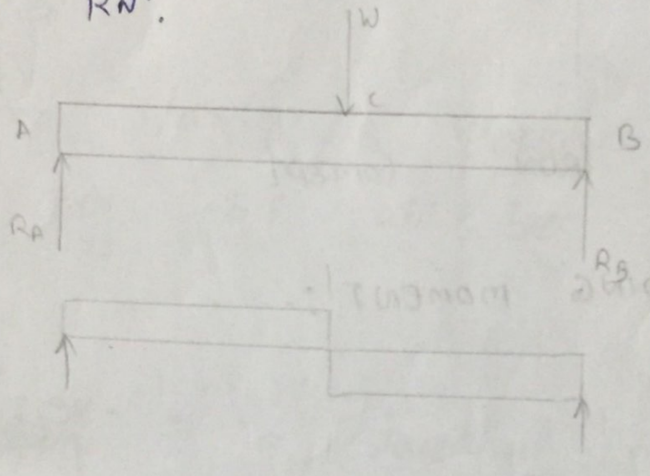
SHEAR FORCE AND BENDING MOMENT

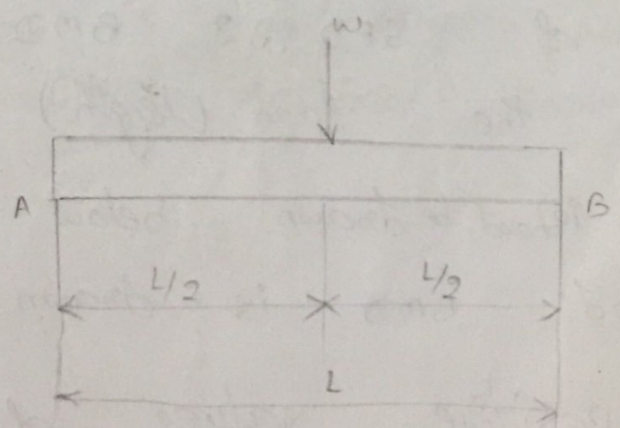
SHEAR FORCE (SF) :-

The algebraic sum of forces at any section of a beam either on left hand side or right hand side of the section is known as shear force. It is expressed in 'N' (or) 'KN'.

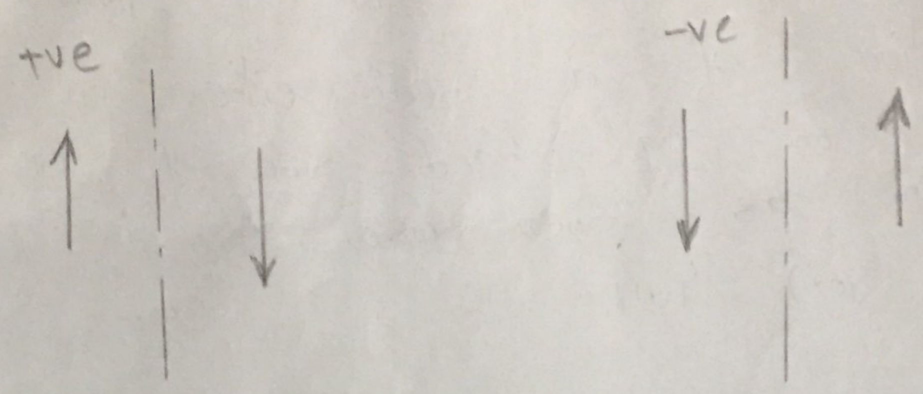


BENDING MOMENT :-

It is defined as the algebraic sum of moments of forces acting on the left side or right side of the section. It is expressed in 'N-m' (or) "KN-m".

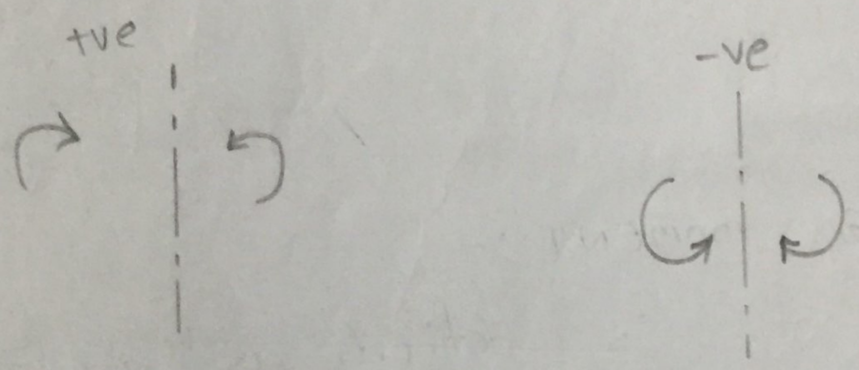


SIGN CONVENTIONS FOR SHEAR FORCE AND BENDING MOMENT.



LUP And RUN Concept.

FOR BENDING MOMENT :-



Sagging BM   Hogging B.M.

IMPORTANT POINTS FOR DRAWING SFD & BMD :-

- Length of SFD & BMD must be equal to the span (length) of the beam.
- SFD should be drawn below the loaded beam and BMD is drawn below the SFD
- The positive values of shear force bending moments are plotted above the

base line, and negative values below the base line.

- For simply supported beam, B.M is zero at the supports.

- For Cantilever beam, B.M will be zero at free end.

- If no load is present between two points then S.F will be constant.

BEAM :-

A beam is a structural member, it is subjected to a system of external forces at right angles to its axis.

- Any section of the beam experiences shearing off by the load and bending by the moment.

- Basically the longitudinal dimension (i.e length) of a beam is too large when compared with its cross-sectional dimensions (i.e b x d).

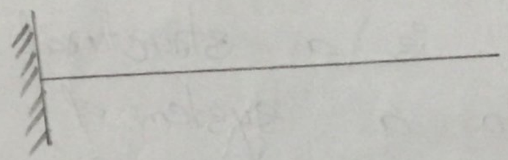
TYPES OF BEAMS :-

The various types of beams are

1) Cantilever beam
2) Simply supported beam.
3) over hanging beam
4) Fixed beam.
5) Continuous beam.

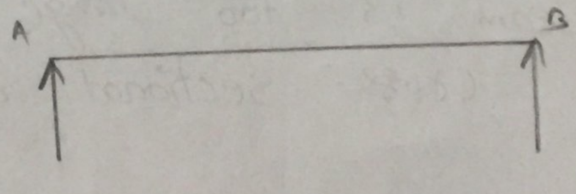
1) CANTILEVER BEAM :-

A beam which is fixed at one end and free at the other end. is known as cantilever beam



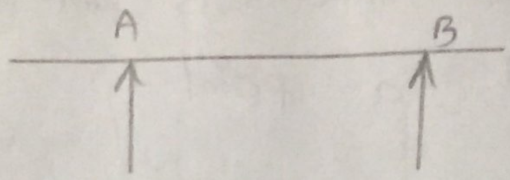
2) SIMPLY SUPPORTED BEAM :-

A beam supported or resting freely on the supports at it both ends. is known as simply supported beam.



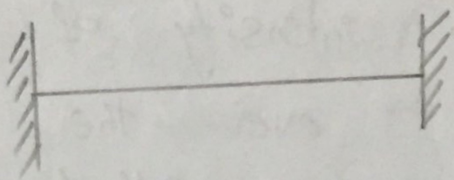
3) OVERHANGING BEAM :-

If the end portion of a beam is extended beyond the support, such beam is known as overhanging beam.



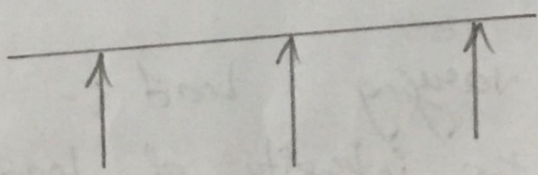
4) FIXED BEAM:-

A beam whose both ends are fixed or built in walls. is known as fixed beams.



5) CONTINUOUS BEAMS:-

A beam which is provided more than two supports is known as continuous beam.



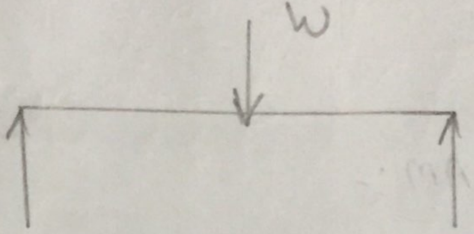
TYPES OF LOADS:-

The various types of loads acting on a beam are

1) Concentrated (or) point load.
2) Uniformly distributed load (udl)
3) Uniformly varying load (UVL)

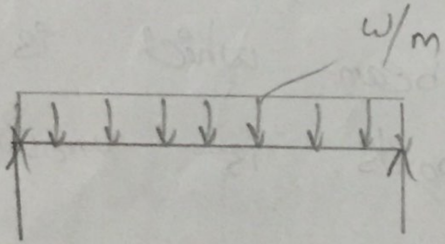
1) Concentrated (or) point load :-

A point load is one which is considered to act at a point.



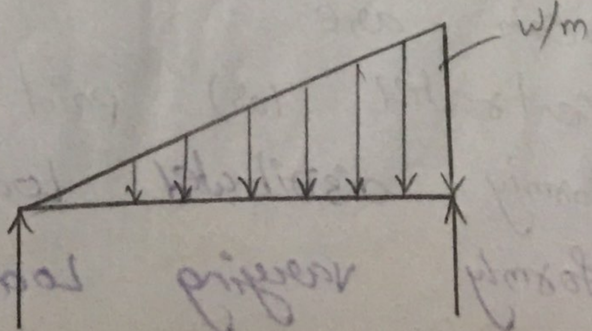
2) Uniformly distributed Load :-

If the intensity of load is uniform (i.e) constant over the entire length of the beam it is called udl. It is expressed in 'N/m'.

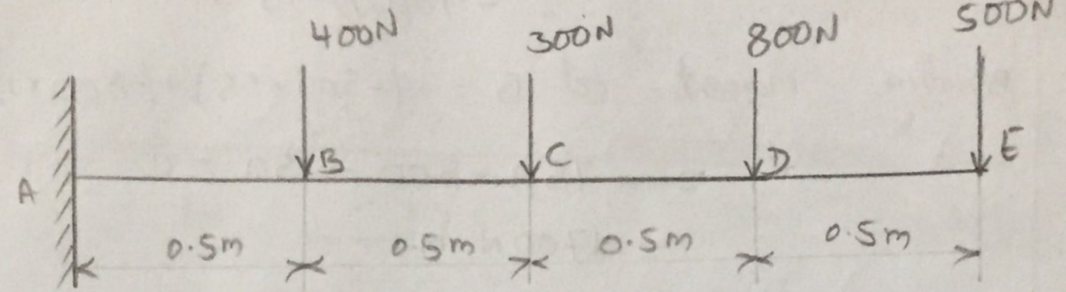


3) Uniformly varying Load :-

If the intensity of load is uniformly varying from one end to another end. it is called UVL. In which the load is zero at one end and increases uniformly to the other end.



3) Draw the shear force and Bending moment diagrams for the Cantilever beam as shown in below fig



Sol: Before drawing the shear force and Bending moment diagrams first we have to calculate shear forces acting at different point and as well as Bending moment at various points

Shear force Calculations :-

Shear force at point E = 500N

Shear force at point D = 500+800N
= 1300N

Shear force at point C = 500+800+300
= 1600N

Shear force at point B = 500+800+300+400
= 2000N

shear force at point A = 500+800+300+400+0
= 2000N

Bending moment Calculations :-

Bending moment at E = 0

Bending moment at D = (-500 × 0.5) + (800 × 0)
= -250N-m.

Bending moment at C $= -(500 \times 1) + (-800 \times 0.5) + (-300 \times 0)$

$= -500 - 400$

$= -900$ N-m

Bending moment at B $= (-500 \times 1.5) + (-800 \times 1) + (-300 \times 0.5) + (-400 \times 0)$

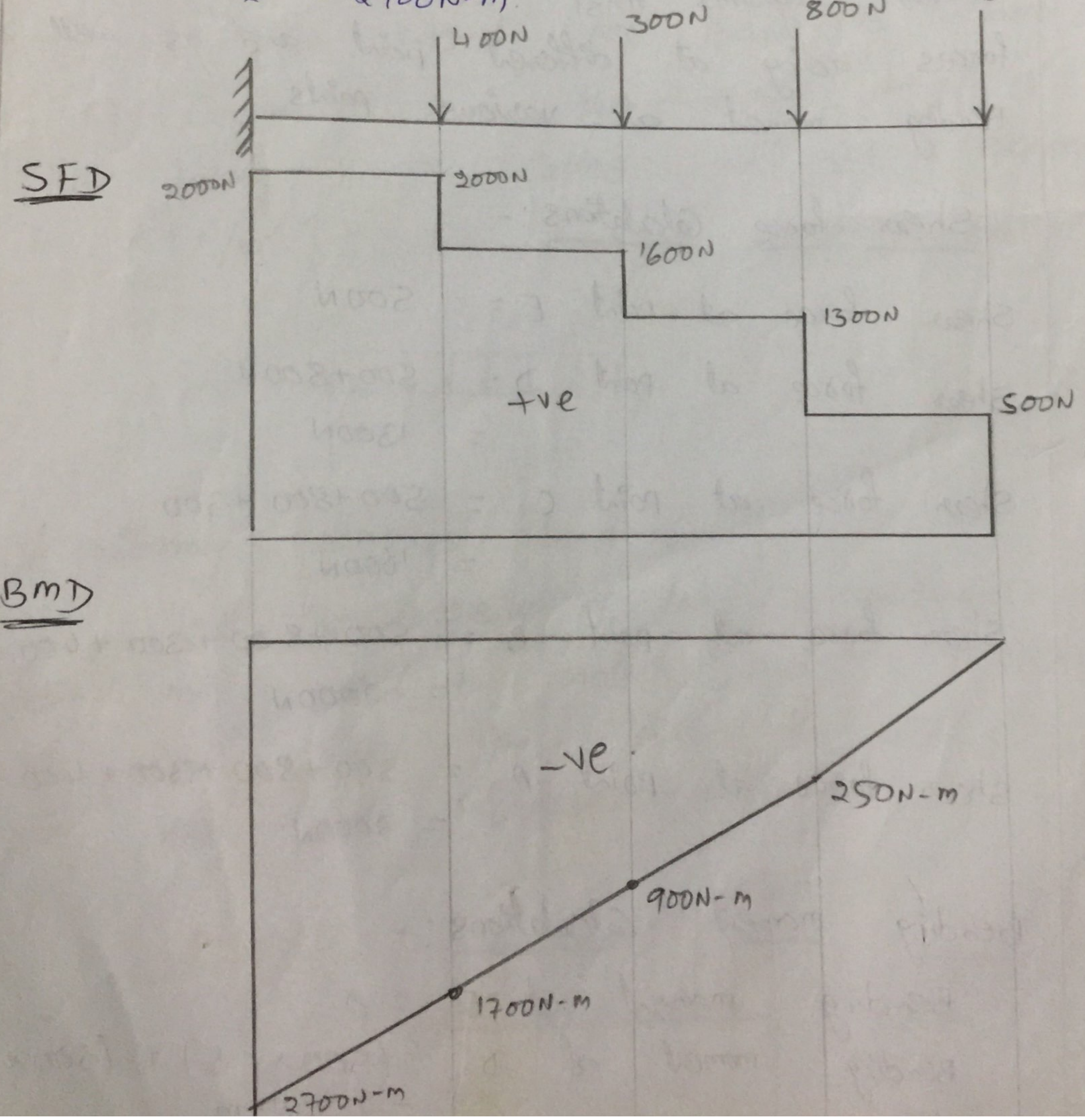
$= -750 - 800 - 150 - 0$

$= -1700$ N-m

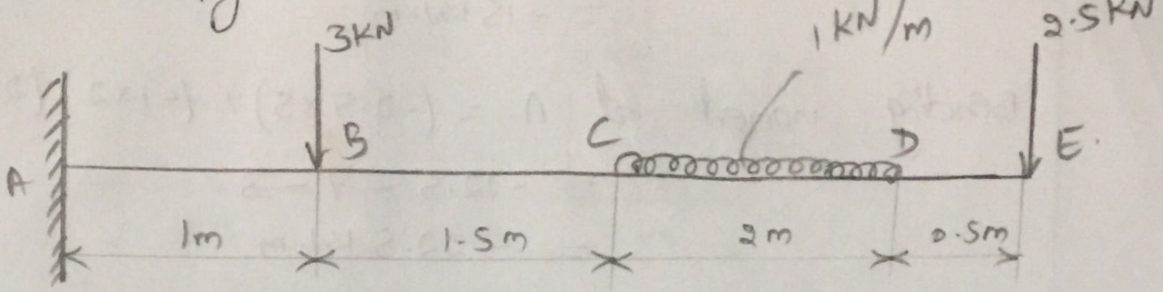
Bending moment at A $= (-500 \times 2) + (-800 \times 1.5) + (-300 \times 1) + (-400 \times 0.5)$

$= -1000 - 1200 - 300 - 200$

$= -2700$ N-m



Draw the SFD & BMD for the beam as shown in figure.



Shear force Calculations :-

Shear force at E = 2.5KN

shear force at D = 0+ 2.5KN

= 2.5KN

shear force at C = 2.5 + 0 + (1×2)

= 2.5 +0 + 2

= 4.5KN

Shear force at B = 2.5 + (1×2) + 3

= 2.5 + 2 + 3

= 7.5 KN

shear force at A = 2.5 + (1×2) + 3 + 0

= 7.5 KN

Bending moment Calculations :-

Bending moment at E = 0

Bending moment at D = (−2.5 × 0.5)

= −1.25 KN-m

Bending moment at C = $(-2.5 \times 2.5) + (-1\times 2) \times \left(\frac{2}{2}\right)$

= −6.25 − 2

= −8.25 KN-m.

Bending moment at B = $(-2.5\times4) + (-1\times2)(1.5+\frac{2}{2})$

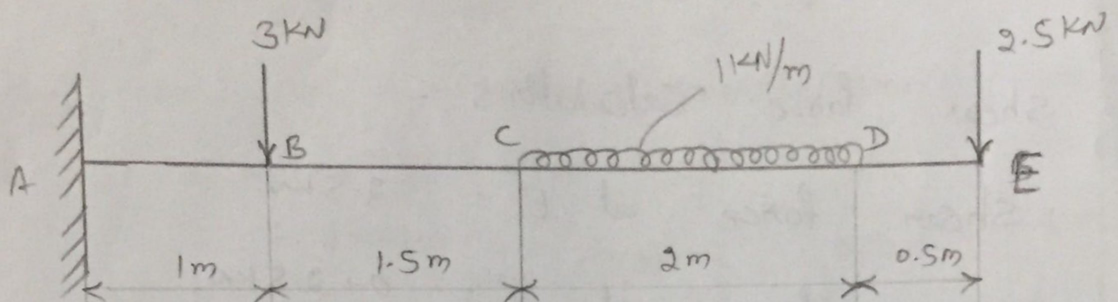
$= -10 - 5$

$= -15$ KN-m

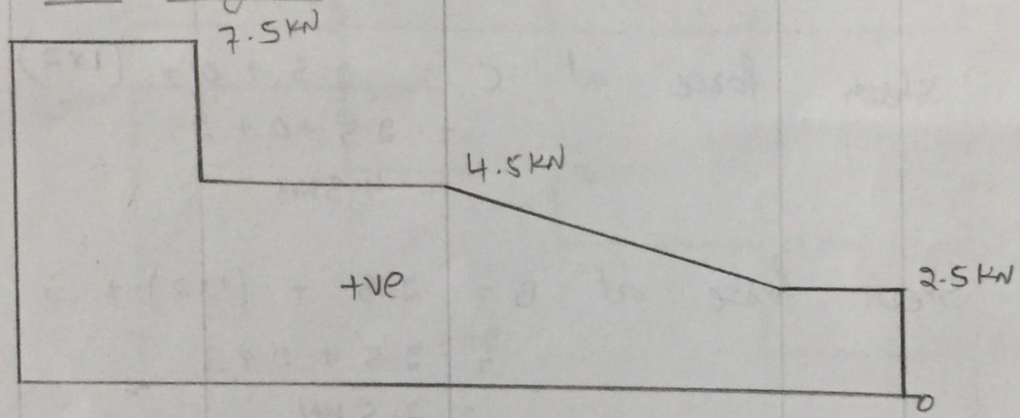
Bending moment at A. $= (-2.5\times5) + (-1\times2)(2.5+\frac{2}{2}) + (-3\times1)$

$= -12.5 - 7 - 3$

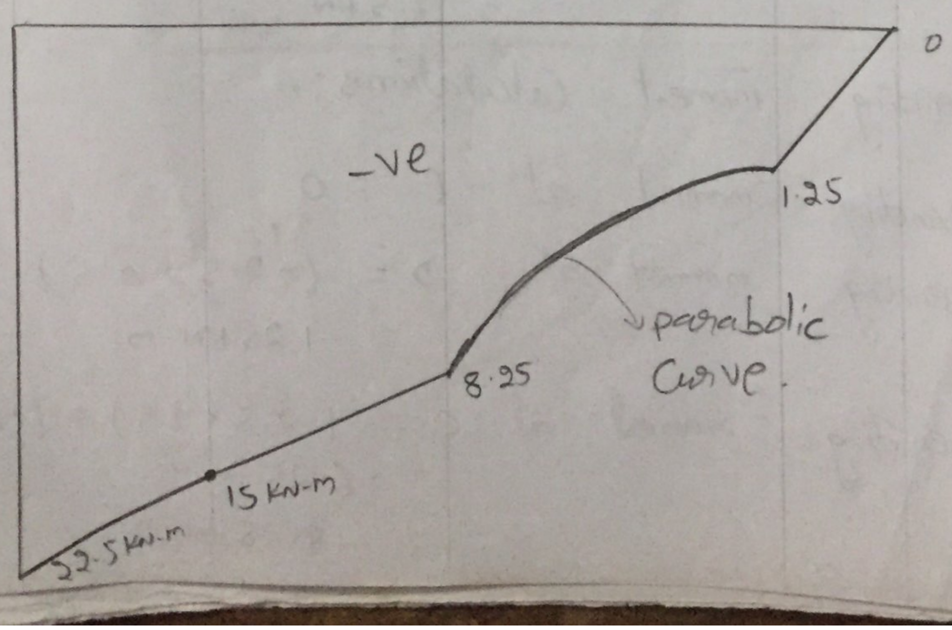
$= -22.5$ KN-m.



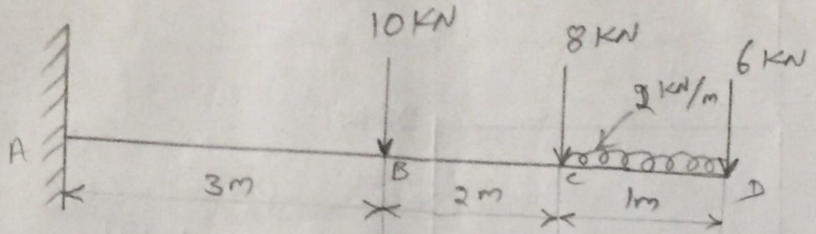
Shear force diagram :-



Bending moment Diagram :-



Draw the SFD and BMD for the beam Loaded as shown in below.



Shear Force Calculations:-

Shear force at D = 6 KN

$6+(2\times1) = 8$ KN-m without point load

$+(2\times1)$

Shear force at C = 6+8 = 16 KN → with point load

" " " B $= (2\times1)+6+8+10 = 26$ KN

shear force at A $= (2\times1)+24+0 = 26$ KN

Bending moment Calculations:-

Bending moment at D = 0

Bending moment at C $= -(6\times1) + \left(-\frac{(2\times1)}{}\times\frac{1}{2}\right)$

$= -6-1 \Rightarrow -7$ KN-m

Bending moment at B $= (-6\times3)+(-8\times2)+(-2\times1)(2+\frac{1}{2})$

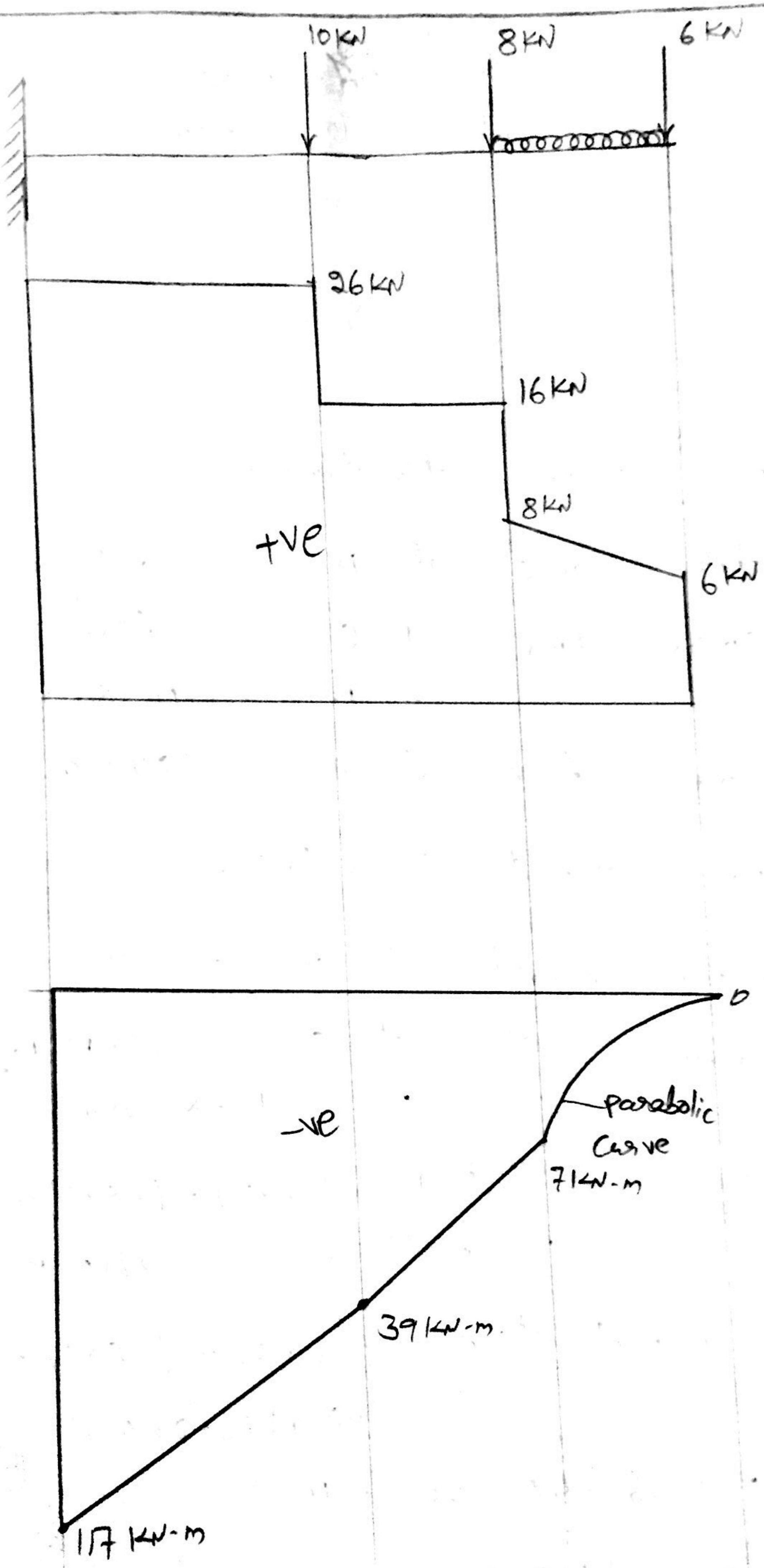
$= -18-16-5 = -39$ KN-m

Bending moment at A $= (-6\times6)+(-8\times5)+(-10\times3)$

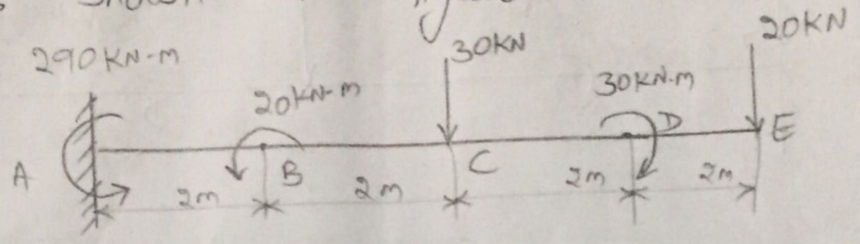
$(-2\times1)(3+2+0.5)$

$= -36-40-30-11$

$= -117$ KN-m


10KN
8KN
6KN
26KN
16KN
8KN
+ve
6KN
0
parabolic
Curve
-ve
71KN-m
39KN-m
117 KN-m

Draw the SFD and BMD for cantilever beam loaded as shown in figure.



Shear force Calculations:

Shear force at point E = 20 KN

D = 20 + 0 = 20 KN

C = 20 + 0 + 30 = 50 KN

B = 50 KN

A = 50 KN.

Bending moment Calculations:

Bending moment at point E = 0

" " " " D = -(20×2) - 30

= -40 - 30

= -70 KN-m

$BM_C = (-20\times4) - 30 \Rightarrow -80-30 \Rightarrow -110$ KN-m

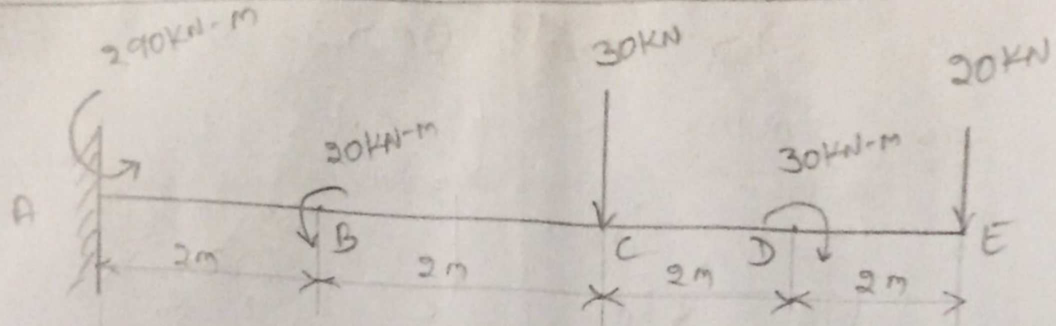
$BM_B = (-20\times6) - 30 - (30\times2) \Rightarrow -120 - 30 - 60$

$= -210$ KN-m $\Rightarrow -210 + 20 \Rightarrow 190$ KN-m

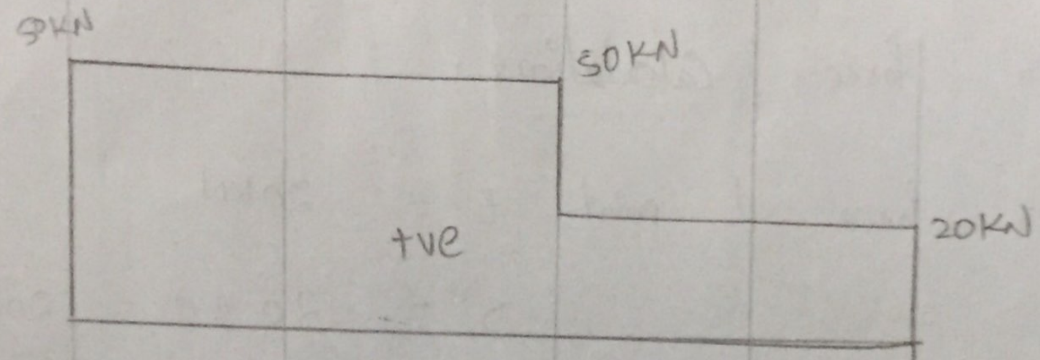
$BM_A = (-20\times8) - 30 - (30\times4) + 20 + 290$

$= -160 - 30 - 120 + 20 + 290$

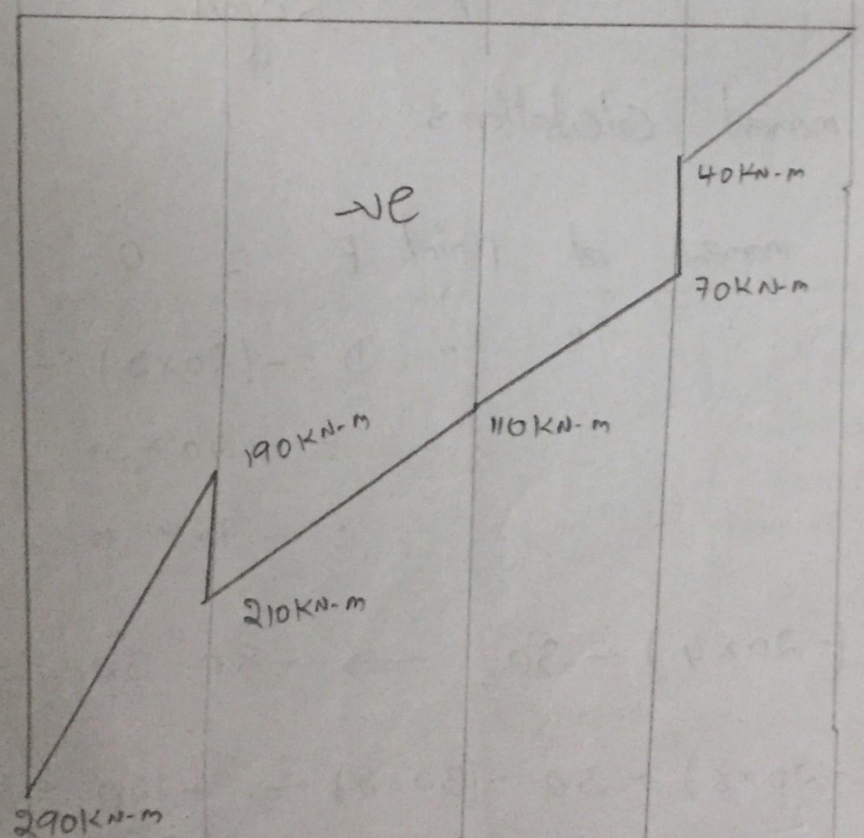
$= -310 + 20 + 290$



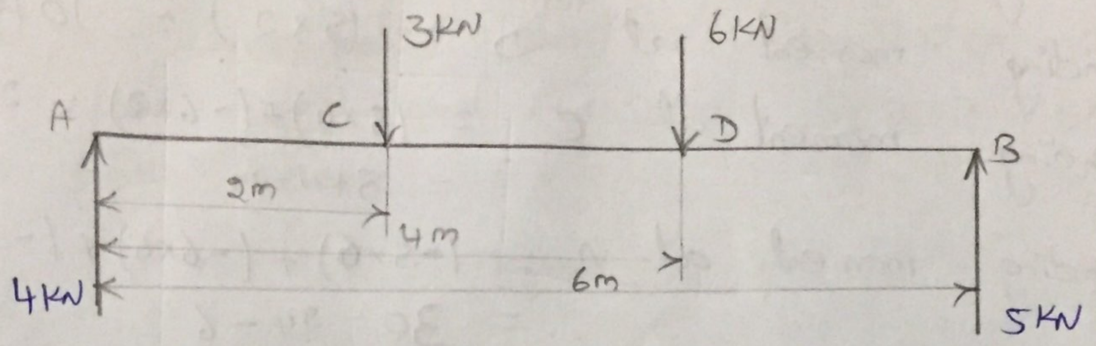
SFD



Bending moment diagram:



A Simply supported beam of length 6m carries a point load of 3KN & 6KN at a distance of 2m & 4m from the left end. Draw SFD & BMD.



whenever the simply supported beam is given the we have to calculate the vertical Reactions we know.

Sum of upward forces = sum of downward forces

$$R_A + R_B = 3+6$$

$$\boxed{R_A + R_B = 9\,KN} \text{ --- (1)}$$

we also know

sum of counter clockwise moments = sum of clockwise mome

Taking moments about A.

$$R_B \times 6 = 6\times4 + 3\times2$$

$$6R_B = 24+6$$

$$R_B = 30/6$$

$$\boxed{R_B = 5\,KN} \text{ --- (2)}$$

substitute R_B value in equation (1) to get R_A.

$$R_A = 9 - R_B \Rightarrow 9-5$$

$$\boxed{R_A = 4\,KN}$$

Shear force calculations:-

Shear force at B = $-5\,KN$

Shear force at D = $-5+6 \Rightarrow 1\,KN$

Shear force at C = $-5+6+3 \Rightarrow 9-5 \Rightarrow 4\,KN$

Shear force at $A = -5+6+3-4 = 9-9 = 0$

Bending moment calculations:-

Bending moment at $B = 0$

Bending moment at $D = (5\times2) = 10\ KN\text{-}m$

Bending moment at $C = (5\times4)+(-6\times2) = 20-12$

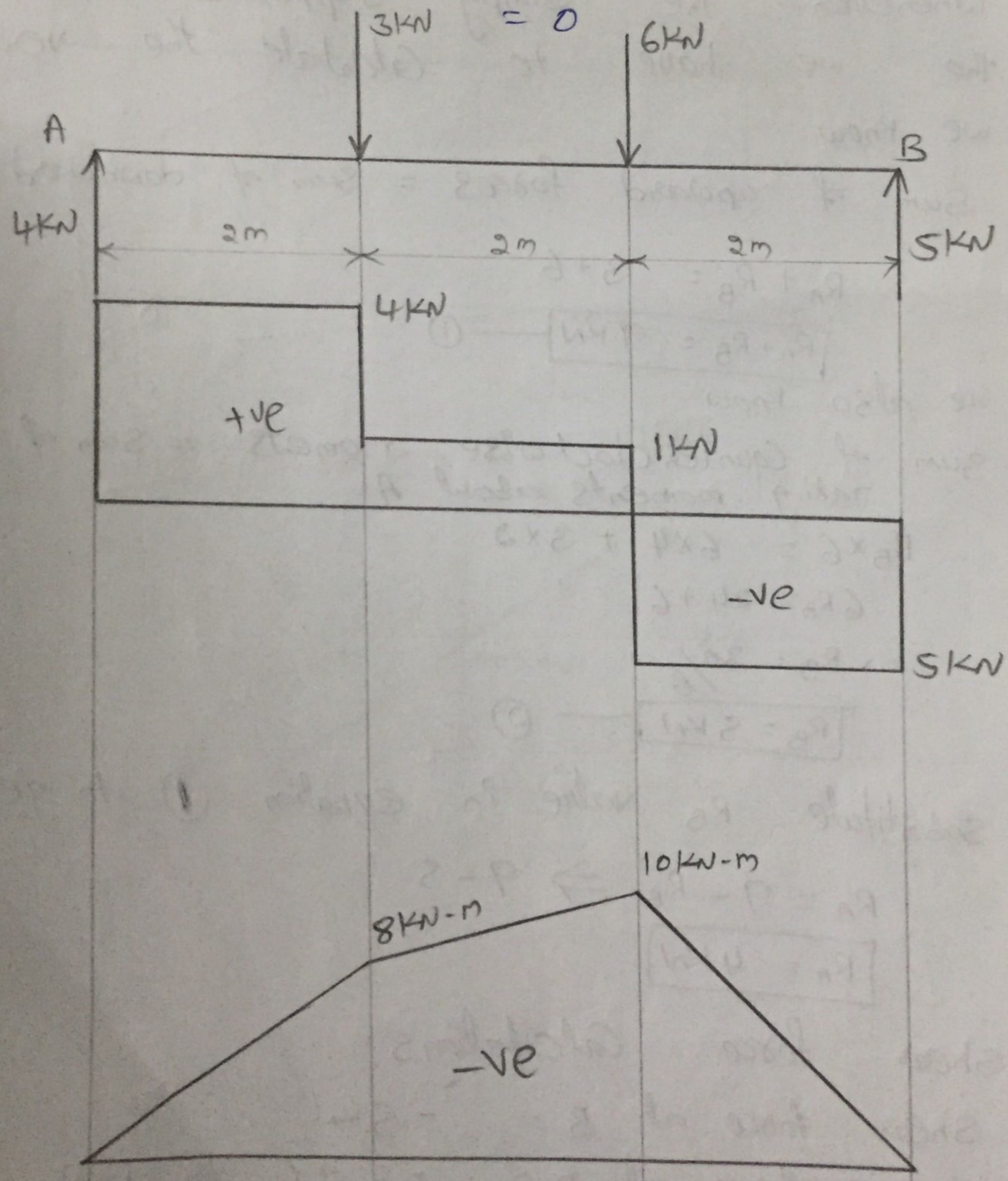
$= 8\ KN\text{-}m$

Bending moment at $A = (-5\times6)+(-6\times4)+(-3\times2)$

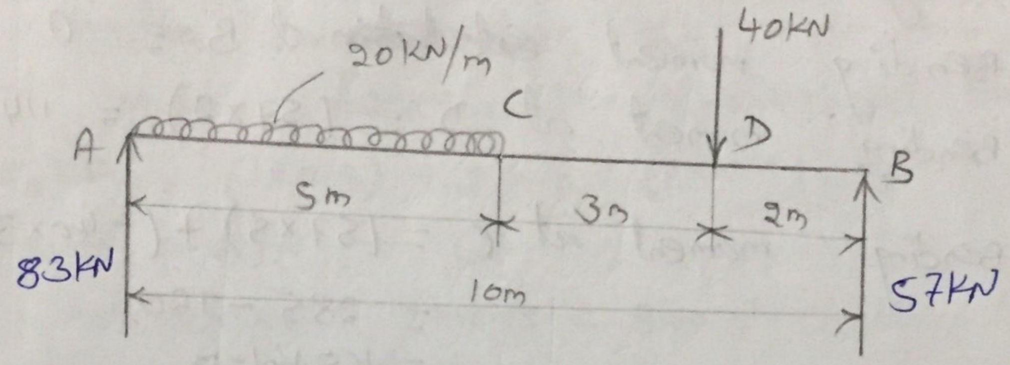
$= 30-24-6$

$= 30-30$

$= 0$



A beam AB 10m long is simply supported beam it carries a uniformly distributed load of 20KN/m for a distance of 5m from the left end, and a concentrated load of 40KN at a distance of 2m from right end B. Draw SFD & BMD for the given beam.



we know

sum of upward forces = sum of downward forces.

$$R_A + R_B = (20\times5) + 40$$

$$\boxed{R_A + R_B = 140\ KN} \quad \text{—(1)}$$

Sum of Anticlockwise moments = Sum of clockwise moments

Taking moments about A.

$$R_B \times 10 = (40\times8) + (\overset{10}{\cancel{20}}\times5)\left(\frac{5}{\cancel{2}}\right)$$

$$10R_B = 320 + 250$$

$$R_B = 570/10$$

$$\boxed{R_B = 57\ KN} \quad \text{—— (2)}$$

Substitute R_B value in eqn (1) to get R_A value

$$R_A = 140 - R_B \Rightarrow 140 - 57$$

$$\boxed{R_A = 83\ KN}$$

Shear force Calculations:-

shear force at B = -57KN

shear force at D = $-57+40 \Rightarrow -17KN$

shear force at C = $-57+40 \Rightarrow -17KN$

Shear force at A $= -57+40+(20\times5)$
$= -57+140$
$= -83$ → without point load.
$= -83+83$
$= 0$ → with point load.

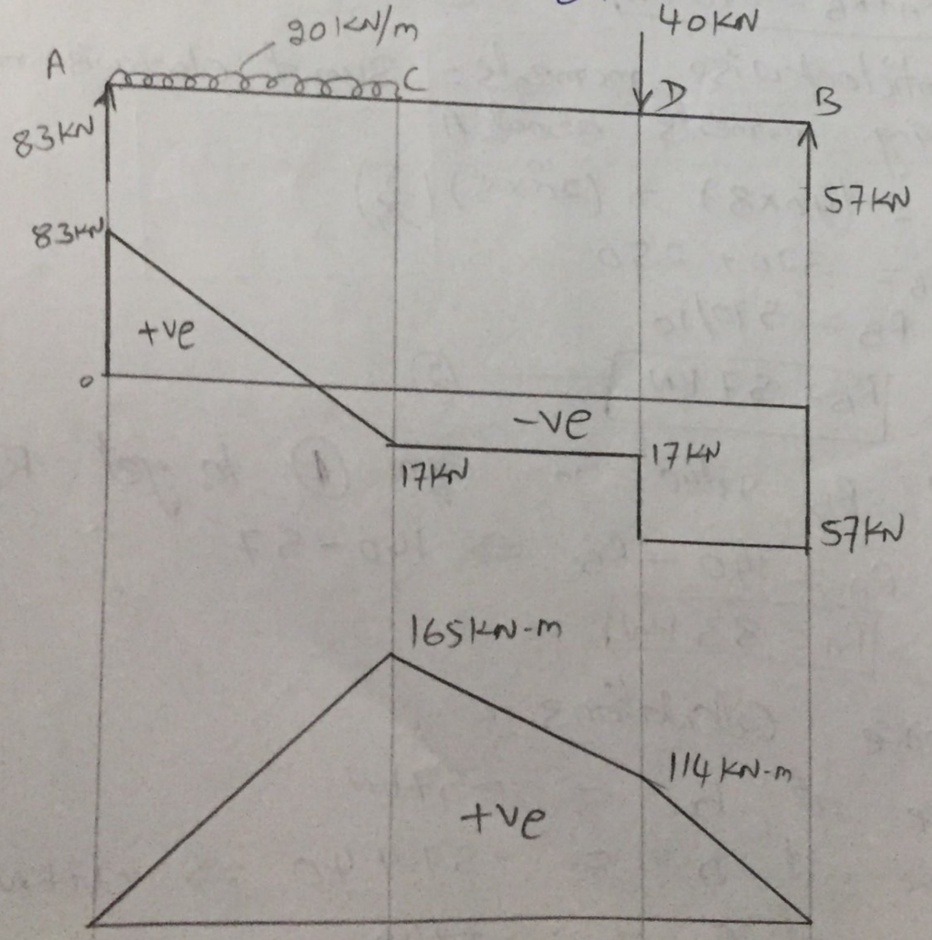
Bending moment Calculations :-

Bending moment ~~calculation~~ at B = 0

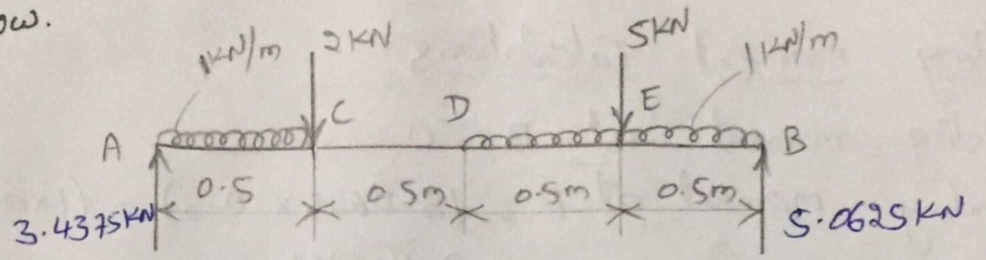
Bending moment at D = (57×2) = 114 KN-m

Bending moment at C = $(57\times5)+(-40\times3)$
$= 285-120$
$= 165$ KN-m

Bending moment at A = $(57\times10)+(-40\times8)+(-20\times5)\times\frac{5}{2}$
$= 570-320-250$
$= 570-570$
$= 0$.



Q) Draw the SFD & BMD for the beam as shown in below.



Sol For the simply supported beam first we have to calculate Reaction. for calculating this we know

Sum of upward forces = sum of downward forces

$$R_A + R_B = (1\times0.5) + 2 + 5 + (1\times1)$$

$$R_A + R_B = 8.5\,KN \quad — (1)$$

Sum of Anticlockwise moments = sum of clockwise moments

Taking moments about A.

$$R_B \times 2 = 2\times(0.5) + (5\times1.5) + (1\times0.5)\left(\frac{0.5}{2}\right) + (1\times1)\left(1+\frac{1}{2}\right)$$

$$= 1 + 7.5 + 0.125 + (1.5)$$

$$R_B = \frac{10.125}{2} = 5.0625\,KN \quad — (2)$$

substitute R_B value in eqn (1).

$$R_A = 8.5 - 5.0625$$

$$\boxed{R_A = 3.4375\,KN}$$

Shear force Calculations:-

Shear force at B = $-5.0625\,KN$

Shear force at E = $-5.0625 + (1\times0.5)$

$= -5.0625 + 0.5$

$= -4.5625\,KN$ — without point load

$= -4.5625 + 5 = 0.4375\,KN$ — with point load

shear force at D = $-5.0625 + 5 + (1\times1)$

$= 0.9375\,KN$

shear force at C = $-5.0625 + 5 + (1\times1) + 2$

$= 2.9375\,KN$

shear force at A = $(-5.0625) + (1\times1) + 5 + 2 + (1\times0.5)$

$= -5.0625 + 8.5 = 3.4375\,KN$ — without point load

Shear force at A $= 3.4375 - 3.4375$ — with point load.
$= 0$

Bending moment Calculations:-

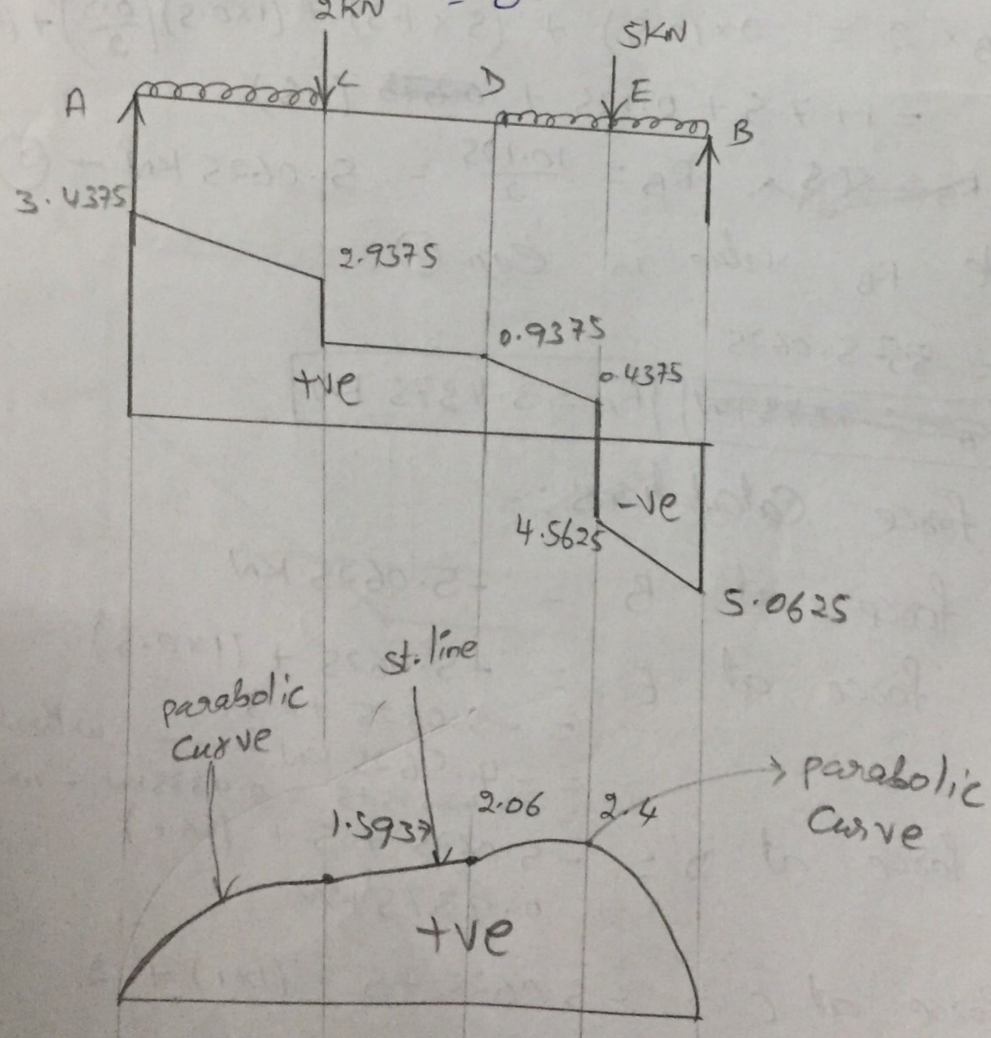
Bending moment at B $= 0$

Bending moment at E $= (5.0625 \times 0.5) - (1 \times 0.5)\left(\frac{0.5}{2}\right)$
$= 2.406$ KN-m

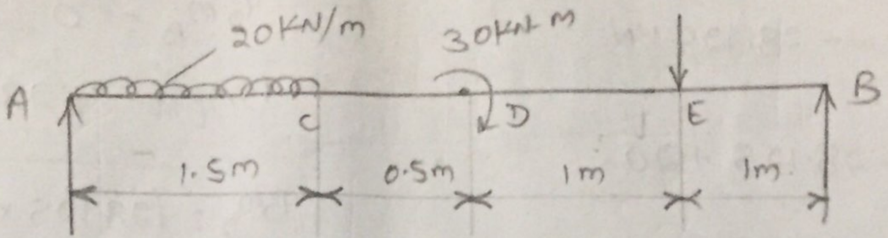
Bending moment at D $= (5.0625 \times 1) - (5 \times 0.5) - (1 \times 1)\left(\frac{1}{2}\right)$
$= 2.0625$ KN-m

Bending moment at C $= (5.0625 \times 1.5) - (5 \times 1) - (1 \times 1)\left(0.5 + \frac{1}{2}\right)$
$= 1.5937$ KN-m

Bending moment at A $= (5.0625 \times 2) - (5 \times 1.5) - (2 \times 0.5) - (1 \times 1)\left(1 + \frac{1}{2}\right) - (1 \times 0.5)\left(\frac{0.5}{2}\right)$
$= 0.$



Draw the SFD and BMD for the loaded beam as shown in below.



we know that whenever simply supported beam is given then we have to calculate Reactions at ends

we also know the equilibrium condition.

Sum of upward forces = sum of downward forces

Here

$$R_A + R_B = 20 + (20 \times 1.5) \Rightarrow 20 + 30$$

$$\boxed{R_A + R_B = 50 KN} \quad (1)$$

we have one more equilibrium condition.

Sum of Anticlockwise moments = sum of clockwise moments

Here Taking moments about A.

$$R_B \times 4 = (20 \times 3) + (20 \times 1.5)\left(\frac{1.5}{2}\right) + 30$$

$$4R_B = 60 + (30)(0.75) + 30$$

$$R_B = \cancel{[illegible]} = \frac{112.5}{4} \Rightarrow \boxed{R_B = 28.125} \quad (2)$$

$$\boxed{\cancel{R_B = 20.625\ KN}} \qquad (2)\ \boxed{R_B = 28.125\ KN} - ($$

substitute eqn (2) in eqn (1) to get R_A value.

$$R_A = 50 - 28.125$$

$$\boxed{\cancel{R_A = 29.375\ KN}} \qquad \boxed{R_A = 21.875\ KN}$$

Shear force Calculations

$SF_B = -28.125\ KN$

$SF_E = -28.125 + 20$

$= -8.125\ KN$

$SF_D = -8.125\ KN$

$SF_C = -8.125\ KN$

$SF_A = -28.125 + 20 + (20 \times 1.5)$

$= -28.125 + 50$

$= -21.875 + 21.875$

$= 0.$

Bending moment Calculations

$BM_B = 0$

$BM_E = +28.125 \times 1 = 28.125\ KN\text{-}m$

$BM_D = (28.125 \times 2) - 20 \times 1 \Rightarrow \cancel{\ \ }$

$= \cancel{\ \ }$

$= 56.25 - 20$

$= 36.25 - 30$

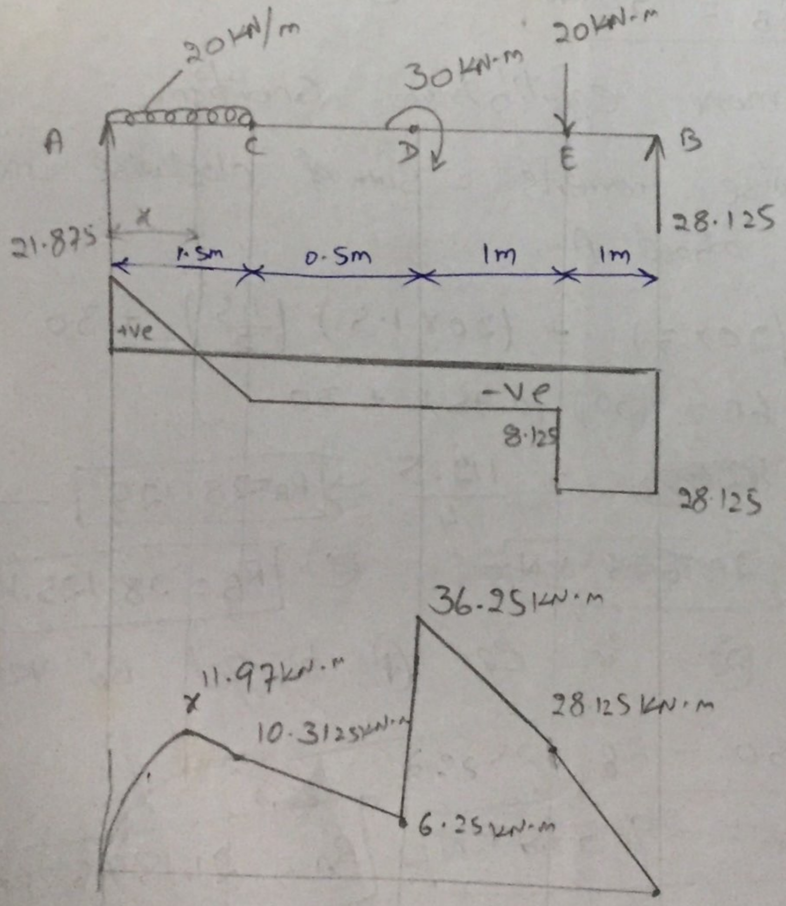
$= 6.25\ KN\text{-}m$

$BM_C = (28.125 \times 2.5) - (20 \times 1.5) - 30$

$= 70.3125 - 30 - 30$

$= 10.3125\ KN\text{-}m$

$BM_A = 0.$



$SF_x = 0$

$-21.875 + 20 \times x = 0$

$x = \frac{21.875}{20}$

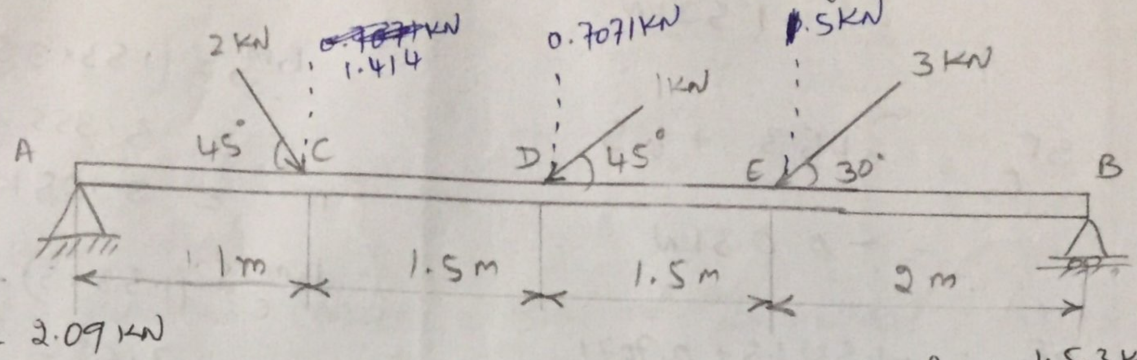
$SF_x = 1.093\ KN.$

$BM_x = R_A \times x - (20 \times x)\left(\frac{x}{2}\right)$

$= 21.875 \times 1.093 - (20 \times 1.093)\left(\frac{1.093}{2}\right)$

$= 11.97\ KN\text{-}m$

A beam is loaded as shown in below fig find the reactions at A and B, and also draw shear force and bending moment diagram



The above fig shows simply supported beam carrying inclined point loads. First we have to resolve into vertical components.

we know

Sum of upward forces = sum of downward force

$$R_A + R_B = 2\sin 45^\circ + 1\sin 45^\circ + 3\sin 30$$

$$= 1.4142 + 0.7071 + 1.5$$

$\boxed{R_A + R_B = \cancel{2.9141}}$ ~~—~~ (1) $\boxed{R_A + R_B = 3.6213 \text{ KN}}$

Sum of Anticlockwise moments = Sum of clockwise moments

$$(R_B \times 6) = (1.5 \times 4) + (0.7071 \times \overset{2.5}{\cancel{2}}) + (\overset{1.4142}{\cancel{0.7071}} \times 1)$$

$\cancel{6.5 R_B = 2.25 + 2.1213 + 1.0606 \; 2.1213}$

$6R_B = 6 + 1.7677 + 1.4142$

$\cancel{R_B = 5.43 / 6.5}$

$$R_B = \frac{9.1819}{6}$$

$\boxed{\cancel{R_B = 0.83 \text{ KN}}}$ ~~—~~ (2)

$\boxed{R_B = 1.53}$

substitute eqn (2) value in (1)

$R_A = \cancel{2.9141 - 0.83}$

$R_A = 3.6213 - 1.53$

$\boxed{\cancel{R_A = 2.0784 \text{ KN}}}$

$\boxed{R_A = 2.09 \text{ KN}}$

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Shear force Calculations

Shear force at point B

$= -1.53\,KN$

$SF_E = -1.53 + 1.5$

$= -0.03\,KN$

$SF_D = -1.53 + 1.5 + 0.7071$

$= 0.6771\,KN$

$SF_C = -1.53 + 1.5 + 0.7071 + 1.414$

$= 2.09\,KN$

$SF_A = 2.09 - 2.09 \Rightarrow 0$

Bending moment Calculations

$BM_B = 0$

$BM_E = (1.53 \times 2) = 3.06\,KN\text{-}m$

$BM_D = (1.53 \times 3.5) - (1.5 \times 1.5)$

$= 5.355 - 2.25$

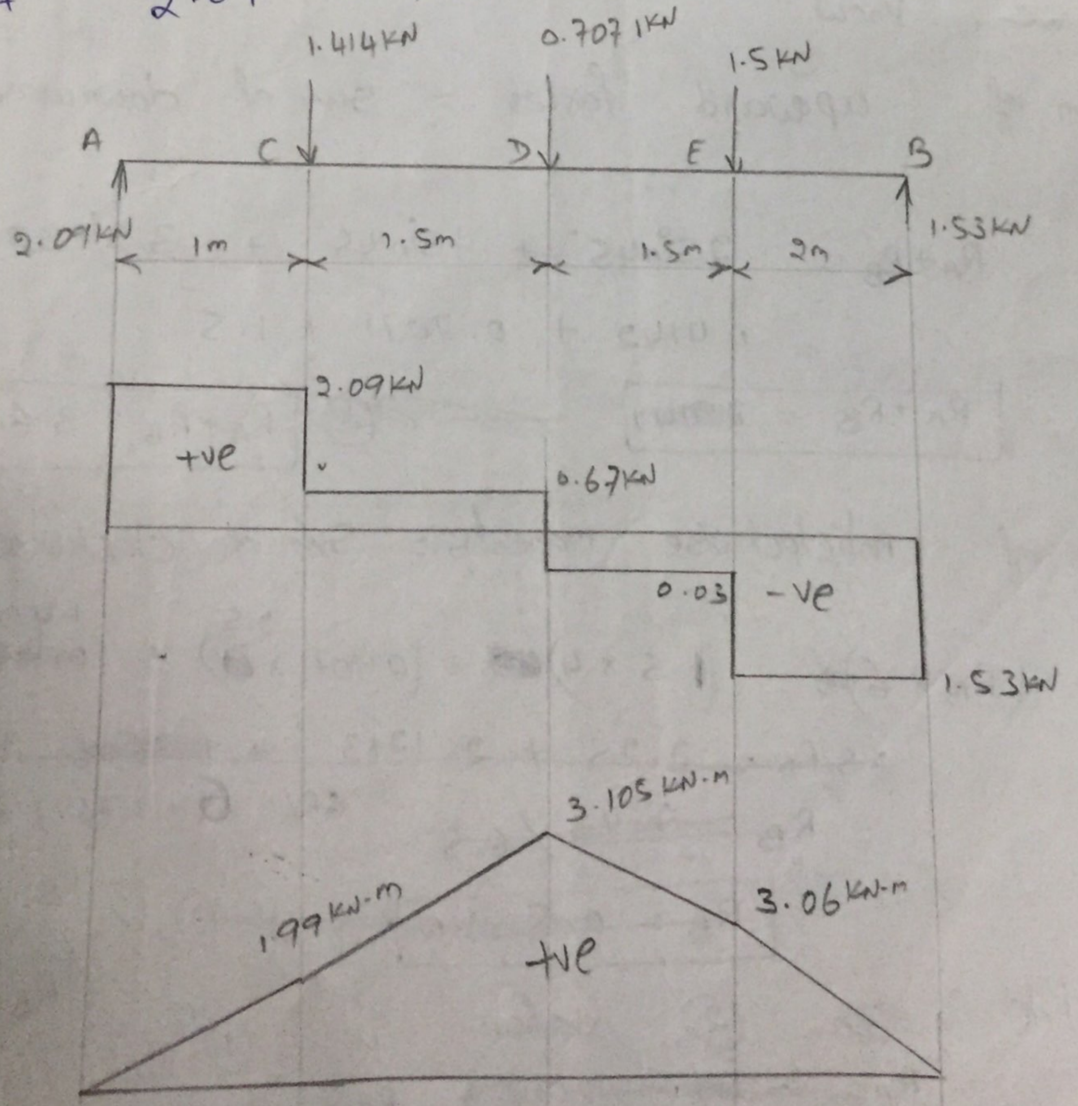
$= 3.105\,KN\text{-}m.$

$BM_C = (1.53 \times 5) - (1.5 \times 3) - (0.7071 \times 1.5)$

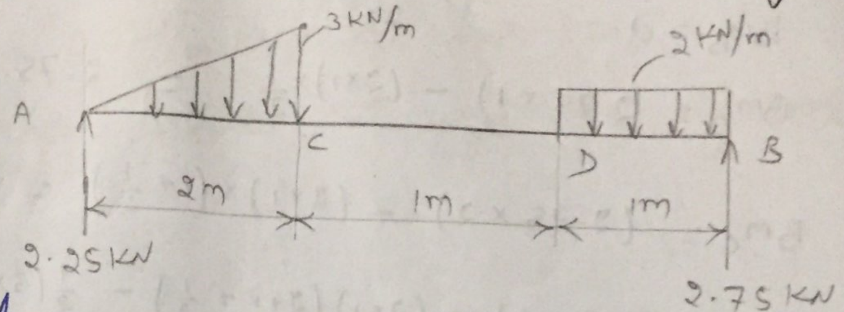
$= 7.65 - 4.59 - 1.0606$

$= 1.99\,KN\text{-}m.$

$BM_A = 0$



Q) Draw the shear force and bending moment diagrams for the beam as shown in fig.



Sol we know that

sum of upward forces = sum of downward forces.

$$R_A + R_B = (2\times1) + (\tfrac{1}{2}\times3\times2)$$

$$\boxed{R_A + R_B = 5\,KN} \text{ --- (1)}$$

we also know

sum of anticlockwise moments = sum of clockwise moments

$$R_B \times 4 = (2\times1)(3+\tfrac{1}{2}) + (\tfrac{1}{2}\times3\times2)\times(2\times\tfrac{2}{3})$$

$$4R_B = 2\times3.5 + \cancel{3}\times\tfrac{4}{\cancel{3}}$$

$$R_B = 11/4$$

$$\boxed{R_B = 2.75\,KN} \text{ --- (2)}$$

Substitute eqn (2) value in eqn (1)

$$R_A = 5 - R_B \Rightarrow 5 - 2.75$$

$$\boxed{R_A = 2.25\,KN}$$

shear force Calculations:-

$$SF_B = -2.75\,KN$$

$$SF_D = -2.75 + (2\times1) = -0.75\,KN$$

$$SF_C = -2.75 + (2\times1) = -0.75\,KN$$

$$SF_A = -2.75 + (2\times1) + \tfrac{1}{2}(3\times2) = 2.25\,KN$$

$$= 2.25 - 2.25$$

$$= 0\,KN$$

Bending moment Calculations:

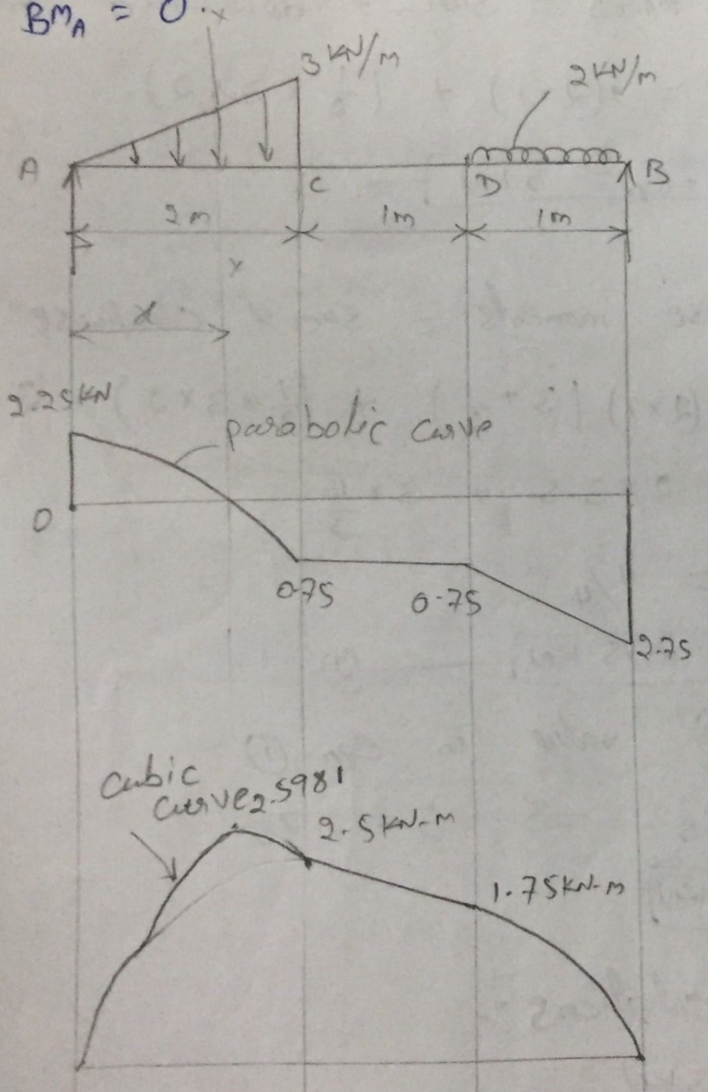
$BM_B = 0$

$BM_D = (2.75 \times 1) - (2\times1)\times\frac{1}{2} = 2.75 - 1 = 1.75\ KN\text{-}m$

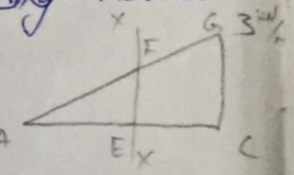
$BM_C = (2.75\times2) - (2\times1)\times(1+\frac{1}{2}) = 5.5 - 3 = 2.5\ KN\text{-}m$

$BM_A = (2.75\times4) - (2\times1)(2+1+\frac{1}{2}) - \frac{1}{2}(3\times2)\times\frac{2\times2}{3} \Rightarrow 11 - 7 - 4$

$BM_A = 0.$



whereever the shear force is zero between two points then there will be maximum bending moment.



$SF_x = 0$

$2.25 -$

from similar Δ^{les}

$\frac{AC}{GC} = \frac{AE}{AF} \Rightarrow \frac{2}{3} = \frac{x}{EF}$

$EF = \frac{3}{2}x.$

$2.25 - \frac{1}{2}(\frac{3}{2}x \times x)$

$2.25 = \frac{3}{4}x^2$

$x^2 = \frac{2.25\times4}{3} = \frac{9}{3} = 3$

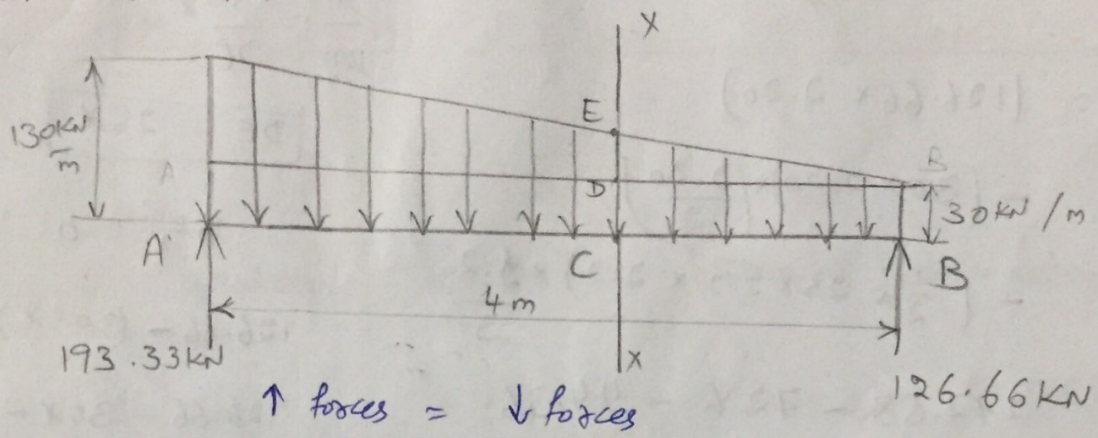
$\boxed{x = 1.732\,m}$

$BM_x = (2.25\times1.73)\times\frac{1}{2}(1.73\times\frac{3}{2}\times1 \times(\frac{1.73}{3})$

$= 3.8925\ (1.2944)$

$= 2.5981\ KN\text{-}m$

The intensity of Loading on a simply supported beam of 4m span increases gradually from 30 kN/m run at one end to 130 run at the other. Draw S.F and B.M diagrams.



↑ forces = ↓ forces

$R_A + R_B = (30 \times 4) + \left(\frac{1}{2} \times 4 \times 100\right)$

$= 120 + 200$

$\boxed{R_A + R_B = 320\ kN}$ —— (1)

Taking moments about A.

$R_B \times 4 = (30 \times 4) \times \left(\frac{4^2}{2}\right) + \left(\frac{1}{2} \times 4 \times 100\right)\left(\frac{4}{3}\right)$

$= 240 + 266.66$

$= 506.66/4 =$

$\boxed{R_B = 126.66\ kN}$ —— (2)

substitute R_B value in eqn (1)

$R_A = 320 - 126.66$

$\boxed{R_A = 193.33\ kN}$

Shear force Calculation :-

$SF_B = -126.66$

$SF_A = -126.66 + (30 \times 4) + \frac{1}{2} \times 4^2 \times 100$

$= -126.66 + 120 + 200$

$= -126.66 + 320$

$= 193.34\ kN.$

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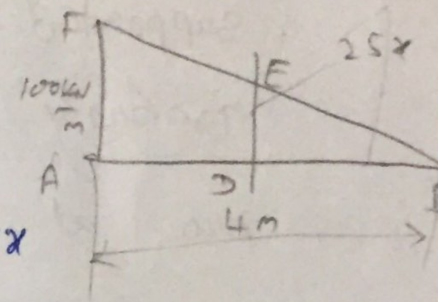
Bending moment Calculations

$BM_B = 0$

$BM_A = 0$

From Similar Δ^{les}



$$\frac{AB}{AF} = \frac{BD}{DF}$$

$$\frac{4}{100_{25}} = \frac{\cancel{BD}\,x}{DE}$$

$$\boxed{DE = 25x}$$

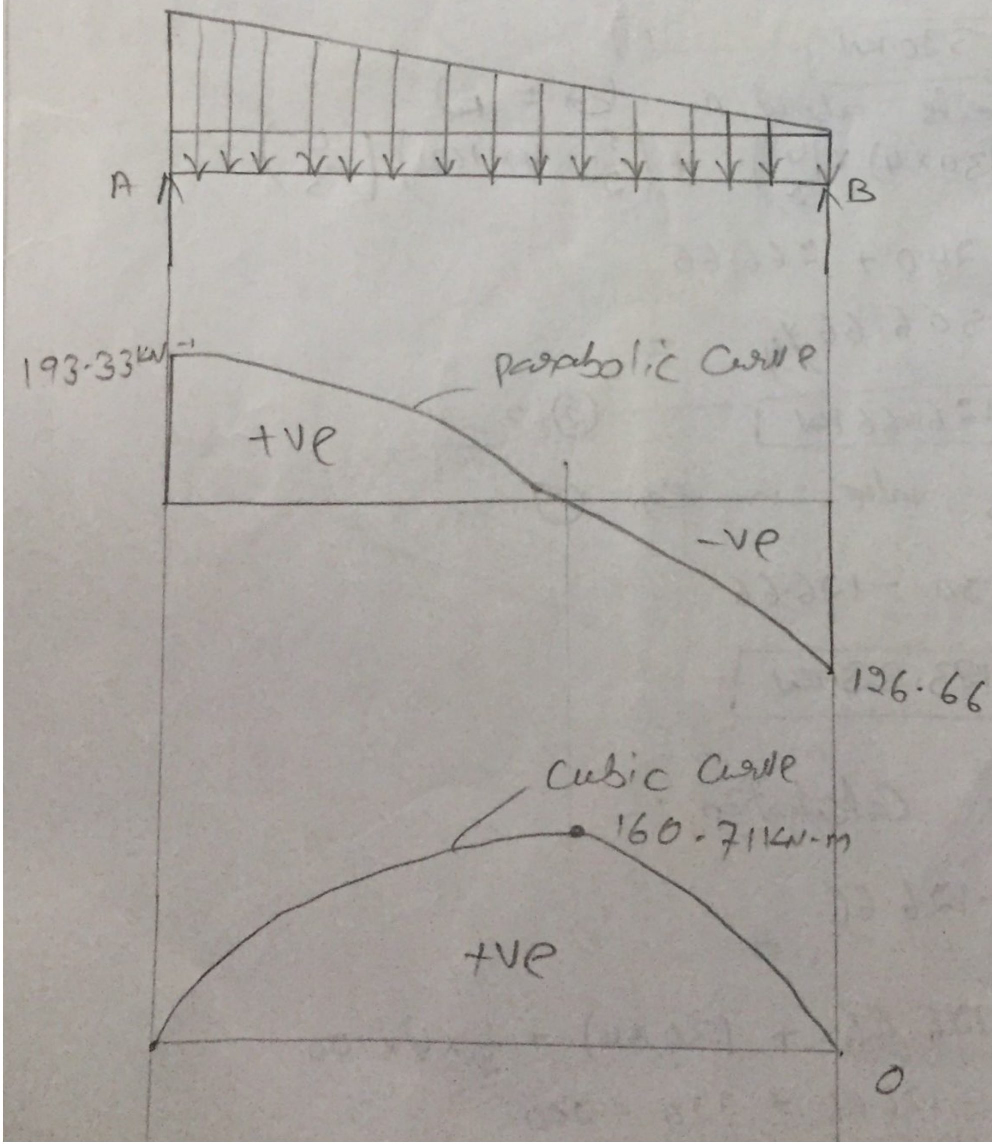
$SF_x = 0$

$126.66 - (30 \times x) - \frac{1}{2} \times x \times 25x$

$126.66 - 30x - 12.5x^2 = 0$

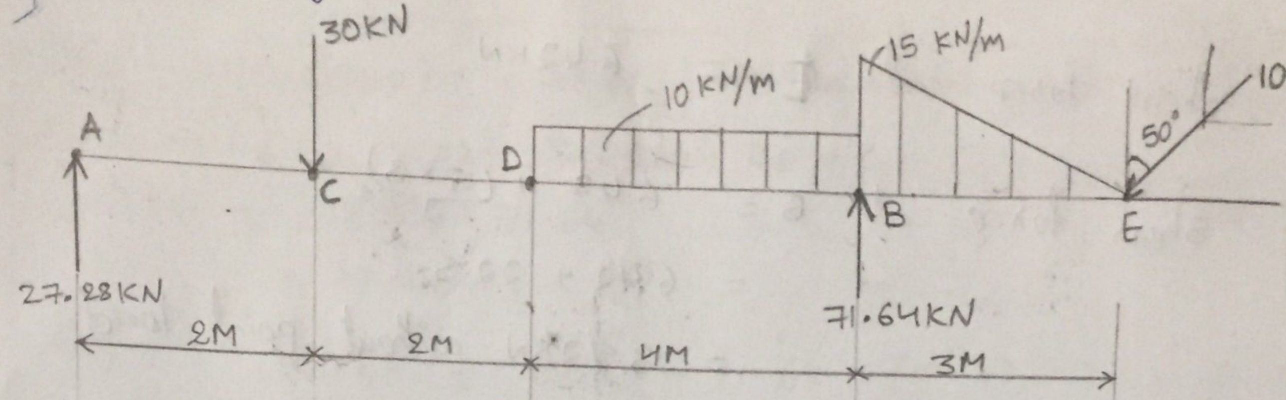
$$\boxed{x = 2.20\ m}$$

$$BM_x = (126.66 \times 2.20)$$
$$- (30 \times 2.20) \times \left(\frac{2.20}{2}\right)$$
$$- \left(\frac{1}{2} \times 25 \times 2.2 \times 2.2\right) \times \frac{2.2}{3}$$
$$= 277.68 - 72.6 - 44.36$$
$$= 277.68 - 116.96$$
$$= 160.71\ kN\text{-}m$$



2-15

Q) Draw SFD and BMD for the loaded beam as shown in below fig



sol we know

Sum of upward forces = sum of downward forces

$$R_A + R_B = 30 + (10 \times 4) + \frac{1}{2}(15 \times 3) + 10\cos 50$$

$$= 30 + 40 + 22.5 + 6.42$$

$$\boxed{R_A + R_B = 94.92\ KN} \text{ --- } (1)$$

we also know

Sum of Anticlockwise moments = sum of clockwise moments

$$R_B \times 8 = (30 \times 2) + (10 \times 4)\left(2+2+\frac{4}{2}\right) + \frac{1}{2}(15 \times 3)\left(2+2+4+\frac{3}{3}\right) + (10\cos 50 \times 11)$$

$$8R_B = 573.20$$

$$\cancel{R_B = 71.64} \quad \boxed{R_B = 71.65\ KN}$$

substitute R_B value in eqn (1) to get R_A

$$R_A = 94.92 - 71.65$$

$$\boxed{R_A = 23.28\ KN}$$

Shear force at E = 6.42 KN

Shear force at $B = 6.42 + \frac{(15 \times 3)}{2}$

$= 6.42 + 22.5$

$= 28.92$ KN without point load.

Shear force at $B = 28.92 - 71.64$

$= -42.72$ KN with point load.

Shear force at point $D = 6.42 + (\frac{1}{2} \times 15 \times 3)$

$+ (10 \times 4) - 71.64$

$= -2.72$ KN

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$$SF_C = (10\cos 50) + (15\times 3\times \tfrac{1}{2}) - \cancel{71.64} + (10\times 4) + 60\sin 3$$

$$= 6.42 + 22.5 - \cancel{71.64} + 40 + 30$$

$$= 98.92 - 71.64$$

$$= 27.28\ \text{KN}$$

$$SF_A = 6.42 + (\tfrac{1}{2}\times 15\times 3) - 71.64 + (10\times 4) + 30 - 27.2$$

$$= 98.92 - 98.92$$

$$= 0.$$

Bending moment Calculations :-

Bending moment at $E = 0$

Bending moment at $B = (-6.42\times 3) + (-\tfrac{1}{2}\times 15\times 3)($

$$= -19.26 - 22.5$$

$$= -41.76\ \text{KN-m}.$$

Bending moment at $D = (-6.42\times 7) + (-\tfrac{1}{2}\times 15\times 3)(4+\tfrac{3}{3}) + (-10\times 4)\times(\tfrac{4}{2}) + (71.64\times 4)$

$$= -44.94 - 112.5 - 80 + 286.56$$

$$= -237.44 + 286.56$$

$$= 49.12\ \text{KN-m}.$$

Bending moment at $C = (-6.42\times 9) - (22.5)\times(6+\tfrac{3}{3}) - (10\times 4)(\tfrac{4}{2}+2) + (71.64\times 6)$

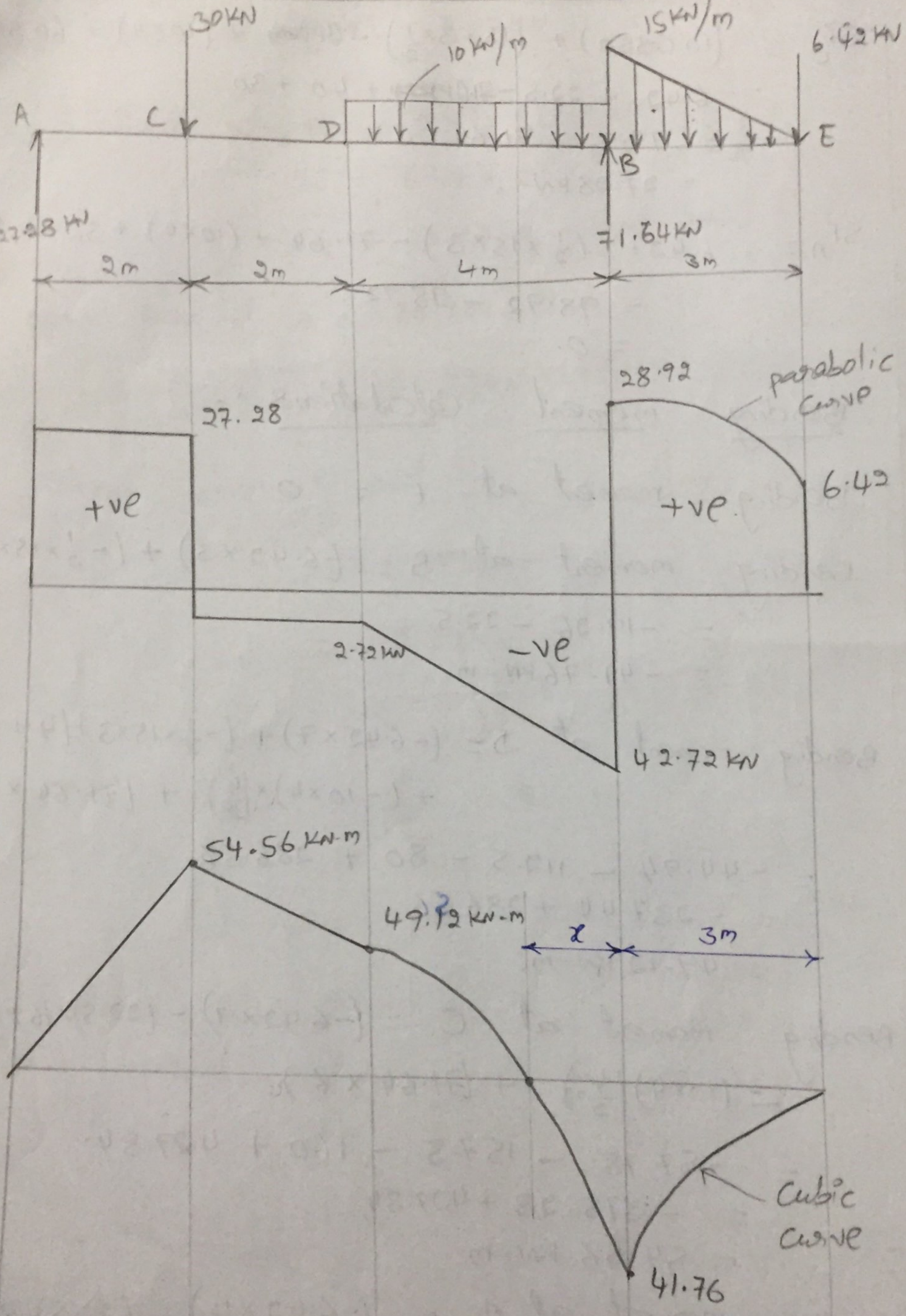
$$= -57.78 - 157.5 - 160 + 429.84$$

$$= -375.28 + 429.84$$

$$= 54.56\ \text{KN-m}.$$

Bending moment at $A = (-6.42\times 11) - (22.5)(8+\tfrac{3}{3}) - (10\times 4)(4+\tfrac{4}{2}) + (71.64\times 8) - (30\times 2)$

$$= -70.62 - 202.5 - 240 + 573.12 - 60$$


30KN
10KN/m
15KN/m
6.42KN
A
C
D
E
B
27.28 KN
71.64KN
2m
2m
4m
3m
28.92
parabolic curve
27.28
6.42
+ve
+ve
2.72KN
-ve
42.72KN
54.56 KN-m
49.12 KN-m
x
3m
Cubic curve
41.76

In the above bending moment diagram we got the point of Contraflexure.
Now we have to locate the point of Contraflexure.

$BM_x = 0$

$$BM_x = [-6.42 \times (3+x)] + (-\tfrac{1}{2} \times 15 \times 3) \times (\tfrac{3}{3}+x) + (-10 \times x)(\tfrac{x}{2}) + (71.64 \times x)$$

$$= -19.26 + (-6.42 \times x) - 22.5 - 22.5x - \frac{10x^2}{2} + 71.64x$$

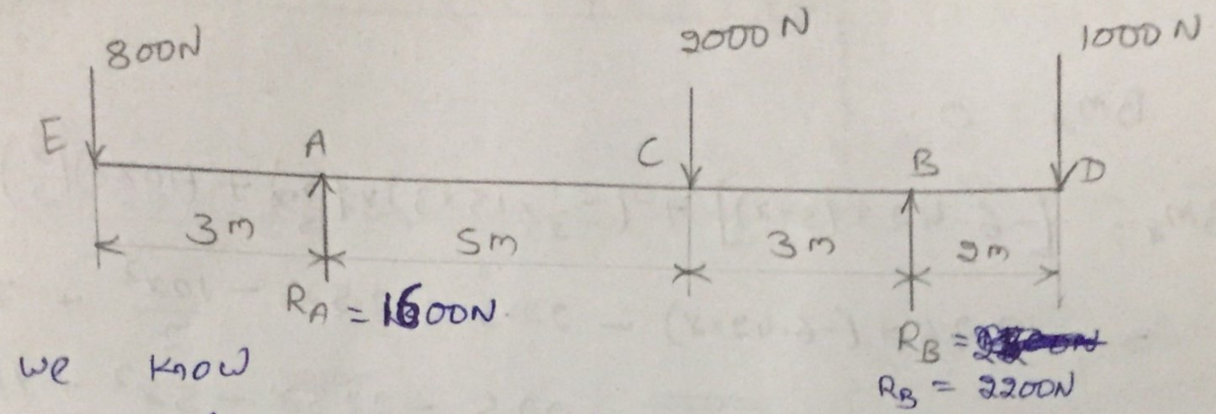
$$= -19.26 - 6.42x - 22.5 - 22.5x - 5x^2 + 71.64x$$

$$= -5x^2 + 42.72x - 41.76$$

$x = 1.125\,m$

The point of Contraflexure is at a distance of $(3+x) \Rightarrow (3+1.125) = 4.125\,m$ from Right support.

Draw the shear force and bending moment diagram for the overhanging beam as shown in below.



we know

Sum of upward forces = Sum of downward forces

$$R_A + R_B = 800 + 2000 + 1000$$

$$\boxed{R_A + R_B = 3800N} \text{ ——— } (1)$$

we Also know

Sum of Anticlockwise moments = Sum of clockwise moments

Taking moments about A.

$$(R_B \times 8) + (800 \times 3) = (2000 \times 5) + (1000 \times 10)$$

$$R_B = \frac{(10{,}000 + 10{,}000)^{-2400}}{8} = \frac{20{,}000 - 2400}{8} = \frac{17600}{8}$$

$$\boxed{R_B = 2200N}$$

substitute R_B value in equation (1) to get R_A

$$R_A = 3800 - 2200$$

$$\boxed{R_A = 1600N}$$

~~Sum of Anticlockwise moments = Sum of clockwise moments~~

<u>SHEAR FORCE CALCULATIONS</u>:-

Shear force at D = 1000N

Shear force at B = 1000 − 2200

$= \cancel{1200\,N} = -1200N$

Shear force at $C = 1000 - 2200 + 2000$
$= -800\,N$

Shear force at $A = 1000 - 2500 + 2000 - 1300$
$= 3000 - 3800$
$= -800$

Shear force at $E = 1000 - 2200 + 2000 - 1600 + 800$
$= 0.$

BENDING MOMENT CALCULATIONS :-

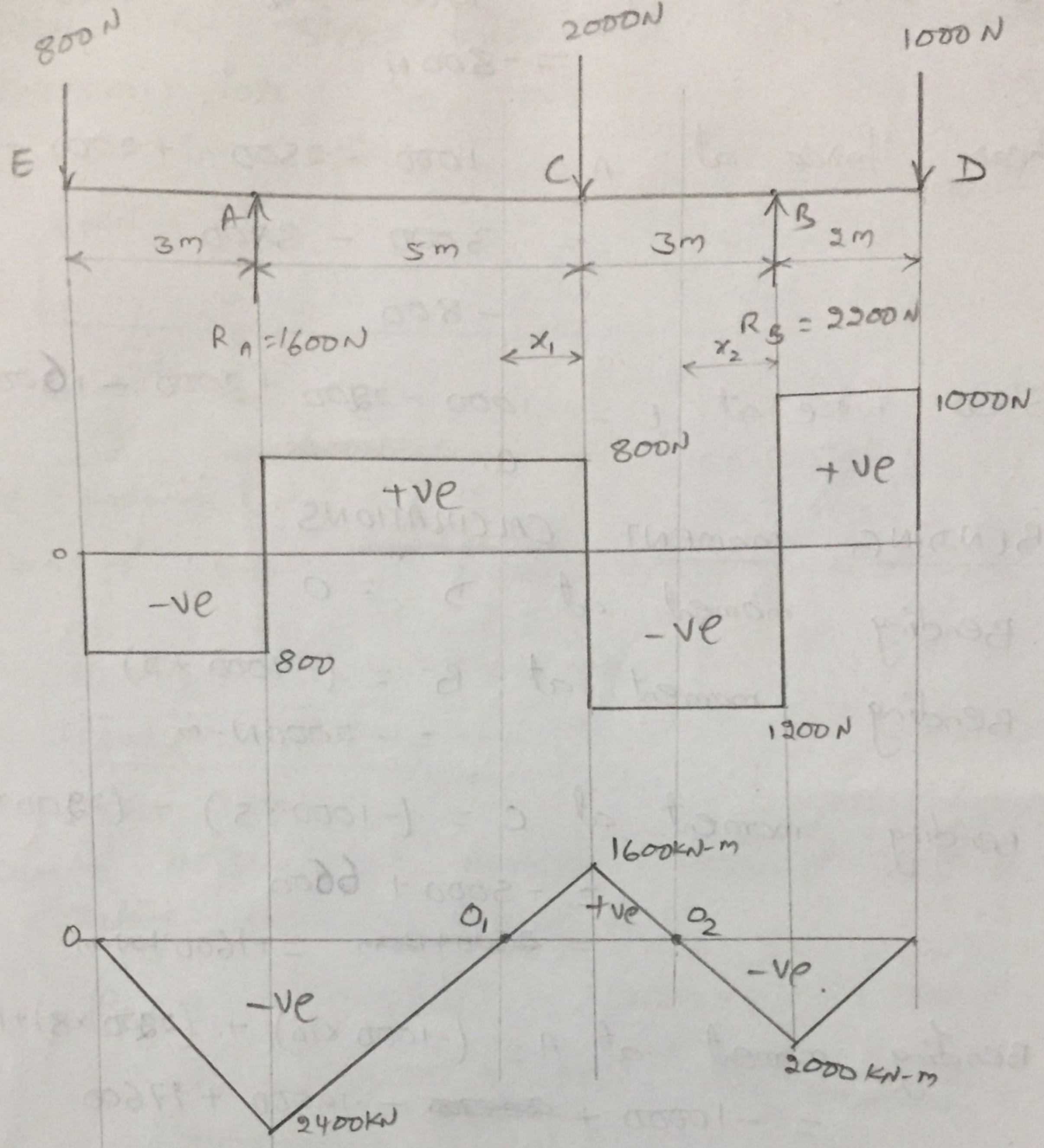
Bending moment at $D = 0$

Bending moment at $B = (-1000 \times 2)$
$= -2000\,N\text{-}m$

Bending moment at $C = (-1000 \times 5) + (2200 \times 3)$
$= -5000 + 6600$
$= \cancel{2800\,KN\text{-}m} = +1600\,KN\text{-}m$

Bending moment at $A = (-1000 \times 10) + (2200 \times 8) + (-2000 \times 5)$
$= -10000 + \cancel{20,000} - 10,000 + 17600$
$= -20,000 + 17600 = -2400\,KN\text{-}m$

Bending moment at $E = (-1000 \times 13) + (2200 \times 11) + (-2000 \times 8) + (1600 \times 3)$
$= -13000 + 24200 - 16000 + 4800$
$= -29000 + 29000$
$= 0.$



Here in the above Bending moment diagram we got two point of Contraflexures. It is shown as O_1 & O_2 in the BMD.

$$BM_{O_1} = [-1000 \times (5+x_1)] + (2200)(3+x_1) + (-2000 \times x_1)$$

$$= -5000 - 1000x_1 + 6600 + 2200x_1 - 2000x_1$$

$$0 = 1600 - 800x_1$$

$$800x_1 = 1600 \Rightarrow x_1 = \frac{1600}{800}$$

$$\boxed{x_1 = 2m}$$

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$$BMO_2 = [-1000 \times (2+x_2)] + (2200 \times x_2)$$

$$= -2000 - 1000x_2 + 2200x_2$$

$$= -2000 + 1200x_2$$

$$2000 = 1200x_2$$

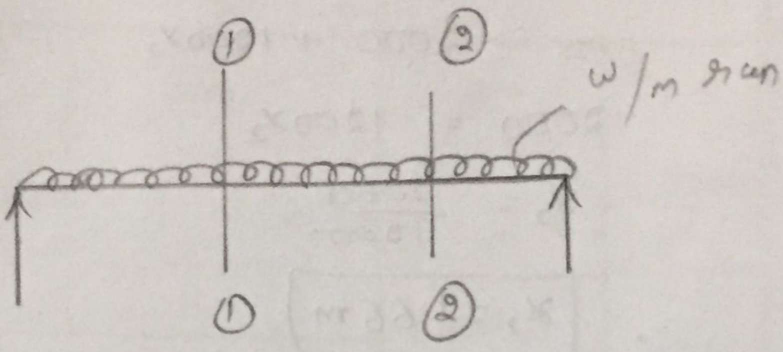
$$x_2 = \frac{2000}{1200}$$

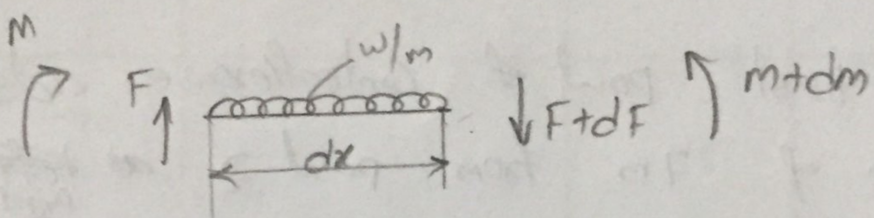
$$\boxed{x_2 = 1.66 m}$$

The first point of Contraflexure acts at a distance of 7m from point D ~~Left~~ Right side. of the beam.

The second point of Contraflexure acts at a distance of 3.66 from point D. Right side.

Relationship between Load, shear force and Bending moment :-





shear force.

$$F - w \cdot dx - (F+dF) = 0$$

$$\cancel{F} - wdx - \cancel{F} - dF = 0$$

$$\boxed{-w = \frac{dF}{dx}}$$

Bending moment :

$$m + F \cdot dx - w \cdot dx \cdot \frac{dx}{2} - (m + dm)$$

$$\cancel{m} + Fdx - \frac{wdx^2}{2} - \cancel{m} - dm$$

neglecting the powers of smaller quantities

$$Fdx - dm = 0$$

$$\boxed{F = \frac{dm}{dx}}$$

PRINCIPAL STRESSES AND PLANES:-

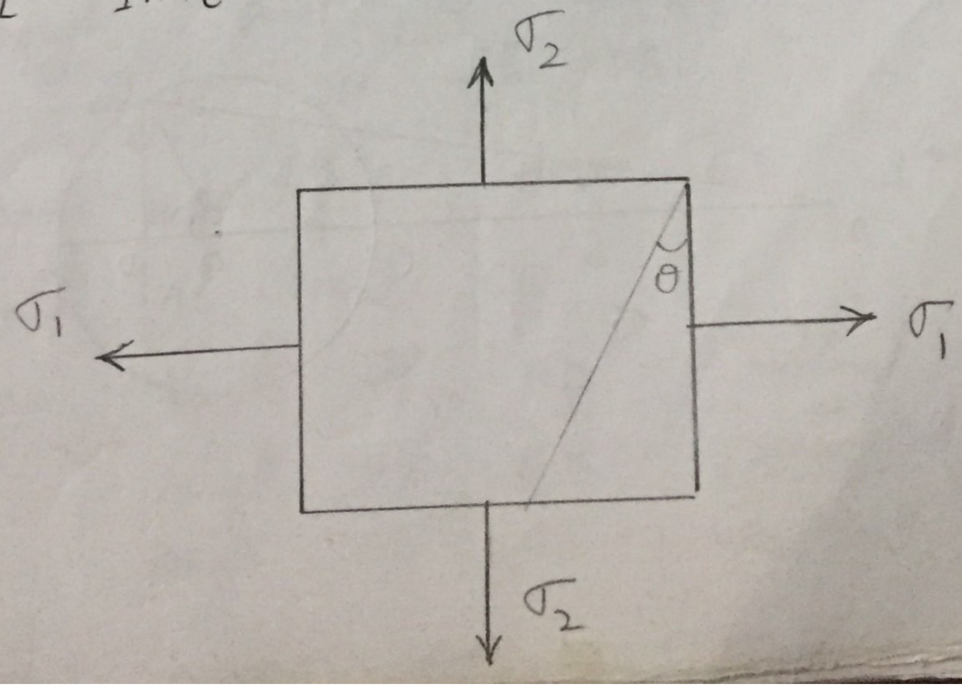
The planes which have no shear stress, are known as principal planes. Hence principal planes are the planes of zero shear stress. These planes carry only normal stresses. The normal stresses acting on a principal plane are known as principal stresses.

MOHR'S CIRCLE:-

mohr's circle is a graphical method of finding normal, tangential and Resultant stresses on an oblique plane. mohr's circle will be drawn for the following cases.

CASE - I

A BODY IS SUBJECTED TO TWO MUTUALLY PERPENDICULAR PRINCIPAL TENSILE STRESSES OF UNEQUAL INTENSITIES :-



PROCEDURE :-

1) Take any point A and draw a horizontal line through point "A".

2) Take $AB = \sigma_1$ and $AC = \sigma_2$ towards right from point A to some suitable scale.

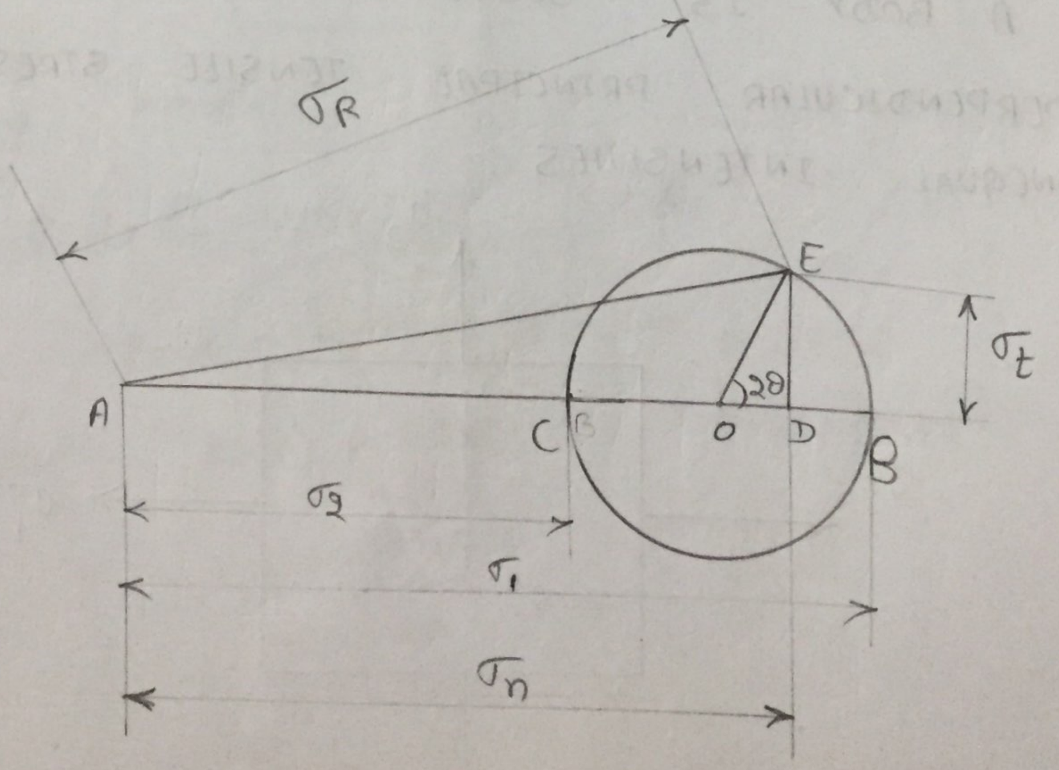
3) with BC as diameter draw a circle.

4) Let "O" is the centre of the circle.

5) Now through point "O" draw a lince "OE" making an angle 2θ with the line "OB".

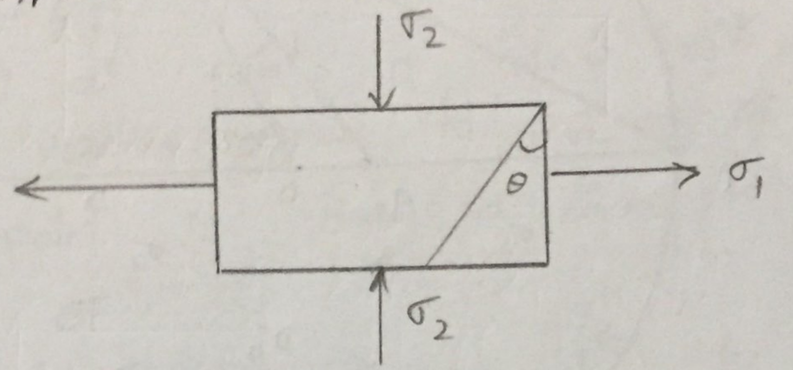
6) From point E draw a line ED i.e $\perp^{lar}$ to AB and join AE.

7) Now length of AD = Normal stress
   length of ED = Tangential stress
   length of AE = Resultant stress.



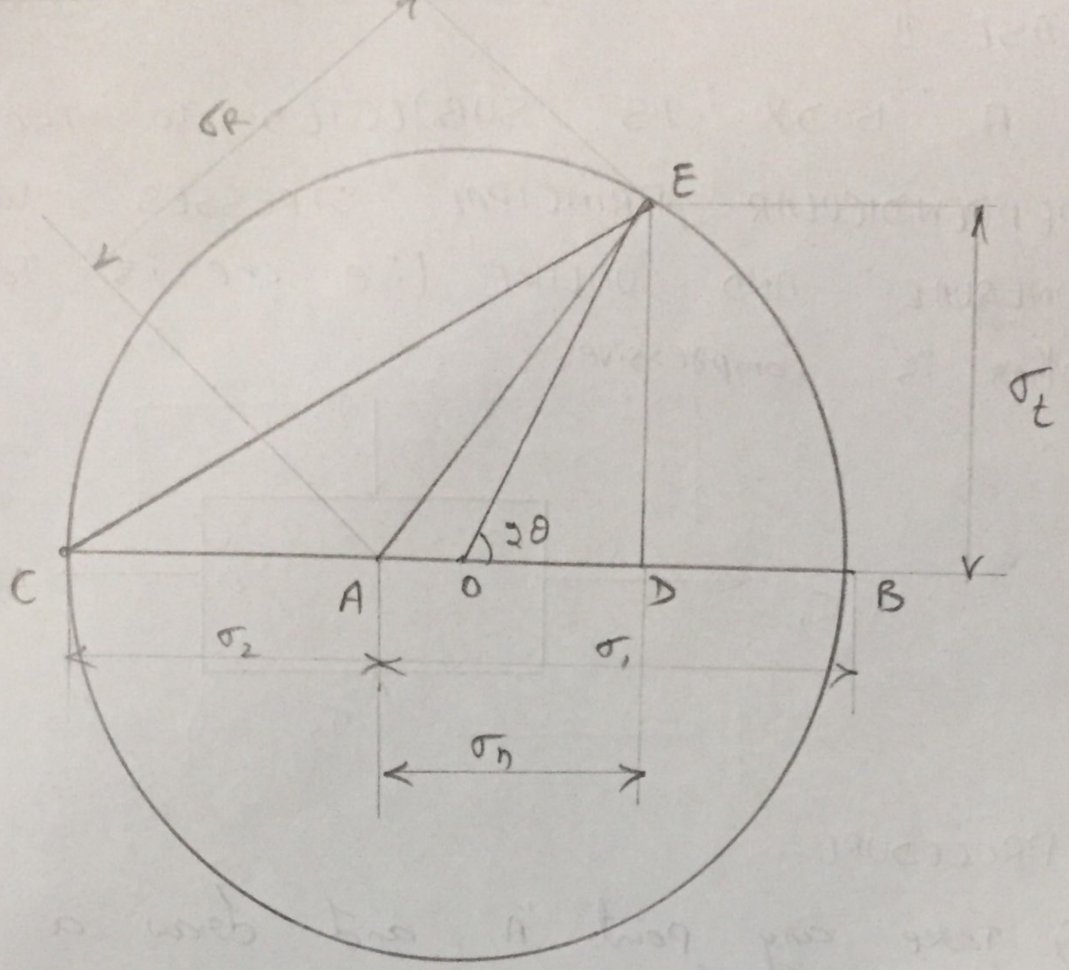
CASE - II

A BODY IS SUBJECTED TO TWO MUTUALLY PERPENDICULAR PRINCIPAL STRESSES WHICH ARE UNEQUAL AND UNLIKE i.e one is Tensile and other is Compressive.



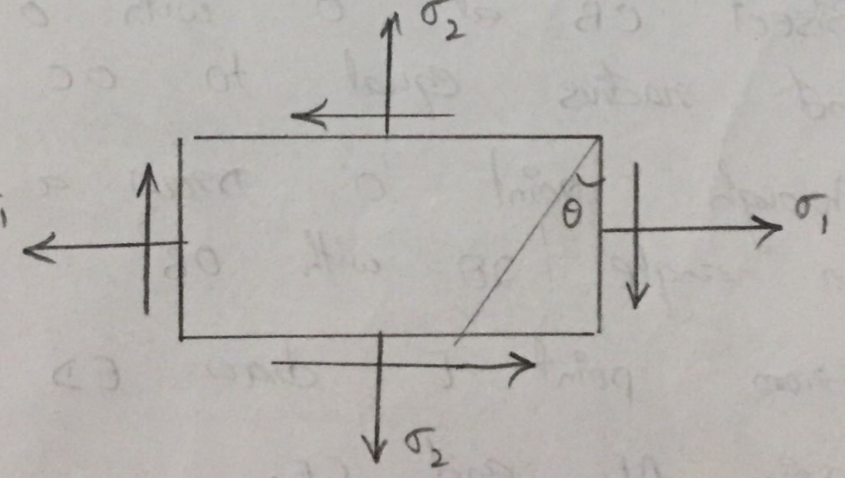
PROCEDURE:-

1) Take any point "A" and draw a horizontal line through point A.
2) Take AB = σ_1 and towards right side of point A and AC = σ_2 towards left side of point A to some suitable scale.
3) Bisect CB at 'O' with O as centre and radius equal to OC (or) OB.
4) Through point 'O' Draw a line OE making an angle 2θ with OB.
5) From point E draw ED $\perp^{lar}$ to AB.
6) Join AE and CE.
7) NOW Length of AD = Normal stress
Length of ED = Tangential stress
Length of AE = Resultant stress



CASE - III

A body is subjected to two mutually perpendicular principal Tensile stresses Accompanied by a simple shear stress.



PROCEDURE:-

1) Take any point 'A' and draw a horizontal line through point 'A'.

2) Take $AB = \sigma_1$ and $AC = \sigma_2$ towards right from 'A' to some suitable scale.

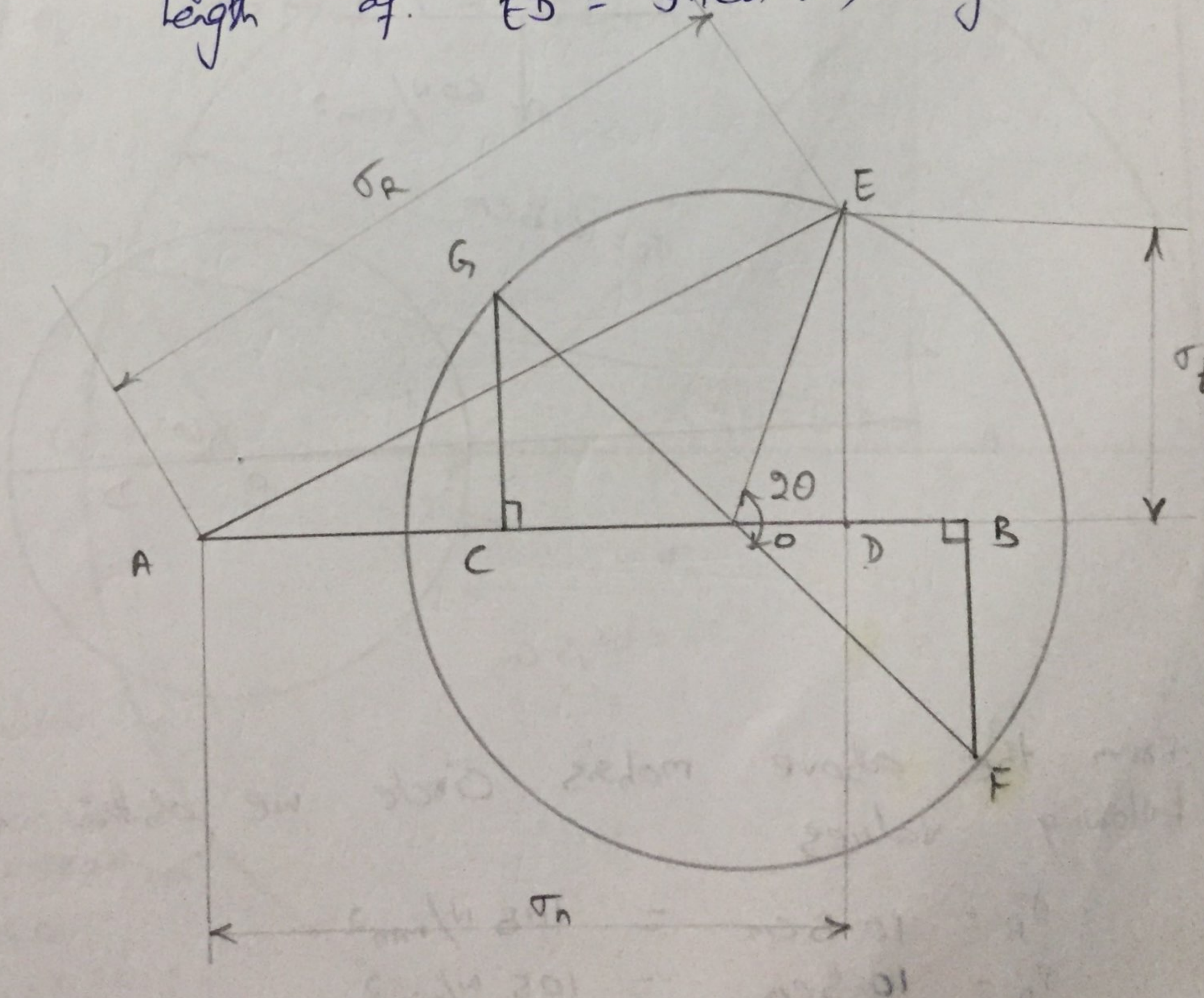
3) Draw $\perp^{lar}$ lines from point B and point C and cut off BF & CG equal to "τ" some suitable scale.

4) Bisect BC at point 'O'. Now with 'O' as centre and radius equal to OG or OF draw a circle.

5) Through point 'O' draw a line making an angle 2θ with OF

6) From point 'E' draw ED $\perp^{lar}$ to CB and Join AE.

7) Now length of AE = Resultant stress
length of AD = Normal stress
Length of ED = Shear (or) Tangential stress.



Q1) The tensile stresses at a point across two mutually perpendicular planes are $120\ N/mm^2$ and $60\ N/mm^2$. Determine the normal, tangential and resultant stresses on a plane inclined at $30°$ to the axis of the minor stress.

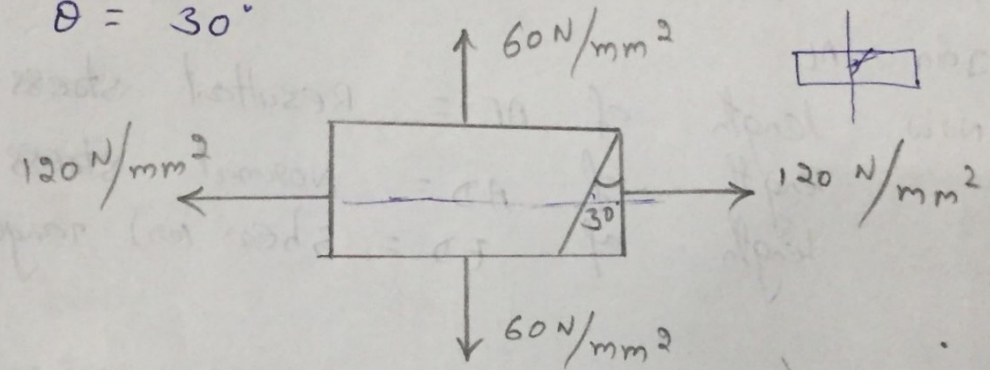
Sol Given data :-

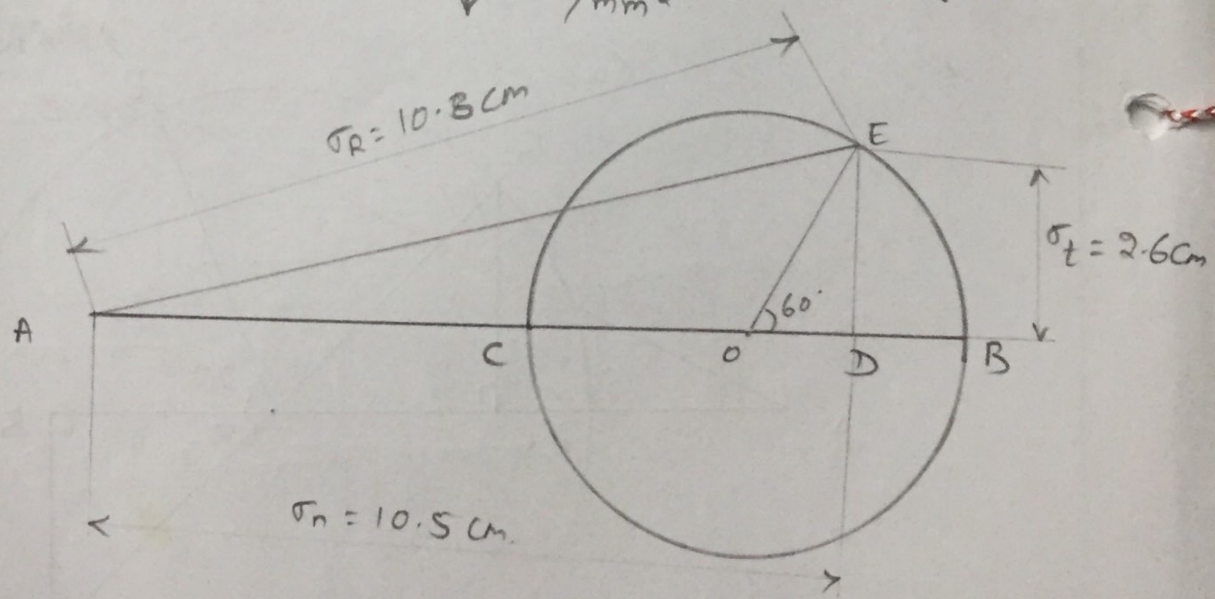
Let $1\ cm = 10\ N/mm^2$

$\sigma_1 = 120\ N/mm^2 = 12\ cm$

$\sigma_2 = 60\ N/mm^2 = 6\ cm$

$\theta = 30°$





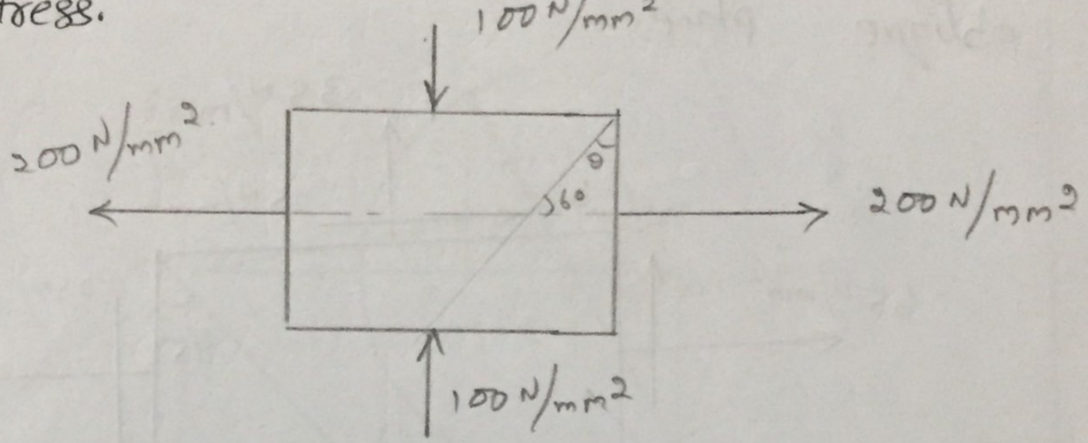
From the above mohr's circle we obtain the following values

$\sigma_R = 10.8\ cm = 108\ N/mm^2$

$\sigma_n = 10.5\ cm = 105\ N/mm^2$

$\sigma_t = 2.6\ cm = 26\ N/mm^2$.

9) The stresses at a point in a bar 200 N/mm² (tensile) and 100 N/mm² (Compressive). Determine Normal, Tangential and resultant stresses on a plane inclined at 60° to the axis of the major stress.

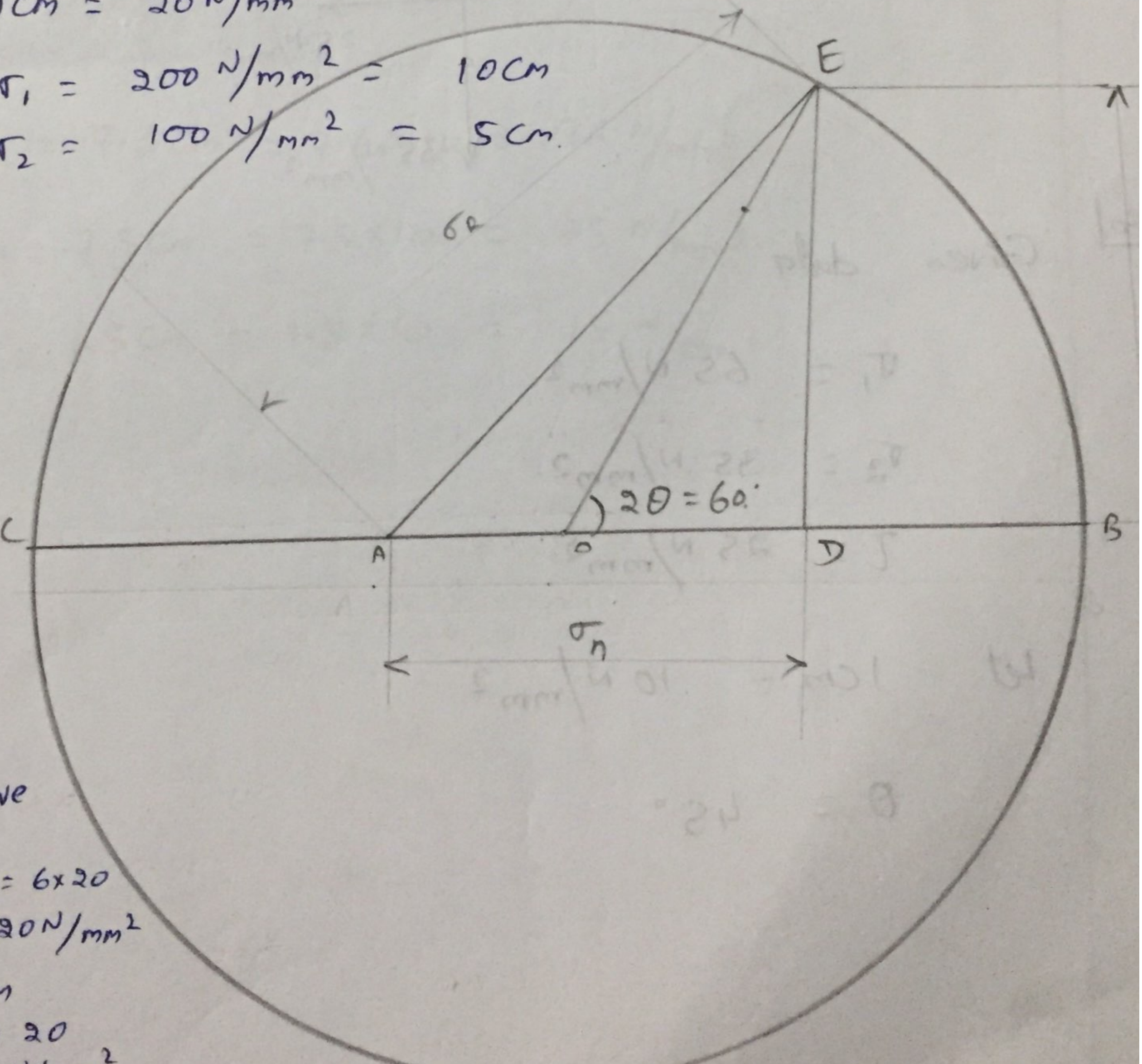


Sol

Let 1 cm = 20 N/mm²

σ_1 = 200 N/mm² = 10 cm

σ_2 = 100 N/mm² = 5 cm.



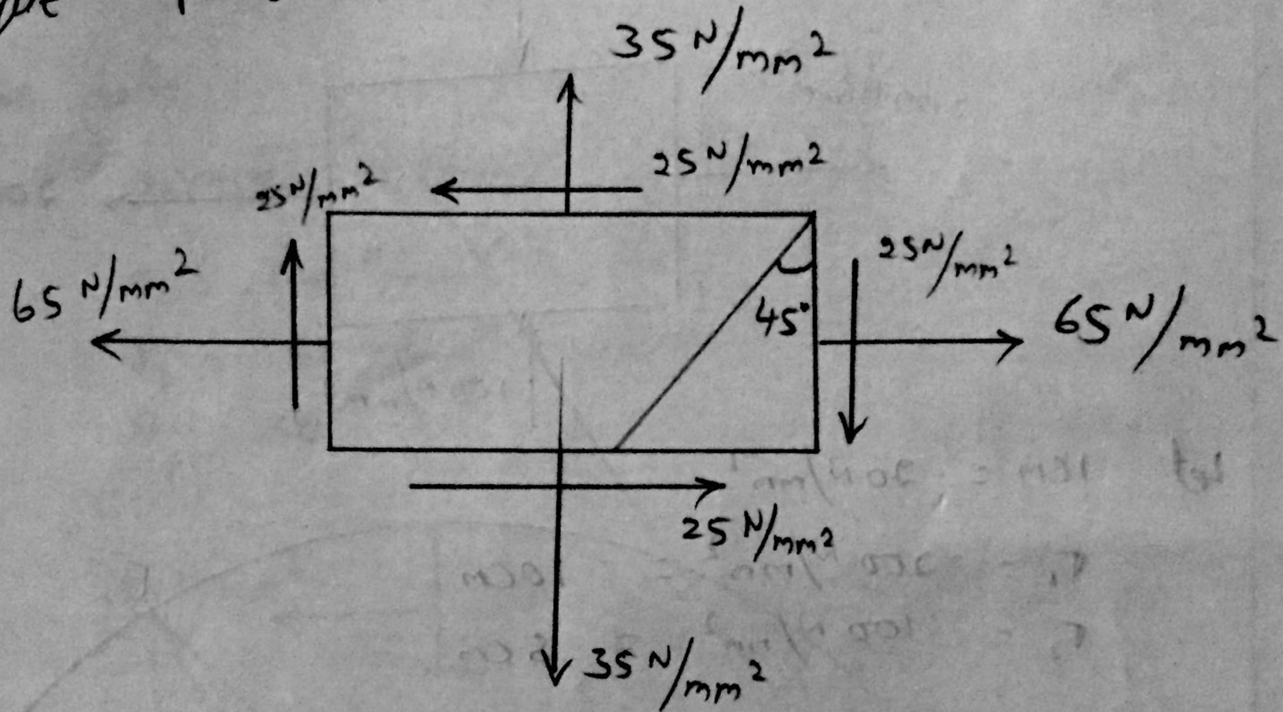
From above

σ_n = 6 cm = 6×20
= 120 N/mm²

σ_t = 6.5 cm
= 6.5 × 20
= 130 N/mm²

9 cm = 9×20 = 180 N/mm²

A point in a strained material is subjected to stresses as shown in below fig. Using Mohr's circle method, determine the normal, ~~and~~ tangential and resultant stresses across the oblique plane.



Given data

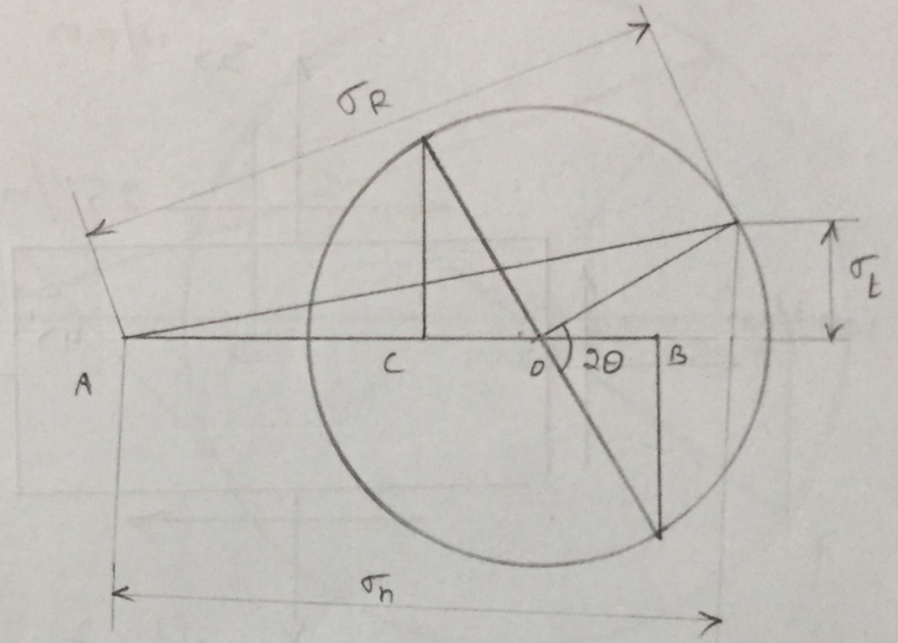
$\sigma_1 = 65\ N/mm^2$

$\sigma_2 = 35\ N/mm^2$

$\tau = 25\ N/mm^2.$

Let $1\ cm = 10\ N/mm^2.$

$\theta = 45°$



$\sigma_R = 7.6\,Cm = 7.6 \times 10 = 76\ N/mm^2$

$\sigma_n = 7.5\,Cm = 7.5 \times 10 = 75\ N/mm^2$

$\sigma_t = 1.5\,Cm = 1.5 \times 10 = 15\ N/mm^2$

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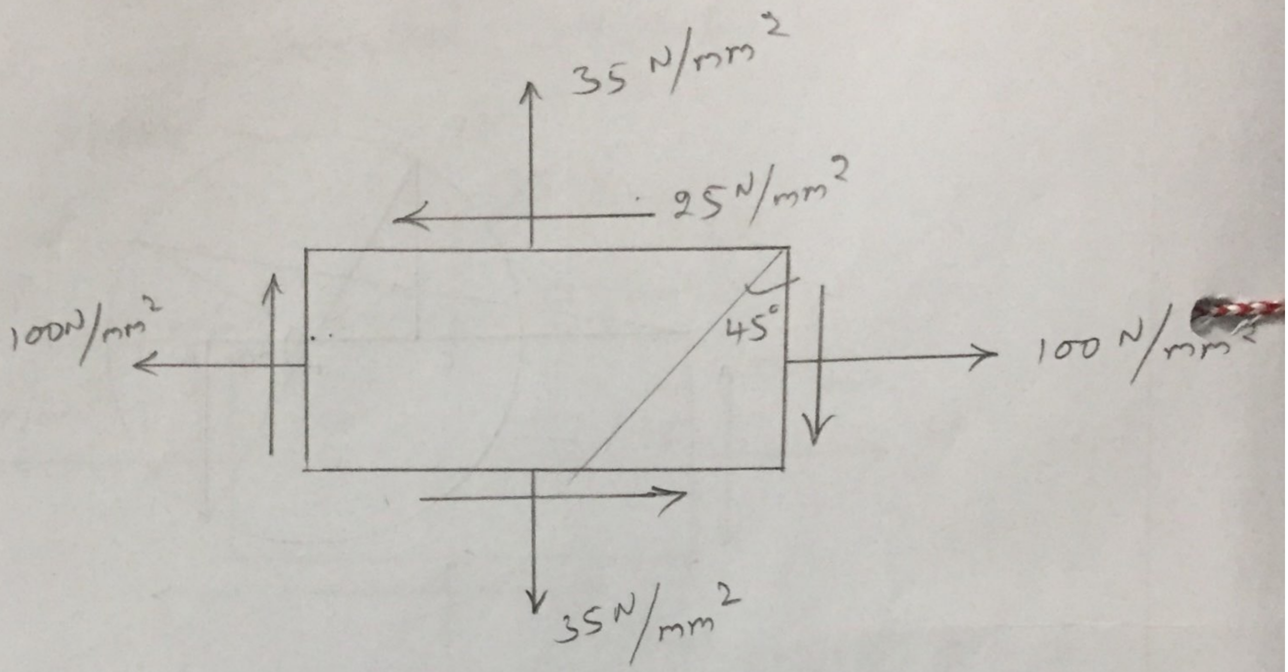
Q) Calculate Normal, Tangential and Resultant stresses on an oblique plane which is 45° to minor stress. as shown in below fig. and also Calculate maximum shear stress.



Sol Given data :-

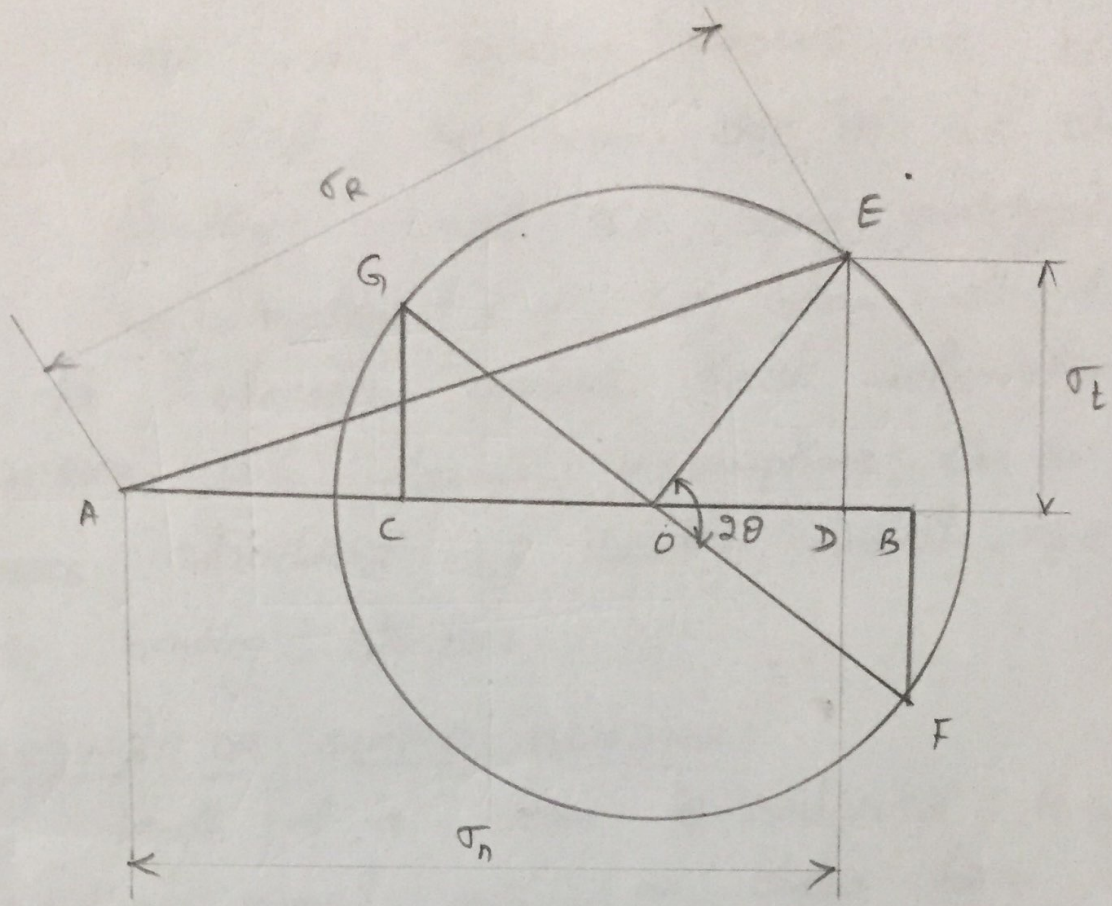
$\sigma_1 = 100 \ N/mm^2$

$\sigma_2 = 35 \ N/mm^2$

$\tau = 25 \ N/mm^2$

$\theta = 45°.$

Let $1cm = 10 \ N/mm^2.$



From the above circle

$\sigma_n = 9.1\ cm = 9.1 \times 10 = 9\cancel{0}\ N/mm^2$

$= 91\ N/mm^2$

$\sigma_t = 3.3\ cm = 3.3 \times 10$

$= 33\ N/mm^2$

$\sigma_R = 9.2\ cm = 9.2 \times 10 = 92\ N/mm^2$

maximum shear stress $= \frac{\sigma_1 - \sigma_2}{2} = \frac{100 - 35}{2} = \frac{65}{2}$

$= 32.5\ N/mm^2$