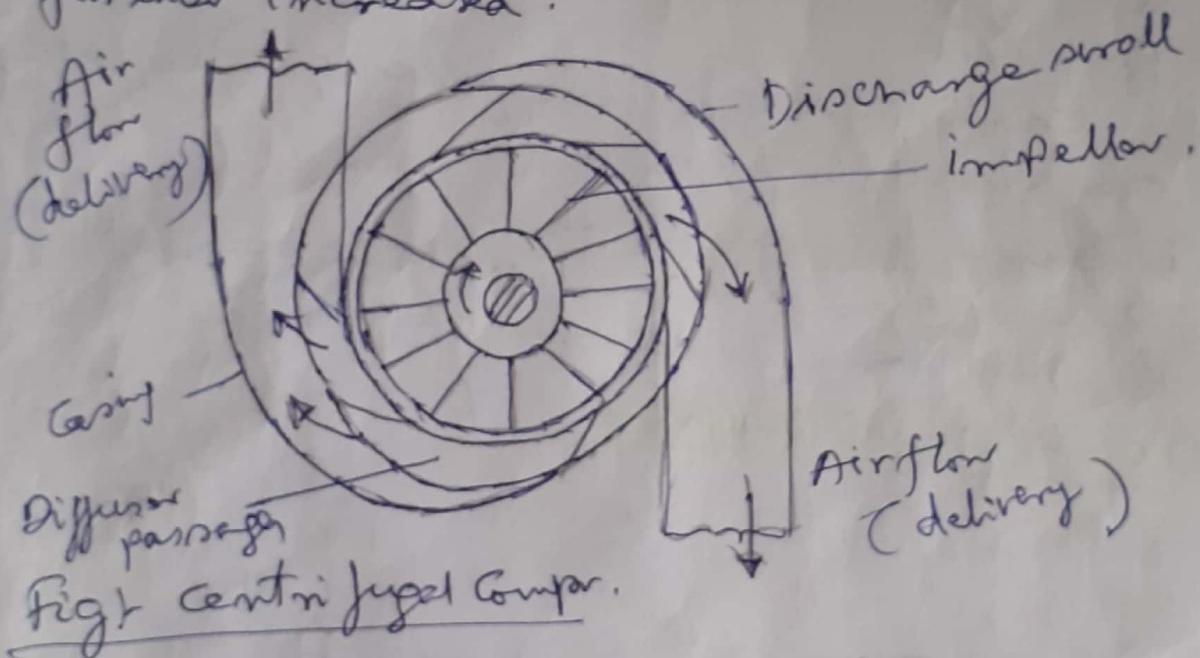
① Centrifugal Compressor



Fig! - Impeller (single eyed) and radial vanes of centrifugal compressor

Centrifugal Compressor consists of an impeller with a series of curved radial vanes.

Air is drawn in near the hub (impeller eye) and is whirled round at high speed by the vanes on the impeller as the impeller rotates at high speed. The static pressure of the air increases from the eye to the tip of impeller. Air leaves the impeller tip & passes through diffuser. K.E of air gets converted to increase in enthalpy and pressure of air is further increased.



The impeller may be double eyed, having an eye on either side of compressor. Air is drawn in on both sides.

In practice nearly half the total pressure is achieved in impeller and remaining half in the diffuser. Pressure ratio of 4:1 can be achieved with single stage and pr ratio of 12:1 can be achieved with multistage.

C_{f1} = vel of flow at inlet

C_{f2} = " " at outlet

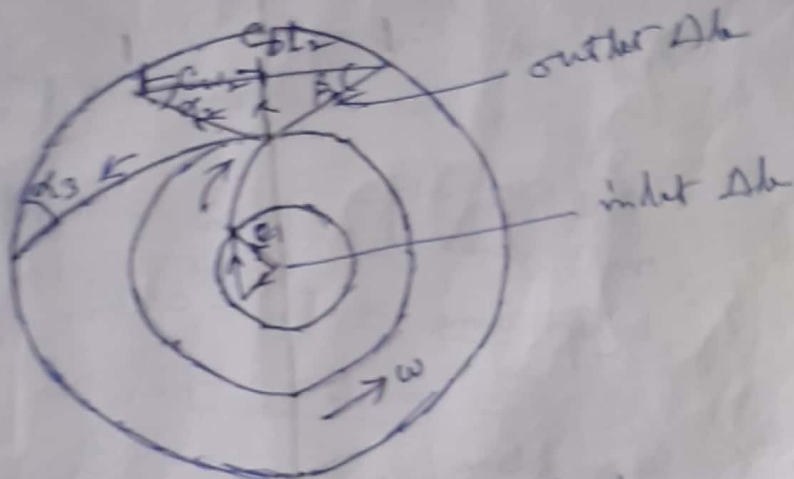
α_1 = ~~inlet~~ angle of guide vane

β_1 = inlet angle to the rotor or impeller

β_2 = outlet " for the " "

α_2 = inlet angle to the diffuser.

Figure shows the vel diagrams for the inlet and outlet of impeller. It is assumed that the entry of the air is axial. So whirl component at inlet $C_{w1} = 0$. \therefore therefore $C_1 = C_{f1}$.



The enlarged views are given below.

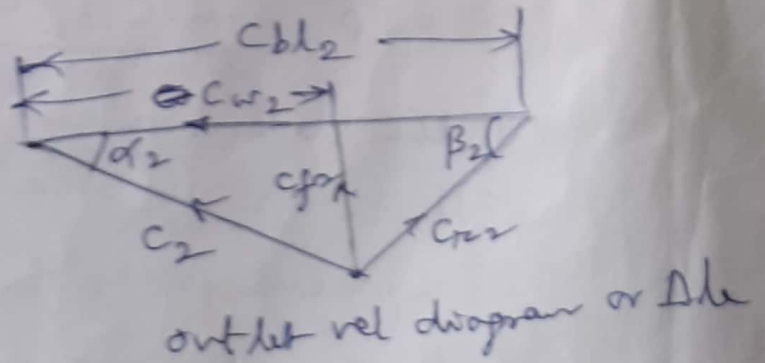
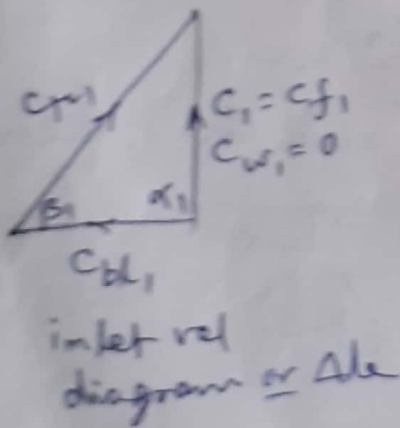


Fig. 1 - velocity diagrams.

P_{01} = stagnation pr at inlet

T_{01} = " Temp " "

P_{02} = stagnation pr at outlet

T_{02} = " Temp at outlet

P_1 = static pr at inlet

T_1 = static Temp "

P_2 = static pr at ~~inlet~~ outlet

T_2 = static Temp at outlet

work done by impeller (Euler's work)

F_t = Tangential force

F_t = Mass of air x rate of change of tangential vel.

$F_t = \dot{m} (C_{w2} - C_{w1})$

$C_{w1} = 0$ as air enters radially (no prewhirl)

$F_t = \dot{m} C_{w2}$

\dot{m} = mass of airflow in kg/sec

Work done = $F_t \times c_{bl2}$

= $\dot{m} C_{w2} c_{bl2}$

work done / kg of air = $C_{w2} c_{bl2}$ (Euler's eqn)

W = Theoretical work x angular vel
 $= (C_{w2} \cdot r_2 - C_{w1} \cdot r_1) \cdot \omega$
 $= C_{w2} \cdot c_{bl2} - C_{w1} \cdot c_{bl1}$
 $= h_{02} - h_{01} = c_p (T_{02} - T_{01})$

W = work done
 ω = angular vel

This work is known as Euler's work. See page 26 or 138

Power reqd for impeller for \dot{m} kg of air

$P = \frac{\dot{m} \cdot C_{w2} \cdot c_{bl2}}{1000}$ Kw.

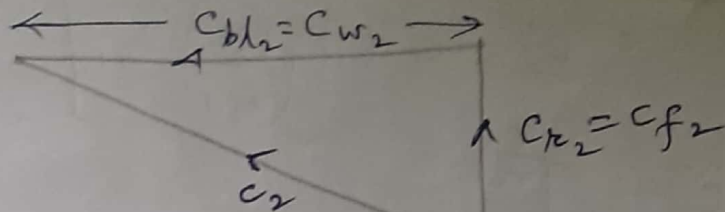
If the blade is radial (ideal case), then

$c_{bl2} = C_{w2}$ (See diagram below)

work done per kg of air/sec

= $W = C_2^2$

since $C_2 = \frac{tip}{tip}$ velocity which is highest.



Considering steady flow, ^{assuming} heat transfer to be zero.

$$h_1 + \frac{C_1^2}{2} + W = h_2 + \frac{C_2^2}{2}$$

$$W = \left(h_2 + \frac{C_2^2}{2} \right) - \left(h_1 + \frac{C_1^2}{2} \right)$$

$$= c_p \left[T_2 + \frac{C_2^2}{2c_p} \right] - c_p \left[T_1 + \frac{C_1^2}{2c_p} \right]$$

$$= c_p \cdot T_{02} - c_p \cdot T_{01}$$

$$W = c_p (T_{02} - T_{01})$$

$$W = c_p \cdot T_{01} \left[\frac{T_{02}}{T_{01}} - 1 \right]$$

$$= c_p \cdot T_{01} \left[\frac{T_2 \left(\frac{p_{02}}{p_2} \right)^{\frac{\gamma-1}{\gamma}}}{T_1 \left(\frac{p_{01}}{p_1} \right)^{\frac{\gamma-1}{\gamma}}} - 1 \right]$$

$$= c_p \cdot T_{01} \left[\left(\frac{p_{02}}{p_{01}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$= c_p \cdot T_{01} \left[(r_p)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

r_p = Press ratio based on stagnation pr.

In most practical problems $C_1 = C_2$

$$\text{Then } W = c_p (T_2 - T_1)$$

$$= c_p T_1 \left[\frac{T_2}{T_1} - 1 \right] = c_p T_1 \left[(r_p)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

We know

$$c_p T + \frac{C^2}{2} = c_p T_0$$

$$c_p \left[T + \frac{C^2}{2c_p} \right] = c_p T_0$$

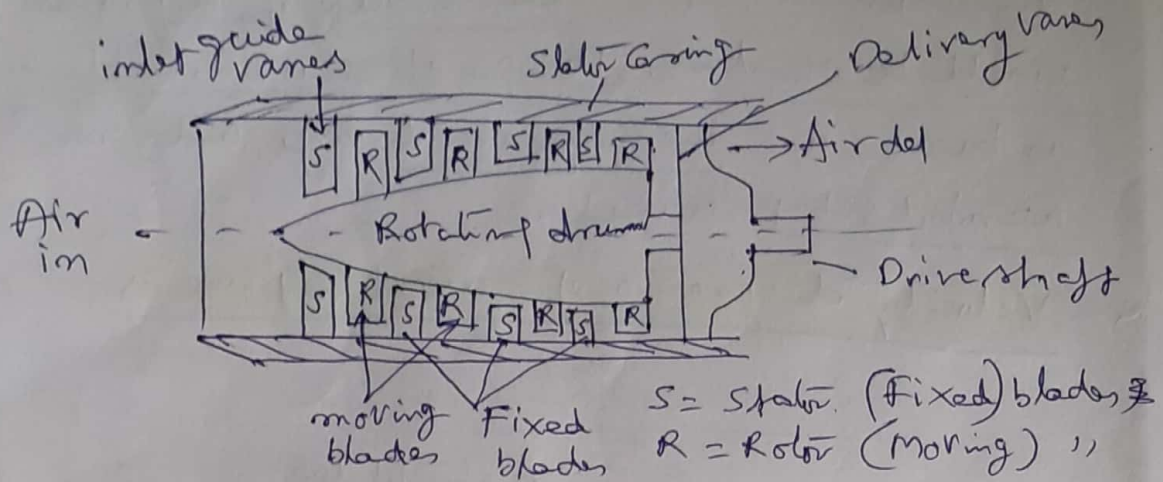
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{p_0}{p} = \left(\frac{T_0}{T} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_0}{T} = \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}}$$

$$T_0 = \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \cdot T$$

Axial Flow Compressor:



An axial flow compression stage consists of a row of moving blades arranged round the circumference of a rotor, and a row of fixed blades arranged round the circumference of a stator.

The air flows axially through the moving and fixed blades in turn. Stationary guide vanes are provided at the entry.

Compression is performed ~~is~~ similar to the centrifugal type. The work input to the rotor shaft is transferred by the moving blades to the air, thus accelerating it.

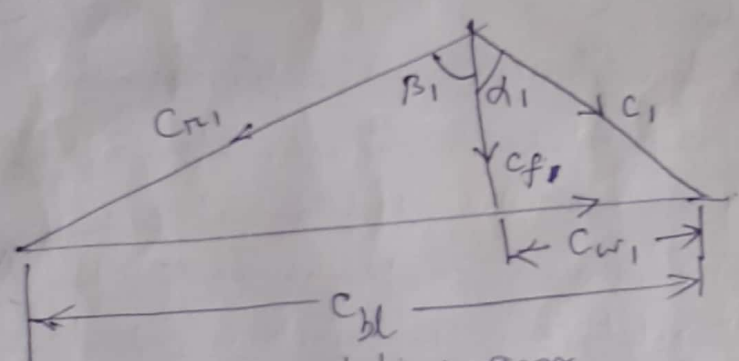
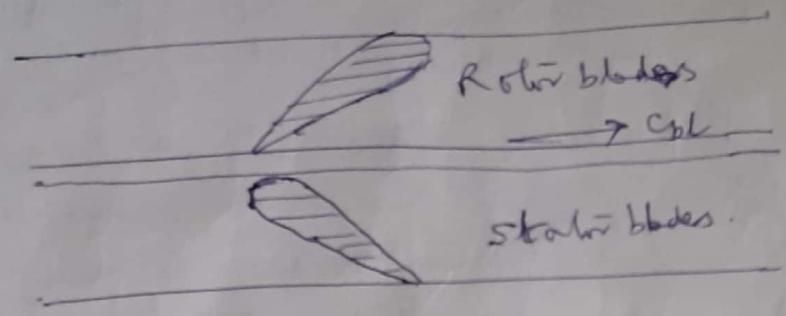
The spaces between the blades form diffuser passages, & hence velocity decreases, and pressure increases.

Large number of stages are possible, & also constant work input per stage is possible. (ex 5 to 14 stages).

In the figure, a flared rotor is shown. This is type helps in reduction in volume. It is usually arranged to have equal long rise in moving & fixed blades and

To keep the axial vel constant throughout the compressor. Thus each stage of compression would be similar. Design of compressor blades is based on aerodynamic theory and an aerofoil shape is used.

Velocity diagrams of axial compressors



* Fig: inlet vel diagram.

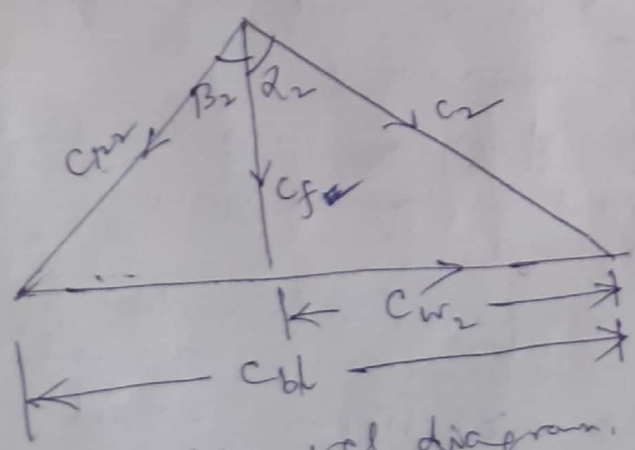
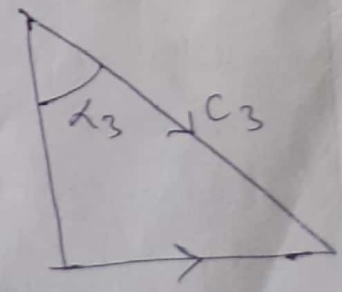


Fig:- outlet vel diagram.



Considering single stage of Compressor,
air enters rotor blades with absolute vel C_1
and at angle α_1 .

C_{bl} , the blade vel is same at entrance and
Exit. C_{r1} , the relative vel is obtained by
vector addition of absolute vel & blade vel.

The pressure rises due to diffusion taking place,
& hence relative vel is reduced &
let it be C_{r2} at outlet. C_{r2} makes an

angle β_2 . During diffusion process, work is
done on rotor, so the air leaves the blades
with increased absolute vel (C_2).

During the passage of air, a part of KE
is converted to pressure energy.

C_2 at the entrance of stator blades,
leaves with reduced vel (C_3), which is

$C_{bl1} = C_{bl2} = C_{bl}$
almost equal to C_1 . Tangential force = $(C_{w2} - C_{w1})$
work done/kg of air = $W = C_{bl} (C_{w2} - C_{w1})$ Nm.

It may be noted that whirl component at
entrance is not zero. (axial flow)

Degree of Reaction :

Degree of reaction (R_d) is defined as the
ratio of pressure rise in the comp. stage

$$R_d = \frac{\text{Press rise in rotor blades}}{\text{Press rise in stage}}$$

$$\begin{aligned} \text{Pr rise in comp. stage} &= \text{work input per stage} \\ &= C_{bl} (C_{w2} - C_{w1}) \end{aligned}$$

$$\text{Pr rise in rotor blades} = \frac{C_{r1}^2 - C_{r2}^2}{2}$$

$$R_d = \frac{C_{r1}^2 - C_{r2}^2}{2 C_{bl} (C_{w2} - C_{w1})}$$

Compressor characteristics - Surging and Choking.

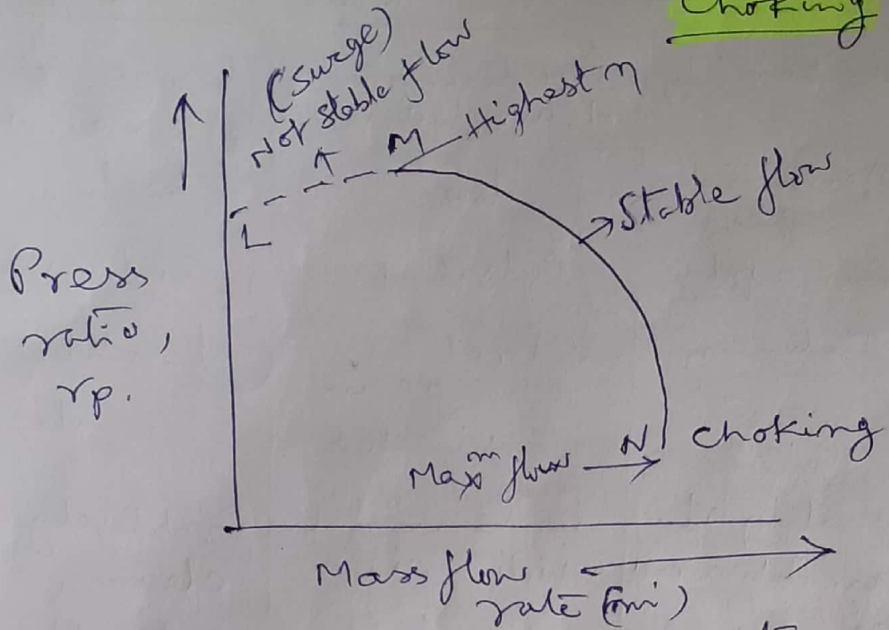


Fig:- Compressor characteristics.

Incidence loss ~~is~~ ^{is} due to incorrect fluid angles and falling of Pressure ratio (r_p). Pressure ratio, Mass flow rate graph is shown above. In this friction loss is also considered.

- (i) Point N :- The Compressor is choked and is passing max^m mass flow rate.
- (ii) Section MN :- Flow is stable. If m reduces, r_p increases and corrects the fall in pressure ratio.
- (iii) Section LM :- Flow is not stable. If m reduces, r_p reduces. In this condition, any small disturbance in mass flow cause reduction in r_p & flow may reverse at some point. When the disturbance is removed, the flow will pick up. Small disturbances cause oscillations in the flow. They are noisy and may cause structural damage. It is called 'Surge'. m is highest at M.

Prewhirl:

The centrifugal compressor employed in gas turbines and jet engines run at very high speed and there is always likelihood of formation of shock wave in the flow passage. To avoid this, Mach No at any point in the flow passage should not exceed more than unity. The maximum value of Mach No. corresponding to relative velocity at the inlet is

$$M_{r1} = \frac{C_{r1}}{\sqrt{\gamma R T_1}}$$

In order to reduce C_{r1} without affecting C_{f1} , the fluid is given an initial pre-rotation called pre-whirl by means of fixed guide vanes attached to casing at inlet of compressor. Pre-whirl reduces the work input by the amount $C_{d1} C_{w1}$ at the cost of additional construction.

Stalling: Choking flow

Theoretically, the mass flow rate becomes maximum when the pressure ratio is unity i.e. there is no compression.

This generally occurs when Mach No. corresponding to relative velocity at inlet becomes sonic. Under this condition the mass flow rate possible from centrifugal or axial flow compressor becomes maximum which is known as choking flow. Choking means fixed mass flow rate regardless of pressure ratio, i.e. the characteristics become vertical.



Fig: choking flow.

Stalling:

Stalling of stage of an axial flow compressor is defined as the aerodynamic stall or the break away of the flow from the suction side of blade aerofoil.

This break away of the flow may occur due to lesser mass flow rate than designed value or due to non uniformity in the blade profile or approaching flow. Thus, stalling is an ahead phenomenon of surging.

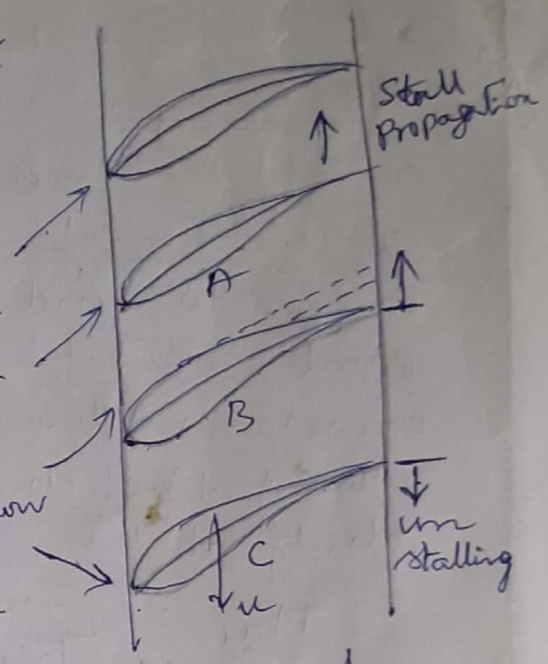


Fig- Stalling in Axial Compressor

Fig shows flow through a cascade. Suppose that ~~in~~ some nonuniformity of ^{approach} flow in a blade profile causes blade B to stall. The air flows on to a blade A ~~at~~ at an increased incident due to blockage of channel AB. The blade A stalls as a result, but the flow on to the blade C is now at lower incidence & blade C may un stall. In this way, a stall cell may move along the cascade opposite in the direction of blade motion or the direction of the lift of the blades. In other words, the phenomenon of reduction of lift force at higher angles of incidence is known as stalling.

If the natural frequency of vibration of the blades coincide with that at which the stall cells pass a blade, the resonance & possible mechanical failure of the blades may occur.

Steam turbine is a prime mover in which potential energy of steam is transformed into KE, and later with its turn is transformed into mechanical energy of rotation of the turbine shaft. It is used for power generation & for transport etc.

Classification:- Different methods of classification.

i) Action of steam (Common Method)

a) Impulse b) Reaction c) Combination of both.

ii) According to no. of stages.

a) Single stage with small power capacity.
They are mostly used for driving centrifugal compressors, blowers etc.

b) Multistage impulse & reaction turbines:-
The power capacities are from small to large

iii) According to direction of steam flow:-

a) Axial turbine → Steam flows parallel to axis.

b) Radial turbine. - Steam flows in a direction perpendicular to axis.

iv) According to No. of Cyls:-

a) Single cyl b) double cyl c) triple cyl.

d) 4 cyls.

single shaft turbines → Multi cyl turbines with same shaft

Multi axial turbines → Turbines with separate rotors shafts parallel to each other.

v) According to method of governing:-

a) Throttle governing: Fresh steam enters through throttle valves

b) Nozzle governing:- Fresh steam enters through nozzles.

Comparison

Particular	Steam Turbine	Steam engine
1) Thermal efficiency	high	low
2) flywheel	not required	required
3) Speed	high speed	low speed
4) output	higher	lower
5) Balancing problem	Minimum	More
6) Internal lubrication	not reqd	required
7) loss due to initial condensation of steam	No loss	loss
8) high vacuum	Can use high vacuum	not reqd
9) over load	possible	not possible

Common Types of Turbines

1) Simple Impulse

2) Reaction Turbine

- i) Steam expands in nozzle,
- ii) Pressure doesn't alter as it moves over blades

- i) Steam expands continuously as it passes over blades.
- ii) gradual fall in pressure during expansion.

1) Simple Impulse Turbine:

Expansion of steam takes place in one set of nozzles, hence it is called Simple impulse turbine.

As the steam flows through the nozzle, its pressure falls from steam chest pressure to condenser pressure. Due to this relatively higher ratio of expansion of steam in nozzle, the steam leaves the nozzle with very high velocity. (See the diagram for pressure, velocity changes).

P.T. 0

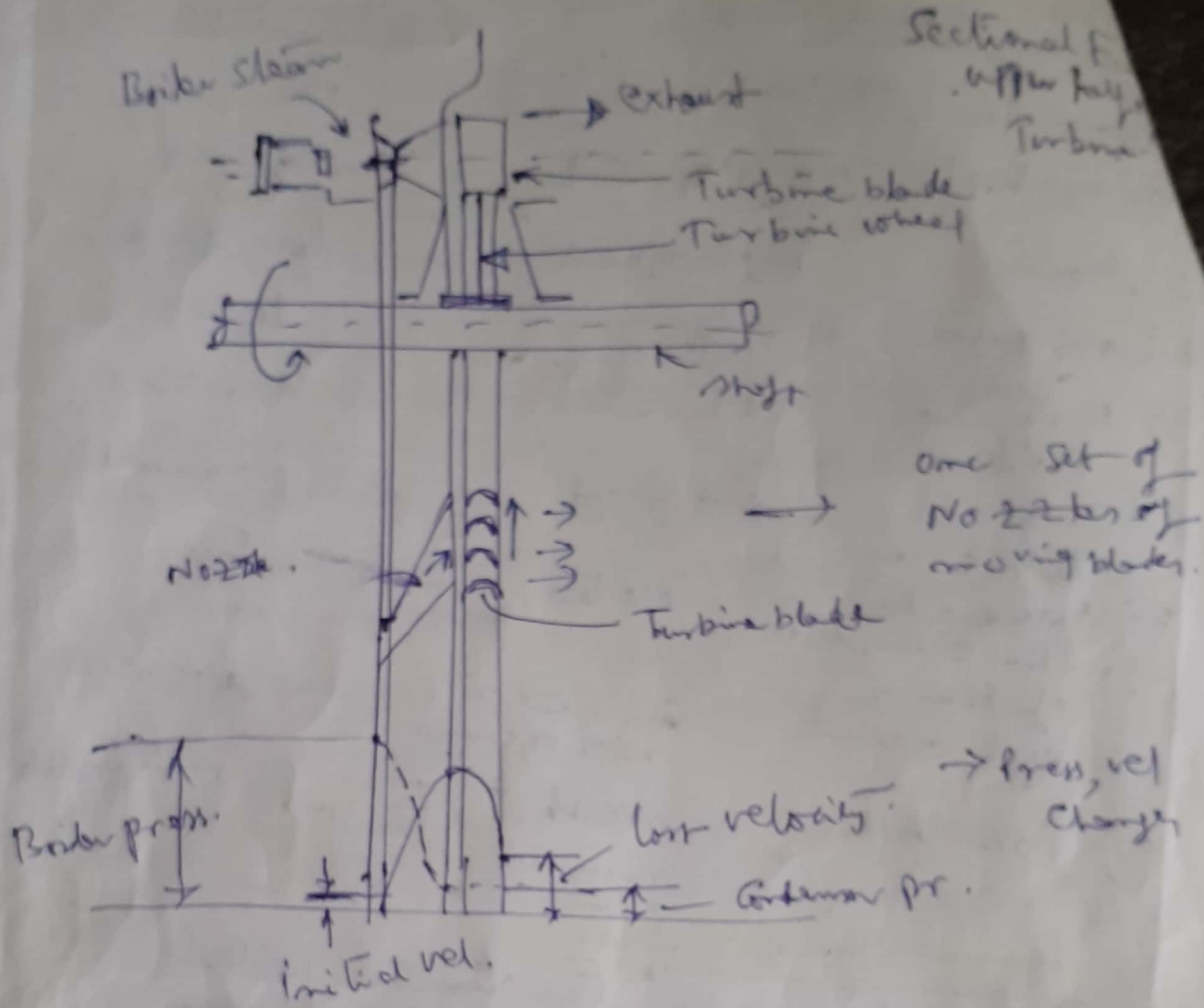


Fig:- Simple Impulse Turbine.

The velocity is high, even at exit of moving blades. (a large portion of $\text{Max}^m \text{ vel of steam}$) This loss of energy due to this higher exit velocity is known as 'Carry over loss' or 'leaving loss'. EX:- De Laval turbine.

The exit velocity is about 3.3% of nozzle outlet velocity. The velocity of wheel is too high (25,000 to 30,000 rpm). The wheel speed is to be reduced by different methods.

II) Reaction turbine.

In this turbine there is a gradual pressure drop and takes place continuously over the fixed and moving blades.

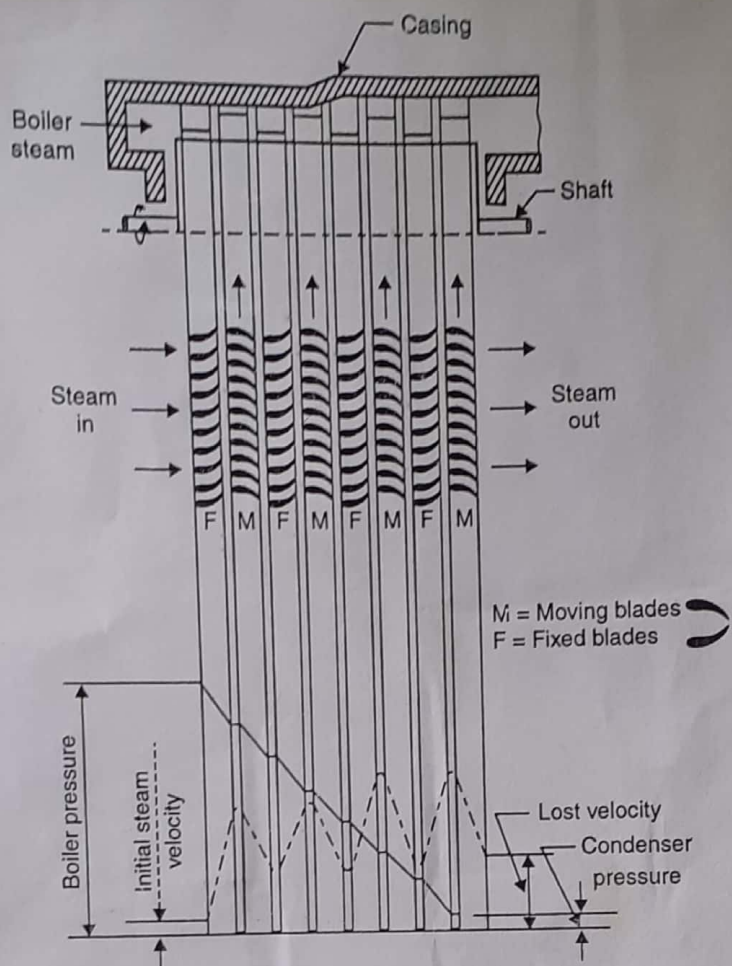


Fig. 19.2. Reaction turbine (three stage).

The function of fixed blade is (same as nozzle) that they alter the direction of ~~steam~~ steam and also allow it to expand to a larger velocity. As the steam passes over moving blades, its KE is absorbed by them. Figure shows 3 stage reaction turbine. Change in press, vel are also shown.

As the volume of steam increases at lower pressure, therefore, the diameter of turbine must increase after each group of blade rings. In this turbine, since the pressure drop per stage is small, therefore the number of stages required is much higher than an impulse turbine of same capacity.

Methods of reducing wheel or rotor speed.:

(Pressure)

Higher speed develops practical complications, There are ~~several~~ several methods of reducing speed. All these methods, utilise a multiple system of rotors in series. The pressure is absorbed in this is known as compounding. Different methods of compounding are (1) velocity compounding (2) Pr. compounding (3) pressure velocity compounding (4) Reaction turbine.

1. Velocity compounding.

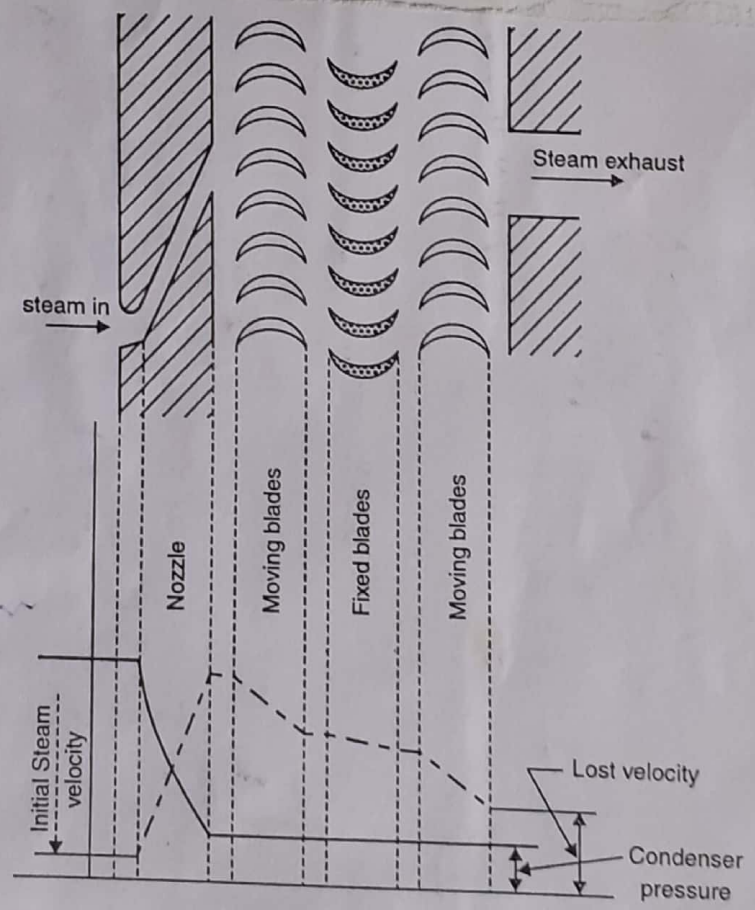


Fig. 19.3. Velocity compounding.

Steam is expanded through a stationary nozzle from inlet pressure to condenser pres.

supplied to the rotor

So the pressure in nozzle drops, KE increases. Moving & fixed blades are alternately kept. A portion of available energy is absorbed by a row of moving blades. The function of fixed blade is to redirect the steam flow without altering its velocity. The steam leaves the turbine with low velocity. Advantages are low cost, less number of stages but efficiency is low.

② Pressure Compounding :

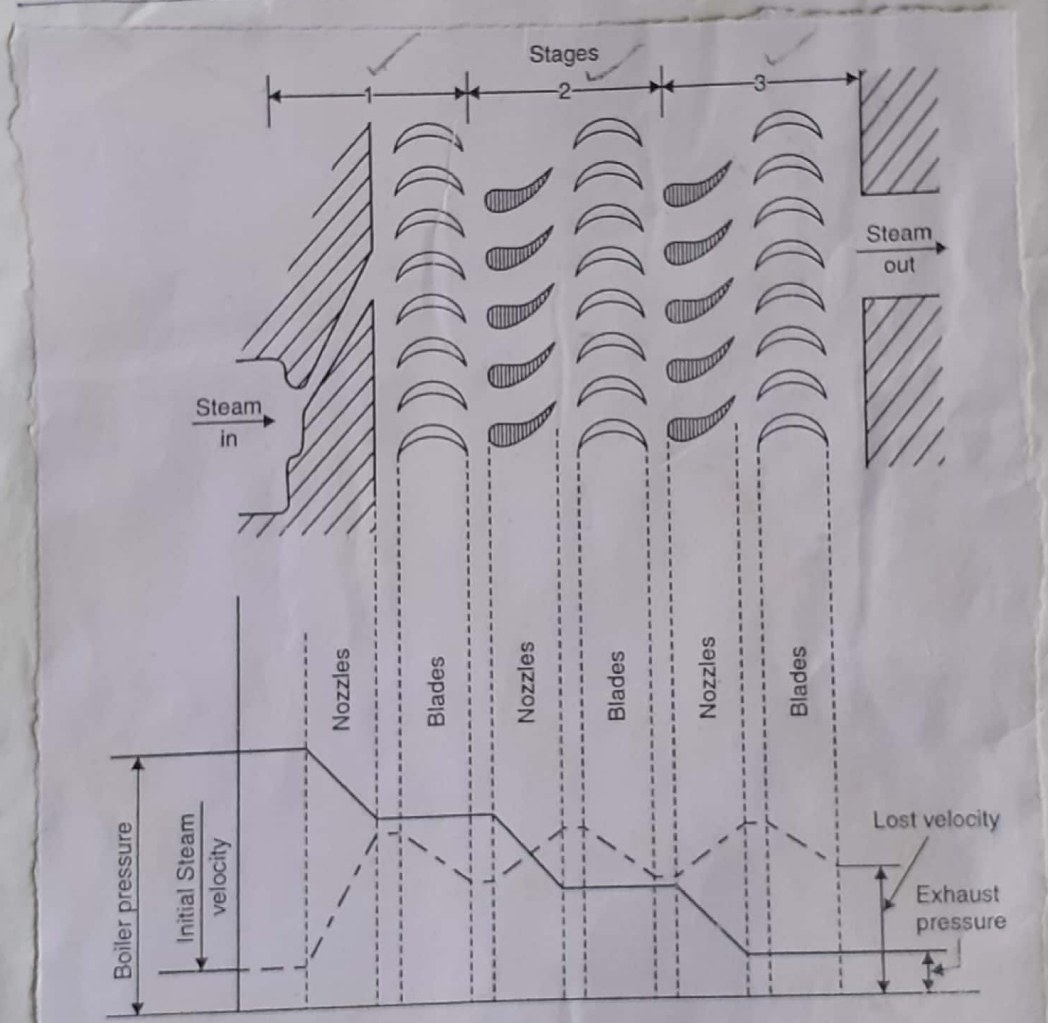


Fig. 19.4. Pressure compounding.

Figure shows that fixed nozzles are incorporated between moving blades. In nozzle, pressure drops and KE increases. In moving blades, the KE is absorbed. Like that number of stages are provided. Steam leaves the turbine at low pressure & low velocity. This method is used in Rateau & Zoelly turbine. Many stages, expensive but most efficient.

3) Pressure velocity Compounding:-

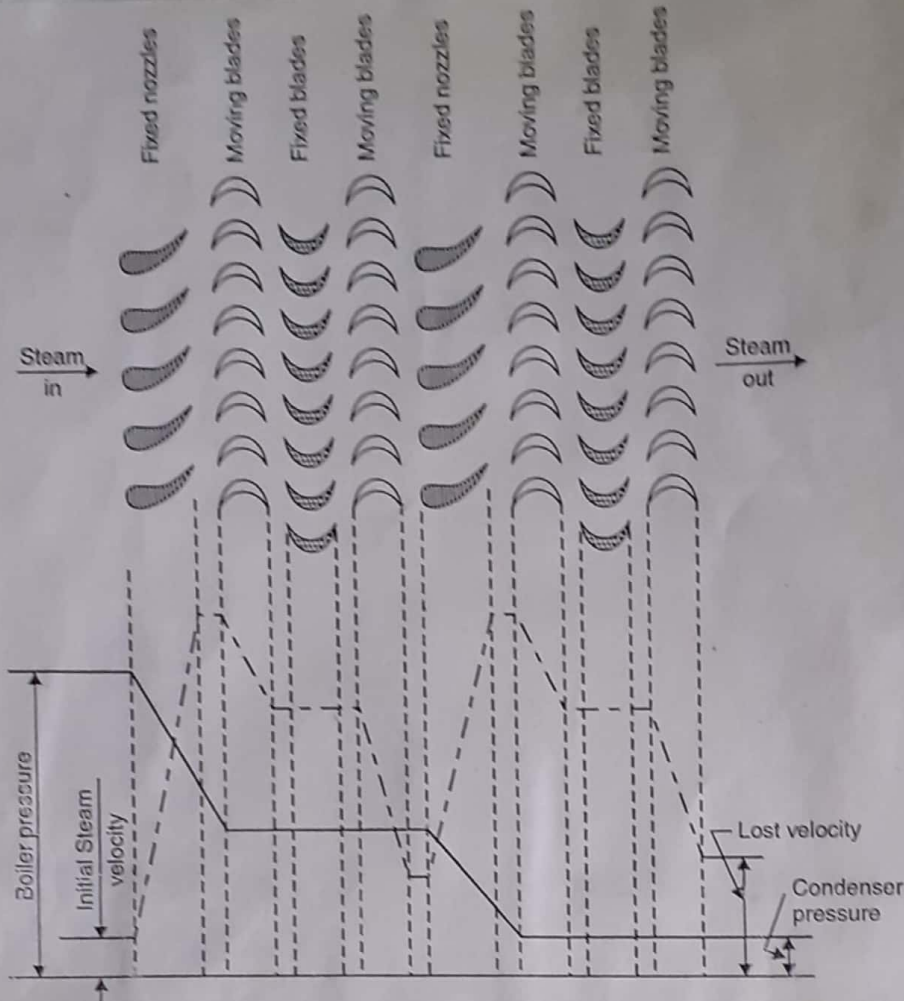


Fig. 19.5. Pressure velocity compounding.

This method of compounding is used in Curits and Moore turbine.

It is a combination of two earlier methods. The total steam pressure drop is divided into stages & velocity obtained is ~~compounded~~ compounded. The nozzles are fixed at the beginning of each stage.

4) Reaction turbine:

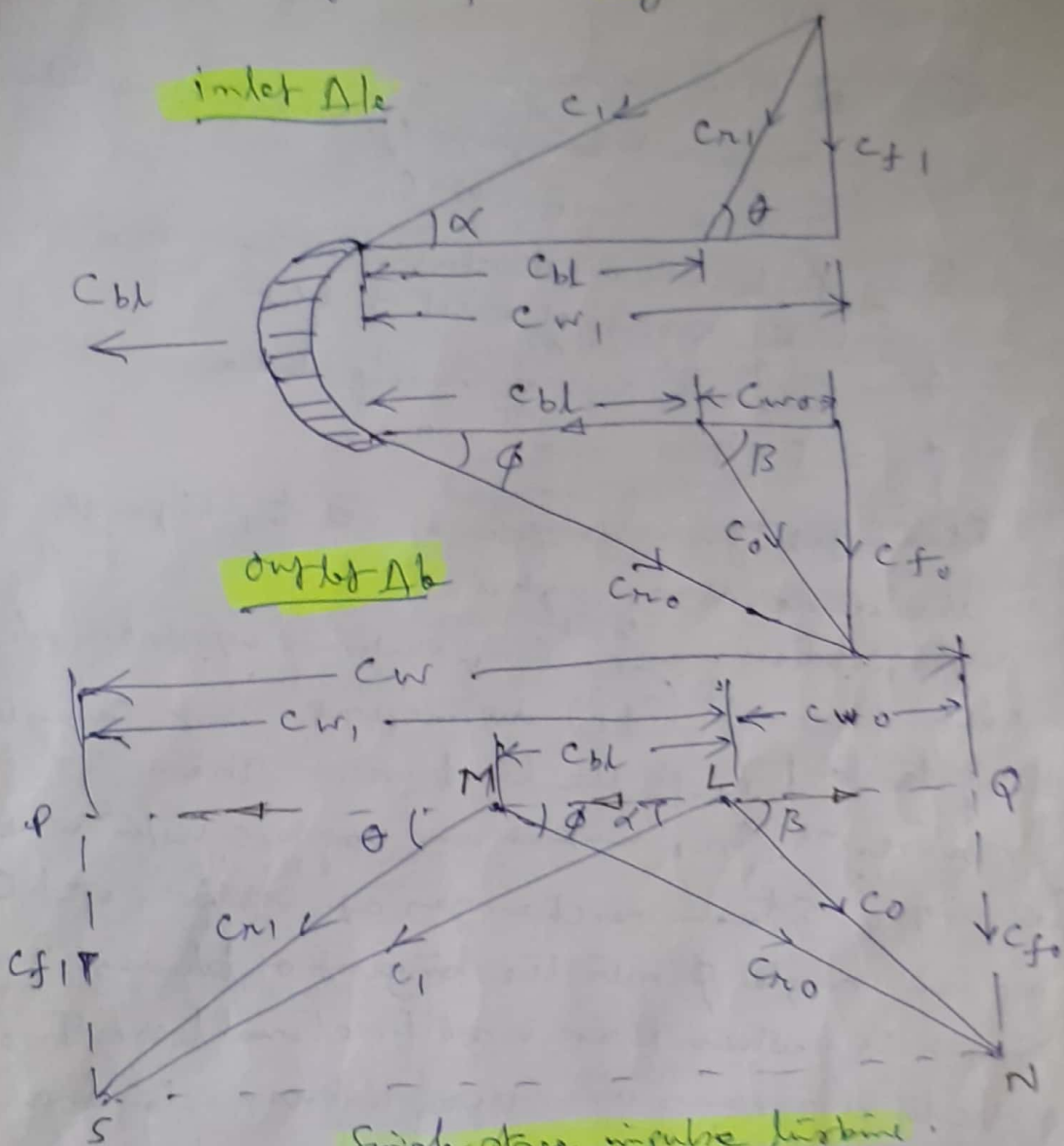
Al ready discussed.

Particulars	Impulse Turbine	Reaction turbine
1) Pr. drop	① Only in Nozzles, not in moving blades	① In Fixed blades & in moving blades
2) Blades	Profile Type	Aerofoil Type
3) Steam Admission	Not all round	All round
4) Power	not much power developed	Much power developed
5) η	low	High
6) Suitable	Small power requirement	Higher power requirement

supplied to the power

Impulse Turbine:

Velocity diagram for moving blade



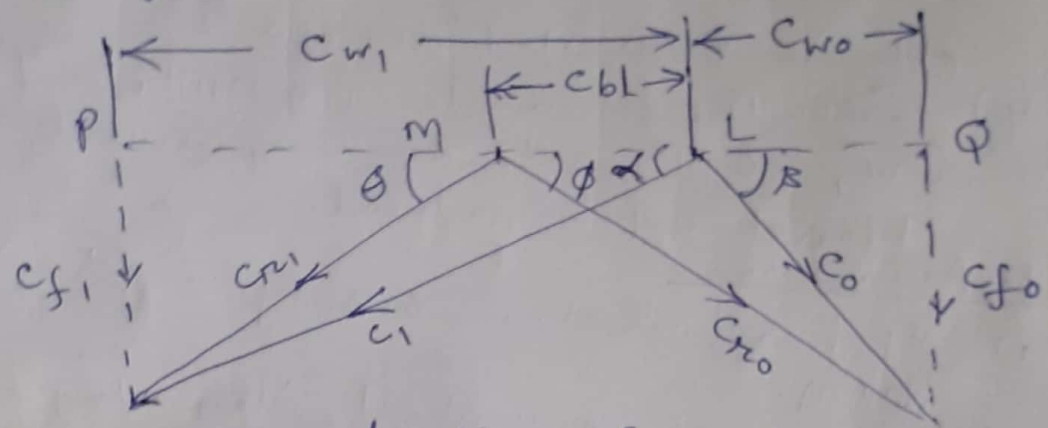
Single stage impulse turbine.

- c_{bl} = linear velocity of moving blade (m/s)
- c_1 = Absolute vel of steam entering moving blade
- c_0 = " " " leaving "
- c_{w1} = vel of whirl at entrance
(tangential component of c_1)
- c_{w0} = vel of whirl at exit
= tangential component of c_0 .
- c_{f1} = vel of flow at inlet
= axial component of c_1
- c_{f0} = vel of flow at exit
= axial component of c_0 .

Reaction Turbine

Pure reaction turbines are not in use. In present days Impulse reaction turbines are used. The expansion of steam & heat drop occur both in fixed & moving blades.

Velocity diagram



In impulse turbine, c_r either remains constant or is reduced due to friction. But in reaction turbine, steam continuously expands and c_r increases.

- $\therefore c_{r2} > c_{r1}$ (reaction).
- $c_{r2} \leq c_{r1}$ (Impulse)

Degree of Reaction (R_d) :-

It is the ratio of heat drop over moving blades to the total heat drop in the stage

$$R_d = \frac{\text{Heat drop in moving blades}}{\text{Heat drop in the stage}}$$

$$= \frac{\Delta h_m}{\Delta h_f + \Delta h_m}$$

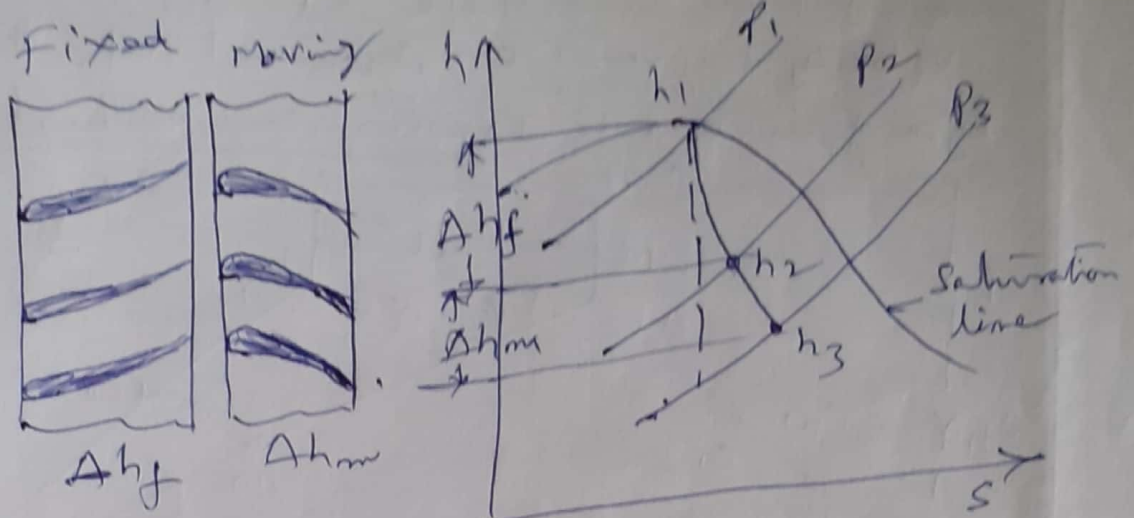
Heat drop in moving blades = increase in relative vel

$$\Delta h_m = \frac{c_{r2}^2 - c_{r1}^2}{2}$$

The total heat drop in the stage ($\Delta h_f + \Delta h_m$) is equal to work done

$$\Delta h_f + \Delta h_m = c_{bl} \cdot (c_{w1} + c_{w2})$$

$$\therefore R_d = \frac{C_{r0} - C_{r1}}{2 \cdot c_{bl} (C_{w1} + C_{w0})}$$



From the diagram

$$C_{r0} = c_{f0} \cdot \text{cosec } \phi, \quad C_{r1} = c_{f1} \cdot \text{cosec } \theta$$

$$\& \quad C_{w1} + C_{w0} = c_{f1} \cdot \cot \theta + c_{f0} \cdot \cot \phi$$

velocity of flow generally remains constant

$$c_{f1} = c_{f0} = c_f$$

Substituting

$$R_d = \frac{c_f (\text{cosec } \phi - \text{cosec } \theta)}{2 c_{bl} \cdot c_f (\cot \theta + \cot \phi)}$$

$$= \frac{c_f}{2 c_{bl}} \left[\frac{(\cot \phi + 1) - (\cot \theta + 1)}{\cot \theta + \cot \phi} \right]$$

$$= \frac{c_f}{2 c_{bl}} \cdot \frac{(\cot \phi - \cot \theta)}{\cot \theta + \cot \phi} = \frac{c_f}{2 c_{bl}} (\cot \phi - \cot \theta)$$

If the turbine is designed for 50% reaction

$$(\Delta h_f = \Delta h_m), \quad R_d = 0.5$$

$$0.5 = \frac{c_f}{2 c_{bl}} (\cot \phi - \cot \theta)$$

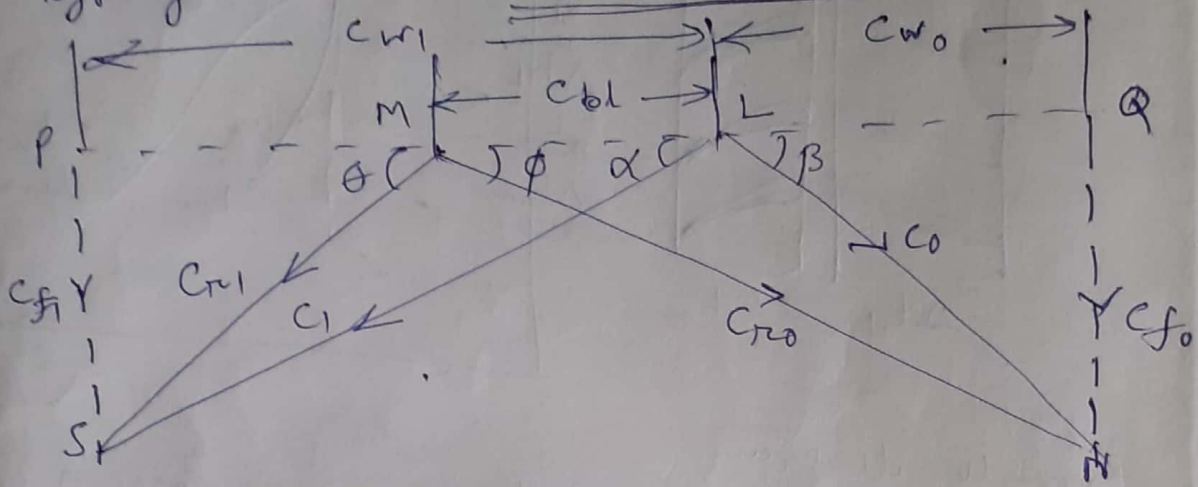
$$c_{bl} = c_f (\cot \phi - \cot \theta)$$

But $c_{f1} = c_{f0} = c_f$, $\theta = \beta$, $\phi = \alpha$.

$$\text{So, } c_{bl} = c_f (\cot \alpha - \cot \beta)$$

$$c_{bl} = c_f (\cot \alpha - \cot \theta)$$

which means that Fixed & moving blades must have same shape, $R_d = 0.5$, velocity diagrams are symmetrical. This type of turbine is "Parsons reaction turbine".



Prob. 19.25, 19.26.

Condition for max^m efficiency:-

Following are the assumptions.

- 1) degree of reaction is 50%.
- 2) Moving & fixed blades are symmetrical.
- 3) Velocity of steam at exit from preceding stage is same as velocity of steam at entrance to the succeeding stage.

$$\begin{aligned} \text{Work done / Kg} = W &= C_{bl} (C_{w1} + C_{w0}) \\ &= C_{bl} [C_1 \cos \alpha + (C_{r0} \cos \phi + C_{bl})] \\ \phi &= \alpha, \quad C_{r0} = C_1 \quad (\text{Assumption}) \end{aligned}$$

$$\begin{aligned} \therefore W &= C_{bl} [2 C_1 \cos \alpha - C_{bl}] \\ &= C_1^2 \left[\frac{2 C_{bl} \cdot C_1 \cos \alpha}{C_1^2} - \frac{C_{bl}}{C_1^2} \right] \\ &= C_1^2 [2 \beta \cos \alpha - \beta^2] \quad \because \beta = \frac{C_{bl}}{C_1} \end{aligned}$$

Multiply & divide by C_1^2

$$\begin{aligned} \text{KE supplied to fixed blade} &= \frac{C_1^2}{2g} \\ \text{KE to moving blade} &= \frac{C_{r0}^2 - C_{r1}^2}{2} \end{aligned}$$

Fields of application

- 1) Aviation
- 2) Power Generation
- 3) oil & gas Industry
- 4) Marine Propulsion.

gas turbine in aviation & Marine fields.

Self contained, light weight, not requiring cooling water.

gas turbine for power generation.

Simple operation, lack of cooling water, quick installation, quick starting.

used oil & gas Industry

Because of cheaper supply of fuel & low installation cost.

Limitations:

- 1) Not self starting
- 2) low efficiencies at part loads,
- 3) non-reversibility
- 4) higher rotor speeds
- 5) low overall η

Classification

- 2 groups ^{Combustion} gas turbine
- 1) Constant pressure gas turbine
 - a) open cycle
 - b) closed cycle
 - 2) Constant Volume combustion gas turbine.
- open cycle gas turbines are used in all the fields. The progress in closed cycle plants is less.

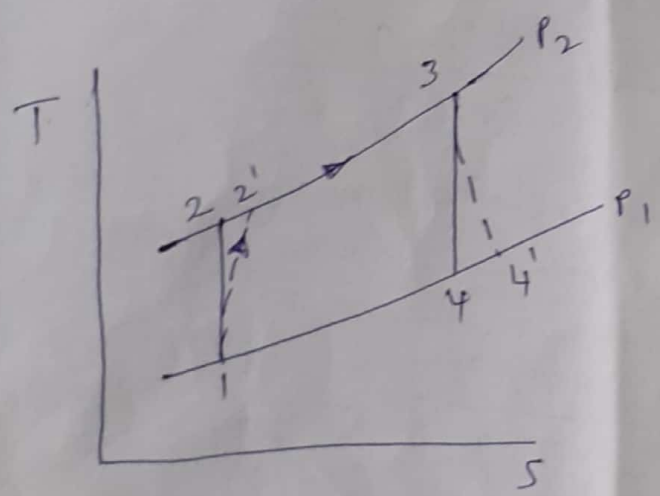
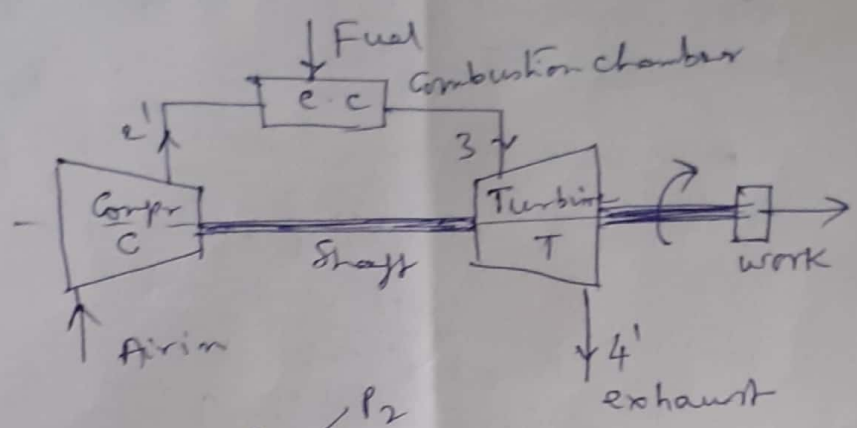
Merits of gas turbine

a) Merits over I.C. engines

- 1) Mechanical η is higher
- 2) Doesn't need a fly wheel
- 3) weight of gas turbine/HP is less
- 4) Can run at 4000 rpm speed
- 5) Work/kg of air is more
- 6) Works at less pressure. Hence lighter components are used.

Constant pressure Combustion gas turbines

(1) Open cycle gas turbine

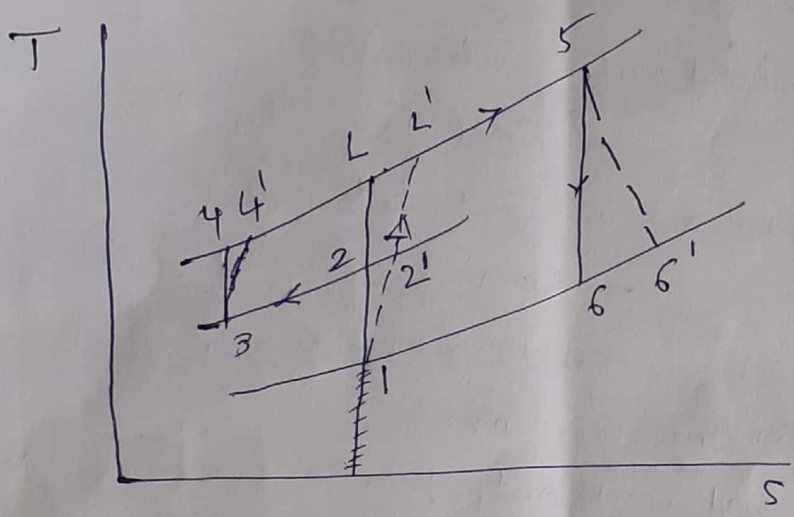
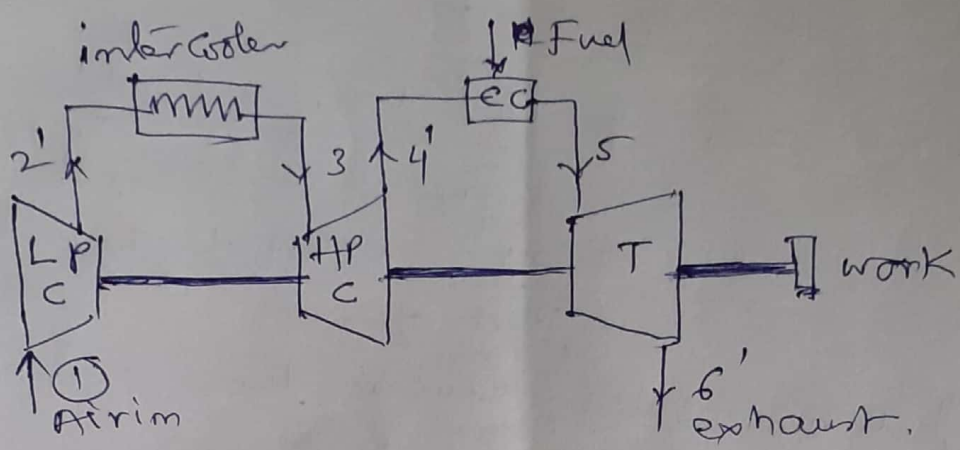


Rotary Compressor & turbine are mounted on common shaft. Air is compressed & goes to C.C. Fuel is sprayed into air stream. Hot gases are formed & expand through the turbine. gases are exhausted to atmosphere. working fluids (air & fuel) are ~~re~~ replaced continuously.

- 1-2' :- irreversible adiabatic compression.
- 2'-3 :- Constant pressure heat supply in C.C
- 3-4' :- irreversible adiabatic expn.
- 1-2 :- ideal ~~is~~ isentropic compression
- 3-4 :- " " expn.

P.T. u

1) Intercooling :-



work reqd by compr can be reduced by compressing in 2 stages & with intercooler in between.

- 1-2' :- L.P. Compr
- 2'-3 :- Intercooling
- 3-4' :- H.P. Compr
- 4'-5 :- c.c. heating
- 5-6' :- Turbine Expn.

Ideal cycle is 1-2-3-4-5-6.

1-L' :- Compr without intercooling.

1-L :- Ideal Isentropic Case.

work input (without intercooling)

$$= c_p (T_{2'} - T_1) + c_p (T_{4'} - T_3)$$

work input (without intercooling)

$$= c_p (T_L' - T_1)$$

$$= c_p (T_{2'} - T_1) + c_p (T_L' - T_{2'})$$

By comparing, it can be found that work (with intercooling) is less.

$$\text{Work ratio} = \frac{\text{Net work output}}{\text{Gross work output}}$$

$$= \frac{\text{Work of expn} - \text{Work of Compn}}{\text{Work of expn}}$$

$(T_4 - T_3)$ is less than $(T_1 - T_2)$ if lines diverge out

If work input is reduced, then work ratio is increased.

$$\text{Heat supplied with intercooling} = C_p (T_5 - T_4')$$

$$\text{Heat supplied without intercooling} = C_p (T_5 - T_L')$$

It can be observed that Heat supplied with intercooling is more.

Although net work output is increased with intercooling, but the thermal η decreases because of extra heat supplied.

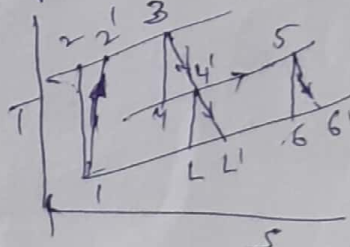
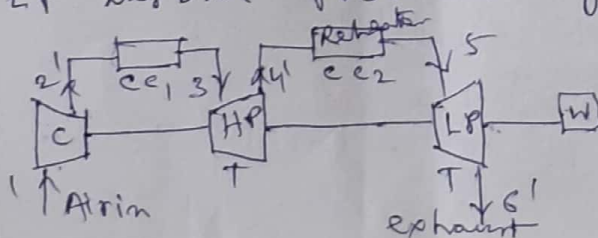
For intercooling, cooling water is also needed. The additional equipment may offset the advantage gained by increasing work ratio.

2) Reheating:

The output of gas turbine can be improved by expanding the gases in 2 stages, with a reheat in between 2 stages.

HP turbine drives the compressor.

LP turbine provides useful power output.



neglecting mechanical losses,
 work output of HP = work input to Compr.

$$C_{pa} (T_2' - T_1) = C_{pg} (T_3 - T_4')$$

Work output (net output) of L.P. turbine :-

with reheat = $C_{pg} (T_5 - T_6')$

without reheat = $C_{pg} (T_4' - T_2')$

Pressure lines diverge to the right, hence, it can be seen that, reheating increases the net work output.

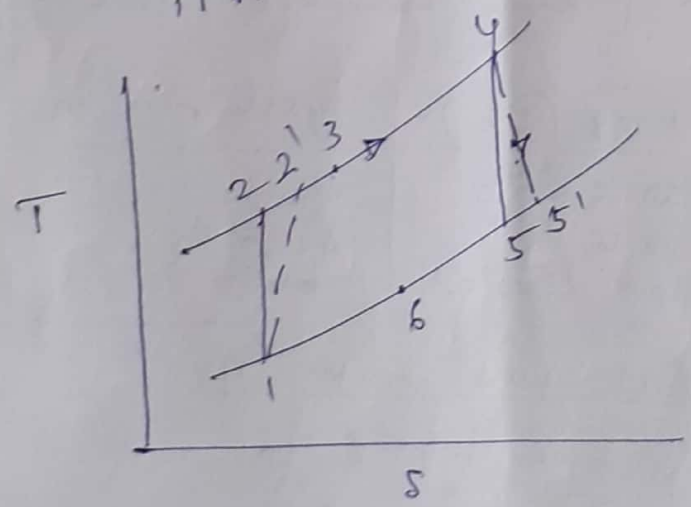
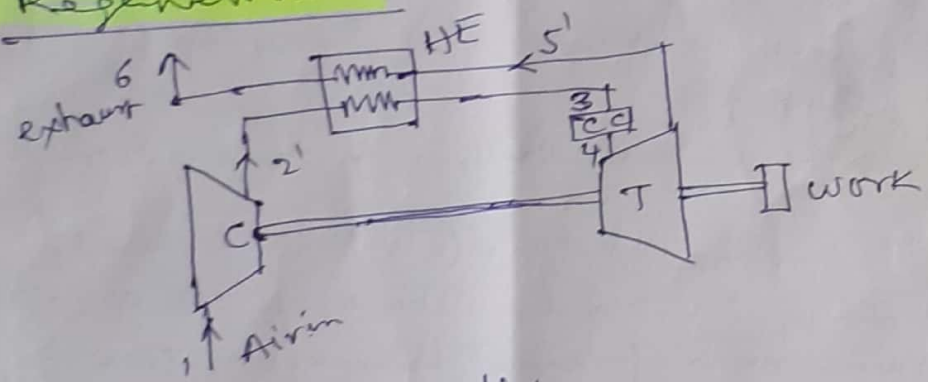
But the heat to be supplied increases,

$$\text{Heat Supplied} = C_{pg} (T_3 - T_2') + C_{pg} (T_5 - T_4')$$

(C_{pa} = spht of air, C_{pg} = spht of gas).

Note effect can be to reduce thermal η .

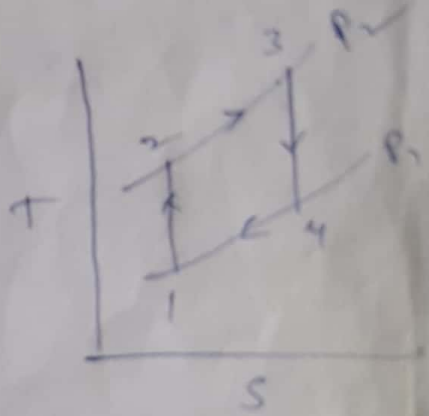
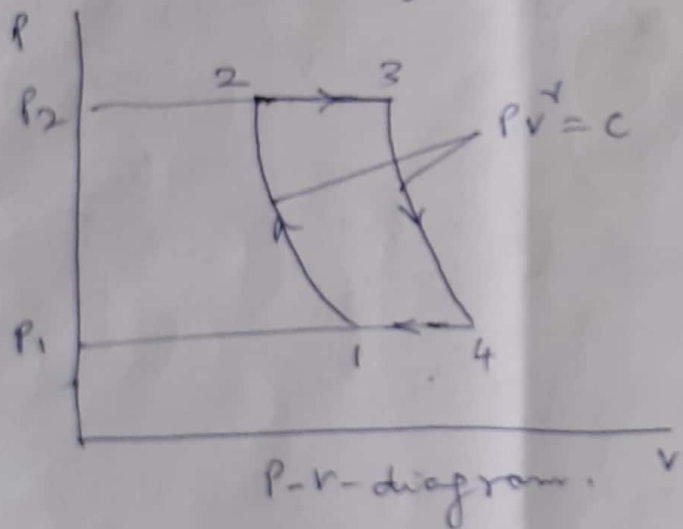
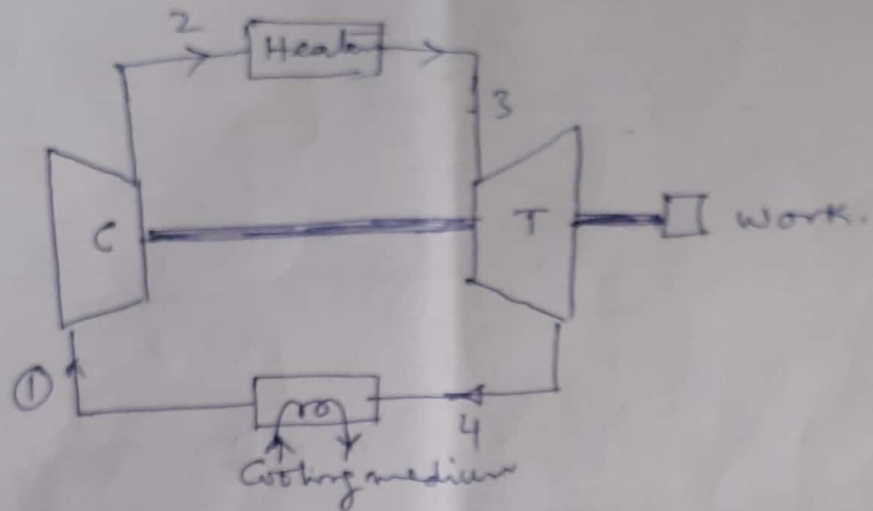
3) Regeneration



Effect of Compressor inlet Temp:-

with the decrease in compressor inlet temp, thermal efficiency of plant increases.

closed cycle gas turbine (Constant Pressure or Joule cycle)



1-2 :- isentropic air compression (P₁ to P₂, T₁ to T₂)

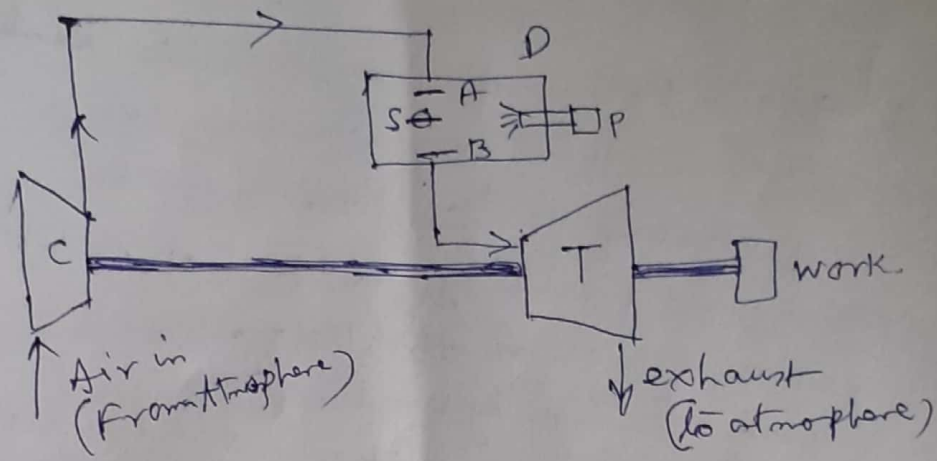
2-3 :- Heat addition (vol V₂ to V₃, P₂ constant, Temp T₂ to T₃)

Heat received = $m c_p (T_3 - T_2)$

3-4 :- air is expanded from P₂ to P₁. Temp falls from T₃ to T₄.

4-1 :- Heat is rejected, vol decreases from V₄ to V₁, Temp decreases from T₄ to T₁. Heat rej = $m c_p (T_4 - T_1)$

Constant volume Combustion turbines



- C - Compressor, T = Turbine
- D - Combustion chamber, A & B are valves.
- P = Fuel pump, S = Spark plug.

Compressed air goes to combustion chamber ~~through valve A~~ through valve 'A'. Fuel is admitted into the chamber through pump 'P'. The mixture is ignited by spark plug 'S'. The combustion takes place at constant volume with increase of pressure. When valve B opens, hot gases flow to turbine T. The Exhaust is discharged to atmosphere. The operations are repeated.

The main demerit is that the press difference velocities of hot gases are not constant & so the turbine speed fluctuates.

Uses of gas turbines

used in following fields

- 1) Supercharging
- 2) TurboJet, Turbo propeller engines
- 3) Marine field
- 4) Railway
- 5) Road Transport
- 6) Electric power generation
- 7) Industry.

The principle of Jet propulsion involves imparting momentum to a mass of fluid, in such a manner that the reaction of imparted momentum provides a propulsive force. It can be achieved by expanding high press gas through a nozzle. High vel of jet is produced that gives propulsive force (in opposite direction due its reaction). For Jet propulsion, open cycle gas turbine is most suitable.

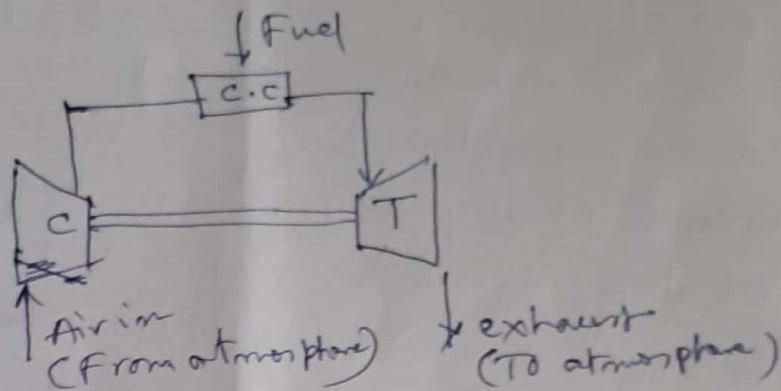
Classification

- 1) Airstream jet engines (Air-breathing engines)
 - a) Steady Combustion systems; Continuous air flow.
 - i) Turbojet ii) Turbo-prop
 - iii) Ramjet. (intermittent flow)
 - b) Intermittent Combustion system: Pulsejet or flying bombs.
- 2) Self contained rocket Engine (~~Not~~ air breathing)
 - i) liquid propellant ii) solid propellant.

In airstream jet engines the oxygen necessary for combustion is taken from the surrounding atmosphere, whereas in a rocket engine the fuel and the oxidiser are contained in the body of the unit which is propelled.

The turbojet and turbo-prop are modified forms of simple open cycle gas turbine. The ramjet & pulsejet are 'athodyds' (aero-thermodynamic ducts) i.e straight duct type of jet engines having no compressor and turbine wheels.

In the past air propulsion was achieved by a screw propeller.



C = Compressor, T = Turbine, C.C. = Combustion Chamber.

Fig:- Powerplant for screw propeller.

By controlling Fuel supply, power supplied is controlled. The rate of increase of η is higher at lower speeds, & η falls rapidly at higher speeds above sonic velocity.

Turbo-Jet

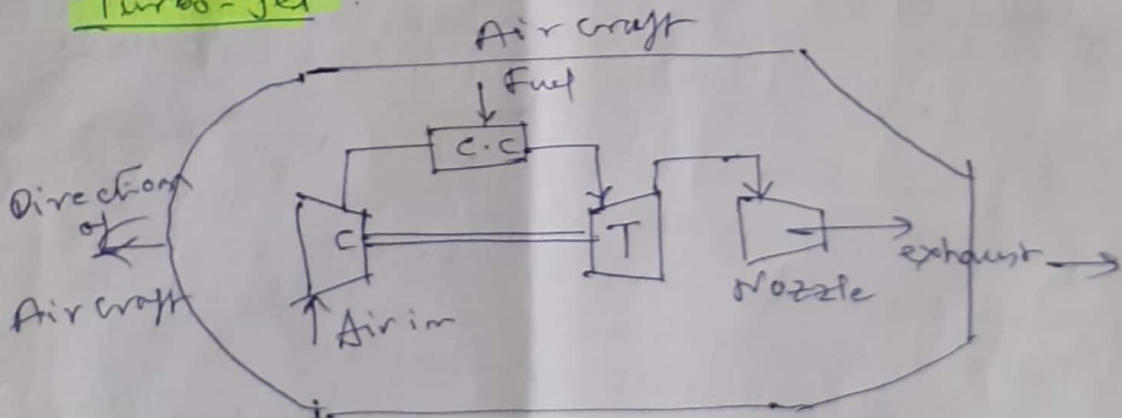


Fig:- Gas turbine plant for turbojet

- 1) Diffuser is kept at entrance which slows down the air velocity. & part of KE is converted into pressure. (This is known as ram compression)
- 2) air is further compressed to 3 to 4 bar by Compr.
- 3) Fuel is added in C.C. Combustion takes place.
- 4) partial expansion takes place in gas turbine.
- 5) Expanded in nozzle high V_{jet} is produced, which provides forward motion to aircraft.

Jet Nozzle

Energy eqn between 5 & 6 gives

$$h_5 + \frac{C_5^2}{2} = h_6 + \frac{C_6^2}{2} \quad \text{--- Ideal}$$

$$h_5' + \frac{C_5'^2}{2} = h_6' + \frac{C_6'^2}{2} \quad \text{--- Actual}$$

If C_5' is very less compared to C_6' , then

$$h_5' = h_6' + \frac{C_6'^2}{2}$$

$$C_6' = \sqrt{2(h_5' - h_6')} \\ = \sqrt{2 \eta_m (h_5' - h_6')}$$

$$C_6' = \sqrt{2 \cdot \eta_m \cdot c_p (T_5' - T_6)}$$

$$\eta_m = \text{Nozzle } \eta$$

$$\text{Thermal } \eta = \eta_{th} = \frac{(h_4 - h_6') (h_3' - h_1)}{(h_4 - h_3')} \\ = \frac{(T_4 - T_6') - (T_3' - T_1)}{(T_4 - T_3')}$$

Thrust, Thrust power, propulsive η , Thermal η ,

Thrust (T)

Let C_a = Forward vel of air-craft through air, m/s.

Assuming ~~air~~ ^{still} air to be ^{the} vel of air, relative to the aircraft, as entry ^{to aircraft} $= C_a$. It is ~~also~~ called as vel of approach of air.

C_j = vel of Jet (gases) relative to ~~air~~ ^{aircraft} nozzle/aircraft, m/s

$$\left[1 + \frac{\text{fuel (mf)}}{\text{air (ma)}} \right] = \text{mass of products leaving nozzle for 1 kg of air}$$

(7)

Thrust = Force produced due to change of momentum.

Absolute^{vel} of gases leaving air craft = $C_j - C_a$

Absolute vel of air entering air craft = 0

Hence $T = \left[1 + \frac{m_f}{m_a} \right] (C_j - C_a) \text{ N/kg of air/s}$ ~~Change of momentum = \left(1 + \frac{m_f}{m_a} \right) (C_j - C_a)~~

~~Thrust~~ Thrust $T = (C_j - C_a) \text{ N/kg of air/s}$
(neglecting mass of fuel)

Thrust Power (T.P) :-

It is the rate at which work must be developed by the engine if the air craft is to be kept moving at constant vel C_a , against friction ~~and~~ drag.

Thrust Power = Forward Thrust \times Speed of air craft.

$$T.P = \left[\left(1 + \frac{m_f}{m_a} \right) (C_j - C_a) \right] \times C_a \text{ w/kg of air}$$

$$= (C_j - C_a) C_a \text{ w/kg of air. (Mass of Fuel neglected)}$$

$$= \frac{(C_j - C_a) C_a}{1000} \text{ KW/kg of air}$$

Propulsive Power (P.P) :-

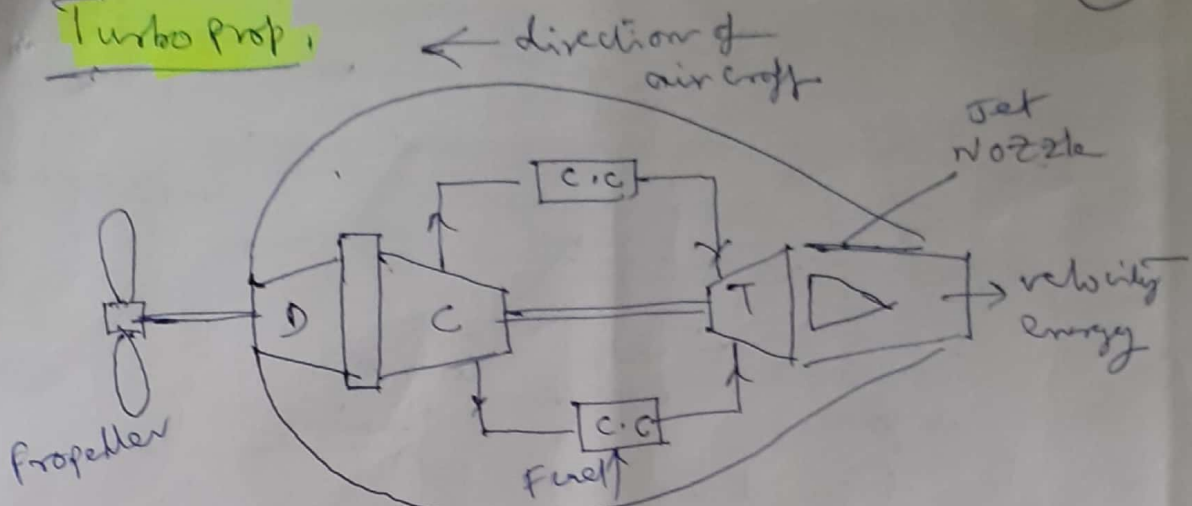
It is energy reqd to change the momentum of the mass flow of gas. It is expressed as difference between the rate of K.E.s of entering air & exit gases.

$$P.P = \Delta K.E = \frac{C_j^2 - C_a^2}{2} \text{ w/kg}$$

$$= \frac{C_j^2 - C_a^2}{2000} \text{ KW/kg of air}$$

(neglecting mass of Fuel)

Turbo Prop.



D = diffuser, C = Compr, C.C. = Combustion Chamber
 T = Turbine.

Here, expansion of gases takes place partly in turbine (80%) and partly in nozzle (20%)

Power developed by turbine is consumed in running the Compr & propeller.

The turbo prop contains the advantages of turbojet (low specific weight) & propeller (high power for takeoff)

- Diffuser improves the efficiency. Press rise takes place in the diffuser. This is due to ~~conversion~~
- Conversion of KE of the incoming air into PE by the diffuser. This type of compression is known "ram effect".

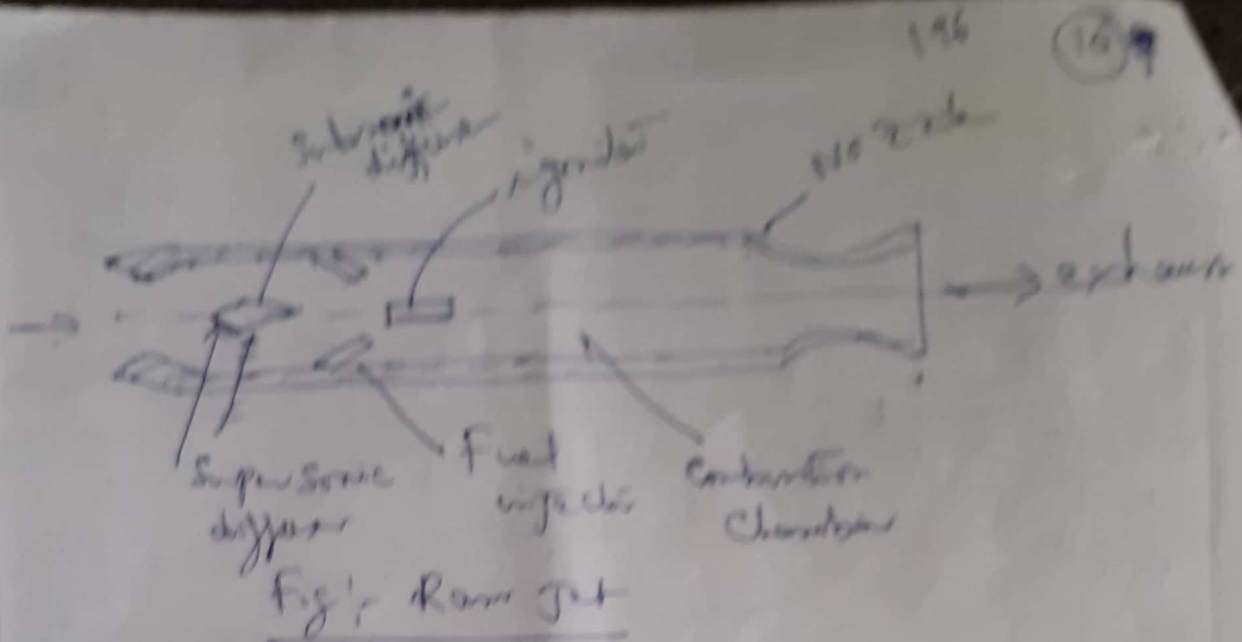
Ramjet:

Ramjet is also called "aerodyd", ^(aerothermodynamic ducts) Loxin tube or flying stove pipe. They have capability of flying at supersonic speeds.

Compressor, turbine are not necessary. Entire compression depends on only ram compression.

It has jet diffuser, C.C., & nozzle.

Air enters at supersonic speed. It is slowed down to sonic velocity in supersonic diffuser.



Pressure suddenly increases & shock wave is formed. Pressure of air is further increased in subsonic diffuser increasing the temp of air. In combustion chamber, fuel is fed through injection nozzle. Fuel, air mixture is ignited by spark plug. Combustion temp of about 2000K are obtained. Hot gas, undergoes expansion in nozzle. PE is converted to KE. High vel gas, leaving nozzle, provides the thrust.

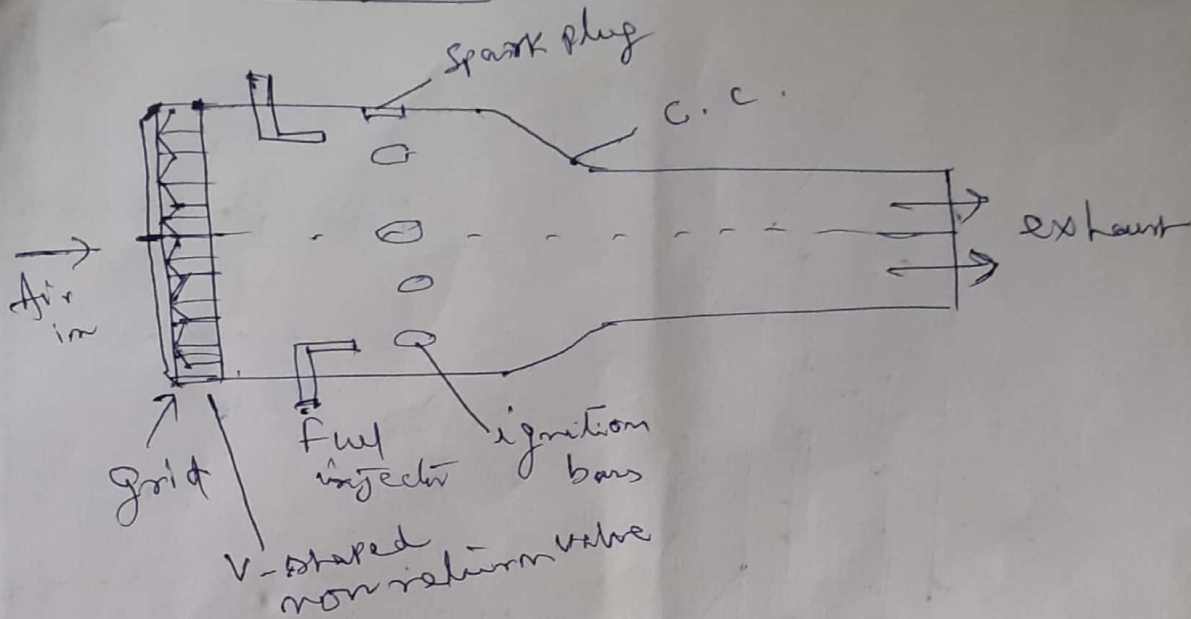
Advantages:

No moving parts, light in weight, wide variety of fuels can be used.

Limitations:

- 1) It cannot be started on its own. It is equipped with a small turbojet, which starts like ramjet.
- 2) Fuel combustion is more at low speed.
- 3) Diffuser needs careful design.
- 4) Some extra devices, like flame holders are required.

Pulsejet engine



It is an intermittent combustion engine. It operates on a cycle similar to a reciprocating engine. Pulsejet engine is an altituded, develops thrust by a high velocity of exhaust gases, without the aid of compr, or turbine. Its development is primarily due to the inability of the ramjet to be self starting.

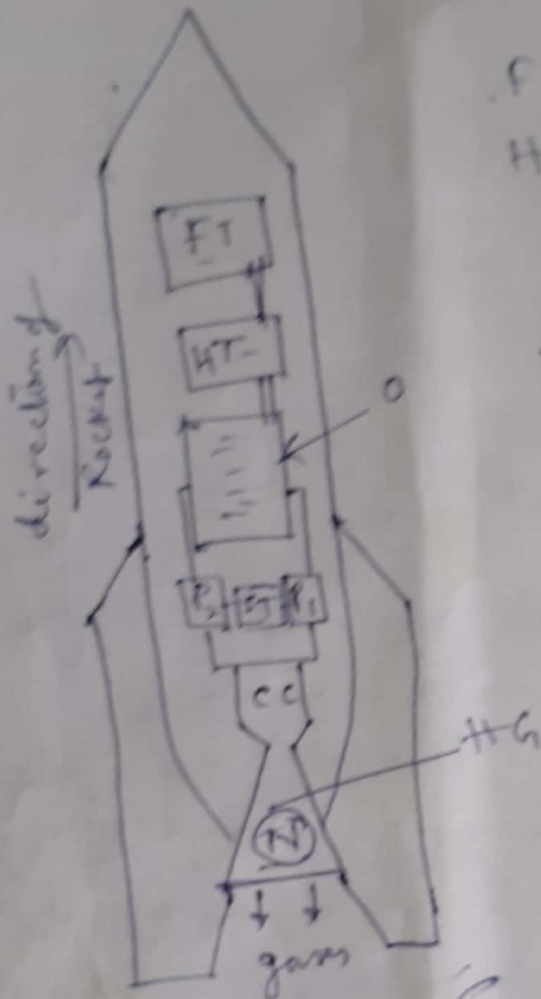
Incoming air is compressed by ram effect in the diffuser section & grid passages.

Fuel is then injected. Spark plug initiates the combustion. Once the engine operates normally, the residual flame is used for combustion. Temp, press of combustion products increase. Hot gases flow out of tail pipe & gives forward thrust.

Advantages : 1) Simple in construction 2) Inexpensive 3) well adapted to pilotless air craft 4) Capable of producing good thrust.

Limitations : 1) High noise 2) Vibrations are more 3) low thrust 4) operating altitudes is limited.

Rocket - Engines



FT = Fuel Tank

HT = Hydrogen peroxide Tank

O = Oxidiser tank

P₁, P₂ = Pumps

ST = Steam turbine

C.C = Combustion chamber

HG = Hot gases

N = Nozzle

Applications

- 1) long range artillery
- 2) lethal weapons
- 3) for satellites
- 4) for space ships etc

Fig:- Rocket (Single stage liquid propellant)

Rocketing is similar to jet propulsion. But in Rocket propulsion fuel & oxidiser both are contained in a propelling body, hence can function in vacuum.

Classification:- 1) Solid propellant rocket
2) Liquid " "

or 1) Single stage rocket 2) Multistage

Fuel is ignited by electrical means.

Pumps are driven by steam turbine. Steam is produced by mixing Hydrogen peroxide & potassium permanganate. Combustion products are discharged through nozzle N. Rocket moves in opposite direction

Thrust work = $C_j \dot{C}_a$, Propulsive work = $\frac{C_j^2 + C_a^2}{2}$

Rocket propulsive $\eta = \frac{2 C_j C_a}{C_j^2 + C_a^2}$