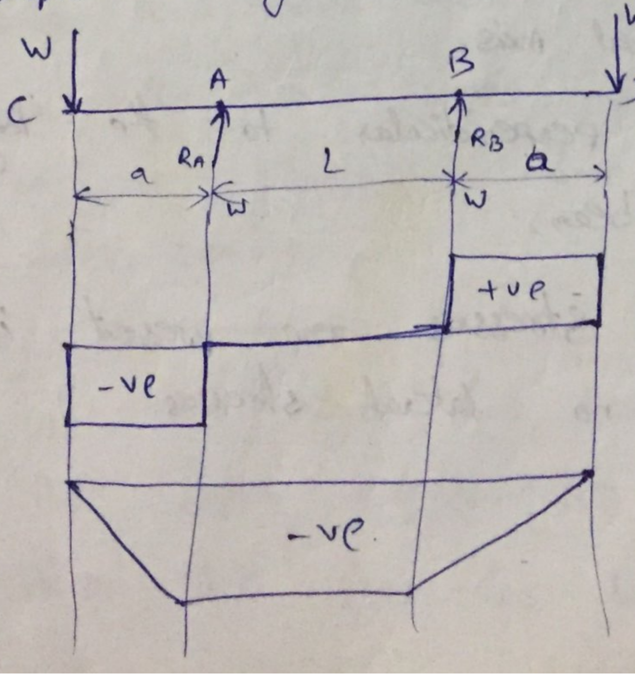
THEORY OF SIMPLE BENDING

Bending stresses:

When some external load acts on a beam, the shear force and bending moments are set up at all sections of the beam. Due to the shear force and bending moment, the beam undergoes some deformation. The material of the beam will offer resistance or stresses against these deformations. These stresses with certain assumptions can be calculated. The stresses introduced by bending moment are known as bending stresses.

PURE BENDING OR SIMPLE BENDING:

The length of a beam is subjected to a constant bending moment and no shear force (i.e zero shear force), then the stresses will be setup in that length of the beam due to B.M. only and that length of the beam is said to be in pure bending (or) simple bending.



$R_A + R_B = w + w$

T.M.A.A

$\circlearrowleft = \circlearrowright$

$w(L+a) = R_B L + w \cdot a$

$wL + \cancel{wa} = R_B \cancel{L} + w \cdot a$

$\boxed{R_B = w}$

$R_A = -R_B + 2w$

$\boxed{R_A = w}$

$BM_D = 0$

$BM_B = -w \times a$

$BM_A = -w(a+L) + wL = -wa$

$BM_C = \cancel{-2w} - 2w(2a+L) + w(L+a) + wa$

$= 0$

ASSUMPTIONS IN THEORY OF SIMPLE BENDING:-

The following assumptions are made in theory of simple bending.

1) The material of the beam is homogeneous (material is same kind throughout) and Isotropic (Elastic properties in all directions are equal).

2) The value of young's modulus of elasticity is same in tension and Compression.

3) The beam is initially straight and all longitudinal filaments bend into circular arcs with a Common Centre of curvature.

4) The radius of curvature is large Compared with the dimensions of the cross section.

5) Each layer of the beam is free to expand or Contract, independently of the layer above or below ~~from~~ to the Neutral Axis.

6) The load acts perpendicular to the longitudinal axis of the beam.

7) only longitudinal stresses are present. in the beam there is no lateral stresses.

Bending stress:-

The internal stress produced in the beam to resist the applied bending moments is known as bending stress. It is denoted by σ_b.

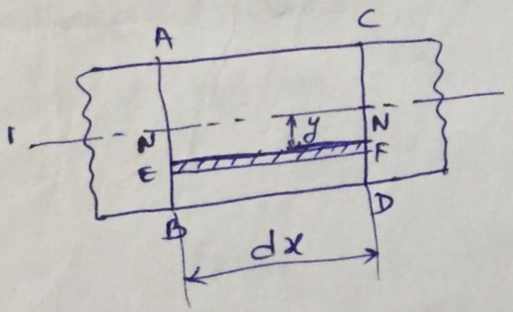
mathematically $\sigma_b = \frac{E}{R} \times y$.

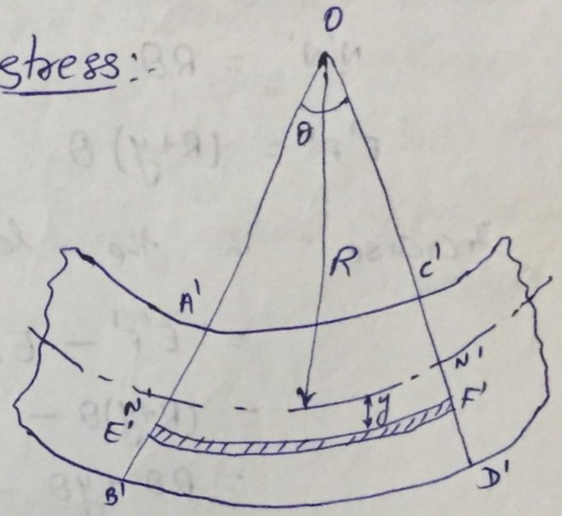
E = Young's modulus of Elasticity.

R = Radius of curvature after bending.

y = Distance of outter most layer from neutral Axis.

Expression for bending stress:-





Consider a small section of beam at a distance 'y' from the neutral axis subjected to a bending moment M. as shown in above fig. before bending and after bending.

The top layer such as AC has deformed to the shape A'C'. This layer has been shortened in its length. The bottom layer BD has deformed to the shape B'D'. This layer has been elongated. At a level b/w the top & bottom of the been, there will be a layer which is neither shortened nor elongated. This layer is known as neutral layer. ~~(ast)~~

From figure.

original length of the neutral Axis (NN) $= \delta x$

After bending the length of Neutral Axis N'N' will remain unchanged. But the length of E'F' will increase.

Hence $N'N' = NN = \delta x$

But from fig (B)

$$N'N' = R\theta$$

$$E'F' = (R+y)\theta$$

Increase in the length of the layer EF

$$= E'F' - EF$$

$$= (R+y)\theta - R\theta$$

$$= \cancel{R\theta} + y\theta - \cancel{R\theta}$$

$$= y\theta$$

strain in layer EF $= \dfrac{\text{Increase in length}}{\text{original length}}$

$$= \frac{y\theta}{R\theta} = \frac{y}{R}$$

we know

modulus of Elasticity $(E) = \dfrac{\text{Stress}}{\text{strain}}$

$$E = \frac{\sigma_b}{\frac{y}{R}} = \frac{R\sigma_b}{y}$$

$$\frac{\sigma_b}{y} = \frac{E}{R}$$

$$\boxed{\sigma_b = \frac{E}{R} \times y}$$

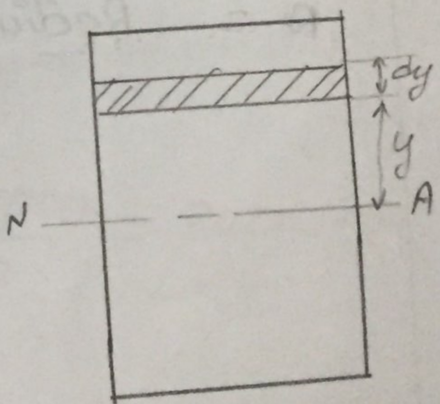
Moment of Resistance :-

Due to the pure bending, the layers above the Neutral axis are subjected to Compressive stress & where as layers below the neutral axis are subjected to tensile stress. due to these stresses the forces will be acting on the layers. these forces have a moment about the N.A. for a section is known as moment of resistance of that section.

Force = stress × Area

$$= \frac{E}{R} \times y \times dA$$

moment = Force × $\perp$lar distance



$$= \frac{E}{R} \times y \times dA \times y$$

$$= \frac{E}{R} y^2 dA$$

Total moment $= \int m$

$$= \int \frac{E}{R} y^2 dA$$

$$= \frac{E}{R} \int y^2 dA$$

$$\boxed{\therefore I = y^2 dA}$$

$$M = \frac{E}{R} \times I$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}} \longrightarrow \text{Bending equation}$$

The above equation is known as Bending equation

where

M = Maximum bending moment (N - mm)

I = Moment of Inertia (mm^4)

σ = Bending stress (N/mm^2)

y = Distance from the neutral axis to the outermost layer (mm)

E = Young's modulus (N/mm^2)

R = Radius of Curvature (mm)

Q) A steel plate of width 120mm and of thickness 20mm is bent into a circular arc of radius 10m. Determine the maximum stress induced and the bending moment which will produce the maximum stress. Take $E = 2\times10^5$ N/mm².

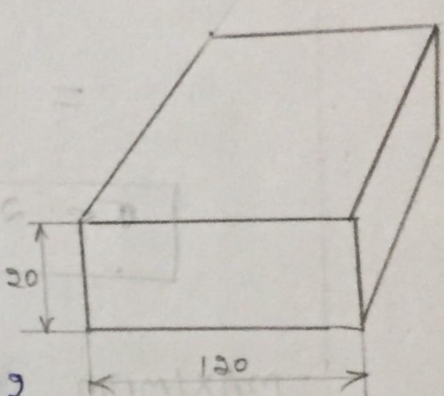
Sol Given data ::

width of steel plate (b) = 120mm

thickness of plate (t) = 20mm

Radius of curvature (R) = 10m

modulus of Elasticity (E) = 2×10^5 N/mm².



we know

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$M = \frac{E}{R} \times I$$

$$I = \frac{bt^3}{12} = \frac{120\times(20)^3}{12} = \frac{960000}{12} = 80{,}000\ mm^4$$

$$M = \frac{2\times10^5}{10\times1000} \times 80{,}000$$

$$= \frac{1.6\times10^{10}}{1\times10^4} = 1600000\ N\text{-}mm = 1600\ N\text{-}m$$

$$\boxed{M = 1.6\ KN\text{-}m}$$

we Also know

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$y = \frac{20}{2} = 10\,mm.$$

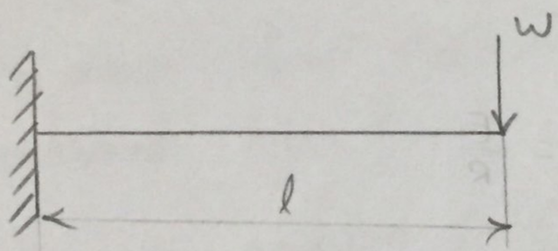
$$\sigma = \frac{2\times10^5}{10\times10^3} \times 10$$

$$= \frac{2\times10^6}{1\times10^4}$$

$$\boxed{\sigma = 200\ N/mm^2.}$$

MAXIMUM BENDING MOMENT FOR DIFFERENT BEAMS:-

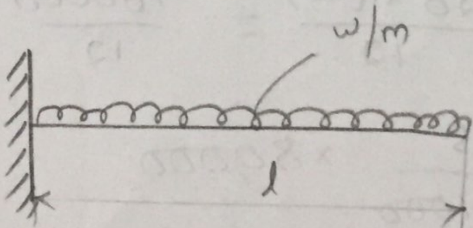
1)



max. bending moment = $w \times l$.

$$\boxed{M = w.l}$$

2)

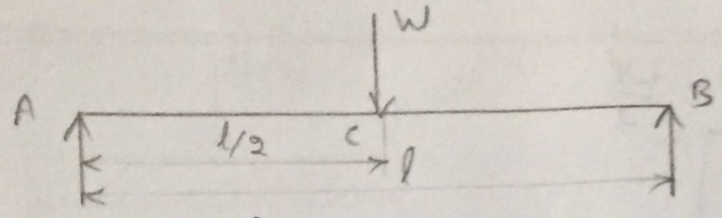


load $\quad$ 1^{lar} distance

maximum bending moment $(M) = (w \times l) \times \frac{l}{2}$

$$\boxed{M = \frac{wl^2}{2}}$$

3)



Sum of upward forces = Sum of downward forces

$R_A + R_B = W$

Sum of Anticlock wise moments = Sum of clockwise moments

Taking moments about A

$R_B \times l = W \times l/2$

$\boxed{R_B = \frac{W}{2}}$

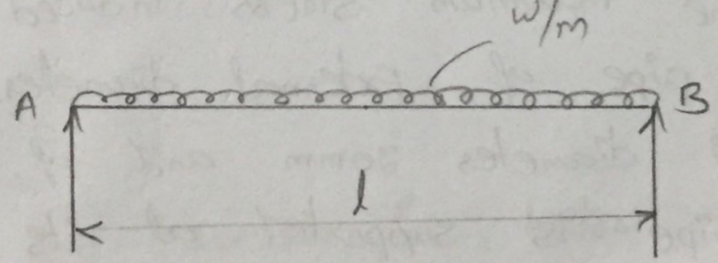
$R_A = W - \frac{W}{2}$

$\boxed{R_A = \frac{W}{2}}$

maximum Bending moment (m) = $\frac{W}{2} \times \frac{l}{2}$

$\boxed{m = \frac{Wl}{4}}$

4)



↑ forces = ↓ forces

$R_A + R_B = w \times l$

↺ moments = ↻ moments

$R_B \times l = w \times l \times \frac{l}{2}$

$R_B = \frac{w l^{\cancel{2}}}{2 \cancel{l}}$

$\boxed{R_B = wl/2}$

$R_A = wl - \frac{wl}{2}$

$$\boxed{R_A = \frac{wl}{2}}$$

maximum bending moment $(m) = R_B \times \frac{l}{2} - \left(\frac{w \times l}{2}\right) \times \frac{\frac{l}{2}}{2}$

$= \frac{wl}{2} \times \frac{l}{2} - \frac{wl}{2} \times \frac{l}{4}$

$= \frac{wl^2}{4} - \frac{wl^2}{8}$

$= \frac{8wl^2 - 4wl^2}{32}$

$= \frac{\cancel{4}^{1} wl^2}{\cancel{32}_{8}}$

$$\boxed{m = \frac{wl^2}{8}}$$

Q) Calculate the maximum stress induced in a cast iron pipe of External diameter 40mm, and internal diameter 20mm and of length 4m when the pipe is supported at its ends and carries a point load of 80N at its centre.

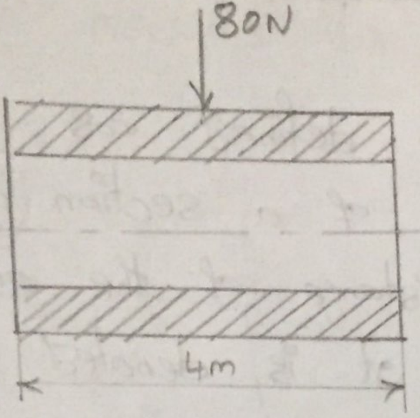
Sol Given data :-

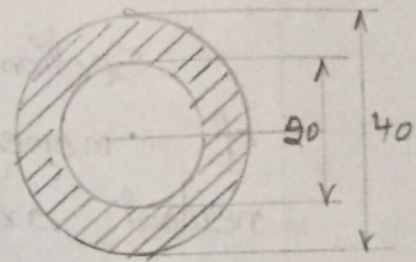
External diameter $(D) = 40mm$

Internal diameter $(d) = 20mm$

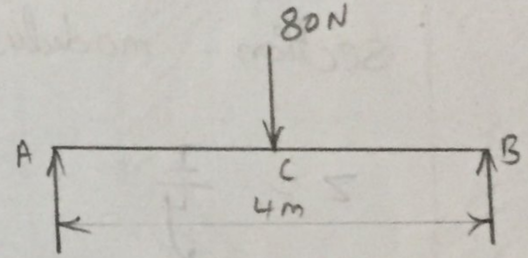
length of pipe $(l) = 4m = 4000mm$

point Load $(w) = 80N$





In the above problem the given beam it is in simply supported in nature and the point load is acting at the centre of the beam.



we know the maximum bending moment for S.S.B carrying point load at the centre is

$$m = \frac{wl}{4} = \frac{80 \times 4000}{4}$$

$$= 80000 \text{ N-mm}$$

$$\frac{m}{I} = \frac{\sigma}{y}$$

moment of Inertia for hollow circular section is given by

$$I = \frac{\pi(D^4 - d^4)}{64} = \frac{\pi(40^4 - 20^4)}{64}$$

$$= \frac{7539822.36}{64} = 117809.72 \text{ mm}^4$$

$$y = \frac{40}{2} = 20 \text{ mm}$$

$$\sigma = \frac{m}{I} \times y \Rightarrow \frac{80000 \times 20}{117809.72}$$

$$\boxed{\sigma = 13.58 \text{ N/mm}^2}$$

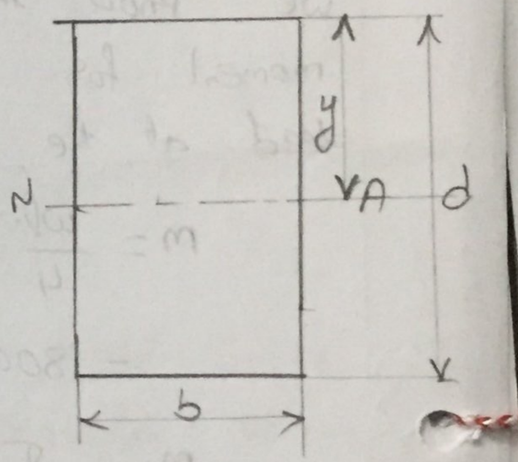
SECTION MODULUS :-

Section modulus is defined as the ratio of moment of Inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis. It is denoted by 'Z'

Hence mathematically.

$$\text{Section modulus } (Z) = \frac{I}{y} =$$

Section modulus for Rectangular section :-

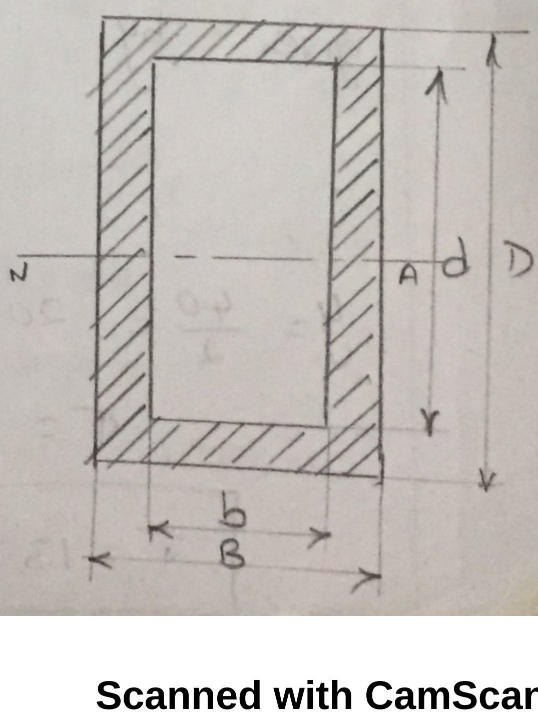


$$Z = \frac{I}{y}$$

$$= \frac{\frac{bd^3}{12}}{\frac{d}{2}} = \frac{b d^{\cancel{3}}}{\cancel{12}_{6}} \times \frac{\cancel{2}^{1}}{\cancel{d}}$$

$$\boxed{Z = \frac{bd^2}{6}}$$

Section modulus for Hollow Rectangular section :-



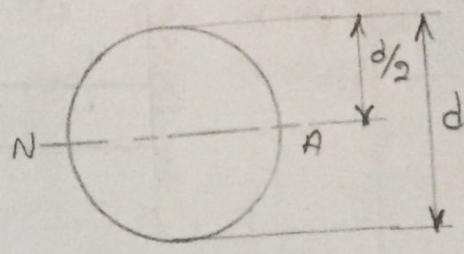
$$Z = \frac{I}{y} = \frac{\frac{BD^3}{12} - \frac{bd^3}{12}}{\frac{D}{2}}$$

$$\frac{BD^3 - bd^3}{\cancel{12}_{6}} \times \frac{\cancel{2}^{1}}{D}$$

$$\boxed{Z = \frac{BD^3 - bd^3}{6D}}$$

Section modulus for circular cross - section :-

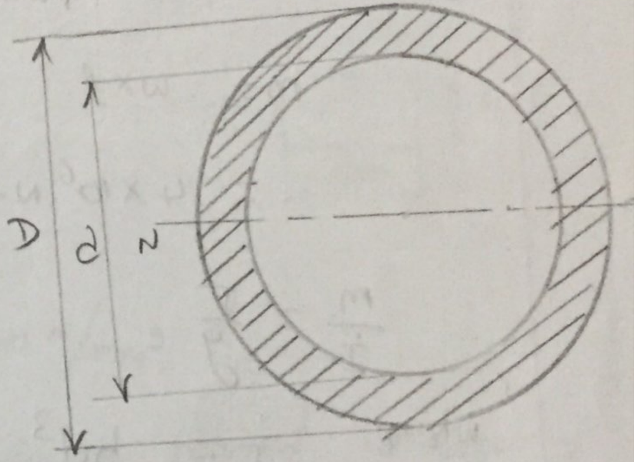
$$Z = \frac{I}{y} = \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^{\cancel{4}\,3}}{\cancel{64}_{32}} \times \frac{2^1}{\cancel{d}}$$



$$\boxed{Z = \frac{\pi d^3}{32}}$$

Section modulus for Hollow circular section :-

$$Z = \frac{I}{y} = \frac{\frac{\pi D^4}{64} - \frac{\pi d^4}{64}}{\frac{D}{2}}$$



$$\frac{\pi (D^4 - d^4)}{\cancel{64}_{32}} \times \frac{2^1}{D}$$

$$\boxed{Z = \frac{\pi (D^4 - d^4)}{32 D}}$$

Q) A Cantilever of length 2m fails when a load of 2KN is applied at the free end. If the section of the beam is 40mm x 60mm, find the stress at the failure.

Sol Given data :-

length (l) = 2m = 2000mm

point Load (W) = 2KN = 2000N

breadth (b) = 40mm

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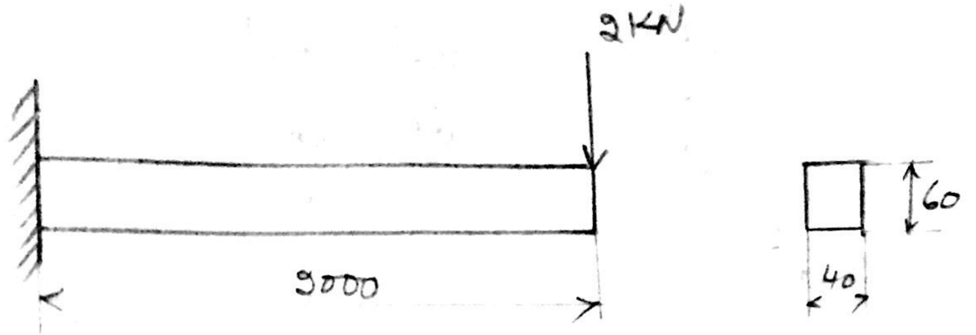
Depth (d) = 60mm



we know maximum bending moment for Cantilever beam when the point load is acting at free end

$$m = w \times l = (2 \times 10^3)(2000)$$

$$= 4 \times 10^6 \text{ N-mm}$$

$$\frac{m}{I} = \frac{\sigma}{y}$$

where $I = \frac{bd^3}{12} = \frac{40(60)^3}{12} = 720000 \text{ mm}^4$

$$y = \frac{60}{2} = 30\text{mm}$$

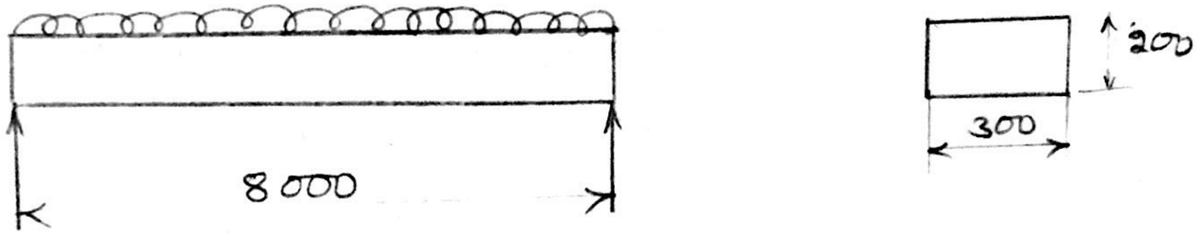
$$\sigma = \frac{m}{I} \times y \Rightarrow \frac{4 \times 10^6 \times 30}{720000}$$

$$= \frac{120000000}{720000}$$

$$\boxed{\sigma = 166.66 \text{ N/mm}^2}$$

Q) A rectangular beam 200mm deep and 300mm wide is simply supported over a span of 8 m. What uniformly distributed load per meter the beam may carry. If the bending stress is not to exceed 120 N/mm².

Sol Given data:



Bending stress (σ_b) = 120 N/mm².

we know the maximum bending moment in case of simply supported beam carries point load it is given by

$$m = \frac{wl^2}{8} = \frac{w(8000)^2}{8} = \frac{64000000\,w}{8}$$

$$m = 8 \times 10^6 \omega.$$

we also know

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$I = \frac{bd^3}{12} = \frac{300(200)^3}{12} = 2 \times 10^8 \text{ mm}^4$$

$$y = \frac{200}{2} = 100 \text{ mm}$$

$$m = \frac{\sigma}{y} \times I$$

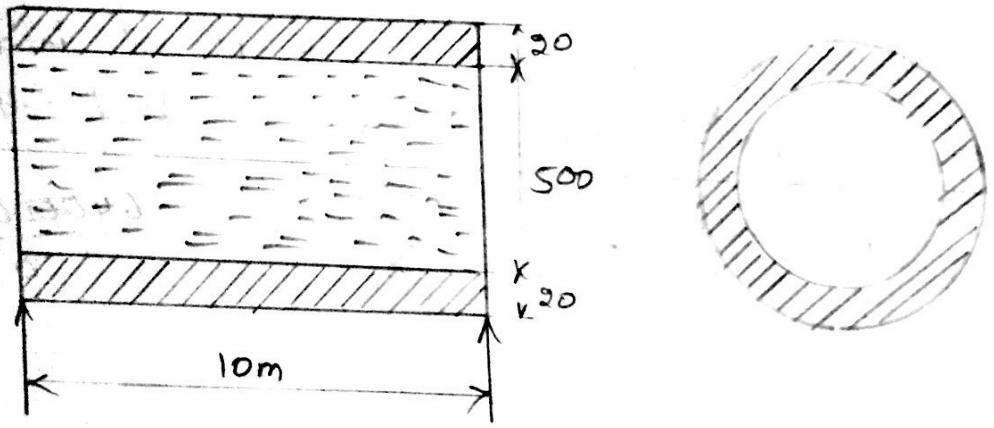
$$8\times10^6\,\omega = \frac{120}{100} \times 2\times10^8$$

$$= \frac{2.4\times10^{10}}{8\times10^6\times100} = \frac{2.4\times\cancel{10^{10}}^{\,2}}{8\times\cancel{10^8}} = \frac{240}{8}$$

$$\boxed{\omega = 30\ N/mm}$$

Q) A water main of 500mm internal diameter and 20mm thick is running full. The water main is of cast iron and is supported at two points 10m apart. Find the maximum stress in the metal. The cast iron and water weigh 72000 N/m^3 & 10,000 N/m^3 resp.

Sol Given data.



Density of cast iron $(\rho) = 72000\ N/m^3$

Density of water $(\rho) = 10,000\ N/m^3$

The above mentioned beam it is simply supported carries with uniformly distribut load in nature. ~~beam~~ because the water is flowing throughout the legth of the beam. For this condition we know

$$m = \frac{wl^2}{8}$$

But in the above eqn we don't know the value of "m" as well as value of "w" ~~the~~ but we have the parameters for calculating w.

we know

$$\text{weight density} = \frac{\text{weight}}{\text{volume}}$$

$$\text{weight} = \text{weight density} \times \text{volume}$$

$$W_{water} = 10000 \times (\text{Area} \times \text{length})$$

$$= 10,000 \times \left[\frac{\pi}{4}(0.5)^2 \times 1\right]$$ ∴ Here consider per meter length only.

$$= 10,000 \times 0.1963$$

$$\boxed{W_{water} = 1963.49\ N}$$

$$\text{weight of Cast iron pipe} = \rho \times \text{volume}$$

$$\cancel{= 72000 \times \left(\frac{\pi}{4}(0.54)^2 \times 1\right)}$$

$$\cancel{= 72000 \times 0.2290}$$

$$\cancel{\boxed{W_{C.I} = 16489.59\ N.}}$$

$$= 72000 \left[\frac{\pi}{4}\left(0.54^2 - 0.5^2\right) \times 1\right]$$

$$= 72000 \left[\frac{\pi}{4}(0.0416)\right]$$

$$= 72000\,(0.0326)$$

$$\boxed{W_{pipe} = 2352.42\ N}$$

Total weight of the pipe = weight of water + wt. of C.I pipe

$= 1963.49 + 2352.42$

$$\boxed{W_{Total} = 4315.91 \text{ N}}$$

we know.

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$I = \frac{\pi (D^4 - d^4)}{64} = \cancel{\pi (540^4)}$$

$$= \frac{\pi (540^4 - 500^4)}{64} = \frac{7.0781 \times 10^{10}}{64}$$

$$I = 1105966278 \text{ mm}^4$$

$$y = \frac{540}{2} = 270 \text{mm}.$$

maximum bending moment for the above mentioned beam

$$M = \frac{wl^2}{8} = \frac{4315.91 (10)^2}{8}$$

$\therefore w$ is in N/m run

$$= \frac{431591}{8} = 53948.87 \text{ N-m}$$

$$= 53948.87 \times 10^3 \text{ N-mm}$$

$$\sigma = \frac{M}{I} \times y = \frac{53948.87 \times 10^3 \times 270}{1105966278} = \frac{1.45 \times 10^{10}}{1105966278}$$

$$\boxed{\sigma = 13.17 \text{ N/mm}^2}$$

Q) A timber beam of rectangular section of length 8m is simply supported. The beam carries a U.D.L of 12 KN/m given over the entire length and a point load of 10KN at 3m from left support. If the depth is two times the width and the stress in the timber is not to exceed $8 \, N/mm^2$. Find the suitable dimensions of the section.

Sol Given data :-

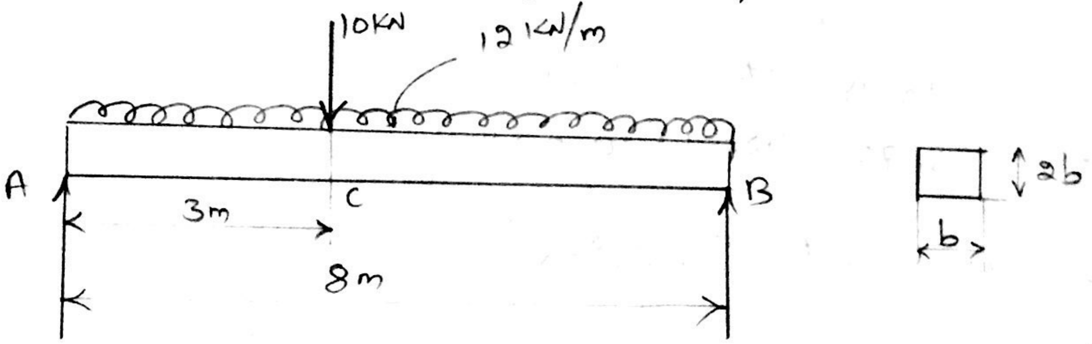
Length of the beam $(l) = 8m$

uniformly distributed Load$(w) = 12 \, KN/m$

point Load $(W) = 10KN$

Depth $(d) = 2b$

stress $(\sigma) = 8 \, N/mm^2$.



we know

$\uparrow$ forces $=$ $\downarrow$ forces

$$R_A + R_B = (12 \times 8) + 10$$

$$= 96 + 10$$

$$R_A + R_B = 106 \, KN \quad \text{—} \; (1)$$

Sum of anticlockwise moments = sum of clockwise moments

Taking moments of A.

$R_B \times 8 = (10 \times 3) + (12 \times 8)\left(\frac{8}{2}\right)$

$8R_B = 30 + 384$

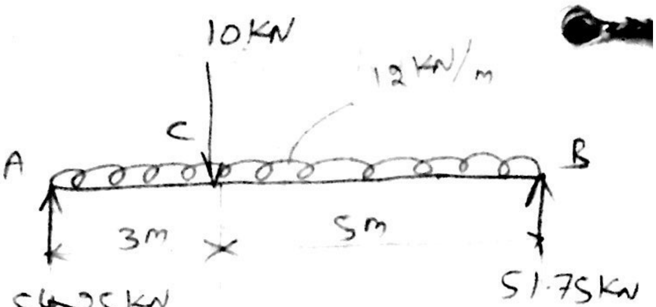
$R_B = 414/8$

$\boxed{R_B = 51.75\,KN}$ — (2)

substitute R_B value in eqn (1) to get R_A

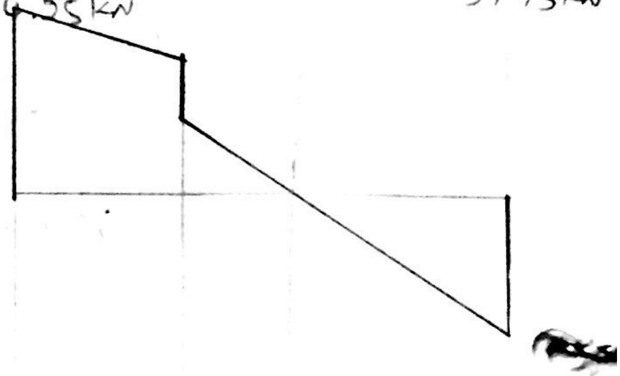
$R_A = 106 - 51.75$

$\boxed{R_A = 54.25\,KN}$



Shear force Calculations:

$SF_B = -51.75\,KN$



$SF_C = -51.75 + (12 \times 5)$

$= 8.25\,KN$ without point Load.

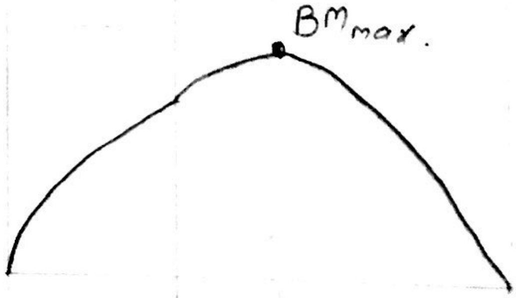
$= 8.25 + 10$

$= 18.25$ with point Load.

$SF_A = -51.75 + (12 \times 8) + 10 + 54.25$

$= 0$.

Bending moment Calculations:-



$BM_B = 0$

$BM_C = (51.75 \times 5) - (12 \times 5) \times \frac{5}{2}$

$= 258.75 - 150$

$= 108.75$

$BM_A = 0$

whenever the shear force is zero there should be max. bending moment.

$$BM_{max} = (51.75 \times x) - (\overset{6}{\cancel{12}} \times x)\left(\frac{x}{\cancel{2}}\right)$$

$$SF_x = 0$$

$$0 = -51.75 + (12 \times x)$$

$$12x = +51.75$$

$$x = \frac{51.75}{12}$$

$$\boxed{x = 4.3125\,m}$$

$$= (51.75 \times 4.3125) - 6\,(4.3125)^2$$

$$= 223.1718 - 111.5859$$

$$\boxed{BM_{max} = 111.5858\ KN\text{-}m}$$

we know

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\boxed{\therefore d = 2b}$$

$$\frac{111.5858 \times 10^3 \times 10^3}{\frac{bd^3}{12}} = \frac{8}{\left(\frac{d}{2}\right)}$$

$$\frac{111.5858 \times 10^6 \times 12}{b\,(2b)^3} = \frac{\cancel{2} \times 8}{\cancel{2}b}$$

$$\frac{1339030350}{8b^4} = \frac{8}{b}$$

$$\frac{1339030350}{64} = \frac{b^{\cancel{4}\,3}}{\cancel{b}}$$

$$b = \sqrt[3]{20922349.22}$$

$$\boxed{b = 275.55\ mm}$$

$$d = 2b = 2 \times 275.55$$

$$\boxed{d = 551.10\,mm}$$

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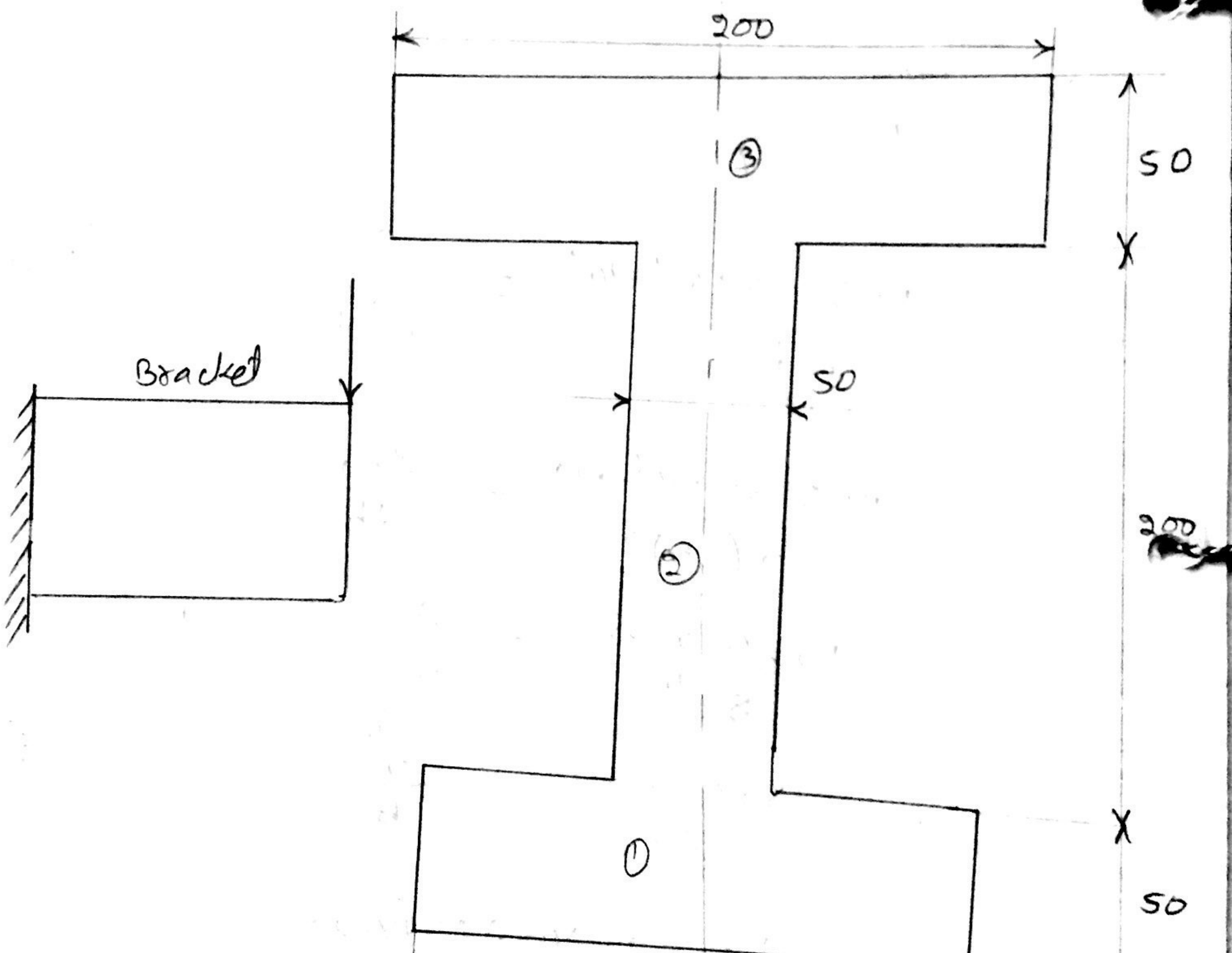
A Cast iron bracket subjected to bending & it has the cross-section of I-form with unequal flanges. The dimensions of the section are shown in fig. Find the position of the neutral axis and moment of inertia of the section about the neutral axis. If the maximum bending moment on the section is 40MN-mm, Determine the maximum bending stress. what is the nature of the stress.



We know

$$\frac{M}{I} = \frac{\sigma}{y}$$

Here except M value all values are unknown.

For Calculating moment of Inertial value first we have to Calculate $\bar{y}$.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$A_1 = 130 \times 50 = 6500\,mm^2$; $y_1 = \frac{50}{2} = 25\,mm$

$A_2 = 200 \times 50 = 10{,}000\,mm^2$; $y_2 = 50 + \frac{200}{2} = 150\,mm$

$A_3 = 200 \times 50 = 10{,}000\,mm^2$; $y_3 = 200 + 50 + \frac{50}{2} = 275\,mm$.

$$\bar{y} = \frac{(6500 \times 25) + (10{,}000 \times 150) + (10{,}000 \times 275)}{6500 + 10000 + 10000}$$

$$= \frac{162500 + 1500000 + 2750000}{26500} = \frac{4412500}{26500}$$

$\boxed{\bar{y} = 166.5\,mm}$ → from bottom.

$\boxed{I = I_{GG} + Ah^2}$ $\boxed{\therefore h = y - \bar{y}}$

$$I_1 = I_{GG_1} + A_1 h_1^2$$

$$= \frac{b_1 d_1^3}{12} + A_1 (y_1 - \bar{y})^2$$

$$= \frac{(130)(50)^3}{12} + 6500(25 - 166.5)^2$$

$$= 1354166.667 + 130144625$$

$$I_1 = 131498791.7\,mm^4$$

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$$I_2 = I_{GG_2} + A_2 h_2^2$$

$$= \frac{50(200)^3}{12} + (10{,}000)(150 - 166.5)^2.$$

$$= 33333333.33 + 2722500$$

$$I_2 = 36055833.33 \text{ mm}^4$$

$$I_3 = I_{GG_3} + A_3 h_3^2$$

$$= \frac{200(50)^3}{12} + (10000)(275 - 166.5)^2$$

$$= 2083333.33 + 117722500$$

$$I_3 = 119805833.3 \text{ mm}^4$$

Total moment of Inertia $(I) = I_1 + I_2 + I_3$

$$= 131498791.7 + 36055833.33 + 119805833.3$$

$$I = 287360458.4 \text{ mm}^4.$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \times y \Rightarrow \frac{40 \times 10^6 \times 166.5}{287360458.4} = \frac{666 \times 10^7}{287360458.4}$$

$$\boxed{\sigma = 23.17 \text{ N/mm}^2}$$

The above stress will be ~~Tensile~~ Compressive because in the Cantilever beam the upper layers are subjected to Tensile whereas the bottom layers are subjected to Compressive stress.

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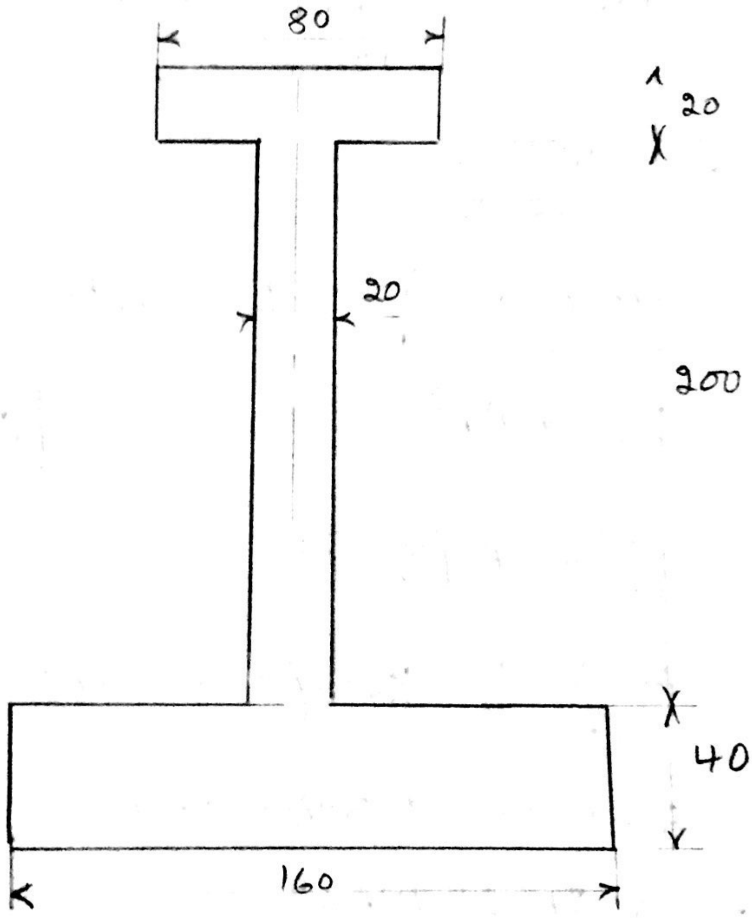
Q) A Cast iron beam is of I- section as shown in fig. The beam is simply supported on a span of 5m. If the tensile stress is not to exceed $20\ N/mm^2$, find the safe uniformly load which the beam can carry. Find also the maximum Compressive stress.

sol

length of the beam (L) = 5m

Tensile stress $(\sigma_t) = 20\ N/mm^2$



we know

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$A_1 = 160 \times 40 = 6400\,mm^2$; $y_1 = \frac{40}{2} = 20mm$

$A_2 = 20 \times 200 = 4000\,mm^2$; $y_2 = 40 + \frac{200}{2} = 140mm$

$A_3 = 80 \times 20 = 1600\,mm^2$; $y_3 = 40 + 200 + \frac{20}{2} = 250mm$

$$\bar{y} = \frac{(6400 \times 20) + (4000 \times 140) + (1600 \times 250)}{6400 + 4000 + 1600}$$

$$= \frac{128000 + 560000 + 400000}{12000}$$

$$= \frac{1088000}{12000}$$

$\boxed{\bar{y} = 90.66\ mm}$ — from bottom

$I = I_{GG} + Ah^2$ $\boxed{\therefore h = y - \bar{y}}$

$$I_1 = \frac{b_1 d_1^3}{12} + A_1 (y_1 - \bar{y})^2$$

$$= \frac{160(40)^3}{12} + 6400(20 - 90.66)^2$$

$= 853333.33 + 31960177.78$

$I_1 = 32813511.11\ mm^4$

$$I_2 = \frac{20(200)^3}{12} + 4000(140 - 90.66)^2$$

$= 13333333.33 + 9737742.4.$

$I_2 = 23071075.73mm^4.$

$$I_3 = \frac{80(20)^3}{12} + 1600(250 - 90.66)^2$$

$$= 53333.33 + 40622776.96$$

$$= 40676110.29 \text{ mm}^4$$

Total M.O.I (I) = $I_1 + I_2 + I_3$

$$= \cancel{853333} = 32813511.11 + 23071075.73 + 40676110.29$$

$$I = 96560697.13 \text{ mm}^4$$

For simply supported beam the layers above the N.A are subjected to Compressive stress. whereas the layers below the N.A are subjected to Tensile stresses.

$$\frac{M}{I} = \frac{\sigma}{y}$$

For simply supported beam carries udl throughout its length. The max ~~beam~~ bending moment it is given by

$$M = \frac{wl^2}{8}$$

$$\frac{wl^2}{8} = \frac{20}{90.66} \times 96560697.13$$

$$= \frac{1931213943}{90.66} \times \frac{8}{(5000)^2}$$

$$= \frac{1.54 \times 10^{10}}{22665 \times 105}$$

$$\boxed{w = 6.79 \text{ N/mm}}$$

$$M = \frac{wl^2}{8} = \frac{6.79\,(5000)^2}{8}$$

$$M = 21233178.91 \text{ N-mm}$$

$$\sigma_c = \frac{M}{I} \times y_c$$

For calculating compressive stress we have to consider y value above the neutral axis, because the layer above the N.A subjected to compressive stress

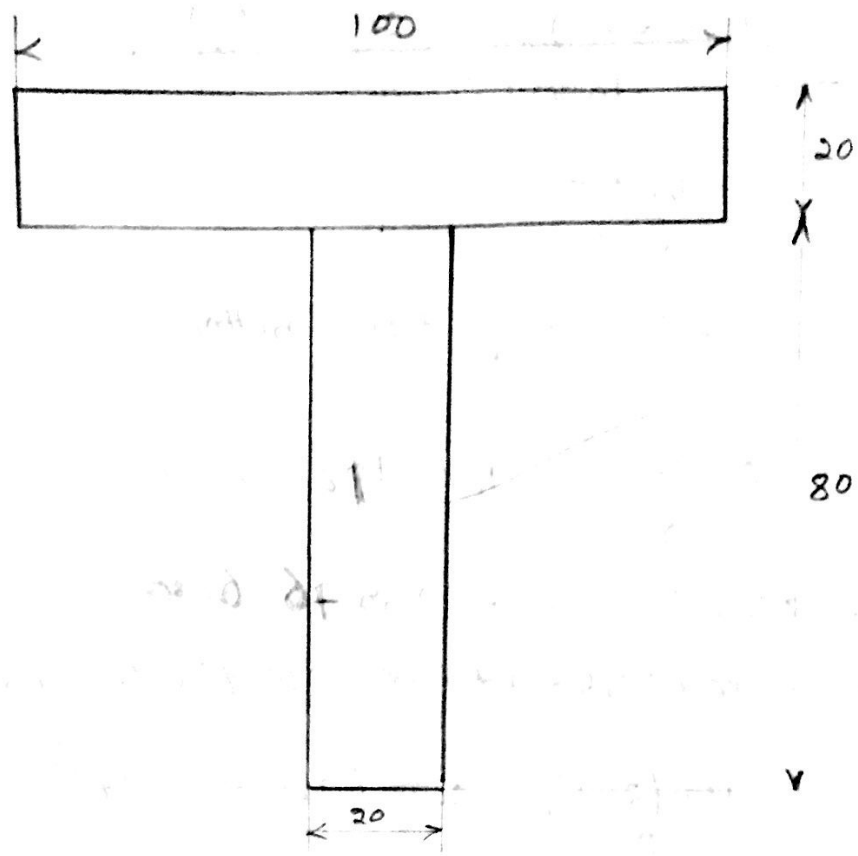
$$= \frac{21233178.91}{96560697.13} \times 169.34$$

$$\boxed{\begin{aligned} y_c &= 260 - 90.66 \\ y_c &= 169.34\text{mm} \end{aligned}}$$

$$= \frac{3595626517}{96560697.13}$$

$$\boxed{\sigma_c = 37.23 \text{ N/mm}^2}$$

A cast Iron beam is of T-section as shown in figure. The beam is simply supported on a span of 8m. The beam carries a uniformly distributed load of 1.5 kN/m length on the entire span. Determine the maximum tensile and maximum compressive stresses.



For simply supported beam carries udl, maximum Bending moment is given by

$$m = \frac{wl^2}{8} = \frac{1.5 \times 10^3 \times (8)^2}{8} = \frac{96000}{8}$$

$$= 12000 \text{ N-m}$$

$$= 12000 \times 10^3 \text{ N-mm}$$

we know $I = I_{GG} + Ah^2$ $\boxed{\therefore h = y - \bar{y}}$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$A_1 = 20 \times 80 = 1600 mm^2$; $y_1 = \frac{80}{2} = 40mm$

$A_2 = 100 \times 20 = 2000 mm^2$; $y_2 = 80 + \frac{20}{2} = 90mm.$

$$\bar{y} = \frac{(1600 \times 40) + (2000 \times 90)}{1600 + 2000} = \frac{64000 + 18 \times 10^4}{3600}$$

$$= \frac{244000}{3600}$$

$\boxed{\bar{y} = 67.77\ mm}$ from bottom

$$I_1 = \frac{20(80)^3}{12} + 1600(40 - 67.77)^2$$

$$= 853333.33 + 1233876.64$$

$$I_1 = \cancel{1092345.67\ mm^4}\ 2087209.97\ mm^4$$

$$I_2 = \frac{100(20)^3}{12} + 2000(90 - 67.77)^2$$

$$= 66666.66 + 988345.8$$

$$I_2 = 1055012.46$$

$$I = I_1 + I_2$$

$$= \overset{2087209.97}{\cancel{1092345.67}} + 1055012.46$$

$\boxed{\cancel{I = 2147358.13\ mm^4}}$ $\boxed{I = 3142222.43\ mm^4}$

we know

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \times y$$

$$= \frac{12 \times 10^6}{\cancel{2147358.13}\ 3142222.43} \times 67.77$$

$$\sigma = 258.81\ N/mm^2.$$

The above Calculated "σ" value is Tensile in nature because we Considered the y value from the bottom. The layer below the N-A subjected to tensile stress whereas the layers above the N-A subjected to Compressive stress in case of simply supported beam.

similarly

$$\sigma_t = \frac{M}{I} \times y_t$$

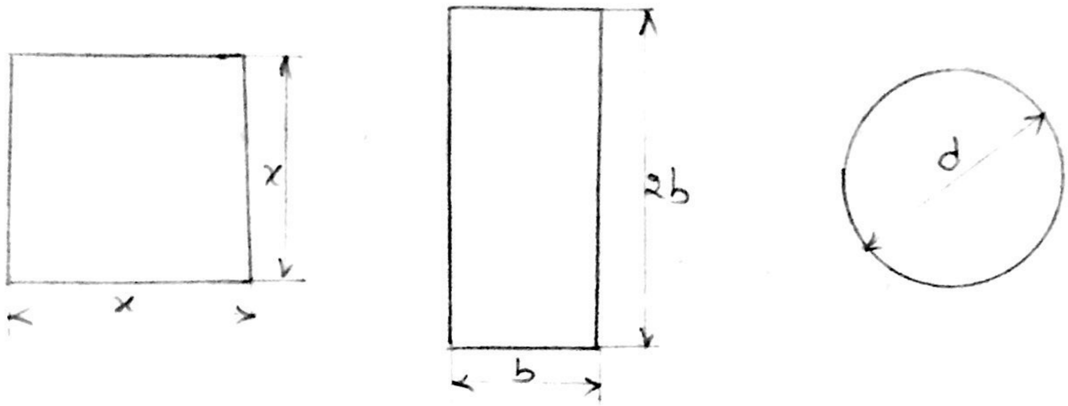
$$y_t = 100 - 67.77 = 32.23$$

$$= \frac{12 \times 10^6}{3142222.43} \times 32.23$$

$$= \frac{386760000}{3142222.43}$$

$$\boxed{\sigma_t = 123.08 \ N/mm^2}$$

Three beams have the same length, same allowable bending stress and same bending moment. The cross section of the beams are a square, rectangle with depth twice the width and a circle. Find the ratios of weights of the circular and the rectangular beams with respect to square beams.



we know $\frac{M}{I} = \frac{\sigma}{y}$

$M_{square} = M_{circular} = M_{Rect}$

$\sigma_{square} = \sigma_{rect} = \sigma_{circular}$

$M_{square} = M_{rectangle}$

$$\frac{I_R}{y_R}\cancel{\sigma_S} = \frac{I_R}{y_R}\cancel{\sigma_R}$$

$$\frac{\frac{x\,x^3}{12}}{\frac{x}{2}} = \frac{\frac{b(2b)^3}{12}}{\frac{2b}{2}} \Rightarrow \frac{x^{\cancel{4}\,3}}{\cancel{12}_{6}}\times\frac{\cancel{2}^{1}}{\cancel{x}} = \frac{\cancel{8}^{2}\,b^{\cancel{4}\,3}}{\cancel{12}_{3}}\times\frac{1}{\cancel{b}}$$

$$\frac{x^3}{6} = \frac{2b^3}{3}$$

$$b^3 = \frac{\cancel{3}^{1}\times x^3}{2\times \cancel{6}_{2}} \Rightarrow b = \sqrt[3]{\frac{1}{4}x^3} = (0.25\,x^3)^{1/3}$$

$$\boxed{b = 0.629\,x}$$

$M_{square} = M_{circular}$

$$\frac{I_B}{y_B} \times \cancel{\sigma_B} = \frac{I_c}{y_c} \times \cancel{\sigma_c}$$

$$\frac{\frac{x x^3}{12}}{\frac{x}{2}} = \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} \Rightarrow \frac{\cancel{x^4}^{3}}{\cancel{12}_{6}} \times \frac{2^1}{\cancel{x}} = \frac{\pi \cancel{d^4}^{3}}{\cancel{64}_{32}} \times \frac{\cancel{2}^1}{\cancel{d}}$$

$$\frac{x^3}{6} = \frac{\pi d^3}{32}$$

$$\frac{32 x^3}{6\pi} = d^3 \Rightarrow (1.6976 x^3)^{1/3}$$

$$\boxed{d = 1.1929 x}$$

we know

$\omega_{square} = \omega_{Rectangle}$

$$\boxed{\omega = \frac{W}{V}}$$

$$\frac{\text{weight of square beam}}{\text{Area of square beam} \times \cancel{l}} = \frac{\text{wt of Rect beam}}{\text{Area of Rect beam} \times \cancel{l}}$$

$$\frac{\text{wt. of square beam}}{\text{wt. of Rect beam}} = \frac{\text{Area of square beam}}{\text{Area of Rect beam.}}$$

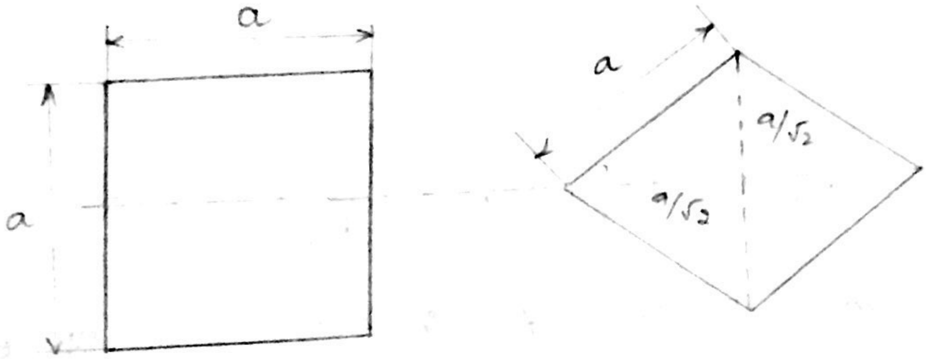
$$\frac{\text{wt. of Rectangular beam}}{\text{wt. of square beam}} = \frac{(0.629 \times x)(2 \times 0.629 x)}{x \times x}$$

$$\boxed{\frac{W_R}{W_S} = 0.7912.}$$

$$\frac{W_{circular}}{W_{square}} = \frac{\frac{\pi d^2}{4}}{x \times x} = \frac{\frac{\pi}{4}(1.1929 x)^2}{x^2} = \frac{1.117 \cancel{x^2}}{\cancel{x^2}}$$

$$\boxed{\frac{W_{circular}}{W_{square}} = 1.117}$$

9) A beam is of square section of the side 'a'. If the permissible bending stress is 'σ' find the moment of resistance when the beam section is placed such that (i) two sides are horizontal, (ii) one diagonal is vertical. Find also the ratio of moments of the resistance of the section in two positions.



$\sin 45 = \frac{opp}{a}$

$\frac{1}{\sqrt{2}} \times a = opp$

moment of resistance :- when two sides are horizontal

$$M = \frac{I}{y} \times \sigma$$

$$= \frac{\frac{a \times a^3}{12}}{\frac{a}{2}} \times \sigma \Rightarrow \frac{\cancel{a^4}^{3}}{\cancel{12}_{6}} \times \frac{\cancel{2}^{1}}{\cancel{a}} \times \sigma$$

$$m_1 = \frac{a^3}{6} \times \sigma$$

$\frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}} = \frac{2a}{\sqrt{2}}$

when one diagonal is vertical.

$$= \frac{I}{y} \times \sigma \Rightarrow \frac{\cancel{2}^{1} \times \frac{bh^3}{\cancel{12}_{6}}}{\frac{a}{\sqrt{2}}} = \frac{1}{6}\left(\frac{2a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right)^3 \times \frac{\sqrt{2}}{a}$$

$$= \frac{1}{6} \times \frac{2a \times a^3}{(\sqrt{2})^3 \times a} = \frac{a^3}{3\sqrt{2}(\sqrt{2})^2} = \frac{a^3}{6\sqrt{2}} \times \sigma$$

$$m_2 = \frac{a^3}{6\sqrt{2}} \times \sigma$$

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Ratio of moment of Resistance of the section in two positions.

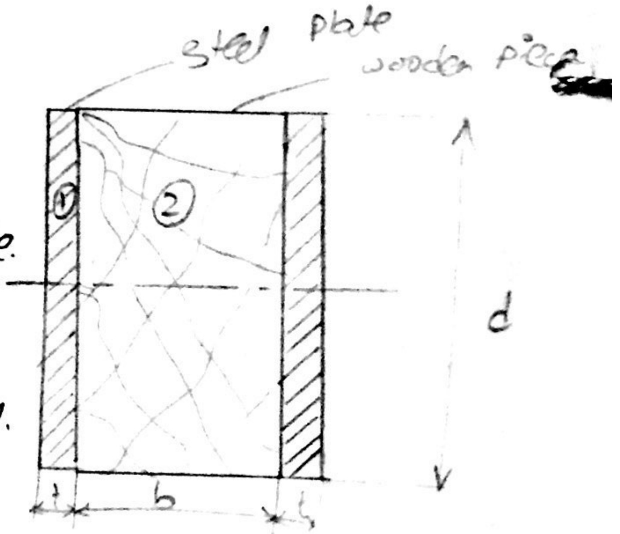
$$\frac{M_1}{M_2} = \frac{\sigma \times \frac{a^3}{6}}{\sigma \times \frac{a^3}{6\sqrt{2}}} = \cancel{\frac{a^3}{6} \times \sigma} \times \frac{6\sqrt{2}}{\cancel{a^3 \times \sigma}} = \sqrt{2}.$$

$$\boxed{\frac{M_1}{M_2} = 1.414}$$

COMPOSITE BEAMS (FLITCHED BEAMS) :-

A beam made up of two or more different materials assumed to be rigidly connected together and behaving like a single piece is known as Composite beam or a wooden flitched beam.

Consider the Composite beam as shown in fig.



Let E_1 = Young's modulus of steel plate.

I_1 = M.O.I of steel about N.A.

M_1 = moment of Resistance of steel.

E_2 = Young's modulus of wood

I_2 = M.O.I of wood about N.A.

M_2 = moment of Resistance of wood.

we know for the Composite section strain should be same in both beams.

$$e_1 = e_2$$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\therefore \boxed{\frac{E_1}{E_2} = m.}$$

modular ratio.

$$\sigma_1 = \frac{E_1}{E_2} \times \sigma_2$$

$$\boxed{\sigma_1 = m\,\sigma_2}$$

$$\boxed{\frac{M}{I} = \frac{\sigma}{y}}$$

moment of Resistance of steel and wood are

$$M_1 = \frac{\sigma_1}{y_1} \times I_1 \qquad M_2 = \frac{\sigma_2}{y_2} \times I_2$$

$$M = M_1 + M_2$$

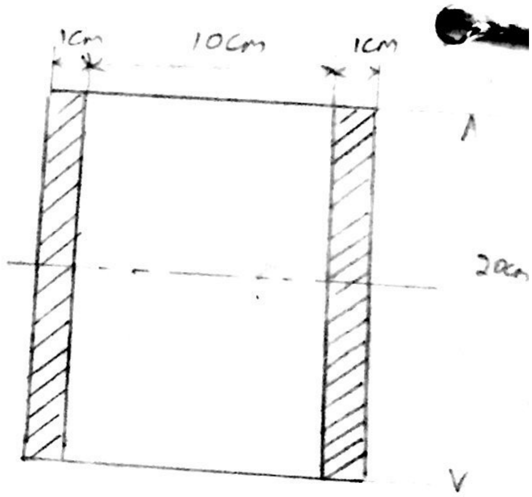
$$= \frac{m\,\sigma_2}{y} \times I_1 + \frac{\sigma_2}{y} \times I_2$$

$$= \frac{\sigma_2}{y}\left[m I_1 + I_2\right]$$

$$\therefore \boxed{I = m I_1 + I_2}$$

$$\boxed{M = \frac{\sigma_2}{y} \times I.}$$

A flitched beam consist of a wooden joist 10cm wide and 20cm deep strengthened by two steel plates 10mm thick and 20cm deep as shown in fig. If the maximum stress in the wooden joist is 7 N/mm², find the corresponding maximum stress attained in steel. Find also the moment of resistance of the composite section. Take young's modulus for steel = 2×10^5 N/mm² and for wood = 1×10^4 N/mm².



$b_2 = 10\text{cm}$

$d_2 = 20\text{cm}$

width of one steel plate $(b_1) = 10\text{mm}$.

depth of " " " $(d_1) = 20\text{cm}$.

No. of plates $= 2$.

max. stress in wood $= 7\ \text{N/mm}^2$

$\sigma_{steel} = ?$

$E_s = 2 \times 10^5\ \text{N/mm}^2$

$E_w = 1 \times 10^4\ \text{N/mm}^2$

we know $\dfrac{M}{I} = \dfrac{\sigma}{y}$

$$I_1 = \frac{bd^3}{12} = \frac{2 \times 10 \times (200)^3}{12} = 1333.33 \times 10^4\ \text{mm}^4$$

$$I_2 = \frac{100\,(200)^3}{12} = 6666.66 \times 10^4\ \text{mm}^4.$$

we know Total moment of inertia for flitched beam

$$I = mI_1 + I_2 \qquad m = \frac{E_1}{E_2} = \frac{2\times10^5}{1\times10^4} = 20$$

$$= 20(1333.33)\times10^4 + 6666.66\times10^4$$

$$= 26666.6\times10^4 + 6666.66\times10^4$$

$$= 33333.2\times10^4\ mm^4.$$

$$M = \frac{\sigma_2}{y}\times I$$

$$= \frac{7\times33333.2\times10^4}{100}$$

$$= \frac{233332.82}{100}$$

$$= 2333.32\times10^4\ N\text{-}mm$$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\sigma_1 = \frac{E_1}{E_2}\times\sigma_2$$

$$= 20\times7$$

$$= 140\,N/mm^2$$